Inventory control in a multistage production-inventory system with stochastic demand

Gerald David Bryant
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INVENTORY CONTROL IN A MULTISTAGE
PRODUCTION-INVENTORY SYSTEM
WITH STOCHASTIC DEMAND

by

Gerald David Bryant

A Thesis
Presented to the Graduate Committee
of Lehigh University
in Candidacy for the Degree of
Master of Science
in
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1973
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

April 17, 1973
Date

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ACKNOWLEDGEMENTS

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ABSTRACT

The interaction between stages and the existence of buffer inventories require that the entire production-inventory system be considered in the development of inventory control models for multistage systems. A number of inventory control models for the multistage system with deterministic demand has been documented in the technical literature. There has been very little documentation for the multistage system when the demand is characterized by a stochastic function. The purpose of this thesis is to investigate a two-stage production-inventory system where the demand is not deterministic but characterized by some known probability distribution. The system under study is a periodic-review type system with the inventory period divided into N cycles of equal length. Raw materials at the production stage are not considered and backlogs are not permitted in this system. Two different inventory policies are examined for the system under consideration.

1) Fixed inventory level policy where the beginning inventory level is constant and the amount of input from the production stage is dependent upon the amount of stock left over from the previous cycle.

2) Fixed lot size policy where the amount produced each cycle is constant and the beginning inventory level is dependent upon the amount of stock left over from the previous cycle and the amount of input at the
Mathematical models are developed using classical optimization techniques when possible for each policy and solved for several hypothetical examples using demand distributions that are normally distributed. A simulation routine for each policy was programmed to simulate the same hypothetical examples using simulated demand data. It was found that the model and simulation routine for the fixed inventory level policy yielded comparable results. A very small difference was observed between the two solutions for large variances in the demand distribution. The fixed lot size policy could not be modeled mathematically using classical optimization techniques due to the fact that the total cost expression for a given inventory cycle is dependent upon the ending inventory level of the previous cycle. This makes the equation state dependent, consequently other techniques such as simulation or dynamic programming must be resorted to.

It was found that the minimum total cost for the optimum lot size policy was greater than the minimum total cost for the optimum inventory level policy. It was also found that the total cost/period for the production-inventory system using independent decision rules for determining the optimum inventory level was greater than the total cost/period obtained using the model developed in this thesis which considers the system as a whole instead of several independent entities. To truly optimize the system under study the optimum inventory level policy should be chosen when possible, and the entire production-inventory system considered in the development
of the inventory control model.
A. General

In order for a business to function successfully inventories of one type or another must be maintained. Inventories, in general, are a means of maintaining balanced production flows, obtaining maximum utilization of capital and facilities, and providing reasonable levels of service to customers.

A manufacturer must maintain inventories of the finished product he sells and also a supply of the raw materials required for production and in some cases, a supply of in-process inventory. Similarly, a service organization must carry an inventory of the items necessary to provide the service. In analyzing inventory problems such as these, the question to be answered is not whether to carry an inventory but what size of inventory to carry and the policy required to maintain the inventory.

Each year the various organizations comprising the business environment invest millions of dollars in inventories in an effort to function successfully. For this reason inventory control is a topic of considerable and widespread interest in today's business environment.

Inherent in any production-inventory system are two types of costs:

1. Those fixed costs associated with manufacturing set-up, placing of an order, and maintaining the inventory system.
2. Those variable costs associated with manufacturing and operation of an inventory system.

Costs included in this category are costs of carrying a unit in inventory for a given period of time, costs of stockouts, and any associated transportation costs. The objective of any production-inventory control model is to provide a decision rule or management policy that will effect a tradeoff in the two types of costs that will minimize the total system costs over a specified planning horizon while simultaneously providing a reasonable service level to the customer.

Production-inventory systems can be classified into four general categories as follows:

1. Single-stage system
2. Parallel-stage system
3. Series-stage system
4. Series-parallel stage system

Inventory control models for the single-stage system are well documented in the technical literature. In this system, each location manufactures or purchases independently of each other and is concerned only with its own financial welfare. The most widely used model for the single-stage system is the economic order quantity model (EOQ). This model is used for systems with deterministic demand to determine the economic order quantity of items to be ordered. Analogous to this model is the economic manufacturing quantity model (EMQ) which is used to determine the manufacturing lot size for items produced and consumed internally by an organization. This model is also used for systems with deterministic demands. The introduction of stochastic demand functions
considerably complicates the development of the inventory control model, particularly when the demand is characterized by certain demand distributions such as the normal distribution. The model for this system contains integral terms which makes solution of the model for the optimum values rather complicated when compared to a deterministic system. Solution of the model is not impossible, however. A number of inventory control models for the multistage production-inventory system has been documented in the technical literature. This system, however, has not been dealt with to the extent the single stage has. Practically all of the multistage inventory models in the literature deal with deterministic systems. There has been very little documentation for the multistage system when the demand is characterized by a stochastic function. This paper will concern itself with a particular case of the stochastic multistage production-inventory system.

There are a considerable number of organizations that exhibit a production-inventory system such as the one shown in Figure 1. In this system, there is a series of production facilities which produce a product and transfer it to a consumer inventory for disposition. Each facility has its own supply of raw materials and subassemblies for use in the manufacture of the finished product. The semi-finished product is shipped to the succeeding stage in predetermined lot sizes. This process is continued until the finished product is delivered to the consumer warehouse.

There are several ways to operate a system such as this. If the various facilities are under different managerial control, it is
FIGURE 1. Schematic Representation of a Typical Series Production-Inventory System.
possible that each facility will determine its own EOQ/EMQ and operate as efficiently as it can. Another way to operate such a system is to use the technique of batch processing wherein a common lot size is determined and the same size lot is processed through all stages.

This paper will concern itself with the type of system whereby the costs for the entire production-inventory system are considered in determining the optimal inventory policy to be followed.

B. Objectives

The interaction between stages and the existence of buffer inventories require that the entire production-inventory system be considered in the development of inventory control models for multistage systems. The purpose of this paper is to investigate the multistage production-inventory system with stochastic demand and develop the methodology for determining economic interstage lot sizes and inventory levels that minimize the total system cost. In particular, the demand distribution to be used in this paper will be the normal distribution. The models developed in this paper should apply equally well to systems whose demands are characterized by other stochastic distributions such as the exponential, etc. Applying the models to other demand distributions is beyond the scope of this paper.
CHAPTER II
BACKGROUND

A. General

Inventory problems have been in existence since the history of mankind. Only recently, since the turn of the century, has any attempt been made to utilize analytical techniques in dealing with inventory problems. Hadley and Whitin [3] state that the real need for analysis was first recognized in industries that had a combination of production scheduling problems and inventory problems, i.e., in situations in which items were produced in lots--the cost of setup being fairly high--and then stored at a factory warehouse.

