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Novel applications in nonlinear crystals based on second-order nonlinear optical interactions

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Novel applications in nonlinear crystals based on second-order nonlinear optical interactions

by

Da Li

Presented to the Graduate and Research Committee of Lehigh University in Candidacy for the Degree of Doctor of Philosophy in Electrical Engineering

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Acknowledgements

“I know that I know nothing.”

-Socrates

It was what I said to myself when I stepped into Rm. 225 of Packard Lab on my first day arriving at Lehigh University to start pursuit of a Ph.D. degree in Electrical and Computer Engineering; and it is what I say to myself now when I am sitting in front of my laptop drafting my Ph.D. dissertation six years later. Undoubtedly, knowledge and skills obtained during the years have been proven useful. Nevertheless, I believe the true meaning of a Ph.D. degree lies in the multiple capabilities she has granted me: for example, the capability of approaching a problem, the capability of collaborating with coworkers, and the capability of overcoming a challenge. She equipped me with a strong mind and made me not fearful of anything on my life path hereafter.

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came up, he and I would work together to figure out the reasons. He is not only my advisor in scientific research, but also my advisor in my life.

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Da Li

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# Table of Contents

Acknowledgements ........................................ iv
List of tables ........................................... xi
List of figures ........................................... xii
Abstract ................................................. 1

## 1 Introduction ........................................ 3
  1.1 Introduction of applications via second-order nonlinear mediums ........................................ 3
  1.2 Overview of the dissertation ........................................... 6

## 2 Single photon counting based on frequency upconversion in PPLN waveguide ........................................ 8
  2.1 Historical overview ........................................... 8
  2.2 Theory of three-wave mixing: sum-frequency generation in periodically poled lithium niobate waveguide ........................................... 12
    2.2.1 Physical Properties of Lithium Niobate ........................................... 12
    2.2.2 Frequency Up-conversion ........................................... 17
    2.2.3 Phase Match and Quasi-Phase-Match for Three Wave Interactions ........................................... 20
    2.2.4 Temperature-dependent Sellmeier Equation ........................................... 26
  2.3 Experimental Setup ........................................... 27
  2.4 Results and discussion ........................................... 31

viii
2.4.1 Waveguide Characteristics..................................................31
2.4.2 Single photon detection.....................................................38
2.5 Analysis of noise photon generation.......................................43
2.6 Novel method: Frequency upconversion in GaP plate..................54
2.7 Sub-conclusion.......................................................................64

3 Evaluation of domain quality in periodically poled structure with non-destructive Method

3.1 Historical review.......................................................................65
3.2 Experimental setup.....................................................................67
3.3 Theory.......................................................................................70
  3.3.1 Perfect QPM grating..........................................................72
  3.3.2 QPM grating with linear taper.............................................76
  3.3.3 QPM grating with periodic errors in boundary position...........80
  3.3.4 Duty cycle error...............................................................82
  3.3.5 QPM grating with random poling period...............................84
  3.3.6 Partially poled waveguide..................................................86
3.4 Analysis on experimental results..............................................89
3.5 Conclusion for Chapter 3.........................................................92

4 Enhancement of forward and backward Anti-stokes Raman signals in lithium niobate waveguides

4.1 Historical Overview...............................................................93
4.2 Theory.......................................................................................94
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3</td>
<td>Experimental Setup</td>
<td>95</td>
</tr>
<tr>
<td>4.4</td>
<td>Results and Discussion</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>4.4.1 Backward-propagating Anti-stokes signals</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>4.4.2 Forward-propagating Anti-stokes signals</td>
<td>102</td>
</tr>
<tr>
<td>4.5</td>
<td>Conclusion for Chapter 4</td>
<td>105</td>
</tr>
<tr>
<td>5</td>
<td>Terahertz generation based on parametric conversion in Gallium Phosphide plate</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>5.1 Overview</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>5.2 Experimental Setup</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>5.3 Results and discussion</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>5.4 Conclusion for Chapter 5</td>
<td>116</td>
</tr>
<tr>
<td>6</td>
<td>Conclusion</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>6.1 Conclusion of Contribution</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>6.2 Outlook and Future Work</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td><strong>Bibliography</strong></td>
<td>122</td>
</tr>
<tr>
<td></td>
<td><strong>Publication List</strong></td>
<td>129</td>
</tr>
<tr>
<td></td>
<td><strong>Vita</strong></td>
<td>132</td>
</tr>
</tbody>
</table>
List of Tables

Table 1  Possible Three-Wave interaction pairs.............................................22
Table 2  Phase-shift with respect to different beam polarization combination.........62
List of Figures

Fig. 1.1  Geometry of second-harmonic generation .................................................3
Fig. 1.2  Geometry of Sum-frequency generation (SFG) ..........................................4
Fig. 1.3  Geometry of Difference-frequency generation (DFG) ...............................4
Fig. 2.1  Hexagonal unit cell of lithium niobate (LiNbO₃) .........................................13
Fig. 2.2  Positions of lithium atoms (double cross-hatched circles) and niobium atoms
         (single cross-hatched circles) with respect to the oxygen octahedral in the
         ferroelectric phase (T<Tc) of lithium niobate ..............................................14
Fig. 2.3  PPXX Bulk or Waveguide ........................................................................16
Fig. 2.4  Wave surfaces of a negative (nₑ<nₒ) uniaxial crystal: full lines, wave surfaces at
         second harmonic frequency; broken lines, wave surfaces at fundamental
         frequency; θ, phase matching angle ...............................................................21
Fig. 2.5  Comparison of Completely Phase Matched condition, Quasi-phase matched
         condition and no phase matched condition. X-axis represents the length in
         crystal; Y-axis represents the output electric field strength .............................25
Fig. 2.6  Experimental setup for single-photon detections based on frequency up-
         conversion .....................................................................................................28
Fig. 2.7  Demonstration of actual setup ....................................................................29
Fig. 2.8 Overall look of the whole detection system……………………………….30

Fig. 2.9 (a) Phase-matching tuning curve for PPLN waveguide, data (dots) & least square fitting (solid line). FWHM=3.74Å. (b) Temperature tuning curve, data (dots) and theoretically calculated curve (solid line)…………………………………………………………………..32

Fig. 2.10 (a) C-band laser spectrum, bandwidth < 0.1 nm (b) 1920 nm laser spectrum, bandwidth = 0.06 nm (c) phase matching curve, and (d) optimum working wavelength at given temperatures…………………………………………….36

Fig. 2.11 (a) (Solid dots) internal conversion efficiency vs. pump power at 980nm under different operating temperatures. (Solid curves) sin²() theoretical fit. (b) (Solid dots) Signal photon rate vs. detected SFG photon rate at different pump levels. (Solid lines) linear least-square fit. …………………………………………………………………………………………………40

Fig. 2.12 Counts spectrum. Pump is always turned on. (Red solid line) Signal is turned off. i.e. dark counts. (Black solid line) Signal is turned on. No other wavelength peaks were found……………………………………………….42

Fig. 2.13 Structure and configuration for SPDC and frequency up-conversion in a PPLN waveguide. A pump wave is coupled from the input facet. Due to nonlinear parametric processes, multiple frequencies can be generated at the output facet…………………………………………………………………….46

Fig. 2.14 Noise photon number at up-converted wavelength vs. propagation distance in the unit of cm………………………………………………………………………48
Fig. 2.15  Phase matching curve for up-conversion process 980nm+1550nm -> 598nm at room temperature ($t=25^\circ$C)…………………………………………………………49

Fig. 2.16  (a) Simulation of phase-matching curve with $\sigma=0.1\*\mu=1.016\ \mu$m………51
Fig. 2.16  (b) Simulation of phase-matching curve with $\sigma=0.01\*\mu=0.1016\ \mu$m………52
Fig. 2.16  (c) Simulation of phase-matching curve with $\sigma=0.001\*\mu=10.16\ nm$………52
Fig. 2.16  (d) Simulation of phase-matching curve with $\sigma=0.0001\*\mu=1.016\ nm$………53

Fig. 2.17  (a) Experimental setup. (b) Input signal within the communication band and pump beam at 979 nm were focused by an aspheric lens on to a GaP plate with the beam waists being located at the exit facet of the GaP crystal with its length of $L \approx 663\mu m$………………………………………………………….55

Fig. 2.18  Pump power dependence for SFG…………………………………………………………..57

Fig. 2.19  Up-converted signal intensity vs. input signal wavelength…………………58

Fig. 2.20  Phase shift for TE-wave with respect to incident angle………………….60

Fig. 2.21  Phase-shift for TM-wave with respect to incident angle………………….61

Fig. 2.22  Incident angle that maximize Eq. 2.30……………………………………..63

Fig. 3.1  Experimental setup for measuring far-field SH pattern………………………..67

Fig. 3.2  Surface-emitting second harmonic generation satisfying condition of $k_{SH}$

$$\sin\theta=\pm 2\pi/\Lambda$$………………………………………………………………..68

Fig 3.3  Perfect domain in periodically poled structure……………………………72

Fig 3.4  1-D Simulation with no domain error………………………………………74

Fig 3.5  Domain period with linear taper…………………………………………..76

Fig 3.6  Simulated far-field SHG patterns for domain period with linear taper error.78
Fig 3.7 Simulation for domain boundaries with periodical errors, η=0.15, Ω=2π/(10Λ₀) ..........................................................81

Fig 3.8 Simulation for domain boundaries with duty cycle Errors.................................83

Fig 3.9 Simulation of domain period with random errors.................................................85

Fig 3.10 Illustration of partially poled waveguide.........................................................86

Fig 3.11 Simulation for 1-D partially poled waveguide.................................................88

Fig 3.12 (Top) Total Diffraction Pattern on CCD (Middle) Right portion (Bottom) Right portion spatial intensity distribution against lateral distance..............90

Fig 3.12 Power Dependence for each fraction...............................................................91

Fig 4.1 Illustration of Stokes and Anti-Stokes Raman Scattering.................................94

Fig 4.2 Experimental setup for forward- and backward- Anti-stokes signal measurement............................................................................................................95

Fig 4.3 Backward Anti-Stokes photon count vs. Raman shift Experimental at different waveguide widths.................................................................98

Fig 4.4 Photon count vs. pump power coupled into waveguide at different widths...99

Fig 4.5 Forward and backward anti-Stokes Raman spectra in LiNbO₃ waveguid....103

Fig 4.6 Forward and backward anti-Stokes Raman spectra in LiNbO₃ bulk crystal. The spectrum for forward configuration in bulk crystal is vertically up-shifted due to presence of strong laser radiation in the rang........................................104

Fig 5.1 Experimental setup for THz generation in GaP stacks.................................108

Fig 5.2 Spectrum of THz radiation generated by reversely-stacked four GaP plates, covering tuning range of 78-300 µm.........................................................110
Fig 5.3  Spectrum of THz radiation generated by forwardly-stacked four GaP wafers, covering tuning range of 135-500 µm…………………………111

Fig 5.4  Photon conversion efficiency vs. input power at 1.0737 µm and 1.0690 µm for reversely-stacked and forwardly-stacked GaP plates, respectively……113

Fig 5.5  Highest THz output power generated by reverse GaP plates versus the number of plates. Blue curve represents nonlinear least square fit to data points using Eq. (5.1)………………………………………………………………………………114
Abstract

Second order nonlinear mediums, like lithium niobate and KTP, have been extensively studied for various applications, such as single photon detection and terahertz generation. In this dissertation, several novel applications in second order nonlinear crystals have been studied in depth, as follows:

1) Highly efficient single-photon level detection at 1.57μm. There exists an absorption peak for carbon dioxide at 1.57 μm, which is extremely desirable for NASA ASCENDS program, due to the fact that it can monitor CO₂ concentration in the outer space. Commercially available detectors at IR wavelengths suffer from high dark counts up to 10⁵/s, as well as low quantum detection efficiencies, usually with values between 10% and 15%. In contrast, single-photon counting in near-IR (600-800nm) can be performed efficiently with silicon APDs. Single photon counting modules with dark counts lower than 25/s and detection efficiency in the range 50% to 70% are commercially available. We took advantage of silicon APDs, implemented efficient conversion from 1.57μm to visible with more than 60% of internal quantum conversion efficiency and demonstrated almost noise-free measurement. As a result, our detection system has reached an ultralow noise level of 25 s⁻¹ and a detectable photon rate of 81 s⁻¹ for communication band.
2) Non-destructive method for evaluating domain errors in PPLN waveguide using surface-emitting geometry (SHG). Periodically-poled lithium niobate (PPLN) waveguide is an efficient nonlinear structure for generation of tunable coherent radiation based on optical parametric oscillators and single-photon detection based on frequency up-conversion. In order to achieve efficient conversion, quasi-phase-matching (QPM) must be satisfied. However, imperfections of periodic domains due to fabrication errors such as linear taper of domain period, duty cycle error, and randomness of domain period significantly affect the performance of the PPLN waveguide. We developed a fast and non-destructive method to statistically evaluate the domain quality of PPLN. The resolution can reach 0.5nm.

3) Anti-Stokes enhancement observation in lithium niobate waveguide. Anti-Stokes signals are much weaker than Stokes counterparts due to significantly reduced phonon occupation numbers. We now measure the forward- and backward- propagating anti-stokes photons from waveguide and observed enhancement factor at least one order of magnitude higher than in the bulk. The required pump power was reduced five orders of magnitude to only a few microwatts. Under two propagation configuration, we observed different layouts of anti-stokes peaks for lithium niobate.

