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# Fatigue crack propagation in polymethylmethacrylate analyzed by the strain energy density theory.

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FATIGUE CRACK PROPAGATION  
IN POLYMETHYLMETHACRYLATE  
ANALYZED BY  
THE STRAIN ENERGY DENSITY THEORY

by

Robert H. Paul

A THESIS

Presented to the Graduate Committee

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in Candidacy for the Degree of

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in

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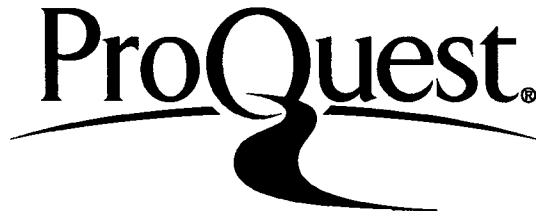
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## ABSTRACT

An analysis is made on the fatigue data of a crack propagating in polymethylmethacrylate in which the combined effect of mean stress and stress amplitude is accounted for. Data are collected from compact tension specimens of PMMA tested at selected mean stress and stress amplitude levels. The Strain Energy Density Theory of crack propagation is proposed to analyze the fatigue data. An expression for predicting the fatigue life is obtained in which the effect of mean stress is also accounted for.

## INTRODUCTION

Traditionally fatigue-crack experimental data has been plotted logarithmically as the incremental crack length per unit cycle versus the range of the mode I stress intensity factor ( $\frac{da}{dN}$  vs  $\Delta k_I$ ). This procedure requires the slope and constant of the resulting curve to be used in the relation  $\frac{da}{dN} = C (\Delta k_I)^n$ , where  $n$  is the slope and  $C$  is the constant. The range of the mode I stress intensity factor  $\Delta k_I$  relates the difference between the maximum and minimum values of the remote applied stress, that is  $\Delta k_I = (\sigma_{\max} - \sigma_{\min}) (a)^{\frac{1}{2}}$ . This relationship for the rate of fatigue-crack growth was first proposed by Paris [1]<sup>1</sup> in 1964. It was to be used with metallic specimens subject to mode I loading where the crack is restricted to propagate in a self-similar fashion. In 1967, Burns and Watts [2] showed that this relationship was equally useful in describing the fatigue behavior of polymethylmethacrylate, a glassy plastic. Later in 1969, further work by Mukherjee, Culver, and Burns [3] again verified the use of the Paris relation with the fatigue behavior of PMMA. However, there are several limitations to the Paris relation that are not immediately obvious. First of all it is limited to mode I loading only which

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<sup>1</sup> Bracketed numbers refer to references listed at the end of the paper.

raises the immediate question of what to do with mode II and mode III loading situations. Secondly, it can be seen that  $\Delta k_I$  does not take into account the remote mean stress applied to the specimen, only the stress amplitude component, where the stress amplitude is defined as  $\sigma_a = (\sigma_{\max} - \sigma_{\min})/2$  and the mean stress as  $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2$ . Ref. [2,3] indicate higher values of  $\frac{da}{dN}$  at higher  $\sigma_m$  values for a given value of  $\Delta k_I^*$ , so certainly the mean remote stress should be a factor in any relation describing the fatigue-crack growth rate. As Figure 1 on the following page indicates, it is quite possible to have the same stress amplitude at large or small mean stress levels, so the above results are quite reasonable.

In 1971 Arad, Radon and Culver [4] were interested in the effect of the mean stress on the fatigue-crack growth rate in PMMA. They concluded the Paris relation was insufficient to fully describe the fatigue behavior in PMMA and suggested a more general relation of the form

$$\frac{da}{dN} = \beta (k_I^2_{\max} - k_I^2_{\min})^n \quad (1)$$

where  $n$  and  $B$  are found by the usual technique. Since  $\beta (k_I^2_{\max} - k_I^2_{\min})$  may be written as  $\frac{1}{4}C$

\* It should be cautioned that a higher  $\sigma_m$  may not always imply a higher crack growth rate should some other quantity other than  $\Delta k_I$  be kept constant.