The earliest development of the simple lot size model was made by Ford Harris of the Westinghouse Corporation in 1915. [5] The formula is often referred to as the "Wilson Model" since it was also derived independently by R. H. Wilson as an integral part of the inventory control scheme which he sold to many organizations.

After World War II, operations research, and the management sciences emerged and as a result increased emphasis was placed on solving inventory problems, both deterministic and stochastic as well. As a result a multitude of information on inventory theory has been documented.

Before discussing inventory models, it is necessary first to explore the basic characteristics of an inventory system which is to be represented by an appropriate mathematical model.

The development of the inventory model for a production-inventory
system involves the following economic parameters:

1. Preparation or Setup Costs - Fixed cost associated with the placement of an order or with the initial preparation of a production system. Setup cost is usually assumed independent of the order quantity.

2. Purchase or Production Costs - Cost of acquiring or producing the product under study. Usually assumed independent of the order quantity.

3. Handling and Storage Costs - Cost of carrying inventory in storage. Holding costs usually are assumed to vary directly with the level of inventory as well as the length of time the item is held in stock.

4. Shortage Cost - Penalty costs incurred as a result of running out of stock during the inventory period. Includes the costs due to loss in customers' good will and due to potential loss in income. In the cases where the unfilled demand can be satisfied at a later date (backlog case), these costs are usually assumed to vary directly with both shortage quantity and delay time. On the other hand, if the unfilled demand is lost (no backlog case), shortage costs become proportional to shortage quantity only.

The demand pattern of an inventory system may be either deterministic or probabilistic. In the deterministic case, it is assumed that the quantities required over subsequent periods of time are known with certainty. The quantities may be
expressed over equal periods of time in terms of known constant
demands or in terms of known variable demands. The first case is
known as a static demand and the second, dynamic demand.

Probabilistic demand occurs when the demand over a certain period
of time is not known with certainty but its pattern can be described
by a known probability distribution. The probability distribution may
be either discrete or continuous; and it maybe either stationary
(constant over time) or nonstationary (variable over time).

The demand for a given period of time may be satisfied instan-
taneously at the beginning of the period or uniformly during the
period, depending on the nature of the inventory system.

The ordering cycle (inventory cycle) is concerned with the time
measurement of the inventory system. An ordering cycle is usually the
time period between two successive placements of orders. The start
of a new ordering cycle can be initiated in one of two ways:

a. Continuous review - A record of the inventory level is
   updated continuously until a certain lower limit is reached
   (reorder point) at which point a new order is placed.

b. Periodic review - Orders are placed usually at equally
   spaced intervals of time. This is the type usually employed
   in a probabilistic inventory system.

When an order is placed, it may be delivered instantaneously
(zero lead time) or it may require some time before delivery is made.
The time between placement of an order and receipt is called lead
time. Lead time may be either deterministic or probabilistic.
Stock replenishment in an inventory system may occur instantaneously or uniformly during the period. In practically all cases the stock replenishment is usually assumed to occur instantaneously at the beginning of the cycle.

The time horizon defines the time period over which the inventory level will be controlled. The horizon may be finite or infinite depending on the nature of the demand.

An inventory system may involve more than one type of item. This case will be of interest usually when some kind of interaction exists between the different items. In most instances only one item will be considered in an inventory model.

The above characteristics are the basic elements for developing inventory models, and a knowledge of them is essential before undertaking the modeling of a particular production-inventory system.

B. Single-Stage Stochastic Models

There are many good references in the literature on the subject of stochastic single stage inventory models [3],[15],[17]. Consequently, no attempt will be made here to repeat them. It is assumed that the reader is familiar with the classical treatment of single stage stochastic systems.

C. Multistage Models

Several authors have treated the problem of multistage production-inventory systems where the demand is deterministic. Taha and Skeith [18] consider a single-product multistage system with static deterministic demand, where the product moves between the stages in
a serial fashion. In this model, any unfilled demand of the finished product is back logged. Overproduction is allowed at the different stages so that each stage $i$ may produce $k_i$ batches once every $k_i$ cycles. The decision variables for each stage $i$ are the number of batches per run $k_i$, the batch size $Q_i$, and the shortage quantity of the finished product $Q_s$. The problem is also considered for the case with storage constraints at the different stages.

Schussel [16] considers the problem of determining lot sizes for the production of subassemblies in a deterministic multistage system. He assumes a delay between the completion of a lot at an operation and its availability for processing by the next operation.

Jensen and Kahn [8] treat the same problem as Taha and Skeith, but without the assumptions of a delay between production and use of a lot and of limits on lot sizes. They present a dynamic programming formulation, but have to use a numerical procedure to evaluate the average in-process inventory levels.

Johnson [9] considers the problem of determining lot sizes at each operation in a serial multistage production system for the case of static, deterministic demand. Expressions for average in-process inventory levels are given as a function of the characteristics of production operations governing an inventory's input and output. A cost model is formulated, with backorders permitted only at the final stage. A direct solution is obtained for the two-stage process and an efficient iterative procedure is developed for the three-stage process.
Bryan, Wadsworth, and Whitin [19] consider a four-stage inventory system with stochastic demand which pertains to seasonal goods. In this system both backlogging and lost sales occur. In addition, there are losses incurred on liquidation of stock left over after the inventory period elapses. The problem considered is that of determining the optimum quantity of inventory to hold at each of the four stages as of the beginning of the period. The production program cannot be modified after the sales season is under way. The optimal levels of inventory at each stage are arrived at by developing four balancing equations which represent the probabilities of there being a demand for at least an additional, or marginal, unit in excess of inventories carried in each stage. Setup, ordering, inventory holding, and shortage costs are not considered in this system. Only unit profit on sales and unit liquidation losses are considered in the development and the objective is to maximize the profit for the period.

Lele [10] has developed an economic lot size model for a two-stage production-inventory system. This simple model contains one production stage and one consuming stage. Raw material inventories at the production stage are not considered. The model is a single product model without competition for facilities. The production facility produces the product, one at a time, until a batch of size $Q$ is completed. The completed product is then shipped to the consuming stage where it is depleted at a rate $D$.