4) Terahertz generation through difference frequency generation of two close wavelength IR beam in Gallium Phosphide (GaP) stacks. By stacking alternatively rotated GaP plates, we reached maximum terahertz photon conversion efficiency of 40%. The corresponding peak power generated inside the four GaP plates approaches 4kW. As the number of plates is increased from four to five, the THz output power is significantly decreased, due to back parametric conversion.
Chapter 1

Introduction

1.1 Introduction of wave interactions in second-order nonlinear mediums

Looking back at the observation of second-harmonic generation by Franken and coworkers [1] in 1961, the field of nonlinear optics is now more than fifty years old. Nonlinear optics is the study of the phenomena that happen as a consequence of the modification of optical properties of a material at the presence of intense light [2]. The response of the material is nonlinear to the applied optical field in a nonlinear optical process. Examples and their potential applications of nonlinear interactions are:

1) Second-harmonic generation

A optical beam with fundamental frequency $\omega$ illustrate on a material with sufficiently large second-order nonlinear susceptibility $\chi^{(2)}$, or second-order nonlinear coefficient $d^{(2)}$. At the output
facet of the material, beam with doubled frequency $2\omega$ can be observed. The conversion efficiency could reach up to 100% under optimum condition. Almost all commercialized green lasers at wavelength of 532 nm are built via second-harmonic generation of 1064 nm emission from Nd:YAG laser crystals.

2) Sum- and Difference- Frequency Generation

If we have two optical beams with distinct frequency components incident on the same material, sum- and difference frequency generation can occur at the same time (assuming $\omega_1 > \omega_2$), being competitive processes depending on phase-matching conditions.
favor. The generation of a new frequency is governed by the coupled-mode equations, which will be introduced in the following chapters. Sum- and Difference- generations are effective and flexible method to build optical sources with various laser-like outputs, whose wavelength is not at the emission lines of any existing laser gain mediums. These techniques are the main focus of this dissertation, as most of our work is based on SFG or DFG.
1.2 Overview of the dissertation

The dissertation is organized as follows.

Chapter 2 demonstrates a single photon detection system based on sum-frequency generation in periodically poled lithium niobate waveguide. We employ plane wave approximation with incorporated waveguide transverse mode profile to describe the three-wave interaction in the nonlinear waveguide for pump beam at 980nm, signal beam at communication band, around 1550nm, and the sum-frequency generation at ~600nm. In order to achieve efficient frequency conversion, a periodically poled structure is employed here to enable the utility of the largest nonlinear coefficient $d_{33}$ of lithium niobate. We then give an overview of the relevant achievements obtained in the field. Upon illustrating our experimental setup, we show corresponding experimental results that support the achievement of single photon detection of communication band photons.

As the noise photons generated in the frequency conversion process are the main factor that limits the sensitivity of the single photon detector, we analyze parametric noise generation on perfect and imperfect lithium niobate gratings and deduce that the noise generation from our waveguide is almost negligible and therefore, we can achieve single photon detection of extremely high sensitivity. Before ending this chapter, we also propose a novel method for potentially achieving single photon detection in GaP-based devices with internal total reflection. A sub-conclusion is presented at the end of this chapter.

Since we mention waveguide imperfection in the previous chapter, we study a non-destructive method to evaluate the periodically poled domain errors in Chapter 3. We
utilize two identical beams counter-propagating in a periodically poled lithium niobate waveguide and analyze the far-field surface-emitting second harmonic generation. Any domain errors deviating from perfect grating structure configuration will be reflected on changes of the far-field patterns. By simulation, we are able to fit some of the imperfection parameters to our waveguide and obtain domain period information in a statistical means.

Also in lithium niobate waveguide, we study anti-Stokes photon enhancement in Chapter 4. By detecting the forward- and backward- propagating anti-Stokes photons from the waveguide, we observe different peaks and identify obvious enhancement than from the bulk, which is also measured. We are able to reduce the pump power to several micro watts and that is five orders of magnitude lower than previous record. Therefore, it is feasible for us to eventually develop a compact and portable Raman probe.

In Chapter 5, we switch our focus from lithium niobate waveguide to GaP stacks, where two close-wavelength infrared beams combine and generate terahertz beam via difference frequency generation (DFG). By alternatively stacking GaP plates, the DFG process meets quasi-phase-matching condition and the terahertz output increases quadratically with the total length of the stack. As we increase the number of GaP stack from four to five, we even observe back parametric conversion, i.e. terahertz beam and one of the infrared beam with lower frequency begins to implement sum-frequency process and convert back to the other IR beam. An overall quantum conversion frequency of 40% has been achieved, with terahertz peak power obtained as high as 4kW.

Chapter 6 concludes the work and contribution mentioned in previous chapters and suggests future works.
Chapter 2

Single photon counting based on frequency upconversion in PPLN waveguide

2.1 Historical Overview

Single-photon detectors [3] play important roles in fiber-based telecommunications, quantum information and computing [4,5], data encryption, imaging [6,7], light detection and ranging, sensing, DNA sequencing, astrophysics, and materials science. For example, the wavelengths in the proximity of 1.55 µm can be used to remotely measure trace levels of CO₂ in the atmosphere, which is considered as an indicator to monitor primary changes in terrestrial ecosystems that are connected with large-scale weather and climate modes. This is extremely important to NASA, especially ASCENDS mission [8].

A perfect single-photon detector is expected to have a detection efficiency of 100%, i.e. the probability for a photon to be successfully detected every time when it hits the detector reaches 100%. In addition, it must have a zero dark count rate. Moreover, it must have a zero reset or dead time. Finally, it must have a zero timing jitter, i.e. zero uncertainty in specifying the instance when a photon arrives. However, for the single-
photon detectors being investigated so far, their performance significantly deviate from the ideal specifications given above. This is especially true at room temperature, which is required to realize certain applications. Therefore, detections of photons at the single-photon levels remain to be challenging. Perhaps, one of the low-cost single-photon detectors is based on a Si-based avalanche photodiode (APD) being operated at room temperature. The detection efficiency, dark count rate, timing jitter, and dead time of the APDs are capable of reaching 70%, 25 s$^{-1}$, ps, and 32 ns at room temperature, respectively. However, the detection wavelength range for these detectors is limited to 400-1060 nm with the detection efficiency peaking at 600 nm.

In the communication band around 1550 nm, however, InGaAs/InP APDs, commercially available, suffer from large dark count rates (i.e. $10^4$-$10^5$ s$^{-1}$) and low detection efficiencies ($\leq 10\%$). In this wavelength range, the best single-photon detectors are perhaps made from superconducting nanowires. Indeed, it was demonstrated that such detectors were capable of reaching the detection efficiency of 93% and dark count rate of $< 10$ s$^{-1}$ at the temperatures up to 2 K [9]. By incorporating a bandpass filter consisting of SiO$_2$/Si bilayers being deposited onto the backside of a thermally oxidized Si substrate, the lowest noise equivalent power was determined to be $2.0\times10^{-19}$ W/Hz$^{1/2}$ [10]. Although these detectors perform extraordinarily, the operating temperatures of up to 2 K may limit their applications.

Besides the detectors based on the superconducting nanowires, frequency upconversion in a nonlinear medium is a viable approach for single-photon detections
This approach builds upon the great performance of the Si-based single-photon detector at the wavelengths of around 600 nm, stated above. By mixing the pump beam at the wavelength of 980 nm from a diode laser with the input signal at 1550 nm in a nonlinear medium, for instance, the output radiation at the wavelength of 600 nm is converted from 1550 nm to 600 nm based on frequency upconversion (i.e. sum-frequency generation) in a nonlinear medium. By measuring the output at 600 nm via a Si-based single-photon detector, one can determine the input signal at 1550 nm, approaching the single-photon levels [14-16]. Besides the classical regime for the upconversion, non-classical intensity correlations between the up-converted photons and pump photons were demonstrated in a quantum frequency-conversion experiment [17].

When a pump beam at the wavelength of 980 nm was used, it can generate the photons at the wavelengths of around 1.55 µm and 2.66 µm through spontaneous parametric down-conversion (SPDC) process in a nonlinear medium. The photons generated around 1.55 µm by SPDC can be up-converted to the photons at 600 nm in the same nonlinear medium. These output photons at 600 nm had dramatically raised the dark count rates of the detection system. Indeed, in the past, it was claimed that the dark counts, typically in the range of $10^4$-$10^5$ s$^{-1}$ originated from SPDC [14-16, 18, 19]. They are still too high for the single-photon detections. To reduce these elevated dark count rates, a pump beam with its wavelength being longer than that for the input signal was used such that SPDC did not generate any photons at the input signal wavelength [20]. As a result, the dark count rates were reduced to $10^3$ s$^{-1}$ only due to anti-Stokes Raman scattering. These rates are still much higher than those from the detectors made from the superconducting...
nanowires. For short pump wavelengths, the upconversion of SPDC photons contribute most to the noise photons, while for long pump wavelengths, the upconversion of the anti-Stokes photons is the main noise generation mechanism. Recently, P. Kuo et al [21] achieved dark count rates of 600cps by using 1.56μm pump for upconversion at 1.3μm. G. Shentu et al [22] implemented ultralow noise up-conversion detector and spectrometer for telecom band with noise equivalent power (NEP) as low as -142 dBm Hz\(^{-1/2}\), almost close to the intrinsic dark count rates of commercial Silicon avalanche photodiode.
2.2 Theory of three-wave mixing: sum-frequency generation in periodically poled lithium niobate waveguide

2.2.1 Physical Properties of Lithium Niobate

Lithium niobate (LiNbO$_3$) is centrally important in integrated and guided-wave optics [23]. It is a human-made dielectric material that does not exist in nature. It was first discovered to be ferroelectric in 1949. It is now one of the most widely used electro-optic materials. This ferroelectric material has a trigonal crystal structure as shown in Fig. 2.1 and is characterized by large pyroelectric, piezoelectric, electro-optic and photo-elastic coefficients. Lithium niobate is naturally birefringent. It has useful acoustic wave properties and a relatively large acousto-optic merit. In addition, it exhibits very strong bulk photovoltaic effect. This effect causes efficient charge migration within the material and in combination with the material’s linear electro-optic effect, can produce a significant photorefractive effect (optically induced refractive index changes). It is transparent for wavelengths between 350 and 5200 nanometers.
Fig. 2.1 Hexagonal unit cell of lithium niobate (LiNbO$_3$) [24]
Lithium niobate’s structures at temperatures below its ferroelectric Curie temperature (approximately 1210 degrees centigrade) consists of planar sheets of oxygen atoms in a distorted hexagonal close-packed configuration (See Fig. 2.2). The octahedral interstices formed in this structure are one-third filled by lithium atoms, one third by niobium atoms, and one-third vacant. In the +c direction, the atoms occur in the interstices in the following order: ..., Nb, vacancy, Li,
Nb, vacancy, Li … In the paraelectric phase above the Curie temperature, the Li atoms lie in an oxygen layer that is c/4 away from the Nb atom, and the Nb atoms are centered between oxygen layers. These positions make the paraelectric phase non-polar. As the temperature decreases from the Curie temperature, the elastic forces of the crystal become dominant and force the lithium and niobium atoms into new positions. The charge separation resulting from this shift of ions relative to the oxygen octahedral causes LiNbO$_3$ to exhibit spontaneous polarization at temperatures below 1210 degrees centigrade. What is more, the shift of Li atoms will be completely consistent, and so do Nb atoms, travelling towards the opposite direction. The displacement of Li atoms and Nb atoms will generate a very strong internal electric field in the crystal. This is a very important property for LiNbO$_3$ to be periodically poled. By applying a series of periodically aligned electrodes on the lithium niobate crystal along the c axis, the domain of lithium niobate can be inversely polarized if the external electric field exceeds its internal electric field. Thus, the periodically poled lithium niobate (See Fig. 2.3) can be used to realize sum-frequency generation by adjusting the phase difference of the three waves if the poling period is properly chosen [23].
The key to producing PPLN is the poling process. The application of an intense electric field can invert the crystal structure, rearranging the crystal at an atomic level. The electric field needed to invert the crystal is very large (~22kV/mm), and is applied for only a few milliseconds to prevent damage to the crystal, after which the periodically reversed structure is permanently imprinted into the crystal structure. The poling voltage must be very carefully controlled so that the poled regions are created with the desired shape.

Fig. 2.3 PPXX Bulk or Waveguide [25]
2.2.2 Frequency Up-conversion

To achieve high efficiency frequency up-conversion, we may utilize an intense pump laser, a very weak input laser, and a bulk crystal periodically poled lithium niobate, quasi-phase-matched to the frequencies of the initial and pump photons [26]. The resulting system up-converts one photon from the input beam and one photon from the pump beam into a single output photon. Due to energy conservation, the output frequency $\omega_0$ is the sum of the input frequency $\omega_i$ and the pump frequency $\omega_p$. The nonlinear interactions in a waveguide should always obey the following Maxwell’s equations [2]:

\[
\nabla^2 E_n - \frac{\varepsilon_n^{(1)}}{c^2} \frac{\partial^2 E_n}{\partial t^2} = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 P_{nNL}^n}{\partial t^2}
\]

(2.1)

where $E_n (n = p, i, s)$is the scalar electric field of the pump/signal/idler waves, $\varepsilon_n^{(1)}$ is the linear permittivity of the nonlinear material for pump/signal/idler waves, $c$ is the speed of light in vacuum, $\varepsilon_0$ is the vacuum permittivity, and $P_{nNL}^n$ is the nonlinear polarization. $E_n$ can be expressed as the product of the slowly varying amplitudes in space ($A_n$) and the normalized waveguide modes [$\varphi_n(x, y)$]:

\[
E_n(z, t) = A_n(z)\varphi_n(x, y) \exp(ik_nz - i\omega_n t) + c.c.
\]

\[
P_{l,s}^{NL}(z, t) = 4\varepsilon_0 d_{eff} E_p E_{s,d}^* \exp(i k_p z - i\omega_p t) + c.c.
\]

(2.2)

where

\[
k_n = \frac{n_p \omega_p}{c}, n_n^2 = \varepsilon_n^{(1)}
\]
Since the fields depend only on the longitudinal coordinate $z$, we can then replace the Laplace operator $\nabla^2$ by $d^2/dz^2$. Note that $\int_{WG} |\varphi_n(x, y)|^2 dS = 1$, and $|d^2A_n/dz^2| < |k_n dA_n/dz|$, we therefore come to the following equation sets that govern the sum-frequency generation process in a periodically poled lithium niobate waveguide, known as the coupled-mode theory [2]:

\[
\begin{align*}
\frac{dE_i}{dz} &= i \frac{\omega_i d_Q}{n_i c} E_o E_p^* \exp(i\Delta k_Q z) \\
\frac{dE_p}{dz} &= i \frac{\omega_p d_Q}{n_p c} E_o E_i^* \exp(i\Delta k_Q z) \\
\frac{dE_o}{dz} &= i \frac{\omega_o d_Q}{n_o c} E_i E_p \exp(-i\Delta k_Q z)
\end{align*}
\]

where $E_i$, $E_p$, $E_o$ are the electric field strengths of the input, pump, and output beams, respectively. $n_i$, $n_p$, $n_o$ are the indices of refraction at the three frequencies; $d_Q$ is the effective nonlinear coefficient; and $z$ is the longitudinal position within the crystal. $\Delta k_Q$ describes the phase mismatch. Here we assume $\Delta k_Q = 0$, i.e. perfect phase match.