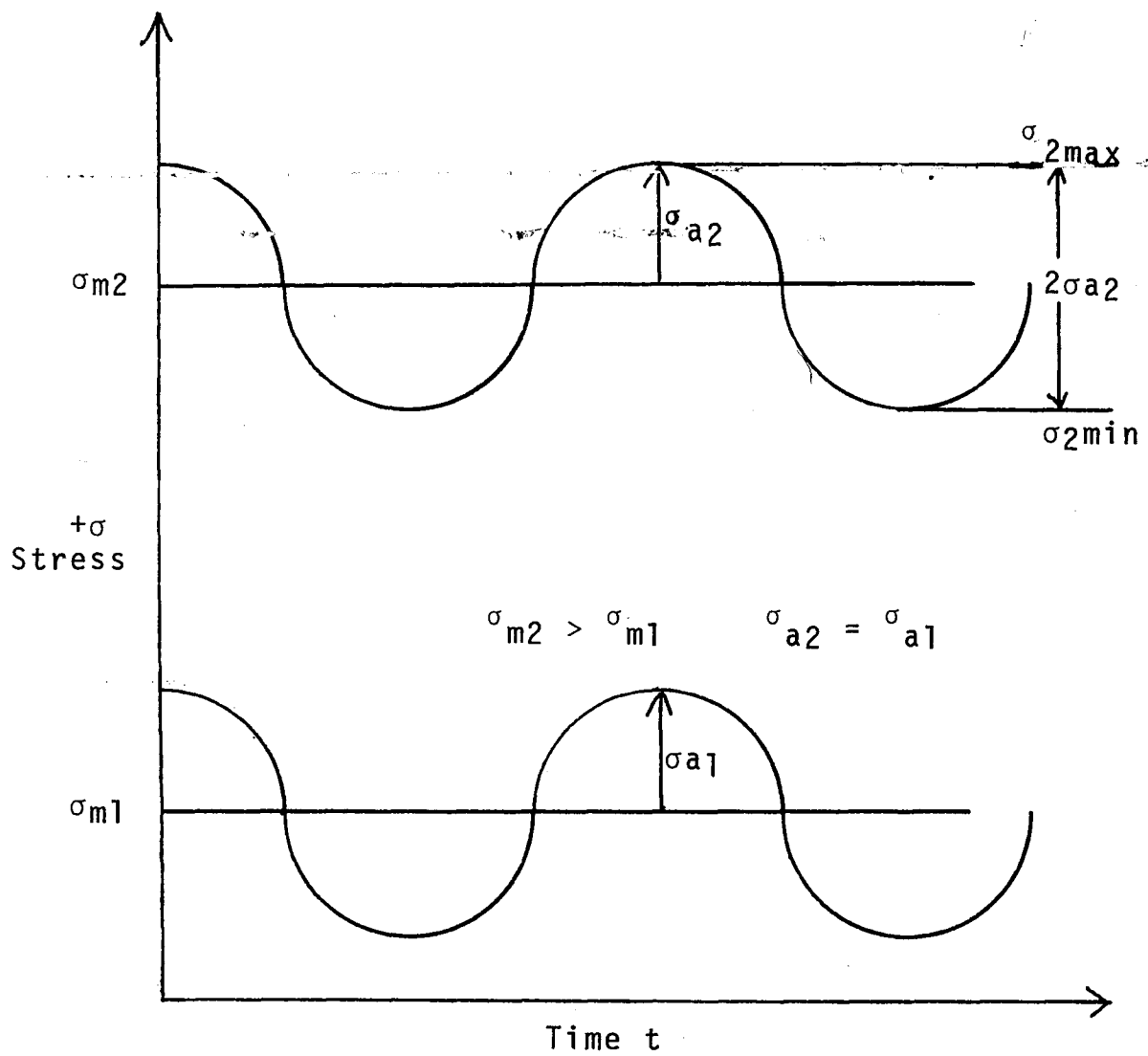


Figure 1: Relationship Between Mean Stress and Stress Amplitude

$(k_1 \text{ max} + k_1 \text{ min})(k_1 \text{ max} - k_1 \text{ min})$  it can be seen that the new relationship contains both the stress amplitude and the mean stress. Since Arad, Radon and Culver used this relationship to fully describe their results, it becomes clear that perhaps a relation containing  $(k_1^2 \text{ max} - k_1^2 \text{ min})$  is a better criterion for predicting fatigue failure.