D. Definition of the Problem

The problem under consideration in this thesis concerns a
multistage production-inventory system similar to the one that Lele developed with the exception that the demand is not deterministic but characterized by some known probability distribution. The system will be a periodic-review type system with the inventory period divided into \(N\) cycles of equal length. Raw materials will not be considered. Two different inventory policies will be examined for the system under consideration.

(1) Fixed inventory level policy where the beginning inventory level is constant and the amount input from the production stage is dependent upon the amount of stock left over from the previous cycle.

(2) Fixed lot size policy where the amount produced each cycle is constant and the beginning inventory level is dependent upon the amount of stock left over from the previous cycle and the amount input at the beginning of the cycle.

Mathematical models will be developed using classical optimization techniques for each policy and when possible solved for hypothetical examples using demand distributions that are normally distributed. The results of the model solutions will be compared with the results of a simulation routine using simulated demand data. The results will be presented and discussed and then conclusions drawn concerning the two policies where possible.
CHAPTER III

THE OPTIMUM INVENTORY LEVEL MODEL

The purpose of this chapter is to develop the necessary formulations and outline the procedure for deriving the mathematical model for the production-inventory system under study. Development of the model is based on the following assumptions:

A. Assumptions

1. Delivery lead time between stages is zero.

2. Replenishment occurs instantaneously at the beginning of each cycle.

3. Demand/inventory cycle is described by the normal distribution \( N(\mu, \sigma) \) and is stationary and independent over time.

4. The model developed is of the single product periodic-review type without competition for facilities.

5. The total interval of time under study (Inventory Period) is divided into cycles of specified length.

6. Production capacity per period is adequate to meet the demand.

7. The various setup, preparation, and order costs are constant over time.

8. Inventory holding costs are a function of average stock level during the cycle.

9. When a stockout occurs, the demand is lost and a shortage cost for each item short is incurred.

10. Inventory holding costs and shortage costs are constant over time.
11. At each inventory review (cycle), demand forecasts for the current cycle are available.

12. The unit manufacturing cost is a constant independent of the quantity ordered.

B. Description of System

The production-inventory system under study consists of a two-stage system, one production stage and one consuming stage. Raw material inventories at the production stage are not considered. The production facility produces the product, one at a time, until a batch of size Q is completed. There is one production run per cycle and N cycles per time period. A time period or planning horizon in this context is meant to be a one year period. The completed product is shipped to the consuming stage at the beginning of the inventory cycle. Since this model is for the fixed inventory level policy, the lot size (Q) will vary from cycle to cycle. The demand on the consuming stage is probabilistic and occurs uniformly throughout the cycle. Any unfilled demand is lost.

C. Notation

The complete production-inventory system is shown in Figure 2. The various parameters of the system are defined below.

\[ X = \text{demand per cycle on the consuming stage, a random variable} \]

\[ f(X) = \text{the probability density function of the demand for } X \text{ per cycle} \]

\[ Y = \text{inventory level after receipt of Q units from production stage (units)} \]
Figure 2. Diagram of Two-Stage Production-Inventory System Using Optimum Inventory Level Policy
Z = inventory level at consuming stage before receipt of a lot from production stage, \( Z = Y - X \) (units)

\( Q \) = size of lot from production stage (varies from cycle to cycle)

CP = setup and preparation costs at the production stage (\$/run)

CO = setup, preparation, ordering and/or interdepartmental transfer costs at the consuming stage (\$/order received)

HP = inventory holding cost for one unit of product per unit time at the production stage

HC = inventory holding cost for one unit of product per unit time at the consuming stage (\( HC \geq HP \))

CS = shortage cost per unit demanded but not filled in a cycle

N = number of inventory cycles per time period

TIC = total incremental cost

P = production rate per period

D. Model Formulation

The setup cost per cycle for the production stage is given by a constant CP since there is only one run per cycle. The setup cost per period is given by \( N(CP) \). The production lot size \( (Q) \) for a given inventory cycle is equal to the difference between the consumption stage optimum inventory level and the consumption stage ending inventory level for that cycle. For the case when the demand for a given cycle \( (X) \) is less than the inventory level \( (Y) \), the production lot size will be equal to the demand for that cycle since \( Q = Y - Z = X \). When the demand for a given cycle is greater than or equal to the inventory level, the production lot size will
be equal to the inventory level quantity since \( Q = Y - X = Y - (0) = Y \).

Using these relationships, the average inventory level per cycle for the production stage is given by

\[
\overline{I} = \frac{X t_p}{2}, \quad \text{when } X < Y, \text{ where } t_p = NX/P
\]

\[
= \frac{NX^2}{2P}
\]

\[
\overline{I} = \frac{Y t_p}{2}, \quad \text{when } X \geq Y, \text{ where } t_p = NY/P
\]

\[
= \frac{NY^2}{2P}
\]

The expected total incremental cost per period for the production stage is (assuming \( Y \) is a continuous variable):

\[
E\{\text{TIC}\} = N(CP) + \frac{N^2(HP)}{2P} \left\{ \int_0^Y X^2 f(x) dx + \int_Y^\infty Y^2 f(x) dx \right\}
\]

The order costs per cycle for the consuming stage is given by a constant \( CO \) since there is only one order receipt per cycle. The order costs per period = \( N(CO) \).

The average inventory per cycle for the consuming stage is:

\[
\overline{I} = (Y - X/2), \quad \text{when } X < Y
\]

\[
\overline{I} = Y t_d /2, \quad \text{when } X \geq Y, \text{ where } t_d = Y/X
\]

\[
= Y^2 /2X
\]
The shortage inventory per cycle is given by

\[ (X-Y) \quad , \quad \text{when } X > Y \]

\[ 0 \quad , \quad \text{otherwise} \]

The expected total incremental cost per period for the consuming stage is given by

\[
E\{TIC\}_C = N(CO) + N(HC) \left\{ \int_0^Y (X-Y)f(y) \, dy + \int_Y^\infty (Y/2X)f(y) \, dy \right\}
\]

\[ + N(CS) \int_Y^\infty (X-Y)f(y) \, dy \]

The expected total incremental costs per period for both stages is,

\[
E\{TIC\}_T = N(CP) + \frac{N^2(HP)}{2P} \left\{ \int_0^Y f(x) \, dx + \int_Y^\infty f(x) \, dx \right\}
\]

\[ + N(CO) + N(HC) \left\{ \int_0^Y (X-Y)f(x) \, dx + \int_Y^\infty (Y/2X)f(x) \, dx \right\}
\]

\[ = N(CS) \int_Y^\infty (X-Y)f(x) \, dx \]