Assume that we will operate in the non-depletion regime ($E_i \ll E_p$), where the pump beam is not significantly depleted from up-conversion of the input beam, we can approximate $dE_p/dz = 0$. Eqs. (2.3)- (2.5) are then reduce to two coupled first-order differential equations. Solving for $E_o$ under the initial condition $E_o|_{z=0} = 0$, we find a sinusoidal oscillation for the output field amplitude, which we then can convert to an efficiency of up-conversion $P_o(z)$:

\[
P_o(z) \propto \sin^2(A_s \sqrt{I_e} z)
\]
where $A$ is a constant. The critical spatial period $L_c$ for this process is given by:

$$L_c = \sqrt{\frac{\pi^2 n_i n_o c^2}{\omega_i \omega_o d_Q^2 |E_p|^2}}$$ \hspace{1cm} (2.7)

For a crystal of length $L_c$, where $L_c$ implicitly depends on the pump intensity, the input light will be completely up-converted to the output frequency, and then down-converted back to the original input frequency before leaving the crystal. For a given crystal length $L$, by choosing $L = 0.5L_c$, we can achieve very high conversion efficiency from $\omega_i$ to $\omega_o$. \hspace{1cm} 19
2.2.3 Phase Match and Quasi-Phase-Match for Three Wave Interactions

2.2.3.1 Phase Matching Techniques for Three-wave Interactions

Giordmaine (1962) and Maker et al. (1962) [27] independently showed that one method of phase matching the fundamental and harmonic waves for a second harmonic generation experiment in potassium dihydrogen phosphate (K.D.P) was to utilize the birefringent and dispersive properties of the crystal.

In a particular cone of directions about the optic axis, the ordinary refractive index of the harmonic wave \( f_2 \) is exactly the same as the extraordinary refractive index of the fundamental wave \( f_1 \). In these directions the two waves travel through the crystal with exactly the same speed. Figure 2.4 shows the wave surface of the \( e \) and \( o \) waves at the fundamental frequency \( f_1 \) and at the second harmonic \( f_2 \), two of the surfaces intersecting at the phase matching angle \( \theta_1 \) measured from the optics axis of the crystal.
Fig.2.4 Wave surfaces of a negative ($n_c<n_o$) uniaxial crystal: full lines, wave surfaces at second harmonic frequency; broken lines, wave surfaces at fundamental frequency; $\theta$, phase matching angle [27]

This technique of phase matching may be extended to the more general case of three-coupled wave interactions in the following way. The frequencies and wave vectors of the three waves are governed by the following equations:

\[ f_3 = f_1 + f_2 \]  \hspace{1cm} (2.8)

\[ k_3 = k_1 + k_2 - \Delta k \]  \hspace{1cm} (2.9)

Phase matching is achieved whenever $\Delta k = 0$.

Now we have

\[ k = \frac{2\pi fn_i}{c} \]  \hspace{1cm} (2.10)

where $i$ is the unit vector parallel to $k$, $n$ the refractive index and $c$ the velocity of light.
in vacuum. Using this relationship Eq. (2.7) may be rewritten, under phase matched condition as

\[ f_3 n_3 i_3 = f_1 n_1 i_1 + f_2 n_2 i_2 \]  

(2.11)

It is experimentally convenient if the three waves are propagated with parallel normals, i.e. \( i_1 = i_2 = i_3 \). This condition reduces Eq. (10) to the form

\[ n_3 = \frac{f_1}{f_1 + f_2} n_1 + \frac{f_2}{f_1 + f_2} n_2 \]  

(2.12)

Two types of phase matching in birefringent crystals are possible in principle.

**Type I.** \( f_1 \) and \( f_2 \) are of the same polarization.

**Type II.** \( f_1 \) and \( f_2 \) are of orthogonal polarization.

Possible interaction pairs are listed in Table 1.

<table>
<thead>
<tr>
<th>Positive uniaxial</th>
<th>Negative uniaxial</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type I</strong></td>
<td><strong>Type I</strong></td>
</tr>
<tr>
<td>( f_1 \equiv \text{e ray}, \ f_2 \equiv \text{e ray}, \ f_3 \equiv \text{o ray} )</td>
<td>( f_1 \equiv \text{o ray}, \ f_2 \equiv \text{o ray}, \ f_3 \equiv \text{e ray} )</td>
</tr>
<tr>
<td>( P(\omega_3) = F_1(\theta, \phi, d) E(\omega_1) E(\omega_2) )</td>
<td>( P(\omega_3) = F_2(\theta, \phi, d) E(\omega_1) E(\omega_2) )</td>
</tr>
<tr>
<td>( n_3^{\text{ord}} = \frac{f_1}{f_3} n_1^{\text{ext}} + \frac{f_2}{f_3} n_2^{\text{ext}} )</td>
<td>( n_3^{\text{ext}} = \frac{f_1}{f_3} n_1^{\text{ord}} + \frac{f_2}{f_3} n_2^{\text{ord}} )</td>
</tr>
</tbody>
</table>

| **Type II**        | **Type II**        |
| \( f_1 \equiv \text{o ray}, \ f_2 \equiv \text{e ray}, \ f_3 \equiv \text{o ray} \) | \( f_1 \equiv \text{e ray}, \ f_2 \equiv \text{o ray}, \ f_3 \equiv \text{e ray} \) |
| \( P(\omega_3) = F_1(\theta, \phi, d) E(\omega_1) E(\omega_2) \) | \( P(\omega_3) = F_2(\theta, \phi, d) E(\omega_1) E(\omega_2) \) |
| \( n_3^{\text{ord}} = \frac{f_1}{f_3} n_1^{\text{ord}} + \frac{f_2}{f_3} n_2^{\text{ext}} \) | \( n_3^{\text{ext}} = \frac{f_1}{f_3} n_1^{\text{ext}} + \frac{f_2}{f_3} n_2^{\text{ord}} \) |

\( f_3 = f_2 + f_1 \) and \( f_2 > f_1 \). We assume that \( dn/d\lambda < 0 \) throughout the range from \( f_2 \) to \( f_3 \).

Table 1 Possible Three-Wave interaction pairs [27]
More generally, if we apply the second-order polarizability tensor $d_{ij}$ on the electric field of incoming waves, we get:

$$
\begin{bmatrix}
    d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\
    d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\
    d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36}
\end{bmatrix}
\begin{bmatrix}
    E_{x_1}E_{x_2} \\
    E_{y_1}E_{y_2} \\
    E_{z_1}E_{z_2} \\
    E_{y_1}E_{y_2} + E_{y_2}E_{y_1} \\
    E_{z_1}E_{z_2} + E_{z_2}E_{z_1} \\
    E_{x_1}E_{x_2} + E_{x_2}E_{x_1}
\end{bmatrix}
= \begin{bmatrix}
    P_x \\
    P_y \\
    P_z
\end{bmatrix}
$$

(2.13)

For lithium niobate, since it belongs to the $3m$ point group, its polarizability tensor has the form of:

$$
\begin{bmatrix}
    0 & 0 & 0 & 0 & d_{15} & -d_{22} \\
    -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\
    d_{15} & d_{15} & d_{33} & 0 & 0 & 0
\end{bmatrix}
$$
2.2.3.2 Quasi-Phase-Match for Three Wave Interactions

In the polarizability tensor of lithium niobate, we should pay special attention to the $d_{33}$ term, which is observably greater than other terms. $d_{33}$ in lithium niobate can be as large as $-27 \text{pm/V}$, while $d_{31}$ ($d_{15}$) is only $-4.64 \text{pm/V}$, almost an order of magnitude smaller than $d_{33}$. No doubt we should take advantage of $d_{33}$ to achieve more efficient photon conversion. Since the length of crystal required to achieve conversion is inversely proportional to the nonlinear coefficient, larger $d$ values mean that people can achieve more conversion in a fixed length of crystals, thus enhancing the conversion efficiency.

However, $d_{33}$ is only related to the $z$ part of the polarization of the two source waves and results in the $z$-direction polarization of the output photon. That is to say we would better use two $z$-polarized waves to get a $z$-polarized output wave. Because all nonlinear crystals are also dispersive (i.e. $n = n(\omega)$), it is impossible to satisfy Eq. (2.10) if all three beams are polarized in the same direction, as in Table 1 there is no such interaction pairs as $o + o \rightarrow o$ or $e + e \rightarrow e$. In order to take advantage of the $d_{33}$ we must have a way to satisfy momentum conservation when all the interacting fields are identically polarized. This can be understood by noting that in Eqs. (2.3)~(2.5), where $\Delta k_\phi$ is taken to be the parameter which describes the regular phase-matching condition, the sign of $dE_o/dz$ changes every time $z$ changes by $\pi/\Delta k_\phi$. Consequently, any increase of $E_o$ in the first $\pi/\Delta k_\phi$ length of crystal is immediately canceled out by the second $\pi/\Delta k_\phi$ length of crystal. In quasi-phase-matching, the crystal is poled to create periodic ferroelectric domains, which contributes to the momentum, thus compensating the mismatched
momentum part. The alternating poled ferroelectric domains have the effect of changing the sign of the nonlinear coefficient (See Fig. 2.5). By properly selecting poling period $\Lambda$, we can create a quasi-phase matching situation which will allow us to take advantage of the larger $d_{33}$ nonlinear coefficient.

The phase match condition is given by:

$$\Delta k \equiv k_o - k_i - k_p - \frac{2\pi}{\Lambda} = 0$$  \hspace{1cm} (2.14)

where $k_o$, $k_i$, $k_p$ are the wave numbers of the three beams in the crystal.

---

**Fig. 2.5** Comparison of Completely Phase Matched condition, Quasi-phase matched condition and no phase matched condition. X-axis represents the length in crystal; Y-axis represents the output electric field strength [27]
2.2.4 Temperature-dependent Sellmeier Equation

The temperature-dependent Sellmeier equation describes the variation for refractive index $n$ with incident wavelength $\lambda$ and crystal temperature, $T$. It can be used to find the poling period and approximate temperature for quasi-phase matching. Jundt et al [28-32] give that:

$$n^2 = a_1 + b_1 f + \frac{a_2 + b_2 f}{\lambda^2 - (a_3 + b_3 f)^2} + \frac{a_4 + b_4 f}{\lambda^2 - a_5^2} - a_6 \lambda^2$$

(2.15)

where $a_1=5.35583$, $a_2=0.100473$, $a_3=0.20692$, $a_4=100$, $a_5=11.34927$, $a_6=1.5334 \times 10^{-2}$, $b_1=4.629 \times 10^{-7}$, $b_2=3.826 \times 10^{-8}$, $b_3=-0.89 \times 10^{-8}$, $b_4=2.657 \times 10^{-5}$, and

$$f = (T - 24.5) \times (T + 570.82)$$

for PPLN crystals. $T$ is the crystal temperature in Celsius Degree.

Sellmeier Equation shows that we can adjust the temperature of PPLN to reach the best quasi-phase match condition. In other words, tuning operation temperature would allow us to tune the working signal wavelength.
2.3 Experimental Setup

In this Section, we demonstrate our experimental setup on the single-photon detections based on frequency up-conversion at the pump wavelength of 980/1920 nm [33,34]. Fig. 2.6 is the experimental setup for the single-photon detections. As shown in Fig. 2.6, a CW tunable signal beam in the communication band (tunable from 1.46μm to 1.58μm) could be attenuated by up to ten orders of magnitude with two fixed fiber attenuators and one variable attenuator. It was combined with a strong CW pump beam at 980/1920 nm by a wavelength division multiplexer (WDM) before being coupled into the 3-cm-long PPLN waveguide, mounted inside a temperature-controlled oven with the temperature accuracy being 0.1°C. The poling period ΛG is 10.16μm. A 40× microscope objective was used to collect the up-converted output beam in the visible or near-infrared range. A set of filters were used to remove the two input beams and their second-harmonic outputs. The up-converted photons generated at 600/803 nm were subsequently coupled into a single-mode fiber using a free-space coupling system, and then coupled into a single-photon counting module (SPCM). The key optical component of the free-space coupling system was an aspherical lens, which was used to focus the up-converted beam and coupled into a single-mode fiber with reduced optical aberrations. The free-space coupling system was capable of blocking the unwanted photons from entering the counting module, such as second harmonic beam generated by the pump wave. The actual setup layout is shown in Fig. 2.7 and Fig. 2.8.
Fig. 2.6. Illustration of experimental setup for single-photon detections based on frequency up-conversion.
Fig. 2.7 Demonstration of actual setup
Fig. 2.8 Overall look of the whole detection system
2.4 Results and Discussion

2.4.1 Waveguide Characteristics

1) Waveguide for interaction 980 nm + 1550 nm -> 600 nm

![Graph showing waveguide characteristics](image)
Fig. 2.9 (a) Phase-matching tuning curve for PPLN waveguide, data (dots) & least square fitting (solid line). FWHM = 3.74 Å. (b) Temperature tuning curve, data (dots) and theoretically calculated curve (solid line).
2) Waveguide for 1920 nm + 1550 nm -> 803 nm
Experimental Data

Theoretical Curve

Normalized SFG Power (a.u.)

Wavelength (nm)
Fig. 2.10 (a) C-band laser spectrum, bandwidth < 0.1 nm (b) 1920 nm laser spectrum, bandwidth = 0.06 nm (c) phase matching curve, and (d) optimum working wavelength at given temperatures.