In summary, what is required in fracture fatigue-crack analysis at a minimum, is inclusion of the mean stress, the stress amplitude, the specimen and crack geometry and the material constants. One such theory that combines all these factors is the Strain Energy Density Theory. It will be the purpose of this study to further investigate the rate of fatigue-crack growth using the Strain Energy Density Concept which turns out to be closely related to the empirical equation (1) proposed in [4].

Fracture fatigue-crack tests are performed with compact tension specimens of polymethylmethacrylate at selected mean stress and stress amplitude levels. The Strain Energy Density Theory is applied to the fatigue test results in order to establish a fatigue-crack growth relationship that combines mean stress, stress amplitude and crack length. Moreover, the concept can also be extended to meet fatigue crack propagation in combined loading situations where the

crack can propagate in any direction as dictated by the assumption of minimum strain energy density.

in which the coefficients  $a_{11}$ ,  $a_{12}$ ,  $a_{22}$ ,  $a_{33}$  for plane strain are given by

$$a_{11} = \frac{1}{16\mu} [(3-4\nu-\cos\theta)(1 + \cos\theta)]$$

$$a_{12} = \frac{1}{16\mu} (2\sin\theta)[\cos\theta - (1-2\nu)]$$

$$a_{22} = \frac{1}{16\mu} [4(1-\nu)(1-\cos\theta) + (1+\cos\theta)(3\cos\theta-1)]$$

$$a_{33} = \frac{1}{4\mu}$$

where  $\mu$  is the shear modulus of elasticity, and  $\nu$  is Poisson's ratio. The stress intensity factors  $k_1$ ,  $k_2$ , and  $k_3$  define the stress intensity for Mode I, II, and III loading respectively.

The critical value of the minimum SED function  $S_{\min}$ ,  $S_{cr}$ , determines the onset of rapid crack extension in the material. The direction of crack extension is found by applying the conditions of minimization:

$$\frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial^2 S}{\partial \theta^2} > 0 \text{ at } \theta = \theta_0 \quad (4)$$

where  $\theta$  is measured positively from the crack plane in the counterclockwise sense. Once  $\theta_0$  is found, the value of  $S_{cr}$  can be found by substituting  $\theta_0$  into the expression for  $S_{\min}$ .  $S_{cr}$  is thus assumed to be a material constant for general loading much the same way  $k_{Ic}$  is assumed to be a material constant for mode I loading.

## Strain Energy Density Theory

The Strain Energy Density concept is a theory of material fracture proposed by Sih [5,6]. It is based on the idea that the strain energy stored in a material element in the immediate vicinity of the crack tip reaches a critical value causing at least local fracture to occur. The Strain Energy Density (SED) Theory assumes crack extension occurs in the direction where the dilatation component dominates. This is determined by minimizing the strain energy density factor or  $S_{min}$ . Thus the SED criterion is not limited to any particular mode of loading as are the classical theories of fracture proposed by Griffith [7] and Irwin [8]. The critical value of  $S_{min}$ ,  $S_{cr}$ , is used as a material parameter and has been measured experimentally for many engineering materials by Gillemot [9] who actually determines  $(\frac{dw}{dv})_{cr} = \frac{S_{cr}}{r_0} =$  constant from which  $S_{cr}$  can be obtained. The distance  $r_0$  determines the location at which failure occurs in front of the crack. The concept is completely general and thus can be applied to all mixed mode problems. For three dimensional loading, the strain energy density factor  $S$  is given as

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2 \quad (2)$$



## SED Concept Applied to the Fatigue Problem

As mentioned previously, fracture fatigue tests will be carried out using the compact tension specimen (Figure 2). Since compact tension specimens involve mode I loading only, the S factor reduces to

$$S = a_{11} k_1^2 \text{ (plane strain)} \quad (5)$$

since  $k_2 = k_3 = 0$ .

However, recall from the introduction that Arad, Radon, and Culver [4] proposed a fatigue-crack growth relationship of the form

$$\frac{da}{dN} = \beta (k_{1 \max}^2 - k_{1 \min}^2)^n.$$

The objective is to modify the S factor for mode I loading into the form  $(k_{1 \max}^2 - k_{1 \min}^2)$ . By doing this an expression containing the mean stress, stress amplitude and crack length will be obtained, along with a general failure criterion. Since the stress applied to the specimen will oscillate between a maximum and minimum value, the strain energy density factor S in a material element ahead of the crack will also oscillate between a maximum and minimum as follows:

$$\Delta S = (S_{\min})_{\max} - (S_{\min})_{\min} \quad (6)$$

Here,  $\Delta S_{\min}$  should be viewed as a quantity of energy which builds at the crack tip as cycling continues until incremental crack extension occurs. The min subscript indicates crack extension occurs in the direction of minimum strain energy density.