We wish to choose \( Y \) to minimize the expected total incremental costs per period. In order to accomplish this objective we take the derivative of TIC with respect to \( Y \), set it equal to zero and solve for the value of \( Y \) which satisfies the relation. Thus

\[
\frac{dE\{TIC\}_T}{dY} = \frac{N^2(HP)}{P} \int_Y^\infty f(x) \, dx + N(HC) \int_Y^\infty f(x) \, dx - N(CS) \int_Y^\infty f(x) \, dx
\]

\[ + N(HC) \int_Y^\infty (1/X)f(x) \, dx = 0 \]
Since
\[ \int_0^\infty f(x)dx = 1 - \int_0^Y f(x)dx \]

The above equation yields
\[ \left\{ \frac{N(HC) - \frac{N^2(HP)}{P}Y^*}{N(CS)} + \frac{N(HC)Y^*}{P} \right\} \int_0^{Y^*} f(x)dx + \frac{N(HC)Y^*}{P} \int_{Y^*}^\infty \frac{1}{X}f(x)dx \]

\[ + \frac{N^2(HP)Y^*}{P} - N(CS) = 0 \]

where the value \( Y^* \) is the optimum value of the inventory level. An evaluation of the above model for various values of \( Y \) reveals that when \( Y=0 \), the value of the first derivative (slope) is negative. When \( Y=Y^* \), the value of the first derivative is zero. When \( Y=\infty \), the value of the first derivative is positive. Since the derivative is negative in the interval from 0 to \( Y^* \), the total cost function is decreasing throughout the interval. Since the derivative is positive in the interval from \( Y^* \) to \( \infty \), the cost function is increasing throughout the interval. Since the derivative is zero at the point \( Y^* \), the total cost function at this point is stationary and represents a minimum point. A plot of \( Y \) versus total cost/cycle will verify that this is true. The above model is valid only if the production rate per period ≥ the expected demand per period. If this occurs the demand satisfied will equal the production rate and the condition will be valid.
E. Test for Minimum Value

The second derivative of the expected total incremental cost with respect to \( Y^* \) is given by

\[
\frac{d^2 E_{\text{LIC}}}{dY^2} = \left\{ N(CS) - \frac{N^2(HP)Y^*}{P} \right\} f(Y^*) + N^2(HP) \int_{Y^*}^{\infty} f(x) \, dx \\
+ N(HC) \int_{Y^*}^{\infty} \left( \frac{1}{x} \right) f(x) \, dx
\]

If the value of the second derivative is positive at the point \( Y^* \), then the point \( Y^* \) corresponds to a minimum point. This will always be true whenever the following condition holds:

\[
CS - \frac{N(HP)Y^*}{P} \geq 0
\]

If this condition does not hold it will be necessary to evaluate the second derivative to determine if \( Y^* \) is a minimum.

F. Solution of Model

In order to evaluate the model for a particular production-inventory system characterized by a continuous demand distribution such as the normal distribution, it is necessary to employ numerical integration procedures to evaluate the integrals since they cannot be readily solved by conventional methods for the value of \( Y^* \). Consequently, a computer program was written which involved the use of an iterative search procedure coupled with a numerical integration routine based on Simpson's rule. This program essentially varies the value of \( Y \) until the value of the first derivative is equal to zero. This value of \( Y \) is the optimum value of \( Y \) and is a minimum. A check for a positive value of the second derivative will insure
that this condition is met. The value of the average lot size/cycle can be determined by the relationship

\[ \bar{Q} = Y^* - \bar{Z} \]

where \( \bar{Z} = \int_{0}^{Y^*} (Y^* - X)f(X)dx \)

The expected total cost/period can then be determined by evaluating the total cost expression presented earlier in this chapter at the point \( Y^* \).

G. Evaluation of Model

The model developed for this inventory policy will be evaluated for different normal demand distributions and compared with a simulation of a comparable system in order to determine the effectiveness of the model. The values obtained from the simulation will be used as a standard in making the comparison.
CHAPTER IV

THE OPTIMUM LOT SIZE MODEL

The purpose of this chapter is to develop the necessary formulations and outline the procedure for deriving the mathematical model for the production-inventory system whereby the lot size is a fixed quantity and the inventory level is permitted to vary. Development of the model is based on the following assumptions.

A. Assumptions

1. Delivery lead time between stages is zero.
2. Replenishment occurs instantaneously at the beginning of each cycle.
3. Demand/inventory cycle is described by the normal distribution $N(\mu, \sigma)$ and is stationary and independent over time.
4. The total interval of time under study (inventory period) is divided into cycles of specified length.
5. Production capacity per period is adequate to meet the demand.
6. The various setup, preparation, and order costs are constant over time.
7. Inventory holding costs are a function of average stock level during the cycle.
8. When a stockout occurs, the demand is lost and a shortage cost for each item short is incurred.
9. Inventory holding costs and shortage costs are constant over time.
10. The unit manufacturing cost is a constant independent of the quantity ordered.

B. Description of System

The production inventory system under study consists of a two-stage system, one production stage and one consuming stage. Raw material inventories at the production stage are not considered. The production facility produces the product, one at a time, until a batch of size \( Q \) is completed. There is one production run per cycle and \( N \) cycles per time period. A time period or planning horizon in this context is meant to be a one year period. The completed product is shipped to the consuming stage in lots of size \( Q \) at the beginning of the inventory cycle. Since this model is for the fixed lot size policy, \( Y \) (inventory level) will vary from cycle to cycle. The demand on the consuming stage is probabilistic and occurs uniformly throughout the cycle. Any unfilled demand is lost.

C. Notation

The complete production-inventory system is shown in Figure 3. The various parameters of the system are the same as discussed in Chapter III and will not be repeated here.

D. Model Formulation

The setup cost per cycle for the production stage is given by a constant \( CP \) since there is only one run per cycle. The average inventory per cycle for the production stage is constant and is given by:
Figure 3. Diagram of Two-Stage Production-Inventory System Using Optimum Lot Size Policy
The expected total incremental cost per cycle for the production stage is

\[ \bar{I} = \frac{Q_p}{2} \quad , \quad \text{where} \quad t_p = \frac{NQ}{P} \]

\[ = \frac{NQ^2}{2P} \]

The expected total incremental cost per cycle for the production stage is

\[ E\{\text{TIC}\}_P = CP + \frac{NQ^2\text{HP}}{2P} \]

The order costs per cycle for the consuming stage is given by a constant CO since there is only one order receipt per cycle.