We first measured the phase-matching tuning curve for the periodically poled lithium niobate waveguide by fixing the pump wavelength at 980nm while scanning the signal wavelength from 1535.5nm to 1536.2nm, and fixing pump wavelength at 1920 nm while scanning the signal wavelength from 1470 nm to 1476 nm, as shown in Fig.2.9 (a) and Fig. 2.10 (c), respectively. Theoretically, one can fit a normalized phase-matching tuning curve by:

\[ P(\lambda_{\text{sig}}) = \text{sinc}^2 \left( \frac{\Delta k(\lambda_{\text{sig}}) L_{\text{eff}}}{2} \right) \]  

(2.16)
Here, $\Delta k$ is the wave vector mismatch among the three interaction waves defined as $\Delta k = k_{\text{SFG}} - k_{\text{Pump}} - k_{\text{Signal}} - \frac{2\pi}{\Lambda}$, where $\Lambda$ denotes the poling period and $L_{\text{eff}}$ stands for the effective interaction length. Least square fitting deduces the effective interaction length to be 29 mm, corresponding to a FWHM bandwidth of 3.74Å, as illustrated in Fig.2.9 (a) and Fig. 2.10 (c). The actual bandwidth is broader than the theoretical value of 3.95 Å deduced from the entire crystal length, 30mm. For pump wavelength of 1920nm, the effective interaction length is almost the same as the waveguide length. Deviation may originate from domain imperfection of the PPLN chip [35].

We also measured the operating temperature tuning data and compared them with theoretically calculated curve based on temperature-dependent Sellmeier equation for LiNbO$_3$ [32]. The results are shown in Fig. 2.9 (b) and Fig. 2.10 (d). The data and the curve coincide well, giving a temperature tuning rate of 0.259 nm/°C at pump wavelength $\lambda_p$=980nm, and 0.273 nm/°C at pump wavelength of $\lambda_p$=1920nm.
2.4.2 Single Photon Detection

We measured the power of up-converted output photons for signal wavelengths from 1535nm to 1556nm under five discrete operating temperatures vs. pump power at 980nm, as illustrated in Fig. 2.11 (a). The injected signal power level is stabilized at 1mW. The similar power dependence curves at different operating temperatures shows good thermal stability of the oven. At operating temperature of 60°C, we observed a maximum internal conversion efficiency of 62.1% with input pump power of 84.02mW. Subsequently, we varied the input signal photon injection rate and recorded the detected SFG photon rates at three different pump power levels, see Fig. 2.11 (b). Linear dependence was expected since the probe signal power levels were extremely low. The slope of each fitted line represents conversion efficiencies, which equal to 3.2%, 12.8% and 62.1%. These values have good agreement with those demonstrated in Fig. 2.11 (a). Generally, the detected photon counts should not exceed 10 Million counts per second, for which might induce detection response nonlinearity in the Si APD. We performed averages for multiple times at which SFG counts are comparable to or even less than the APD noise level. The lowest SFG photon count rates we observed from the SPCM were 14.7s⁻¹. By using typical detection efficiency of SPCM at 600nm of 65%, measured filter transmission of 60%, maximum internal conversion of 62.1%, and free-space coupling efficiency of 75%, we deduced minimum detectable signal photons at C-band of 81s⁻¹ in the waveguide. This is a record-low detectable photon rate ever achieved.
Internal Conversion Efficiency vs Pump Power (mW)

- 35°C
- 60°C
- 85°C
- 110°C
- 135°C

Graph shows the relationship between internal conversion efficiency and pump power at different temperatures.
Fig. 2.11 (a) (Solid dots) internal conversion efficiency vs. pump power at 980nm under different operating temperatures. (Solid curves) $\sin^2()$ theoretical fit. (b) (Solid dots) Signal photon rate vs. detected SFG photon rate at different pump levels. (Solid lines) linear least-square fit.
To determine the dark count rate affected by pump beam only, we switched off the weak signal probe, leaving only the pump on. We tuned the pump power from 2mW to 80mW, and we did not see an increase in the dark count rates, which remained steady at 45 counts per second. In order to be more convincing, we recorded the dark count rates spectrum by inserting a monochromometer in front of the free-space coupler. Fig. 2.12 demonstrated the counts spectrum at full response range for the Si APD, from 450nm to 1100nm, with or without signal beam at 1550nm band. The dark counts, when the signal was turned off, showed a nearly flat line (red line) at the level of 45 counts per second for all responsive wavelengths. This could even be further reduced since we confirmed that these dark counts were coming from 980nm photons scattered into free space coupler by surrounding objects. When the signal beam was switched on, we see only the up-converted photons at around 600nm. The figure demonstrate clearly that we have completely blocked the noise photons, including un-phase-matched SHGs of both pump and signal photons, as well as parametric fluorescence caused by spontaneous parametric down conversion (SPDC) of the pump photons.
Fig. 2.12 Counts spectrum. Pump is always turned on. (Red solid line) Signal is turned off. i.e. dark counts. (Black solid line) Signal is turned on. No other wavelength peaks were found.
2.5 Analysis of noise photon generation

The noise sources for sum-frequency generation based single photon detector are re-upconversion of pump-induced Raman scattering and parametric down-conversion. J. S. Pelc [36] studied the Raman scattering and discovered that for pump of 980nm, the portion shifted to 1550nm due to Raman scattering is negligible. Therefore, in this section, we only consider and investigate spontaneous parametric down-conversion and its subsequent frequency upconversion. We then calculate the dark count rates and make comparison between our theoretical results with the measured values. We investigate the generation of the signal photons by SPDC from the pump beam as well as the subsequent re-upconversion process. The theory for estimating the signal photon numbers generated by SPDC has been well developed [37,38] and observed [39]. However, there has been no theory published for the specific configuration and structure used in our experiment [33]. The commonly accepted method for calculating the SPDC photon counts is semi-classic, i.e. using the classic governing equations to describe the down-conversion process denoted as $\omega_3$ (pump) -> $\omega_1$ (idler) + $\omega_2$ (signal), while finding the total power of mode with frequencies near the idler mode ($\omega_1$) by putting one photon into each mode and use it as effective inputs for obtaining the output power of driven signal mode ($\omega_2$) [37,38]. The pump field ($E_p$) is assumed to be un-depleted and thus remains constant. We therefore come to the following equation sets that govern the noise photon generation process, as developed from section 2.2:
\[
\begin{align*}
\frac{dA_i}{dz} &= \frac{2i\omega_i^2 d_{NL}}{k_i c^2} U_1 A_p A_i^* \exp(i\Delta k_N z) \\
\frac{dA_s}{dz} &= \frac{2i\omega_s^2 d_{NL}}{k_s c^2} U_1 A_p A_s^* \exp(i\Delta k_N z) + \frac{2i\omega_s^2 d_{NL}}{k_s c^2} U_2 A_p A_s \exp(-i\Delta k_s z) \\
\frac{dA_u}{dz} &= \frac{2i\omega_u^2 d_{NL}}{k_u c^2} U_2 A_p A_s \exp(i\Delta k_s z)
\end{align*}
\] (2.17)

where \(A_i\) and \(A_s\) are defined as before. \(A_u\) is the slowly varying electric field amplitude in space for the sum-frequency output, here mainly responsible for the noise. The two phase mismatch vectors, \(\Delta k_N\) and \(\Delta k_S\), for the down-conversion and upconversion process respectively, are defined as:

\[
\Delta k_N = k_p - k_i - k_s = \frac{2\pi n_p}{\lambda_p} - \frac{2\pi n_i}{\lambda_i} - \frac{2\pi n_s}{\lambda_s}
\]

\[
\Delta k_S = k_u - k_p - k_s = \frac{2\pi n_u}{\lambda_u} - \frac{2\pi n_p}{\lambda_p} - \frac{2\pi n_s}{\lambda_s}
\] (2.18)

\(U_1\) and \(U_2\) are the mode coupling coefficients:

\[
U_1 = \iint_{WG} \varphi_i \varphi_s \varphi_p dS / \iint_{WG} dS
\]

\[
U_2 = \iint_{WG} \varphi_u \varphi_s \varphi_p dS / \iint_{WG} dS
\] (2.19)

The initial conditions are \(A_s(0)=A_u(0)=0\), and \(A_i(0)\) is derived later. In fact, the periodically inverted domain along the propagation direction caused the nonlinear coefficient to flip sign accordingly. The nonlinear coefficient \(d_{NL}\) is then a function of propagation distance, and can be expressed as:
\[ d_{NL}(z) = \begin{cases} 
    d_{33} & m\Lambda \leq z \leq m\Lambda + \Lambda / 2 \\
    -d_{33} & m\Lambda - \Lambda / 2 \leq z \leq m\Lambda 
\end{cases} \]  
\quad (2.20)

where we assume the PPLN chip has \(2m\) or \(2m+1\) periods and \(m\) is an integer. Although we may expand the nonlinear space-varying coefficient into Fourier series, we should include all terms and should not neglect the cross terms since the grating periods are not designed to enhance the first SPDC process.

We use the zero-point energy of the electromagnetic modes as an effective input field with an intensity equivalent to one photon per mode, denoted here as \(A_i(0)\) [37,38]. Consider a PPLN waveguide structure, where its width \(W\) and depth \(d\) are much shorter than its length \(L\), see Fig. 2.13. In such a case, it allows us to use the one-dimensional density of states to calculate the input idler power density in the range of \(k_i\) to \(k_i+dk_i\):

\[
dI_i(0) = \frac{dk_i}{2\pi/L} \frac{\hbar \omega_i}{Vn_i} \frac{c}{d} \frac{\lambda_i}{\lambda_i^2} \]

\[
dk_i = d\left(\frac{n_i\omega_i}{c}\right) = \frac{2\pi n_i}{\lambda_i^2} d\lambda_i \]  
\quad (2.21)

Since \(dI_i(0) = 2n_i\epsilon_0\epsilon|A_i(0)|^2\), we achieved an explicit expression for \(|A_i(0)|^2\):

\[ |A_i(0)|^2 = \frac{\hbar c}{2n_i\epsilon_0\lambda_i\lambda_i^2 S} d\lambda_i \]  
\quad (2.22)
Fig. 2.13. Structure and configuration for SPDC and frequency up-conversion in a PPLN waveguide. A pump wave is coupled from the input facet. Due to nonlinear parametric processes, multiple frequencies can be generated at the output facet.
We run numerical simulations based on (2.17) and illustrate the noise power evolution against propagation distance $z$. A typical solution curve for the noise photon count rate is shown in Fig. 2.14. As can be seen from the figure, the noise SFG power experienced a sinusoidal oscillation with regard to the propagation distance, and reached its peak at around $z = 3$ cm. The numerical simulation was run by assuming that $d\lambda_s = 0.357$ nm when calculating the value for $|A_i(0)|^2$. In fact, the total noise counts should be the summation of the SFG generation at the output facet over wavelengths adjacent to the target wavelength, i.e. 1550 nm. Here, we take the integration for $\lambda_s$ from 1549.2 nm to 1550.8 nm (which is broad enough to cover the main sinc function peak of the phase matching curve, see Fig. 2.15, to reach total noise counts of 397 s$^{-1}$. Based on measured collection efficiency of 90.0%, the filter transmittance of 60.0%, the coupling efficiency of 75.0% of into the Si APD, and the detection efficiency of 65.0% for the photons around 600 nm in the Si APD, we have obtained an effective detection efficiency of 26.3% and dark count rate of 104.5 s$^{-1}$. 
Fig. 2.14. Noise photon number at up-converted wavelength vs. propagation distance in the unit of cm.
As mentioned in [35], fabrication errors of the periodically poled structure, such as random duty cycle (RDC) errors, can result in an extension up to hundreds of nanometers from the phase-matching peak. We now investigate how to deduce the average standard deviation of domain period.

The phase matching curve is a function of both the signal wavelength $\lambda_s$ and domain period $d$:
\[
Y(\lambda_s, \Lambda) = C \frac{\sin^2[\Delta k(\lambda_s, \Lambda)L/2]}{[\Delta k(\lambda_s, \Lambda)L/2]^2}
\]  
(2.23)

where:

\[
\Delta k(\lambda_s, \Lambda) = \frac{2\pi n_u - 2\pi n_p}{\lambda_a} - \frac{2\pi n_s}{\lambda_s} - \frac{2\pi}{\Lambda}
\]  
(2.24)

To find out the expected phase matching curve for some certain domain period distribution \(f(\Lambda)\), the following expression is used:

\[
\bar{Y}(\lambda_s) = \frac{\int_{-\infty}^{+\infty} f(\Lambda)Y(\lambda_s, \Lambda) \, d\Lambda}{\int_{-\infty}^{+\infty} f(\Lambda) \, d\Lambda}
\]  
(2.25)

Here we assume the probability density function (pdf) to be a standardized Normal Distribution:

\[
f(\Lambda) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\left(\frac{\Lambda - \mu}{2\sigma^2}\right)^2\right]
\]  
(2.26)

\(\mu=\)mean value of domain period, \(\sigma=\)standard deviation of domain period. Since it is a standardized pdf, the integration in the denominator:

\[
\int_{-\infty}^{+\infty} f(\Lambda) \, d\Lambda = 1
\]  
(2.27)

So finally:
Simulations are shown below:

$$\bar{Y}(\lambda_s) = \int_{-\infty}^{\infty} f(\Lambda) Y(\lambda_s, \Lambda) d\Lambda$$

(2.28)

$$\mu = 10.16 \mu m$$

Fig. 2.16 (a) Simulation of phase-matching curve with $\sigma = 0.1*, \mu = 1.016 \mu m$
Fig. 2.16 (b) Simulation of phase-matching curve with $\sigma=0.01*\mu=0.1016\ \mu m$

Fig. 2.16 (c) Simulation of phase-matching curve with $\sigma=0.001*\mu=10.16\ nm$
Fig. 2.16 (d) Simulation of phase-matching curve with $\sigma=0.0001*\mu=1.016$ nm

Compare the bandwidth of the phase-matching curve we measured (0.357 nm) in the actual experiment with simulations in Fig. 2.16 (a)-(d), we deduced that the average standard deviation (STD) of domain period to be 0.02%, i.e. $\sigma=0.02\%l$, where $l=A_G/2=5.08\mu$m. Such an STD value would modify the total noise photon generation rate to be 294.42cps and a detected rate to be 77.43cps. It is true that randomness of domain period will increase the photon numbers in the parametric down-conversion process. However, the random distribution of domain lengths will drastically decrease the subsequent up-conversion process and as a result, the noise SFG photons will decrease essentially. This theoretical value is quite close to our observation and therefore supports our experimental findings.
2.6 Novel method: Frequency upconversion in GaP plate

III-V semiconductors exhibit large second-order nonlinear coefficients [40]. For example, for a gallium phosphide (GaP) crystal, one of the element of the second-order nonlinear susceptibility tensor, i.e. $d_{14}$, can be as large as 100 pm/V [40]. This value is about three times greater than $d_{33}$ for lithium niobate, which has been considered as one of the most widely used efficient nonlinear crystals for single photon detection at the communication band based on frequency upconversion [14, 20, 33]. The high nonlinearity and wide transparent range of GaP in infrared (0.5-11 µm) and THz regions makes it attractive for achieving efficient frequency conversion [41]. Moreover, GaP is free from photorefractive effect which can be detrimental in frequency conversion, limiting conversion efficiency and causing permanent damage [42].