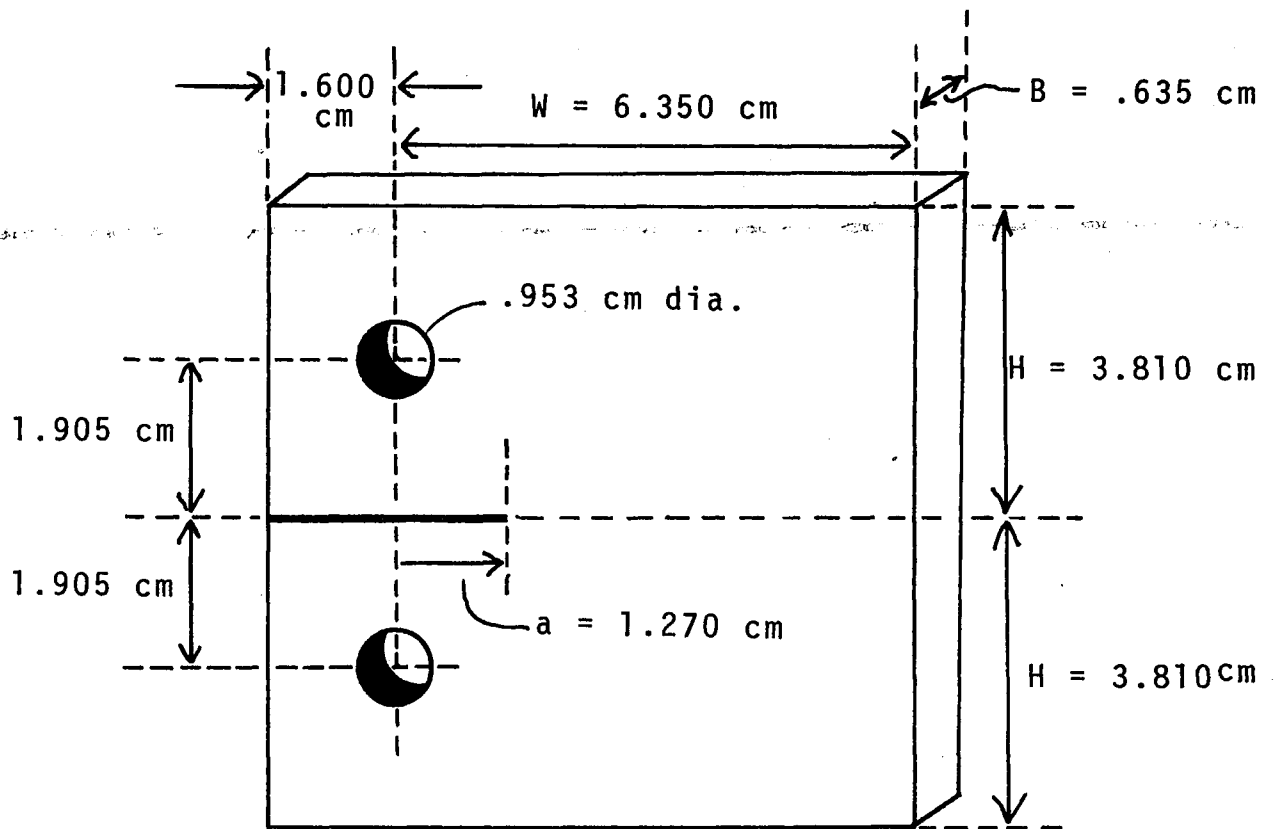


Figure 2: Compact Tension Specimen

For compact tension specimens this extension occurs in the plane of the crack.  $\Delta S_{\min}$  may be written more conveniently as

$$\Delta S_{\min} = a_{11}(k_{1\max}^2 - k_{1\min}^2) \quad (7)$$

where  $k_{1\max}$  and  $k_{1\min}$  are the maximum and minimum values of the mode I stress intensity factor.