The average inventory for cycle (t) for the consuming stage is

\[ \bar{I} = \begin{cases} (Y_t - X_t)/2 & , \quad \text{when} \quad X_t < Y_t \\ Y_t t_d/2 & , \quad \text{when} \quad X_t \geq Y_t, \quad \text{where} \quad t_d = Y_t / X_t \end{cases} \]

\[ = \frac{Y_t^2}{2X_t} \]

The expected shortage per cycle (t) is given by

\[ (X_t - Y_t) \quad , \quad \text{when} \quad X_t > Y_t \]

\[ 0 \quad , \quad \text{otherwise} \]
The expected total incremental costs for cycle \((t)\) for both stages is

\[
E_t \left\{ \text{TIC} \right\}_T = CP + \frac{NQ^2(HP)}{2P} + (HC) \int_{0}^{\infty} (Y_t - X_t/2) f(x)dx + \int_{Y_t}^{\infty} (Y_t^2/2X_t)f(x)dx + (CS) \int_{Y_t}^{\infty} (X_t - Y_t)f(x)dx + CO
\]

Since we wish to determine the lot size which minimizes the expected total incremental costs per period, we substitute the expression \(Y_t = Q + Z_{t-1}\) in the above expression yielding the following expression

\[
E_t \left\{ \text{TIC} \right\}_T = CP + \frac{NQ^2(HP)}{2P} + CO + (HC) \int_{0}^{(Q+Z_{t-1})} \int_{(Q+Z_{t-1})}^{(Q+Z_{t-1})} (Q + Z_{t-1} - X_t/2)f(x)dx + (CS) \int_{(Q+Z_{t-1})}^{(Q+Z_{t-1})} (X_t - Q - Z_{t-1})f(x)dx + CO
\]

As can be seen the expected total cost relation for a given inventory cycle \((t)\) is state dependent since the expected cost equation is dependent upon both the lot size \((Q)\) and the ending inventory level of the previous cycle \((Z_{t-1})\). Instead of a single expression for the expected cost of an inventory period, we now have a series of costs, one for each cycle of the period. Since the expected total cost relation for a given inventory cycle \((t)\) is a function of \(Z_{t-1}\) and \(Q\), the total cost/period is given by

\[
\left\{ \text{TIC} \right\}_T = \sum_{t=1}^{N} G(Z_{t-1}, Q), \text{ where } Z_{t-1} = Z_t - Q + X_t
\]
As a result, the conventional mathematical optimization technique used in Chapter III cannot be used. Other methods such as dynamic programming or simulation must be resorted to in order to find the optimum value of the lot size. The above total cost relationship is valid only if the production rate per period \( \geq \) the expected demand per period. If this happens the demand satisfied will equal the production rate and the condition will be valid.
CHAPTER V
SIMULATION STUDIES

A. SIMULATION OF FIXED INVENTORY LEVEL POLICY

1. General

The purpose of this chapter is to discuss the methodology used in the evaluation of the fixed inventory level model developed in Chapter III. In order to provide a method of evaluating the model, an iterative search technique using simulated demands was chosen as the standard from which to make the comparison. A comparison of the results of the model and the iterative search technique was made using several normal demand distributions and is presented in Chapter VI.

2. Description of Iterative Search Technique

The iterative search technique essentially starts out with a value of the inventory level which is less than the optimum inventory level. Using this starting value, the total incremental cost per cycle for the production-inventory system is computed over a fixed horizon using simulated demand data for each cycle. The system costs that are computed for each cycle are setup costs, inventory holding costs for stage one (production stage), order processing costs, inventory holding costs for stage two (consuming stage), and shortage costs. Each of these costs are accumulated over the simulation horizon and then an average per cycle computed. In addition to the cost data, statistics are obtained on average lot size per cycle, average leftover stock per cycle, and average units short per cycle. The starting
value of the inventory level is then incremented and the simulation run repeated using the same simulated demand data. As the value of the inventory level is increased and approaches the optimum value, the total incremental cost approaches the minimum cost. When the inventory level passes the optimum inventory level, the total incremental cost will begin to increase. By observing the value of the inventory level and the behavior of the total cost per cycle, the optimum inventory level can be determined for that run. By making several runs using different seeds for the simulated demands and averaging the results, reasonably accurate values for the variables in question can be obtained and compared with the results of the model. A flow chart of the computer program for the simulation technique is shown in Appendix A. An explanation of the parameters used in the program and flow chart are also presented in Appendix A.

3. Determining Size and Number of Runs

The number of observations (cycles) required for a simulation run was determined in the following manner. Five different seeds were chosen at random for use in the normal demand generator. For each of the five seeds chosen, demand data for a common mean and standard deviation was generated. In the first case 250 demands were generated for each seed. An inventory level was selected arbitrarily for test purposes. The iterative search technique was employed using the test inventory level and each of the five sets of simulated demands. The average total cost per cycle for each seed was obtained and the variance computed. The procedure was repeated for 500, 750, and
1000 observations. A comparison of the variances revealed a significant change between 250 and 500 observations and between 500 and 750 observations. There was a slight change in the variance between 750 and 1000 observations, therefore 750 observations per run was chosen for the simulation of the system.

Having determined the number of observations per run, the next step was to determine the number of runs required to insure valid results. This was done by selecting additional seeds at random and generating 750 demands for each using the same mean and standard deviation as before. The same test inventory level was used and the iterative search technique employed for each of the additional seeds. Average total costs/cycle for each seed were obtained. Each of the new average costs/cycle were combined one at a time with the original five and the variance computed each time. The results of this analysis revealed that the variance appeared to reach a stationary level at seven runs, therefore this was the number of runs selected for the iterative search technique. An average of the results of the seven runs of 750 observations each should provide valid data which can be used as a standard in evaluating the model developed in Chapter III.

The computer programs used in the iterative search technique were written in Fortran language for use on the PDP-10 computer and employ the use of the system subroutines RANDU and GAUSS.
B. Simulation of Fixed Lot Size Policy

1. General

The purpose of this chapter is to discuss the methodology used in the simulation of the fixed lot size policy discussed in Chapter IV. In order to provide a method of evaluating the policy, an iterative search technique using simulated demands was chosen. The results of the iterative search technique using several normal demand distributions are presented in Chapter VI.