However, due to the cubic symmetry of the zincblende lattice, GaP has no birefringence. Therefore, phase matching is not possible. As a result, GaP has not been considered as an efficient nonlinear material for frequency conversion in the past. By periodically inverted <111> direction of the GaP plates in a stack, efficient conversion can be achieved in the THz region based on quasi-phase-matching [41]. However, in the infrared region, such a scheme is difficult to realize since the required thickness is too thin in order to satisfy the quasi-phase-matching condition. Recently, enhanced sum-frequency generation in doubly resonant GaP photonic crystal nanocavities was demonstrated [43]. However, for practical applications an alternative scheme must be utilized. It is worth noting that total-internal-reflection quasi-phase-matching (TIR-QPM)
can be used to enhance the efficiency of frequency conversion. In such a scheme, each
TIR induces a phase shift, which can be exploited to compensate the phase mismatch for
nonlinear frequency conversion [44,45].

Here, we report our results of achieving frequency up-conversion in a GaP plate, i.e.
up-converting the incoming photons within the communication band to those within the
visible band.

![Diagram](image)

**Fig. 2.17.** (a) Experimental setup. (b) Input signal within the
communication band and pump beam at 979 nm were focused by an
aspheric lens on to a GaP plate with the beam waists being located at the
exit facet of the GaP crystal with its length of $L \approx 663 \ \mu m$.

As shown in Fig. 2.17 (a), a CW C-band semiconductor laser (Agilent) with its output
wavelength tunable from 1.48 $\mu m$ to 1.58$\mu m$ is used as the input signal. It was combined
with a pump diode laser at the wavelength of 979 nm by a dichroic mirror. The two
beams were focused by an aspheric lens having the focal length of 5 mm (Thorlabs). The
beam diameter measured at its waist is about 4 µm. A pinhole was placed in the beam path to ensure that both pump and signal beams spacially overlap at the focal plane which is located at the exit facet of the crystal, see Fig. 2.17 (b). The undoped GaP plate was used for achieving upconversion. The thickness of the crystal is 663 µm. The upconverted output generated via sum-frequency generation (SFG) was subsequently collected by a 40× objective. After passing through a set of optical filters, the sum-frequency was coupled into the single photon counting module (Perkin Elmer). It has a dark count rate of 25 s⁻¹.

To ensure the recorded counts originate from SFG rather than noise photons from the pump source or from ambient air, we turned off both the laser sources, turned on only the pump, and turned on only the input signal only. For these three cases, we took the average of the signal output from the single photon counting module for 200 times. The average signals from the counter were measured to be 27 s⁻¹, 29 s⁻¹ and 26 s⁻¹, respectively. Since these count rates are essentially the same as the dark count rate of the counter, any count rates above these levels originate from upconversion. We subsequently measured the pump power dependence, i.e. the upconverted photon counter rate as a function of the pump power when the incident power of the C-band source was set to 2 mW. As shown in Fig. 2.18, the power dependence curve demonstrated good linearity, which is consistent with our theory. We would like to note that the coherent length of the SFG process can be calculated to be about 3 µm, which is much shorter than the crystal thickness (663 µm). Therefore, only a small portion of the GaP crystal effectively contributed to the power conversion for SFG. Since the conversion efficiency
was relatively low, depletion of the input signal is negligible. The maximum count rate we have achieved in our experiment was 10000 s$^{-1}$ under the pump power of 250 mW. The normalized conversion efficiency is determined to be about 100%/kW·cm$^2$, which is much higher than that for LiNbO$_3$. As we tuned the input signal wavelength, the upconverted output power exhibited oscillations, see Fig. 2.19. This is due to the fact that the coherence length oscillates as the input wavelength is increased, similar to the behavior in CdSe [46].

![Graph showing the relationship between pump power and SFG counts.](image)

**Fig. 2.18** Pump power dependence for SFG
Fig. 2.19 Up-converted signal intensity vs. input signal wavelength.

To further improve the conversion efficiency for the upconversion, we can exploit TIR-QPM [44]. Generally speaking, the sum frequency generation for total internal reflection assisted quasi-phase matching is governed by the following equation:

$$I_3 = \frac{8\pi^2 d_{eff}^2}{\eta_1 \eta_2 \eta_3 \lambda_1 \lambda_2 \epsilon_0} \left[ \frac{\sin(\Delta kL/2)}{\Delta kL/2} \right]^2 \left[ \frac{\sin(N\epsilon/2)}{\sin(\epsilon/2)} \right]^2$$

(2.29)

To maximize the above equation, we need to maximize the product of the last two brackets:
\[
\left[ \frac{\sin(\Delta kL/2)}{\Delta k/2} \right]^2 \left[ \frac{\sin(N\varepsilon/2)}{\varepsilon/2} \right]^2
\]  

(2.30)

where \( \varepsilon = \Delta kL + \Delta \phi + \phi_s - 2\pi M \). \( \Delta \phi = \phi_3 - \phi_2 - \phi_1 \). \( \phi_s \) is either 0 or \( \pi \), depending on the incident waves geometry and combination, e.g. pps or sss.

In isotropic crystals like GaP, it is not generally possible to make \( \Delta k = 0 \) where the first bracket reaches its maximum for a given set of wavelengths. However we can make \( \sin(\Delta kL/2) = 1 \). This requires \( \Delta kL = (2N + 1)\pi \). The second term reaches its maximum at \( \varepsilon = 0 \) and this time, we can achieve the goal with the help of phase shift upon total internal reflection.

For TE wave: (s wave or \( \sigma \) wave), it’s phase shift upon TIR as regard to TIR angle is shown below:
Fig. 2.20 Phase shift for TE-wave with respect to incident angle

And for TM wave (p wave, π wave):
Fig. 2.21 Phase-shift for TM-wave with respect to incident angle

The expressions for the phase shift are:

\[
TE = \begin{cases} 
0 & \theta < \theta_c \\
-2 \tan^{-1} \left( \frac{n^2 \sin^2 \theta_i - 1}{n \cos \theta_i} \right) & \theta > \theta_c
\end{cases}
\]

\[
TM = \begin{cases} 
0 & \theta < \theta_B \\
\pi & \theta_B < \theta < \theta_c \\
\pi - 2 \tan^{-1} \left( \frac{n^2 \sin^2 \theta_i - 1}{\cos \theta_i} \right) & \theta > \theta_c
\end{cases}
\]  

(2.31)  

(2.32)

A summary for $\phi_s$ with regard to combinations of beam polarization is shown below:
Therefore, to fulfill $\epsilon = 0$, $\Delta \phi$ will need to satisfy the following conditions (note that $\Delta kL = (2N + 1)\pi$):

By plotting $\Delta \phi$ against incident total reflection angles, we found that only pps combination resulted in a valid incident angle:

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</tbody>
</table>

Table 2.2 Phase-shift with respect to different beam polarization combination
Fig. 2.22 Incident angle that maximize Eq. 2(30)

From Fig. 2.22 we see that at internal incident angle of 20.56°, the phase shift reaches \(-\pi\) and compensates the phase mismatch caused by normal dispersion.

From above calculation, we see that when each leg of the zigzag path in the crystal plate is an odd integer multiple of the coherent length, the net phase shift on each TIR bounce can result in the accumulative growth of the generated field [44], thus enhancing the up-converted signal intensity. At the input signal wavelength of 1550 nm, we have calculated the incident angle to be \(\theta = 20.56^\circ\). The required plate thickness \(t\) can be determined from \(t/cos\theta = (2N+1)L_c\), where \(N\) is an integer and \(L_c\) is the coherence length. GaP-based devices have potential to be utilized in single photon detection as an alternative to lithium niobate based devices.
2.7 Conclusion for Chapter 2

Commercially available detectors at IR wavelengths suffer from several drawbacks. InGaAs/InP avalanche photodiodes (APDs) are commonly used. They have high dark counts up to $10^5$/s, as well as low quantum detection efficiencies, usually with values between 10% and 15%. Their long dead-time will cap the detection rate to no more than 400 kHz. In contrast, single-photon counting in near-IR (600-800nm) can be performed efficiently with silicon APDs. Single photon counting modules with dark counts lower than 25/s and detection efficiency in the range 50% to 70% are commercially available. We can take advantage of silicon APDs if efficient conversion from C-band to near-IR can be realized. In our experiment, we have implemented single photon detection in periodically poled lithium niobate waveguide with 980nm pump, with negligible noise photon counts. We then theoretically investigated the generation of noise counts by considering both perfect and imperfect grating structures for lithium niobate. The key is that for a given grating period, the gain peak for spontaneous parametric down-conversion process and the gain peak for designed up-conversion process usually do not overlap, resulting in intrinsic low noise photon generation. The simulation agreed well with our experimental results. At last, we explored GaP as an alternative second-order nonlinear material to implement single photon detection. Experimental results have been demonstrated and we investigated total-internal-reflection assisted phase-matching in GaP plate, where by other means perfect phase match cannot be achieved.
Chapter 3

Evaluation of domain quality in periodically poled structure with non-destructive method

3.1 Historical review

Periodically-poled lithium niobate (PPLN) waveguide is an efficient nonlinear structure for generation of tunable coherent radiation based on optical parametric oscillators [20] and single-photon detection based on frequency up-conversion [33]. In order to achieve efficient conversion, quasi-phase-matching (QPM) must be satisfied. However, imperfections of periodic domains due to fabrication errors such as linear taper of domain period, duty cycle error, and randomness of domain period [35] significantly affect the performance of the PPLN waveguide. When a bulk PPLN crystal under an electric field was illuminated with a laser, domain quality [47] and duty cycle randomness [48,49] were studied by analyzing far-field diffraction patterns. However, these optical techniques suffer from poor spatial resolutions on the order of 10 μm. For reaching a higher resolution, surface treatment techniques must be employed. However, these techniques are destructive. There is no effective technique for evaluating domains within PPLN waveguides.
Here, we demonstrate that surface-emitting second-harmonic generation is indeed an effective technique for evaluating the domain quality of a PPLN waveguide.
3.2 Experimental setup and results

Fig. 3.1 Experimental setup for measuring far-field SH pattern
Fig. 3.2 Surface-emitting second harmonic generation satisfying condition of $k_{\text{SH}}$

$$\sin \theta = \pm 2\pi / \Lambda$$
In our experiment, a compact and portable Nd:YLF laser was used as a pump beam for investigating the surface-emitting second-harmonic generation (SHG), see Fig. 1(a). The pump spectrum centered at 1.053 μm had a linewidth of 0.5 nm (135 GHz). An acoustic-optic Q-switch was used to modulate the pump beam at the repetition rate of 5 kHz. The pulse width of the laser was 10 ns. The beam was split by a 50/50 beam splitter and coupled into the PPLN waveguide using aspheric lenses. The \( y \)-propagating surface-emitting SH beam was focused along \( z \) direction by a cylindrical lens to avoid divergence due to a 6-μm-wide waveguide. The final spatial profile of the SH beam was observed directly on a white screen being placed at a distance of 1 m away from the waveguide. It was also imaged by a CCD beam profiler.
3.3 Theory

Under an ideal condition, the counter-propagating pump beams having exactly the same wave vector generate uniform surface-emitting SH beams [50] with their wave-vector component along x direction being equal to $\pm 2\pi/\Lambda$, where $\Lambda$ is the domain period of the QPM grating. The SH beam emitting from the waveguide surface is refracted by LiNbO$_3$, resulting in a relation given by $\sin \theta = \lambda/\Lambda$, where $\theta$ is the emitting angle of the SH beam (Fig. 3.2) and $\lambda$ is the SH wavelength. The 1-D spatial profile, $F_z(x)$, is given by

$$F_z(x) = \left| f_z(x) \right|^2 = \frac{\exp(ikz)}{\sqrt{i\lambda z}} \exp\left(i \frac{kx^2}{2z} \right) \int_{-\infty}^{\infty} f_0(x_0) \exp\left(i \frac{kx_0^2}{2z} \right) \exp\left(-i \frac{kxx_0}{\lambda z}\right) dx_0 \right|^2 \quad (3.1)$$

where $k$ is the wave vector of the SH, $z$ is the distance from waveguide chip to the projection screen, $\lambda$ is the SH wavelength, $x_0$ is the coordinates on the waveguide, and $f_0(x_0)$ is the waveguide pupil function whose value alternating between 1/2 and -1/2 at the opposite domains. In the actual experiment, the distance $z=1.06\text{m}$, which was far more less than the critical distance $z_{\text{critical}}=z^2/\lambda=1711\text{m}$. Therefore, the term $\exp(ikx_0^2/2z)$ cannot be neglected. Thus the projected spatial profile was not simply the Fourier transfer of the pupil function $f_0(x_0)$. In other words, it was not *Fraunhofer* diffraction. As can be seen from Eq. (3.1), the SH spatial profile projected on the screen depend heavily on the QPM grating pupil function $f_0(x_0)$, i.e. the SH spatial profile contains direct information of the grating shape. By substituting different $f_0(x_0)$ into Eq. (3.1), we are able to obtain corresponding SH far-field spatial profile. We then analyze our experimental results and
compare with theoretical values to unveil the actual waveguide pupil function.