At this point let

$$\bar{k}_1 = \frac{1}{2}(k_{1\max} + k_{1\min}) \quad (8)$$

be defined as the mean value of the mode I stress intensity factor and let

$$\Delta k_1 = (k_{1\max} - k_{1\min}) \quad (9)$$

be defined as the range of the mode I stress intensity factor. By multiplying together  $\bar{k}_1$  and  $\Delta k_1$ , the relationship for  $\Delta S$  (dropping subscript) can be written as

$$\Delta S = 2a_{11}\bar{k}_1\Delta k_1$$

where it can be clearly seen that a parameter involving the mean stress and stress amplitude has been obtained.

The proposal now is to replace  $\Delta k_1$  in the Paris expression by  $\Delta S$ , so that

$$\frac{da}{dN} = C(\Delta S)^n \quad (10)$$

where  $C$  and  $n$  are no longer the same constants as in the equation for  $\Delta k_1$ , but must be determined separately.

## The Solution Procedure

Polymethylmethacrylate was used as the material for the specimens because of its' optical properties, brittle behavior at room temperature, ease of fabrication, and relatively low cost. The compact tension specimen configuration was prepared in accordance with Figure 2. The cracks in the specimens were initially cut 1.270 cm in length with a fine band saw. Prior to testing, each specimen had a razor blade crack introduced into the crack tip of the band saw cut. In addition, a grid pattern was laid along the line of expected crack extension of each specimen. Increments of .127 cm were used in order to closely monitor the length of the crack as the fatigue test proceeded.

The fatigue tests were performed with a haversign wave form at a frequency of 10HZ. Mean load and load amplitude levels were chosen to provide a range of data over which the results could be compared. The load levels with English Unit conversion are listed in Table 1 on the following page.

Two separate ways were employed in preparing the specimen cracks to start the fatigue tests. In the first group of specimens, the length of the band saw cut plus the increment of crack length added by the

Table 1 - Load Levels Used for Tests

	Mean Load (Newtons)	Load Ampl (Newtons)	Applied Load (Newtons)	Mean Load (Pounds)	Load Ampl (Pounds)	Applied Load (Pounds)
1	177.92	111.20	66.72-289.12	40.0	25.0	15.0-65.0
2	177.92	88.96	88.96-266.88	20.0	20.0	20.0-60.0
3	155.68	111.20	44.48-266.88	35.0	25.0	10.0-60.0
4	155.68	88.96	66.72-244.64	35.0	20.0	15.0-55.0
5	133.44	111.20	22.24-244.64	30.0	25.0	5.0-55.0
6	133.44	88.96	44.48-222.40	30.0	20.0	10.0-50.0
7	133.44	66.72	66.72-200.16	30.0	15.0	15.0-45.0
8	111.20	88.96	22.24-200.16	25.0	20.0	5.0-45.0
9	111.20	66.72	44.48-177.92	25.0	15.0	10.0-40.0

razor cut was measured. The cycling of the specimen was begun at the measured crack length and continued until fracture. Crack length and number of cycles were recorded at .127 cm intervals. For the second set of specimens, each specimen was fatigued to a crack length of 1.524 cm. This crack length was then defined as the zero value at which point the cycle counter was set equal to zero and the test continued until unstable fracture. Again crack length and number of cycles were recorded at .127 cm intervals.

Approximately fifty percent of the specimens were fatigued using the first way of preparing the cracks. However, it should be pointed out that when preparing a crack in this manner, it is not assured once cycling has begun the crack will start to grow immediately. Quite often there is an incubation period in which the crack remains at rest while load cycles accumulate before it starts to grow. Apparently the razor crack tip is relatively blunt. This notch effect tends to accumulate energy from the fatigue process until a new sharp crack tip is formed. Then the specimen crack will grow with further cycling and hence the two methods of preparing the cracks should yield the same results once fatigue crack propagation has initiated.

At the early stage of fatigue life, this

pre-crack-growth cycling can be a source of considerable discrepancy when comparing the lives of two specimens fatigued at identical load levels. The period of pre-crack-growth cycling may be thousands of cycles in length and at this stage presents a considerable amount of uncertainty. Fatiguing specimens with cracks prepared the second way eliminates this problem since the test is begun after the crack has started to grow. However, it seems reasonable when comparing two specimens run at identical load levels that the crack growth rates at a given crack length should be the same no matter what way was used to prepare the initial cracks. A comparison of the results can be made in Table 2.