2. Description of Simulation Routine

The iterative search technique discussed in Part A was modified for purposes of simulating the fixed lot size policy. The procedure is the same with the exception that a starting value of the lot size is specified in lieu of an inventory level. Using the starting value, the total incremental cost per cycle for the production-inventory system is computed over a fixed horizon using simulated demand data for each cycle. The system costs computed for each cycle are the same as in the fixed inventory level case. Instead of computing average lot size/cycle as before, the average inventory level/cycle is now computed. The method of locating the optimum lot size is the same as was discussed for the fixed inventory level policy in Part A. A flow chart of the computer program for the simulation technique is shown in Appendix B. The parameters that apply to the program and flow chart for this policy are also presented in Appendix B.
3. Determining Size and Number of Runs

The same size and number of runs used in the simulation of the fixed inventory level policy were used in the simulation of the fixed lot size policy.
CHAPTER VI

EXPERIMENTAL RESULTS AND DISCUSSION

A. General

The two simulators discussed in Chapter V were used to evaluate the two-stage production-inventory systems described in Chapters III and IV. The parameters used in the simulations are shown in Table 1. The system under consideration consists of a consuming stage and a production stage without regard for raw materials. The evaluation time used in each simulator was 750 cycles or 62.5 periods.

The objective of simulator number 1 was to locate the optimum value of the inventory level and obtain the average total cost/cycle, average inventory costs/cycle for each stage, average shortage cost/cycle, average leftover stock/cycle, average lot size, and average number of units short/cycle for the fixed inventory level policy. Using this policy, only the maximum inventory level is fixed and not the inventory level at any given time.

The objective of simulator number 2 was to locate the optimum value of the lot size and obtain the average total cost/cycle, average inventory costs/cycle for each stage, average shortage cost/cycle, average leftover stock/cycle, average inventory level, and average number of units short/cycle for the fixed lot size policy.

B. Results of Fixed Inventory Level Simulator

The simulation for the fixed inventory level policy was first run using a normally distributed demand with a mean of 400 units/
### TABLE 1

PARAMETERS FOR SIMULATION OF THE TWO-STAGE SYSTEM

<table>
<thead>
<tr>
<th></th>
<th>Simulator #1</th>
<th>Simulator #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1 Setup Cost</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Stage 1 Inventory Holding Cost</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Stage 2 Ordering Cost</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Stage 2 Inventory Holding Cost</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Shortage Cost</td>
<td>5.0</td>
<td>5.0</td>
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<tr>
<td>Production Rate/Period</td>
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<td>6000.0</td>
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<tr>
<td>Number of Cycles/Run</td>
<td>750</td>
<td>750</td>
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<tr>
<td>Number of Cycles/Period</td>
<td>12.0</td>
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<td>Demand Distribution</td>
<td>N(400,100)</td>
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</tr>
<tr>
<td></td>
<td>N(400,50)</td>
<td>N(400,50)</td>
</tr>
</tbody>
</table>
cycle and a standard deviation of 100. The variable system costs versus inventory level for this demand distribution are plotted in Figures 4 and 5. It can be seen that the total cost curve is definitely convex over the region of possible values and contains a global minimum at some value of \( Y \) (i.e., \( Y^* \)). The optimum value of the inventory level and its associated components are presented in Table 2.

The second simulation for the fixed inventory level policy was run using a normally distributed demand with a mean of 400 units/cycle and a standard deviation of 50. The system costs versus inventory level are plotted in Figures 6 and 7. Again the total cost curve is convex over the region of possible values and possesses a minimum value at some optimum inventory level. The optimum value and associated components are presented in Table 2.

C. Results of Fixed Lot Size Simulator

The first simulation for the fixed lot size policy was run using a normally distributed demand with mean of 400 units/cycle and standard deviation of 100. The variable system costs versus lot size for this demand distribution are plotted in Figures 8 and 9. The total cost curve is definitely convex over the region of possible values and possesses a minimum value at some optimum lot size \( (Q^*) \). The optimum value of the lot size and its associated components are presented in Table 3. The second simulation for the fixed lot size policy was run using a normally distributed demand with a mean of
NORMAL DEMAND
N(400,100)

Figure 4. Total Costs/Cycle Vs. Inventory Level
NORMAL DEMAND
\( N(400,100) \)

Figure 5. Variable Cost Functions for Fixed Inventory Level Policy.
TABLE 2
RESULTS OF SIMULATIONS FOR FIXED INVENTORY LEVEL POLICY

<table>
<thead>
<tr>
<th></th>
<th>DEMAND</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>N(400,100)</td>
<td>N(400,50)</td>
</tr>
<tr>
<td>Inventory Level (Y*)</td>
<td>478.0</td>
<td>439.0</td>
</tr>
<tr>
<td>Average Total Cost/Cycle (TIC*)</td>
<td>$480.10</td>
<td>$409.03</td>
</tr>
<tr>
<td>Average Lot Size (Q)</td>
<td>389.0</td>
<td>395.0</td>
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<tr>
<td>Average Surplus Inventory/Cycle (Z)</td>
<td>89.0</td>
<td>44.0</td>
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<tr>
<td>Average Units Short/Cycle</td>
<td>13.0</td>
<td>6.0</td>
</tr>
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</table>
NORMAL DEMAND
N(400,50)

Figure 6. Total Costs/Cycle Vs. Inventory Level
A: Stage Two Inventory Costs
B: Stage One Inventory Costs
C: Shortage Costs

NORMAL DEMAND
N(400,50)

Figure 7. Variable Cost Functions for Fixed Inventory Level Policy
NORMAL DEMAND

$N(400, 100)$

Figure 8. Total Costs/Cycle Vs. Lot Size
Figure 9. Variable Cost Functions for Fixed Lot Size Policy

NORMAL DEMAND
\(N(400, 100)\)
### RESULTS OF SIMULATIONS FOR FIXED LOT SIZE POLICY

<table>
<thead>
<tr>
<th>Lot Size (q*)</th>
<th>N(400,100)</th>
<th>N(400,50)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Total Cost/Cycle (TIC*)</td>
<td>$555.24</td>
<td>$446.58</td>
</tr>
<tr>
<td>Average Inventory Level ((\bar{Y}))/Cycle</td>
<td>454.0</td>
<td>427.0</td>
</tr>
<tr>
<td>Average Surplus Inventory/Cycle ((\bar{Z}))</td>
<td>87.0</td>
<td>43.0</td>
</tr>
<tr>
<td>Average Units Short/Cycle</td>
<td>34.0</td>
<td>17.0</td>
</tr>
</tbody>
</table>
400 units/cycle and standard deviation of 50. The system costs versus lot size for this demand distribution are plotted in Figures 10 and 11. Again the total cost curve is convex over the region of possible values and possesses a minimum value at some optimum lot size. The optimum value of the lot size and its associated components are presented in Table 3.

