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ESTIMATION OF PRESTRESS LOSSES IN
CONCRETE BRIDGE MEMBERS

by

Ti Huang

Research Project No. 74-3
Prestress Losses in Post-Tensioned Members

Fritz Engineering Laboratory Report No. 402.4
PRESTRESS LOSSES IN
POST-TENSIONED MEMBERS

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IN POST-TENSIONED MEMBERS
Rimbos, P. and Huang, T., F. L. Report 402.1,
March 1976

PREDICTION OF PRESTRESS LOSSES
IN POST-TENSIONED MEMBERS
Huang, T. and Hoffman. B., F. L. Report 402.3,
December 1979

ESTIMATION OF PRESTRESS LOSSES
IN CONCRETE BRIDGE MEMBERS
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Prepared in cooperation with the Pennsylvania Department of Transportation and the U. S. Department of Transportation, Federal Highway Administration. The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Pennsylvania Department of Transportation or the U. S. Department of Transportation, Federal Highway Administration. This report does not constitute a standard, specification or regulation.

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January 1980
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A procedure for the estimation of prestress losses has been developed previously, based on the linking of experimentally established stress-strain-time relationships for the concrete and steel materials used commonly for prestressed concrete bridge members. This procedure is expanded to apply to all types of prestressed concrete members: pretensioned, post-tensioned, pre-post-tensioned, as well as segmental structures. A simplified procedure suitable for manual usage is also applicable to all prestressed concrete members. The simplified procedure is based on a linear semi-logarithmic growth of prestress losses. Also presented are results from an experimental study which shows that excursion into high temperature during steam curing causes no long-term thermal loss of prestress.
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ABSTRACT

A procedure for the estimation of prestress losses has been developed previously, based on the linking of experimentally established stress-strain time relationships for the concrete and steel materials used commonly for prestressed concrete bridge members. This procedure is expanded to apply to all types of prestressed concrete members: pretensioned, post-tensioned, pre-post-tensioned, as well as segmental structures. A simplified procedure suitable for manual usage is also applicable to all prestressed concrete members. The simplified procedure is based on a linear semi-logarithmic growth of prestress losses. Also presented are results from an experimental study which shows that excursion into high temperature during steam curing causes no long-term thermal loss of prestress.
1. INTRODUCTION

1.1 Background

One of the main problems in the analysis and design of prestressed concrete structural members is the estimation of the loss of prestress over an extended period of time. It has long been recognized that while underestimation of prestress losses may result in premature cracking and excessive deflection of the member, an overestimation could lead to wasteful use of prestressing tendons and excessive camber, which are less serious but also undesirable. An accurate knowledge of the change of prestress through the member's service life is essential to a satisfactory engineering design.

The study of prestress losses at Lehigh University was started in 1966, under the financial sponsorship of Pennsylvania Department of Transportation and the U. S. Federal Highway Administration. In earlier projects, (2,3) a general procedure has been developed for the estimation of prestress losses in pretensioned members based on the linking of stress-strain-time relationships of the concrete and steel materials. PennDOT Research Project 74-3 (Fritz Engineering Laboratory Project 402) entitled "Prestress Losses in Post-Tensioned Members", was started in 1974 for the purpose of extending the previously developed methods to post-tensioned as well as pre-post-tensioned bridge members.

1.2 Objectives

The objectives of research project 74-3 are the following:

1. To develop a method for the prediction of prestress losses in post-tensioned structural members.
2. To develop a method for the prediction of prestress losses in pre-post-tensioned structural members.

3. To study the effect of the maturation of concrete (gaining of strength with time) on the prestress losses.

4. To determine the effect of ambient environmental conditions on the overall prestress losses.

5. To determine the effect of elevated temperature during curing on the prestress losses in pretensioned and pre-post-tensioned members.

6. To modify the general procedure developed in PennDOT Research Project 66-17 to cover the above mentioned parameters.

7. To prepare and document a general computer program for the purpose of predicting prestress losses for prestressed concrete structural members.