After a fatigue test is run, a crack length versus number of cycles graph is plotted for each specimen. At specified crack lengths up to the critical crack length (the crack length at which  $k_{I\max}$  reaches  $k_{Ic}$ ), the slope of the  $a$  versus  $N$  curve is obtained. The slope of the  $a$  versus  $N$  curve is  $\frac{da}{dN}$ . Knowing the load levels and the crack length, a value of  $\Delta S$  is obtained for each  $\frac{da}{dN}$ . Then  $\frac{da}{dN}$  versus  $\Delta S$  is plotted on log-log paper. The slope of this curve is the exponent  $n$  in equation (10).

For each specimen tested, the above procedure is

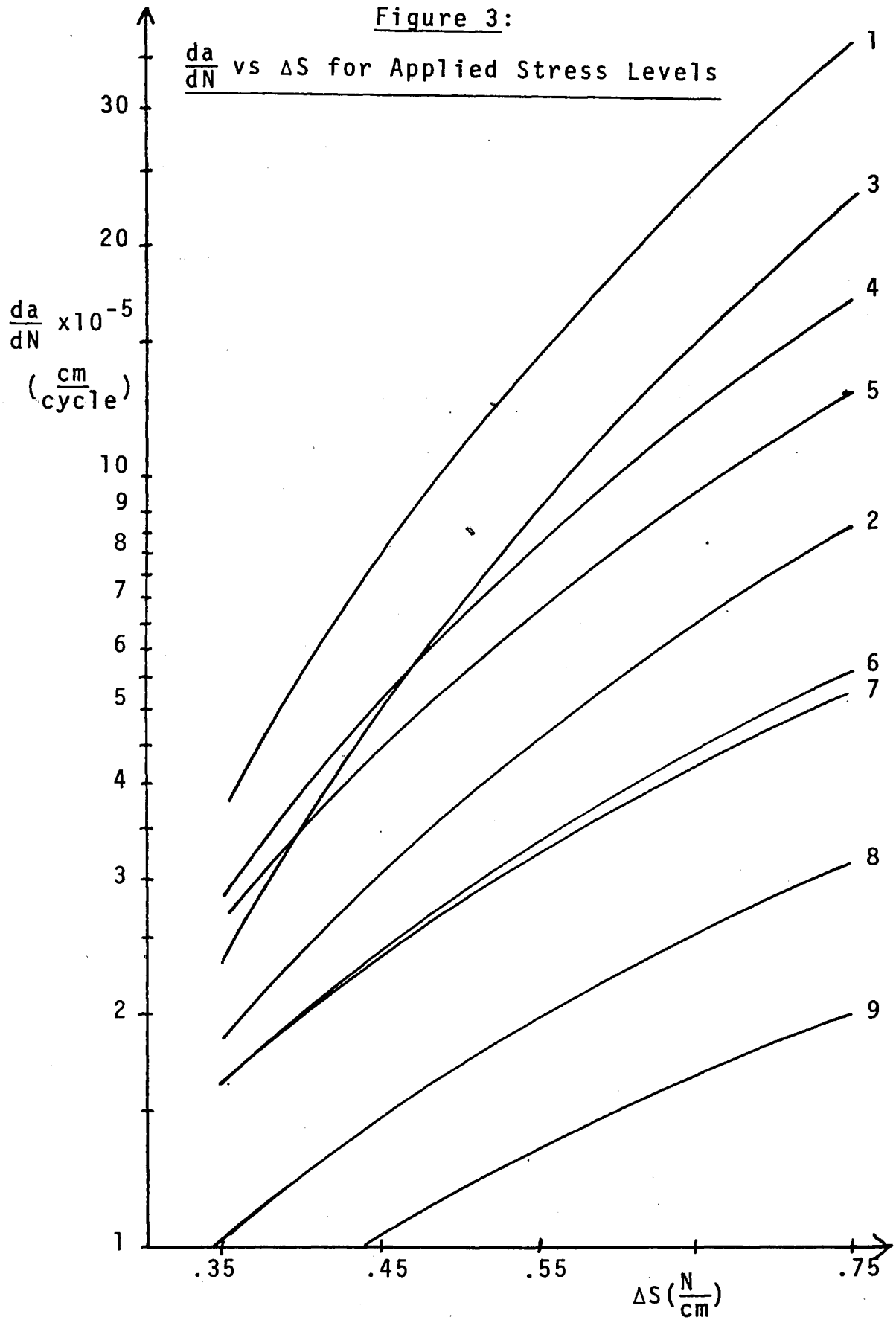
carried out yielding a value of C and n. By running a number of specimens at each set of load levels, an average set of C and n values is obtained. Table 2 on the following page presents the results of this study showing the results for both ways of preparing the initial cracks and the number of specimens tested at each applied stress level. Figure 3 is a plot of  $\frac{da}{dN}$  vs  $\Delta S$  using the average results of Table 2 and equation (10). The numbers next to the applied stress levels in Table 2 and the applied load levels in Table 1 correspond to their respective plots on Figure 3.