The expression for \( F_z(x) \) can be readily extended to 2-D case by integrating the \( y_0 \) coordinate. However, since the SH intensity remained constant along \( y_0 \) direction, the integral would simply be a constant and would impose no impact on the lateral spatial profile projected on the screen plane. Therefore, in the following sections we will only consider 1-D cases.
3.3.1 Perfect QPM grating

We first consider the ideal case, i.e. the domain width $w = \Lambda_0/2$ and keeps a constant. The optical axis polarization in adjacent domains alternate accordingly. We number the central two domains of the waveguide with integer $m=0$ and mark them as benchmark. We then number those domains left to the benchmark with negative integers ($m=-1, -2\ldots$) and those right to the benchmark with positive integers ($m=1, 2\ldots$), as illustrated in Fig. 2. We assume that the whole waveguide has $2N+1$ domain periods. For the actual waveguide we were using, $N=1500$.

![Diagram of perfect domain in periodically poled structure](image)

Fig 3.3 Perfect domain in periodically poled structure
Following notations defined above, the waveguide pupil function, $f_0(x_0)$, at the $m$th period, can be expressed explicitly as:

$$f_{0,m}(x_0) = \begin{cases} 
\frac{1}{2}, & m\Lambda_0 \leq x_0 \leq m\Lambda_0 + \frac{1}{2}\Lambda_0 \\
-\frac{1}{2}, & m\Lambda_0 - \frac{1}{2}\Lambda_0 < x_0 < m\Lambda_0 
\end{cases}$$

(3.2)

Substituting the expression of $f_{0,m}(x_0)$ into Eq. (1), we obtain the far-field integral for $m$th domain:

$$f_{z,m}(x) = \frac{\exp(ikz)}{i\lambda z} \cdot \exp\left(i\frac{kx^2}{2z}\right) \left[ \int_{m\Lambda_0}^{m\Lambda_0 + \frac{1}{2}\Lambda_0} \exp\left(-\frac{i2\pi}{\lambda z} x x_0\right) dx_0 + \int_{m\Lambda_0 - \frac{1}{2}\Lambda_0}^{m\Lambda_0} \exp\left(i\frac{kx^2}{2z}\right) \exp\left(-\frac{i2\pi}{\lambda z} x x_0\right) dx_0 \right]$$

(3.3)

Here, $x_0$ is the coordinate on the waveguide surface, and $x$ is the coordinate on the projection plane. $erfz(x)$ is known as the error function with complex arguments, which is usually discussed in scaled form as the Faddeeva function [51].

The total far-field integral can now be obtained by summarizing domain period number $m$ from $-N$ to $N$:

$$f_z(x) = A \cdot \left[ \frac{1}{2} g(N + \frac{1}{2}) + \frac{1}{2} g(-N - \frac{1}{2}) + \sum_{p=-N}^{N} g(p + \frac{1}{2}) - \sum_{p=-N}^{N} g(p) \right]$$

(3.4)
where

\[ A = -\frac{i\sqrt{\pi}}{2\sqrt{2z}} \exp(ikz), \quad g(m) = \text{erfz}\left[\frac{i^2k}{2z}(m\Lambda_0 - x)\right] \]

Finally, the 1-D spatial profile is given by:

\[ F_\varepsilon(x) = |f_\varepsilon(x)|^2 \]  

and is numerically calculated. The simulation curve is shown in Fig. 3.4, for N=1500.

![Simulation Curve](image)
As illustrated in the figures, there existed two peaks centered at ±3.5395mm, which agreed with the theoretically predicted value at $x_{1,2}=±\lambda z/\Lambda_0$. The width of each peak was 29.6mm, very close to the actual waveguide length.
3.3.2 QPM grating with linear taper

In the growth of a periodically poled lithium niobate crystal, a drift in the poling period would occur along the length of a sample due to drift in the crystal pull rate or in the modulation frequency of the boundary positions. [35]. We can then define a linear taper coefficient \( c_t = (l_N - l_1) / (2N) \), where \( l_N \) and \( l_1 \) are the domain widths of the last and first period, and \( N \) is the period number, see Fig. 3.5.

![Fig. 3.5 Domain period with linear taper](image)

We still assume that the waveguide has \( 2N+1 \) periods, from \( m = -N \) to \( m = N \). Since each domain period length varies, we now assume the period with notation number \( m = 0 \) has the “correct” length, \( \Lambda_0 \). We may now write domain length for the \( m \)th period:

\[
\Lambda_m = \Lambda_0 + mc_i
\]  

(3.7)

And the central coordinate for each period is given by:
\[ x_m = m \Lambda_0 + \frac{m^2}{2} c_t \]  

(3.8)

Following notations defined above, the waveguide pupil function, \( f_0(x_0) \), at the \( m \)th period, can be expressed explicitly as:

\[
 f_{0,m}(x_0) = \begin{cases} 
 \frac{1}{2}, & (x_m \leq x_0 \leq x_m + \frac{1}{2} \Lambda_m) \\
 -\frac{1}{2}, & (x_m - \frac{1}{2} \Lambda_m < x_0 < x_m) 
\end{cases}
\]  

(3.9)

Similar integration to Eq. (3) was performed:

\[
 f_{z,m}(x) = \frac{\exp(ikz)}{\sqrt{i \lambda z}} \left( -i \sqrt{\pi} \right)^2 \frac{1}{2} \text{erf} \left[ \frac{i k}{2 z} (x_m + \frac{\Lambda_m}{2} - x) \right] - \text{erf} \left[ \frac{i k}{2 z} (x_m - x) \right] + \frac{1}{2} \text{erf} \left[ \frac{i k}{2 z} (x_m - \frac{\Lambda_m}{2} - x) \right] 
\]

(3.10)

The final 1-D spatial far-field profile can now be obtained as:

\[
 F_z(x) = |A| \left[ \frac{1}{2} \text{erf} \left[ \frac{i k}{2 z} (x_m + \frac{\Lambda_m}{2} - x) \right] + \frac{1}{2} \text{erf} \left[ \frac{i k}{2 z} (x_m - \frac{\Lambda_m}{2} - x) \right] + \sum_{n=-N}^{N} \text{erf} \left[ \frac{i k}{2 z} (x_n + \frac{\Lambda_m}{2} - x) \right] - \sum_{n=-N}^{N} \text{erf} \left[ \frac{i k}{2 z} (x_n - \frac{\Lambda_m}{2} - x) \right] \right] 
\]

(3.11)

The corresponding simulation curves are demonstrated in Fig. 3.6, for \( N=1500 \) against various linear taper coefficient values from \( c_t=1\,\text{Å/period} \) to \( c_t=9\,\text{Å/period} \).
Fig. 3.6 Simulated far-field SHG patterns for domain period with linear taper error
As it can be seen from Fig. 3.6, the larger the linear taper coefficient, the larger the pattern distortion, and the wider of widths of the two legs.
3.3.3 QPM grating with periodic errors in boundary position

Due to oscillation in the positions of the freezing boundary during growth of a periodically poled crystal, a periodic domain drift in the position of the boundaries would occur [35]. Such domain period defect can be modeled with the following equation:

\[ x_m = m\Lambda_0 + \eta\Lambda_0 \sin(\Omega m\Lambda_0) \]  

In Eq. (3.12), we notice that the first term, \( m\Lambda_0 \), is considered as the equivalent position for each domain boundaries. The second term can be viewed as a perturbation imposed on the “ideal” situation. \( \eta \) is the relative amplitude of the perturbation, and \( \Omega \) is the oscillation frequency. Since it is obvious that:

\[ x_m - x_{m-1} = (\Lambda_m + \Lambda_{m-1})/2 \]  

(3.13)

We can deduce the expression for \( \Lambda_m \):

\[
\Lambda_m = \begin{cases} 
\Lambda_0 + 2\eta\Lambda_0 [\sin(m\Omega\Lambda_0) + 2\sum_{p=1}^{m-1} (-1)^{m-p} \sin(p\Omega\Lambda_0)] & (m \geq 2) \\
\Lambda_0 + 2\eta\Lambda_0 \sin(\Omega\Lambda_0) & (m = 1) \\
\Lambda_0 & (m = 0) \\
\Lambda_{-m} & (m \leq -1) 
\end{cases}
\]  

(3.14)

We plug the expression for \( x_m \) and \( \Lambda_m \) into Eq. (3.11) and get the following simulation curve for this case.
Fig. 3.7 Simulation for domain boundaries with periodical errors, $\eta=0.15$, $\Omega=2\pi/(10\Lambda_0)$
### 3.3.4 Duty cycle error

In an ideal periodically poled crystal, the direction of optical axis in one complete period is pointing upward for exactly half of the period and downward for the other half of the period. If we consider a rectangular wave of period \( \Lambda_0 \) taking values of ±1/2, with the positive section of \( l \) corresponding to the part of crystal whose optical axis is pointing upward, we can then define duty cycle as \( a = l/\Lambda_0 \). Therefore, the duty cycle for an ideal periodically poled crystal is 0.5. However, in an actual poling process, due to the poling electric field distribution on the crystal, or the mask itself, the duty cycle would deviate from 0.5 and lead to some new features on the far-field spatial profile.

The waveguide pupil function is now modified to:

\[
f_{0,m}(x_0) = \begin{cases} 
  \frac{1}{2}, & (m\Lambda_0 \leq x_0 \leq m\Lambda_0 + (1-a)\Lambda_m) \\
  -\frac{1}{2}, & (m\Lambda_0 - a\Lambda_0 < x_0 < m\Lambda_0)
\end{cases}
\]  

(3.15)

When \( a = 0.5 \), it reduces to the ideal case.

The far-field integral for the \( m \)th period is:

\[
f_{z,m}(x) = \frac{\exp(ikz)}{\sqrt{i\pi z}}\left(-\frac{i\sqrt{\pi}}{ik}\frac{1}{2z}\text{erf}\left[\frac{ik}{2z}(m\Lambda_0 + (1-a)\Lambda_0 - x)\right] - \text{erf}\left[\frac{ik}{2z}(m\Lambda_0 - x)\right] + \frac{1}{2}\text{erf}\left[\frac{ik}{2z}(m\Lambda_0 - a\Lambda_0 - x)\right]\right)
\]

(3.16)

By summing up \( m \) from \(-N\) to \( N \) (\( N = 50 \)), we are able to achieve the simulation curve for duty cycle error:
When $a=0$, i.e. no periodically poled structure, we would observe only ONE peak at the center. As $a$ goes from 0 to 0.5, the side peaks gradually enhance while the center one is suppressed.
3.3.5 QPM grating with random poling period

We now consider a periodically poled structure with random domain period. We assume that the domain periods statistically follow normal distribution, with mean \( \Lambda_0 \) and standard deviation \( \sigma \), i.e. \( \Lambda \sim N(\Lambda_0, \sigma) \). Under such assumption, we can calculate expected values of the spatial profile.

The normalized probability density function for variable \( \Lambda \) (period) can be expressed as:

\[
h(\Lambda) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\frac{(\Lambda - \Lambda_0)^2}{2\sigma^2}\right]
\]

It can be observed that \( \int_{-\infty}^{+\infty} h(\Lambda)d\Lambda = 1 \). The spatial profile function as regard to variable \( \Lambda \) can be written as:

\[
F_z(\Lambda) = |A|^2 \cdot \left| \frac{1}{2} g(N + \frac{1}{2}) + \frac{1}{2} g(-N - \frac{1}{2}) + \sum_{p=-N}^{N} g(p + \frac{1}{2}) - \sum_{p=-N}^{N} g(p) \right|^2
\]

(3.18)

where \( h(m) \) is defined as:

\[
g(m) = \text{erf}_z\left[\frac{ik}{2z}(m\Lambda - x)\right]
\]

(3.19)

Finally, the expected 1-D spatial profile is calculated to be:

\[
E[F_z(x)] = \int_{-\infty}^{+\infty} h(\Lambda)F_z(\Lambda)d\Lambda
\]

(3.20)

The results are shown in Fig. 3.9.
Fig. 3.9 Simulation of domain period with random errors

As the standard deviation varies from 0.1 μm to 1 μm, the strength of the far-field pattern decreases drastically.
3.3.6 Partially poled waveguide

In the fabrication of proton-exchange periodically poled lithium niobate waveguide, the waveguide is made, followed by the periodic poling by applying alternate strong electric field across the chip. In such cases, the periodically poled part may not fill the whole volume of the waveguide, which will lead to some interesting changes in the far-field pattern for surface-emitting second-harmonics. See Fig. 3.10.