D. Model Results

The mathematical model developed in Chapter III was solved using the same parameters as were used in simulator number 1, using the numerical integration routine developed especially for this purpose. The results of the model solution for demand distribution \( N(400, 100) \) and \( N(400, 50) \) are presented in Table 4. A comparison of the model results versus simulator results is presented in Table 5. It can be seen that the model provides very accurate results when compared to the simulator for normal demand distributions with large variances. As the variance is decreased the model performance improves as can be seen in Table 5 for the demand distribution \( N(400, 50) \).

In order to compare the results obtained from the model developed for the entire production-inventory system with the results obtained from a model developed using independent decision rules, it was necessary to develop a model utilizing the expected cost function for the consuming stage only. The model developed is given as:

\[
(N_{HC} + N_{CS}) \int_{0}^{Y^*} f(x)dx + N_{HC}Y^* \int_{0}^{\infty} \frac{1}{x}f(x)dx - N_{CS} = 0
\]

This model was solved for the value of \( Y^* \) using the same parameters.
as were used in the model discussed above. The results of the model solution for demand distributions \( N(400,100) \) and \( N(400,50) \) are presented in Table 6. It can be seen that the total expected cost/period using the total system concept is less than the total expected cost/period using the independent decision rule model developed from the consuming stage only.
NORMAL DEMAND
N(400,50)

Figure 10. Total Costs/Cycle Vs. Lot Size
A: Stage Two Inventory Costs
B: Stage One Inventory Costs
C: Shortage Costs

NORMAL DEMAND
N(400,50)

Figure 11. Variable Cost Functions for Fixed Lot Size Policy
TABLE 4

RESULTS OF OPTIMUM INVENTORY LEVEL MODEL

<table>
<thead>
<tr>
<th>DEMAND</th>
<th>(N(400,100))</th>
<th>(N(400,50))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum Inventory Level ((Y^*))</td>
<td>479.0</td>
<td>439.0</td>
</tr>
<tr>
<td>Average Lot Size ((Q))</td>
<td>387.0</td>
<td>394.0</td>
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<tr>
<td>Average Surplus Stock ((Z))</td>
<td>91.0</td>
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<tr>
<td>Average Shortage/Cycle</td>
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<tr>
<td>Cost/Cycle</td>
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### TABLE 5

**COMPARISON OF MODEL RESULTS VS. SIMULATION RESULTS FOR FIXED INVENTORY LEVEL POLICY**

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<tr>
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<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Model</td>
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<tr>
<td>Optimum Inventory Level (Y*)</td>
<td>478.0</td>
<td>479.0</td>
</tr>
<tr>
<td>Expected Lot Size (Q)</td>
<td>389.0</td>
<td>387.0</td>
</tr>
<tr>
<td>Expected Surplus Stock (Z)</td>
<td>89.0</td>
<td>91.0</td>
</tr>
<tr>
<td>Expected Shortage/Cycle</td>
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<td>12.0</td>
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<tr>
<td>Expected Cost/Cycle</td>
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<td>Expected Cost/Period</td>
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</tr>
<tr>
<td></td>
<td>T.S. Model</td>
<td>I.D.R. Model</td>
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<td>Optimum Inventory Level (( Y^* ))</td>
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<td>Expected Lot Size (( Q ))</td>
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<td>Expected Surplus Stock (( Z ))</td>
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<td>Expected Shortage/Cycle</td>
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<td>Expected Cost/Cycle</td>
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<td>Expected Cost/Period</td>
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CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

A. Conclusions

When dealing with multistage production-inventory systems, the total system costs should be considered in the development of the inventory control models. If the manufacturing concern is large enough, it is possible that the different stages will be under different managerial control and operated independently of each other, with each manager trying to optimize its operating costs. Since considerable amounts of a firm's capital is invested in the production-inventory system, the operations of the various stages should be viewed collectively instead of individually.

It was shown in the previous chapter that the model developed in this thesis for the multistage production-inventory system described in Chapter III using the fixed inventory level policy, performed well when compared to a simulation of the same system with a normal demand with large variance. Since the model is accurate for normal demand distributions with variances around 10,000, the model should perform as well or better for normal distribution with variances less than 10,000. This was verified when the model was solved for a normal demand distribution which had a variance of 2,500. The model yielded almost identical results as the simulation.

For production-inventory systems having normally distributed demands and the characteristics discussed in Chapter III, the model developed in Chapter III provides solutions almost identical to
simulation solutions in a fraction of the time required to run the simulation. The model should perform equally well with other demand distributions.

The fixed lot size policy could not be modeled using the classical optimization techniques employed in the fixed inventory level policy. The total cost for a given inventory cycle is dependent upon both the lot size \((Q)\) and the ending inventory level of the previous cycle \((Z_{t-1})\) which makes the system state dependent. As a result other techniques such as simulation or dynamic programming must be resorted to. The method of simulation was used to evaluate this policy as was shown in the previous chapter.

It can be seen from the previous chapter that the minimum total cost for the optimum lot size policy is greater than the minimum total cost for the optimum inventory level policy. This is due to the marked increase in shortage cost that occurs when the lot size is fixed and the inventory level permitted to vary. To use the fixed lot size policy for this production-inventory system would be to suboptimize the system. It can also be seen that the total cost/period for the production-inventory system using independent decision rules for determining the optimum inventory level is greater than the total cost/period obtained using the total system model developed in this thesis.

The true optimization of the system is obtained by operating under the optimum inventory level policy, using the total system concept and this policy should be chosen whenever possible if the
goal is to minimize total costs.

Throughout the development of this thesis, no concern has been given to safety stocks or service levels. If a service level is specified, then the system may not be operating under true minimum cost conditions. Specified service levels can be obtained however by simply adjusting the value of the optimum inventory level upwards to that value which will provide the required service level.

B. Recommendations for Further Study

An obvious extension of the work presented in this thesis is to evaluate the model for demand distributions other than the normal distribution. A similar approach to the one used in this thesis could be used in the evaluation.

Another extension of this thesis will be to investigate the multistage production inventory system for systems with various constraints imposed, i.e., floor space restrictions, sales volume restrictions, etc.