Table 2 - Results of PMMA Fatigue Tests

	Applied Stress ( $\frac{N}{cm^2}$ )	1st way (crack started with razor)			2nd way (crack fatigued to 1.524 cm)		
		No. Spec.	Average C ( $\frac{cm^2}{N-cycle}$ )	Average n	Average n	Average C ( $\frac{cm^2}{N-cycle}$ )	No. Spec.
1	110.31-478.01	0	-	-	3.01	$86.45 \times 10^{-5}$	4
2	147.08-441.24	0	-	-	2.02	$15.37 \times 10^{-5}$	5
3	73.54-441.24	0	-	-	3.01	$54.68 \times 10^{-5}$	5
4	110.31-404.47	4	$33.65 \times 10^{-5}$	2.58	2.15	$33.49 \times 10^{-5}$	4
5	36.77-404.47	4	$22.77 \times 10^{-5}$	2.10	2.06	$24.08 \times 10^{-5}$	6
6	73.54-367.70	12	$11.31 \times 10^{-5}$	1.78	1.49	$6.53 \times 10^{-5}$	4
7	110.31-330.93	9	$11.02 \times 10^{-5}$	1.53	1.55	$5.37 \times 10^{-5}$	6
8	36.77-330.93	4	$5.08 \times 10^{-5}$	1.53	1.48	$4.64 \times 10^{-5}$	4
9	73.54-294.16	4	$3.19 \times 10^{-5}$	1.15	1.44	$2.61 \times 10^{-5}$	2

Figure 3:



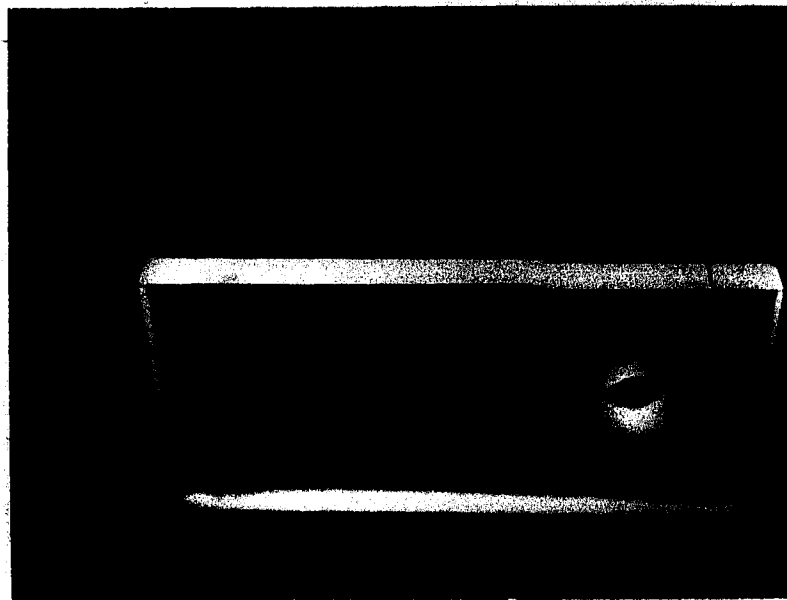
## Experimental Observations

During the fatiguing of the specimens a number of observations were made concerning the fatigue lives. It was noted that all the specimens exhibited flat surfaces of fatigue crack propagation. In addition, once unstable crack propagation was reached the fracture surface continued to be flat. This behavior is representative of a brittle failure and is one of the reasons PMMA was chosen for this study (Figure 4).