![Illustration of partially poled waveguide](image)

Fig. 3.10 Illustration of partially poled waveguide

We start by considering the 1-D case with perfect QPM grating structure. Assuming that depth of the total waveguide is $d_2$, and that of the periodically poled portion is $d_1, d_2>d_1$. The far-field pattern integral is given by:

$$f_z(x) = \int_{-\infty}^{\infty} A^* \left[ \frac{1}{2} g(N + \frac{1}{2}) + \frac{1}{2} g(-N - \frac{1}{2}) + \sum_{p=1}^{N} g(p + \frac{1}{2}) - \sum_{p=1}^{N} g(p) \right] dx_0 + \int_{-\infty}^{\infty} A^* \left[ \text{erf} \left( \frac{1}{2} \left( \frac{L}{2} - x \right) \right) - \text{erf} \left( \frac{1}{2} \left( \frac{L}{2} - x \right) \right) \right] dx_0$$

(3.21)
where

\[ A' = \frac{-i\sqrt{\pi}}{2} \exp[i k(z + x_0)] \exp[\frac{i k}{\sqrt{2(z + x_0)}}], \quad g'(m) = \text{erfz}[\frac{i k}{2(z + x_0)}(m\Lambda_0 - x)] \]  

(3.22)

It is not difficult to apply the results to the 3-D case for a real waveguide. Applying the Cartesian coordinate in Fig. 3.6.1 (a), the far-field integral is given by:

\[
f_z(x, y) = \exp[i k(z + z_0)] \int_{yz} dy dz \frac{1}{\lambda(z + z_0)} \left[ g'(N + \frac{1}{2}) + \frac{1}{2} g'(-N - \frac{1}{2}) + \sum_{p=1}^{N} g'(p + \frac{1}{2}) - \sum_{p=-N}^{N} g'(p) \right] \cdot \mathcal{Q}(y_0, z_0, y)
\]

\[
+ \exp[i k(z + z_0)] \int_{y_{\text{pp}}} dy dz \frac{A' d y_0 d z_0}{\lambda(z + z_0)} \left[ \text{erfz}[\frac{i k}{2(z + x_0)}(L - x)] - \text{erfz}[\frac{i k}{2(z + x_0)}(-L - x)] \right] \cdot \mathcal{Q}(y_0, z_0, y)
\]

(3.23)

where

\[
\mathcal{Q}(y_0, z_0, y) = \exp[i k y_0^2] \exp[-i \frac{2\pi y y_0}{\lambda(z + z_0)}]
\]

(3.24)

As a matter of fact, Eq. (3.23) is the general expression for all cases as long as we substitute the pupil function \( f_0(x_0) \) with desired domain distribution. The simulation curves for the simplified 1-D version are shown in Fig. 3.10.
Fig. 3.11 Simulation for 1-D partially poled waveguide

The regions without periodically poled structure is equivalent to the case where \( a=0 \) in section 3.3.5. Therefore, the more volume of un-poled region exists in the waveguide, the more prominent of the center peak it is.
3.4 Analysis on experimental results

By measuring the distances among the centers of these three beams, we have determined \( \sin \theta \) and found that the experimental values agree well with the calculated values.

Using Eq. (3.1), the simulated results are illustrated in Fig. 3.4. As one can see, there are only two peaks instead of the three peaks observed in our experiment. The third peak in the middle is due to the fact that the depth of periodic poling is less than the depth of the waveguide, as shown in Fig. 3.11. The SH beam generated by monodomain, i.e. the region which was not periodically poled makes up the center beam (see Fig. 3.10. with the different poling depths). Thus, we can determine the ratio of the poling depth and the waveguide depth by measuring the powers within three distinct beams (poling depth ratio). Fig. 3.12 confirms the linear power dependence on the pump power from one of the counter-propagating beams. We have determined the linear taper coefficient, defined as \( c = (l_N - l_1)/N \). Indeed, according to Fig. 3.6, the left and right SH beams may exhibit an unsymmetrical power distribution when linear taper exists.
Fig. 3.11. (Top) Total Diffraction Pattern on CCD (Middle) Right portion (Bottom) Right portion spatial intensity distribution against lateral distance
For our experiment, the poling depth ratio is deduced to be 40%±3%. The linear taper coefficient is determined to be $(5.5\pm0.2)\times10^{-7}$mm. This indicates that on an average the adjacent domain period difference is around 0.5 nm, which reaches a nanoscale level. Many other domain quantities can be determined by giving a perturbation to the QPM grating and investigating the resulting spatial pattern. It is worth noting that our 1-D model can be readily extended to a 3-D model to evaluate characteristics of bulk and 2-D periodically-poled domains.
3.5 Conclusion for Chapter 3

In summary, we have developed an effective nondestructive technique to evaluate the quality of periodically-poled ferroelectric domains in a waveguide. Using such a technique, we have determined average poling period, degree of linear tapering, and poling depth ratio. We also calculated the far-field pattern under other possible domain imperfections. Our results show that such a technique has reached a nanoscale resolution for evaluating domain period.
Chapter 4

Enhancement of forward and backward Anti-stokes Raman signals in lithium niobate waveguides

4.1 Historical Overview

Anti-Stokes Raman is an effective technique to investigate phonon dynamics [52]. In addition, combining with Stokes Raman, anti-Stokes Raman can be used to investigate the hot phonon effects [53]. However, anti-Stokes signals are much weaker than Stokes signals due to inherently-low phonon occupation numbers. Therefore, in the past, relatively high pump powers were required to probe anti-Stokes signals. Indeed, the pump power of around 100 mW was used to generate measurable anti-stokes and stokes photons [54,55]. As a result, there could be a strong heating effect. Such an effect induces Raman frequency shifts and modifies phonon decay rates, resulting in broadening of Raman peaks [55]. Since the elevated temperatures are unknown, such an effect causes large errors in determining phonon decay rates and phonon temperatures. Therefore, it is important for us to improve the signal strength of anti-Stokes Raman.

Here, we present our new result on the forward and backward anti-Stokes Raman scattering. For the first time, we identified five and three propagating anti-Stokes peaks from the waveguide for the forward and backward configurations, respectively. These two different sets of the peaks are unique for the two configurations. Moreover, we have not only observed an enhancement factor of 30 for the waveguide but also reduced the pump power by five orders of magnitude compared with the previous value [54,55].
4.2 Theory

The spontaneous Raman Effect was discovered by C. V. Raman in 1928. A beam of light with certain frequency ($\omega_p$) illuminates on a material sample and the scattered light is measured with spectrometer. The incident light (called pump here) interacts with optical photons (with frequency $\omega_s$) in the material and could result in scattered photons with altered frequency, i.e. $\omega_p - \omega_s$ (Stokes) and $\omega_p + \omega_s$ (Anti-stokes), see Fig. 4.1.

Typically, the strength of Anti-stokes lines will be much weaker than Stokes lines, as in thermal equilibrium, the population of the ground level is greater than that of the vibrational states, or in other words, the optical photon level [2].
4.3 Experimental Setup

In our experiment, a compact and portable Nd:YLF laser was used as a pump source for the generation of the anti-Stokes Raman signals. As shown in Fig. 1, a 4×4×10 mm³ Nd:YLF crystal was pumped by a 808-nm diode laser in a plano-concave laser cavity. Such a laser generated two orthogonally-polarized output beams at 1053 nm and 1047 nm, respectively. In the laser cavity, a Brewster plate was served as a wavelength selector whereby the 1053-nm wave was oscillating whereas the 1047-nm beam was transparent and will not return to the laser cavity. The reflectivity for 1053-nm wave is 99.5%. An acousto-optic Q-switch was used to modulate the laser output at 5 kHz. The pulse width of the laser was 10 ns. A flip mirror was placed in front of the photon collection system to measure the forward-propagating photons or backward-propagating photons. A 95% beam splitter was placed in the optical path to collect the backward-propagating anti-stokes photons generated by the LiNbO₃ waveguide. The photon collecting system consisted of an edge filter to cut off the pump beam, a convex lens, and a monochromator.

Fig. 4.2. Experimental setup for forward- and backward- Anti-stokes signal measurement. LD - laser diode at 808 nm, IM - input mirror, OC - output coupler, BP - Brewster plate, AO - acousto-optic Q-switch, BS - Beam Splitter, OBJ – objective, WG – waveguide, FLT - filter, CL-convex lens, MC –
The photons were then measured by a Si APD with dark count rates of $< 25 \text{ s}^{-1}$. 
4.4 Results and Discussion

4.4.1 Backward-propagating Anti-stokes signals

Our first sample, which was used to study the properties of backward-propagating anti-Stokes signal, was an x-cut LiNbO$_3$ chip with six Ti-diffused waveguides of lateral widths from 5 μm to 10 μm with an increment of 1 μm. The coupling efficiencies were measured to be 47.1% (10 μm), 38.9% (9 μm), 24.5% (8 μm), 10.0% (7 μm), 3.6% (6 μm), and 2.7% (5 μm), respectively. The polarization of 1053 nm beam was along the optical z axis of the LiNbO$_3$ waveguide.
Fig. 4.3 Backward Anti-Stokes photon count vs. Raman shift Experimental at different waveguide widths.
Fig. 4.4 Photon count vs. pump power coupled into waveguide at different widths
According to the Raman spectra measured by us, see Fig. 4.3, four Raman shift peaks at 256.64 cm\(^{-1}\), 282.04 cm\(^{-1}\), 341.00 cm\(^{-1}\), and 644.66 cm\(^{-1}\) were identified. In LiNbO\(_3\), phonon branches are labeled by 5\(A_1\), 5\(A_2\) and 10\(E\) at a nearly zero wave vector. Out of these phonon branches, the five \(A_2\) fundamentals and 4\(A_1\) and 9\(E\) optical branches are Raman active [59]. The four peaks are then identified as \(E(TO3)\) mode, \(E(TO3)+E(LO3)\) combination mode, \(E(LO4)\) mode, and \(A_1(TO)\) mode [57], respectively. The positions of the Raman peaks indicate that the orientation of the crystal during measurement should be notated by \(Y(Z,Z)\bar{Y}\) based on Porto notation [60], based on the crystal orientation, propagation and polarization of the pump beam and Raman scattering signal.

In order to efficiently generate anti-Stokes signals, the following equations must be satisfied:

\[
\begin{align*}
\hbar \omega_p &= \hbar \omega_{as} - \hbar \omega_q \\
\mathbf{k}_p &= \pm (\mathbf{k}_{as} - \mathbf{q})
\end{align*}
\]

where \(\omega_p\) denotes the angular frequency of pump, \(\omega_{as}\) the angular frequency of anti-Stokes signals, \(\omega_q\) the angular frequency of phonon, \(\mathbf{k}_p\) the wave vector of the pump, \(\mathbf{k}_{as}\) the wave vector of anti-Stokes signals, and \(\mathbf{q}\) the wave vector of phonon. The plus/minus sign denotes forward/backward propagating configuration. The fact that the value of \(q\), which can literally take any number between 0 and \(2\pi/c\) (Brillouin zone boundary), where \(c\) is the crystal constant and is usually several Ångström (Å), is more than enough to satisfy the momentum conservation requirement. Therefore, we can always pick a \((\omega_q, q)\) pair to fulfill a given Raman scattering configuration, or in other words, the process is always phase-matched if spatial overlap of pump and anti-Stokes signal is guaranteed. In the case of waveguide, both the pump and Raman beams propagate in the waveguide. As a result, the presence of the waveguide allows the spatial accumulation of the Raman
signals, resulting in the large enhancements over that from the bulk crystal. In our experiment, the highest enhancement factor obtained by us is 30. In such a case, the pump power of as low as 230 nW is sufficiently for probing the anti-Stokes Raman signal. Therefore, our observation can be considered as the first step for developing a compact and portable anti-Stokes Raman probe.

We also measured the backward anti-stokes photon count by using the APD, generated by each waveguide having a different width, as a function of the pump power, see Fig. 4.4. One can see that the dependence was linear under relatively low powers. However, at relatively high pump powers the data deviated from the linear dependence. This was probably caused by the local heating effect [58]. This result illustrates the importance of using low pump powers.
4.4.2 Forward-propagating Anti-stokes signals

In order to investigate the difference between backward and forward propagating anti-Stokes signals, we alternated the sample to a reverse proton exchanged x-cut LiNbO\textsubscript{3} waveguide chip, which has higher photorefractive effect damage threshold and polarization restriction that allows only beams polarized along optical z direction to have the maximum transmission. We chose the 10-\textmu m-wide waveguide to conduct the experiment. We measured the forward-propagating and backward-propagating anti-Stokes photons generated by the waveguide and the bulk LiNbO\textsubscript{3} crystal, see Fig. 4.5, which correspond to $Y(Z,Z)Y$ and $Y(Z,Z)\bar{Y}$ as denoted by Porto notation, respectively. According to Fig. 4.5, we identified five anti-stokes peaks for the forward configuration at the Raman shifts of 200.96 cm\textsuperscript{-1}, 258.68 cm\textsuperscript{-1}, 292.64 cm\textsuperscript{-1}, 348.61 cm\textsuperscript{-1} and 565.90 cm\textsuperscript{-1}, respectively. Based on Fig. 4.6, the count rates in the waveguide are obviously much higher than those in the bulk crystal. We assigned 200.96 cm\textsuperscript{-1}, 258.68 cm\textsuperscript{-1}, 292.64 cm\textsuperscript{-1}, 348.61 cm\textsuperscript{-1}, and 565.90 cm\textsuperscript{-1}, the Raman scattering from the phonon modes of E(LO1), E(TO3), E(LO3), E(LO4), and E(TO8) [57], respectively. It is worth noting that for the forward-propagating configuration the Raman peak is located at 565.90 cm\textsuperscript{-1} whereas the peak for the backward-propagation configuration is blue-shifted to 654.19 cm\textsuperscript{-1}. This is due to the differences of the phonon wave vectors required to satisfy the phase-matching condition for the Raman scattering explained earlier in the paper. Indeed, for the backward-propagating configuration the wave vector required to satisfy the phase-matching condition is much larger. Due to the phonon dispersion, the Raman frequency shift for the backward configuration is significantly different from that for the forward configuration.
Fig. 4.5 Forward and backward anti-Stokes Raman spectra in LiNbO₃ waveguide
Fig. 4.6 Forward and backward anti-Stokes Raman spectra in LiNbO₃ bulk crystal. The spectrum for forward configuration in bulk crystal is vertically up-shifted due to presence of strong laser radiation in the range.
4.5 Conclusion for Chapter 4

In this chapter, we have demonstrated that a compact and portable Raman probe device is achievable by employing sending pump beam into a waveguide structure, which has more than 30 times enhancement compared to signals in bulk material. By detecting using Si-based single photon detector, we can readily reduce the pump power to a few microwatts, which is capable of greatly relieving the local heating effect. We have also discovered that the forward and backward propagating anti-stokes signals have exhibited significant difference in spectral features due to the fact that the phonon wave vectors required to satisfy these two configurations are different. Backward propagating anti-Stokes has application in correction of phase-distorted images through phase conjugation.
Chapter 5

Terahertz generation based on parametric conversion in

Gallium Phosphide plates

5.1 Overview

Terahertz (THz) wave refers to electromagnetic waves with wavelengths from 30 μm to 3000 μm [61]. In the frequency domain, it lies between microwave and infrared wave. Terahertz wave has promising utilities in security screening and therefore, extensive study towards the generation of terahertz wave has been performed. Quantum cascade lasers are capable of emitting THz via the proper design of multiple quantum wells. However, such emitters will need cryogenic temperatures for operation [62]. Broadband THz pulses can be generated from electro-optic crystals pumped by picoseconds laser pulses based on optical rectification [63], photocurrent surge [64], and Cherenkov radiation [65]. Applications in chemical sensing and detections of biological and chemical agents would prefer a tunable THz source, which can be implemented by difference frequency generation (DFG) of two close-wavelength infrared beams in Gallium Phosphide (GaP) plates. Compared with other ways of generating THz waves, DFG approach benefits from room temperature operation, various choices of IR sources,
and wide tunability. In this Chapter, we investigated THz generation in alternatively stacked GaP plates and as a result, we achieved maximum photon conversion efficiency of 40% and observed back parametric conversion [41].
5.2 Experimental Setup

Fig. 5.1 Experimental setup for THz generation in GaP stacks
We investigated the THz generation from two stacking configurations of the GaP plates. Each plate has a diameter of 48.5 mm and a thickness of 663 μm. In the reversely-stacked geometry, every other plate was rotated around z axis of the crystal by 180°. As a result, effective second-order nonlinear coefficient changes its sign from one plate to the next, and therefore, such a configuration can be used to achieve quasi-phase matching (QPM) for the THz generation based on DFG at each designated set of the mixing and THz output wavelengths. On the other hand, in the forwardly-stacked geometry, all the plates were stacked in the way that each second-order nonlinear coefficient has the same sign from one plate to the next.