An investigation of the fixed lot size policy using a dynamic programming approach would also be logical extension of this thesis.
### TABLE A1. PARAMETERS USED IN FLOW CHART

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>Manufacturing setup cost/setup</td>
</tr>
<tr>
<td>HP</td>
<td>Inventory holding cost/unit (Stage 1)</td>
</tr>
<tr>
<td>CO</td>
<td>Order processing costs/order</td>
</tr>
<tr>
<td>HC</td>
<td>Inventory holding cost/unit (Stage 2)</td>
</tr>
<tr>
<td>CS</td>
<td>Shortage cost/unit</td>
</tr>
<tr>
<td>P</td>
<td>Production rate/period</td>
</tr>
<tr>
<td>N</td>
<td>Number of cycles in simulation run</td>
</tr>
<tr>
<td>A</td>
<td>Number of cycles/period</td>
</tr>
<tr>
<td>X</td>
<td>Demand/cycle (simulated)</td>
</tr>
<tr>
<td>Y</td>
<td>Inventory level at beginning of cycle (fixed)</td>
</tr>
<tr>
<td>M</td>
<td>Increment value of Y (used in runs subsequent to first)</td>
</tr>
<tr>
<td>Z</td>
<td>Leftover stock at end of cycle</td>
</tr>
<tr>
<td>SHORT</td>
<td>Number of units demanded but not available</td>
</tr>
<tr>
<td>Q</td>
<td>Lot size</td>
</tr>
<tr>
<td>OCC</td>
<td>Order cost</td>
</tr>
<tr>
<td>ICP</td>
<td>Inventory holding costs (Stage 1)</td>
</tr>
<tr>
<td>TOTP</td>
<td>Total cycle costs (Stage 1)</td>
</tr>
<tr>
<td>ICC</td>
<td>Inventory holding costs (Stage 2)</td>
</tr>
<tr>
<td>SCC</td>
<td>Shortage cost</td>
</tr>
<tr>
<td>TCOST</td>
<td>Total cycle cost</td>
</tr>
<tr>
<td>SUM</td>
<td>Cumulative total cost</td>
</tr>
</tbody>
</table>
TABLE A1. (cont'd.)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMQ</td>
<td>Cumulative sum of lot size</td>
</tr>
<tr>
<td>SUMZ</td>
<td>Cumulative sum of leftover stock</td>
</tr>
<tr>
<td>SUMS</td>
<td>Cumulative sum of shortages</td>
</tr>
<tr>
<td>SUMP</td>
<td>Cumulative sum of stage 1 inventory costs</td>
</tr>
<tr>
<td>SUMC</td>
<td>Cumulative sum of stage 2 inventory costs</td>
</tr>
<tr>
<td>SUMSH</td>
<td>Cumulative sum of shortage costs</td>
</tr>
<tr>
<td>ASUM</td>
<td>Average total cost/cycle</td>
</tr>
<tr>
<td>ASUMQ</td>
<td>Average lot size/cycle</td>
</tr>
<tr>
<td>ASUMZ</td>
<td>Average leftover stock/cycle</td>
</tr>
<tr>
<td>ASUMS</td>
<td>Average shortage/cycle</td>
</tr>
<tr>
<td>ASUMP</td>
<td>Average inventory cost/cycle (stage 1)</td>
</tr>
<tr>
<td>ASUMC</td>
<td>Average inventory cost/cycle (stage 2)</td>
</tr>
<tr>
<td>ASHOR</td>
<td>Average shortage cost/cycle</td>
</tr>
<tr>
<td>YMAX</td>
<td>Largest value of Y desired in a run</td>
</tr>
</tbody>
</table>
Figure A1. Flow Chart of Simulation Program For Fixed Inventory Level Policy
Figure A1. (continued)
ASUM = SUM/N
ASUMQ = SUMQ/N
ASUMZ = SUMZ/N
ASUMS = SUMS/N
ASUMP = SUMP/N
ASUMC = SUMC/N
ASHOR = SUMSH/N

Figure A1. (continued)
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<tr>
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<th>DESCRIPTION</th>
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<tr>
<td>SUM</td>
<td>Cumulative total cost</td>
</tr>
<tr>
<td>PARAMETER</td>
<td>DESCRIPTION</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td>SUMY</td>
<td>Cumulative sum of inventory level</td>
</tr>
<tr>
<td>SUMZ</td>
<td>Cumulative sum of leftover stock</td>
</tr>
<tr>
<td>SUMS</td>
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<td>ASUM</td>
<td>Average total cost/cycle</td>
</tr>
<tr>
<td>ASUMY</td>
<td>Average inventory level/cycle</td>
</tr>
<tr>
<td>ASUMZ</td>
<td>Average leftover stock/cycle</td>
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<tr>
<td>ASUMS</td>
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<tr>
<td>ASHOR</td>
<td>Average shortage cost/cycle</td>
</tr>
<tr>
<td>QMAX</td>
<td>Largest value of Q desired in a run</td>
</tr>
</tbody>
</table>
Figure B1. Flow Chart of Simulation Program For Fixed Lot Size Policy
Figure Bl. (continued)
ASUM = SUM/N
ASUMY = SUMY/N
ASUMZ = SUMZ/N
ASUMS = SUMS/N
ASUMP = SUMP/N
ASUMC = SUMC/N
ASHOR = SUMSH/N

K < N

NO

WRITE Q
WRITE ASUMY
WRITE ASUMZ
WRITE ASUMS
WRITE ASUMP
WRITE ASUMC
WRITE ASHOR

YES

K = K + 1

Figure Bl. (continued)
Figure Bl. (continued)
BIBLIOGRAPHY


10. Lele, P. T., personal communication.


PERSONAL HISTORY:

Name: Gerald D. Bryant
Birth Place: Anniston, Alabama
Birth Date: February 5, 1940
Parents: Felix and Annie Bryant
Wife: Mable Bryant
Children: Sherry and Melanie Bryant

EDUCATIONAL BACKGROUND:

Anniston High School
Anniston, Alabama
Graduated 1958

University of Alabama
Bachelor of Science in Industrial Engineering
Graduated 1964

Lehigh University
Candidate for Master of Science in Industrial Engineering
1971-1973

HONORS:

Tau Beta Pi, National Engineering Honorary
Alpha Pi Mu, National Industrial Engineering Honorary

PROFESSIONAL EXPERIENCE:

Anniston Army Depot
Anniston, Alabama
Student Trainee (Industrial Engineering) 1958-1963
PROFESSIONAL EXPERIENCE (cont'd.)

Monsanto Company
Decatur, Alabama
Industrial Engineer 1964-1966

United States Navy
Civil Engineer Corps Officer
Public Works Management 1966-1970

Western Electric Company, Inc.
Atlanta, Georgia
Planning Engineer 1970-1971

Western Electric Company, Inc.
Princeton, New Jersey
Development Engineer 1971-1973