Another characteristic of some of the specimen crack surfaces was the tendency for the crack to twist out of plane in the mode III direction. This effect is believed due to the improper drilling of the loading pin holes in some of the compact tension specimens. Specimens that grew cracks grossly off plane were disregarded because the twisted crack plane had the effect of considerably increasing the fatigue life of the specimen.

Finally, all other variables seemingly equal, specimens run at identical load levels with similar crack surfaces tended to have different fatigue lives. However, by fatiguing four or five specimens at a given load level it was hoped a reasonable average of the results would be obtained.

Figure 4: Fatigue and Fracture  
Surface of PMMA Compact Tension  
Specimen



Note bandsaw cut on right hand portion of specimen, grid pattern directly to the left with flat fatigue surface above it.

## Discussion of Results

As mentioned previously, the results of this study were obtained by preparing the starting cracks of the specimens in two different ways. The first way involved introducing a razor crack into the tip of the band saw cut and then fatiguing the specimen to fracture while monitoring crack length and number of cycles as the test proceeded. The second way involved fatiguing the razor crack to a length of 1.524 cm, which was then defined as the zero value (thus eliminating pre-crack-growth cycling), setting the cycle counter to zero and continuing the test to fracture while monitoring the crack length and number of cycles. The end result sought was how did the crack growth rates compare between specimens loaded identically, but with their starting cracks prepared differently. Table II indicates for the applied stresses compared that generally the starting method has little effect on the crack growth rate after the fatigue process has begun.

As Table II also indicates, both the constants  $C$  and  $n$  show an increasing trend with an increase in the applied mean stress and stress amplitude. However, caution should be exercised in labeling the stress amplitude as the dominant factor in the fatigue process

(as is often done). As Figure 3 indicates, both the mean stress and stress amplitude play important roles in the fatigue-crack growth rate process. With the exception of plot 2, the crack growth rate for a given  $\Delta S$  decreases with decreasing mean stress and stress amplitude with no one factor exhibiting more influence than the other (no explanation can be given for plot 2 being out of order). The results indicate the crack growth rate is dependent not only on the stress amplitude, but also on the applied mean stress, thus the family of curves exhibited in Figure 3.

Figure 3 is plotted between the limits of  $\Delta S = .35$  and  $\Delta S = .75$  because they represent the range within which the data was collected. Outside these limits, extrapolation would be necessary. The results should be used only for estimating the life of cracks that are growing under the fatigue process between the limits specified in this study. In addition, the results should be viewed as estimates of the crack growth rates due to the averaging of the data and the graphical calculation of the C and n values.

## Conclusions and Future Work

What has been determined in this study is a relationship for the rate of fatigue crack growth in PMMA for mode I loading and selected applied stress levels. The relationship provides a new way of interpreting fatigue data and suggests the energy density (as opposed to stress) approach in predicting material failure. Current research by Gillemot [9] has given further evidence that the energy density or the energy absorbed per unit volume is the controlling material parameter of material failure under monotonic loading. Further investigation is needed to test the validity of the  $\Delta S$  prediction on fatigue life of materials.

The next step in completing a study of this kind is to obtain a complete set of  $C$  and  $n$  values for all useable stress levels of PMMA. Further work would include fatigue crack propagation studies for mixed mode loading. Since the Strain Energy Density criterion is completely general, further application of the theory would only involve adding the  $k_2$  and  $k_3$  expressions in  $\Delta S$ . For mixed mode problems the SED Theory would also be capable of predicting the direction of fatigue growth as given by  $\theta_0$ . For the mode I loading case studied in this paper,  $\theta_0$  was simply  $0^\circ$  since the crack always

grew in a self similar manner.

As mentioned previously, the fatigue crack growth rate equation proposed is only valid for cracks that are propagating under fatigue conditions. That is, a crack that is being subjected to an oscillatory stress must be through the pre-crack-growth cycling stage before it can have its' growth rate predicted. This restriction is simply a consequence of the way the problem was set up and the data obtained.



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## VITA

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The author expects to be awarded the degree of Master of Science in Mechanics in May 1977.