Two coherent beams used for DFG were generated by a Nd:YAG laser at 1.064 μm (≤ 15.0 mJ, 10 ns) and a β-BaB\(_2\)O\(_4\)-based master oscillator/power oscillator (≤ 20.4 mJ, 5 ns). The beam radius for both of the mixing beams was 1.0 mm upon entering the stacked GaP plates. These coherent beams collinearly propagated in a direction opposite to the surface normal of the stacked GaP plates, with their polarizations being parallel to the [1-10] and [001] directions, respectively. Therefore, we can access the large effective second-order nonlinear coefficient of each GaP plate through the QPMed DFG [66]. The THz output beam was then generated by DFG.
5.3 Results and discussion

Fig. 5.2 Spectrum of THz radiation generated by reversely-stacked four GaP plates, covering tuning range of 78-300 µm [41];
Fig. 5.3 Spectrum of THz radiation generated by forwardly-stacked four GaP wafers, covering tuning range of 135-500 µm [41].
We first measured the spectra of the THz radiation generated by the reversely-stacked four GaP plates (the total thickness of 2.65 mm), see Fig. 5.2. According to Fig. 5.2, the output wavelength of the THz radiation was tuned continuously from 78 µm to 300 µm. At the output wavelength of 120.3 µm, DFG was QPMed. Therefore, the output power at this wavelength was dramatically increased by reversely stacking the four GaP plates. The highest output peak power was measured to be 2.73 kW. This is the highest peak power reported so far for the nanosecond THz pulses generated by DFG. At such a power, the input powers for the two mixing beams were measured to be 1.5 MW and 4.1 MW, respectively. The corresponding conversion efficiency, defined in terms of the peak powers is determined to be 0.18%. Taking into consideration the Fresnel reflection losses at the input and at the output facets, the internal photon conversion efficiency is 39.6%. To the best of our knowledge, this is the highest photon conversion efficiency which has ever been achieved for the THz generation based on DFG. As shown in Fig. 5.4, the generated THz is approaching saturation. The individual peaks in the spectrum with the same frequency separation were caused by the Fabry-Pérot interference effect inside the GaP plates. As the number of the GaP plates was increased from 4 to 5, the output power and conversion efficiency were reduced, see Fig. 5.5. This is likely due to the back parametric conversion of the THz wave inside the GaP plates.

We also measured the spectrum of the THz radiation from the forwardly-stacked four GaP plates (the thickness of 2.65 mm), see Fig. 5.3. Based on Fig. 5.3, the output wavelength of the THz radiation was from 135 µm to 500 µm. The maximum output peak power reached 510 W at the wavelength of 236 µm, corresponding to a power
conversion efficiency of 0.034%. The internal photon conversion efficiency is 14.6%, which is significantly lower than that under the reversely-stacked geometry.

Fig. 5.4 Photon conversion efficiency vs. input power at 1.0737 μm and 1.0690 μm for reversely-stacked and forwardly-stacked GaP plates, respectively. For both of the geometries, input power at 1.0642 μm was set to 150 mW. Solid line – nonlinear least square fit to data points using Eq. (1). Dashed line – linear fit to data points [41].
Fig. 5.5 Highest THz output power generated by reverse GaP plates versus the number of plates. Blue curve represents nonlinear least square fit to data points using Eq. (5.1) [41]
Fig. 5.4 illustrates the dependence of the internal photon conversion efficiency on the average input power. Under the reversely-stacked geometry, the conversion efficiency increases linearly below the input power of 50 mW. However, as the input power is increased further, the saturation of the conversion efficiency becomes stronger and stronger. When the input power approaches 200 mW, the conversion efficiency reaches a plateau. The highest conversion efficiency achieved in our experiment is 39.6% at the input power of 204 mW (130 MW/cm²). To the best of our knowledge, this is the first report on the strong saturation in the conversion efficiency for the THz pulses generated by DFG. According to Ref. [67], the photon conversion efficiency can be written in the following form:

\[ \eta_p = \eta_{max} \sin^2 \left( \frac{\pi}{2} \sqrt{\frac{P_i}{P_{opt}}} \right) \]  

(5.1)

where \( \eta_{max} \) is the maximum conversion efficiency, \( P_i \) and \( P_{opt} \) are the input and optimal powers, respectively. Following the nonlinear least square fitting to the data points by using Eq. (5.1) above, we have obtained \( \eta_{max} \approx 40.0\% \) and \( P_{opt} \approx 209 \) mW. Therefore, we have indeed achieved the maximum photon conversion efficiency from our experiment.

In comparison, for the forwardly-stacked GaP plates no obvious saturation was observed, see Fig. 5.5. The highest conversion efficiency is 14.6%. Such a comparison illustrates the importance of QPM for the THz generation.
5.4 Conclusion for Chapter 5

In this chapter, we investigated THz generation in reverse and forward stacked GaP plates, and have reached maximum conversion efficiency for THz generation in reversely stacked GaP plates. The highest output power generated inside the GaP plates reached 3.77kW at the wavelength of 120 μm in correspondence of the Quasi-phase matching condition. The internal photon conversion efficiency reached 39.6%. When we added one more GaP plate in the stacks from 4 to 5, we saw the output power decrease and that was evidence that back parametric conversion has taken place.
Chapter 6

Conclusion

6.1 Conclusion of Contribution

In this dissertation, we investigated several intriguing and novel applications in second-order nonlinear medium via optical multiple wave interactions, specially focusing on periodically poled lithium niobate waveguide, for its high second-order nonlinear coefficient \(d_{33}=27\) pm/V and the waveguide feature to allow tight confinement of transverse modes for all interacting beams along the entire crystal length. Sum-frequency generation can be implemented in such a crystal to convert photons at communication band to visible band, which can be efficiently detected by Silicon based avalanche photodiode. We achieved noise-free detection and greatly enhanced the sensitivity of such a frequency up-conversion detector. Also, we proposed theoretical analysis to identify the noise photon numbers and it agreed well with our experimental results. Waveguide imperfection was analyzed to better support the data.

In the following, we summarize the contributions presented in this dissertation.

In Chapter 2, we demonstrated a single photon detector based on frequency upconversion in periodically poled lithium niobate waveguide. Since commercially available detectors at IR wavelengths, such as InGaAs/InP avalanche photodiodes (APDs)
suffer from drawbacks of high dark counts up to $10^5$/s, as well as low quantum detection efficiencies, usually with values between 10% and 15%, they are not suitable for single photon detection operating at CW mode. In contrast, single-photon counting in near-IR (600-800nm) can be performed efficiently with silicon APDs. Single photon counting modules with dark counts lower than 25/s and detection efficiency in the range 50% to 70% are commercially available. Obviously, we can take advantage of silicon APDs by efficiently converting C-band photons to near-IR or visible photons. In our experiment, we have implemented single photon detection in periodically poled lithium niobate waveguide with 980nm pump, with negligible noise photon counts. The internal quantum efficiency has reached 62%, and the minimum incident number of photons we can detect is measured to be as low as 81 counts per second. We then theoretically investigated the generation of noise counts by considering both perfect and imperfect grating structures for lithium niobate. The key is that for a given grating period, the gain peak for spontaneous parametric down-conversion process and the gain peak for designed up-conversion process usually do not overlap, resulting in intrinsic low noise photon generation. The simulation gives noise photon generation rate of 77 counts per second, and it agreed well with our experimental results. At last, we explored GaP plates as an alternative second-order nonlinear material to implement single photon detection. Experimental results have been demonstrated and we investigated total-internal-reflection assisted phase-matching in GaP plate, where by other means perfect phase match cannot be achieved.

In Chapter 3, we have developed an effective nondestructive technique to evaluate the
quality of periodically-poled ferroelectric domains in a waveguide, by measuring the far-field pattern of second harmonics generated by two identical counter propagating IR beams in the waveguide. The diffraction angle is linked to the poling period with a simple expression. Using such a technique, we have determined average poling period, degree of linear tapering, and poling depth ratio. We also simulated the far-field pattern under other possible domain imperfections. We then compared our experimental results with the simulation to determine the imperfection parameters. Our results show that such a technique has reached a nanoscale resolution (0.5 nm) for evaluating domain period linear taper.

In Chapter 4, we have demonstrated that a compact and portable Raman probe device is achievable by employing sending pump beam into a waveguide structure, which has more than 30 times enhancement compared to signals in bulk material. By detecting using Si-based single photon detector, we can readily reduce the pump power to a few microwatts, which is capable of greatly relieving the local heating effect. We have also discovered that the forward and backward propagating anti-stokes signals have exhibited significant difference in spectral features due to the fact that the phonon wave vectors required to satisfy these two configurations are different. Backward propagating anti-Stokes has application in correction of phase-distorted images through phase conjugation.

In Chapter 5, we investigated THz generation in reverse and forward stacked GaP plates, and have reached maximum conversion efficiency for THz generation in reversely stacked GaP plates. The highest output power generated inside the GaP plates reached 3.77kW at the wavelength of 120 μm in correspondence of the Quasi-phase matching
condition. The internal photon conversion efficiency reached 39.6%. When we added one more GaP plate in the stacks from 4 to 5, we saw the output power decrease and that was evidence that back parametric conversion has taken place.
6.2 Outlook and future work

Single photon detectors based on frequency up-conversion have extensive use in gas detection and quantum communications. It is of great interest to develop this single photon detector to a gas concentration detection system. As we know, gases have their unique absorption lines suitable for detection, for example, 1.27 μm for O₂, 1.57 μm for CO₂, and 1.651 μm for CH₄. Same pump wavelength can be used as long as the poling period is designed to match the specific up-conversion process. The multiple waveguides with different poling period can even be fabricated on the same lithium niobate chip. By using reference gas cell and measuring the photon counts difference between reference gas cell and sample gas cell, the concentration of a specific gas can be precisely determined at a extremely low level (less than ppb) since the noise counts are almost zero. This gas detector can be useful to monitor the CO₂ gas concentration change in the atmosphere for climate purposes, or to monitor the CH₄ gas concentration change at natural gas production sites.
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Journal and Conference Publications list

Journal Papers:


Conference Papers:


2. **Da Li**: Pengda Hong; Zhaojun Liu; Yujie J. Ding; Lei Wang, “Enhanced Forward and Backward Anti-Stokes Raman Signals in Lithium Niobate Waveguides”, JW2A.103, CLEO 2014

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**Vita**

Da Li is a Ph.D. student under the supervision of Prof. Yujie Ding. He is an active member of Optical Society of America. His research interests include novel applications by exploiting second-order nonlinear media, especially single photon detection, THz generation, image correction, and probing of nanostructures. During the last several years, Mr. Li has investigated single photon detection in lithium-niobate waveguide within the communication band. As a result, he has achieved the world-record results. Single photon detection in the communication band has variety of applications such as quantum information and computing and sensing of CO$_2$. InGaAs/InP-based detectors suffer from several drawbacks such as high dark count rates and low detection efficiencies, preventing them from reaching single-photon levels. In comparison, frequency upconversion devices are capable of reaching ultralow dark counts and high detection efficiencies, and therefore, making the single-photon detection a reality. In his research, Mr. Li has designed a waveguide consisting of periodically-poled lithium niobate as a second-order nonlinear medium. Using such a waveguide as a frequency-upconversion detector, Mr. Li has reached the single-photon detection level within the communication band. Due to his unique design, Mr. Li has completely eliminated the noise photon counts.
induced by the pump beam by taking advantage of a free-space coupler for collecting the photons. In his most recent work, for the first time, to the best of his knowledge, Mr. Li has established a theoretical model based on a clear concept for noise generation processes to support his experimental findings.

In addition to single photon detection, Mr. Li has made contributions to the research projects such as generation of THz waves in reversely-stacked Gallium Phosphide (GaP) wafers. These THz waves have applications in chemical sensing and detection of biological and chemical agents. He has also developed a nondestructive technique to probe the periodically-poled ferroelectric structures by studying far-field pattern based on sum-frequency generation from two counter-propagating beams. Mr. Li has demonstrated that such a technique is capable of reaching the nanoscale resolution.

Mr. Li has published articles in some of the top journals in his area, such as Applied Physics Letters and Optics Letters. He has presented his key findings in Conference on Laser and Electro-Optics (CLEO) annually since 2010. He is an active reviewer for Optics Express and IEEE JSTQE.

Mr. Li earned his B.S. degree in Materials Science from Fudan University, Shanghai, China, where he was awarded People’s Scholarship and Exxon Mobil Scholarship for his excellent academic performances. He has devoted himself to conduct scientific research when pursuing his Ph.D. degree at Lehigh University. He is the recipient of Packard Fellowship and RCEAS Emeritus Fellowship in 2012 and 2009, respectively.