Of the above list, item 1 was fully dealt with in Interim Report FL 402.3. (1) item 4 was dealt with in the earlier report FL 382.5. (2) A documented computer program PENDOT had been submitted to PennDOT in 1975. In this final report, the general procedure is modified to allow either prestressing or post-tensioning operations, and the method is extended to cover pre-post-tensioned fabrication and segmental construction as well. In addition, the other items in the objective listing will be addressed to.
1.3 Definitions

A number of terms used extensively in this report are given specific definitions for the purpose of clarification, as follows.

**Prestress:** Prestress is defined as the stress introduced into concrete and steel prior to the application of loads. At any time after transfer, prestress is evaluated as the difference between the total stress in the material under load, and the theoretical stress caused by the applied loads. Thus, the prestress may be viewed as the stress remaining in the material if all applied loads, including the self-weight of the member, were temporarily and imaginarily removed. By this definition, the application of load to a member will change the internal stresses, but not the prestress. However, the long-term effect of any sustained loading on prestress is recognized.

**Losses:** Loss of prestress is evaluated with reference to the tensioning stress in steel just before anchoring. For pretensioned fabrication, the frictional and anchorage losses are generally negligible, and the reference datum may be taken as the steel stress immediately after anchoring to the prestressing bed. The major components of prestress losses are those due to elastic shortening, creep, shrinkage and relaxation.

For post-tensioned members, the frictional and anchorage seating losses are not negligible, and must be properly included. Total prestress loss is therefore directly referenced to the jacking stress. For the sake of convenience, an intermediate prestress value, at the critical location immediately after anchoring, is introduced. It is equal to the jacking
stress reduced by the frictional and anchorage losses. The remaining losses can then be calculated based on this intermediate stress value using a procedure similar to the one used for pretensioned members.

**Stage:** Forces causing variations in the prestress loss of a prestressed concrete member may be applied to the member in a number of increments. In particular, post-tensioning may be done sequentially at significant intervals, and permanent loads may be applied to the bridge member some time after prestress has been partially or completely introduced. Each introduction of additional permanent stress will be referred to as one stage. This may reflect pretensioning of strands, the post-tensioning of tendons or activating of permanent load to the member. In particular, a procedure where post-tensioning of tendon is done at several distinct times is referred to as multi-stage post-tensioning.

1.4 **Units**

Unless specifically indicated, all quantities in this report are given in consistent kip-inch-day units. Strains are dimensionless, and are expressed in absolute values in calculations. In discussion, it is often convenient to express strains in micro-inches per inch, or micro-strains, which are synonyms and equal to $10^{-6}$. 


2. REVIEW OF PREVIOUS WORK

2.1 Basic Concept of General Procedure

The basic procedure was developed initially for pretensioned members\(^{(3)}\) and later expanded to include post-tensioned members as well\(^{(1)}\). The procedure makes use of experimentally established stress-strain-time characteristics equations of steel and concrete materials. These equations are linked by time and strain compatibility relationships, the equilibrium conditions, and a linear equation defining the distribution of concrete stresses across the member section. A direct solution of the stress and strain conditions at any given time is then possible. Details of the general procedure were presented in several earlier reports\(^{(1,2,3)}\). A very brief resume of the procedure for post-tensioned member is presented here to facilitate discussion. It will be shown later that with a few assignments, this general procedure can be made to deal with pretensioned members as well. This procedure forms the basis of the general computer program which deals with several types of prestressed concrete members.

The stress-strain-time relationship for prestressing steel is:

\[
\sigma_s = f_{pu} \left[ A_1 + A_2 \sigma_s + A_3 \sigma_s^2 \right] - \left[ B_1 + B_2 \log(t + 1) \right] \sigma_s \left[ B_3 + B_4 \log(t + 1) \right] \sigma_s^2 \]  

(2-1)

where:

- \( \sigma_s \) = steel stress
- \( f_{pu} \) = specified ultimate tensile strength of steel
- \( \sigma_s \) = steel strain, in \( 10^{-2} \)
\[ t_s = \text{steel time, starting from tensioning.} \]

The coefficients A and B, which were obtained by a regression analysis of experimental data, are shown in Table 1. The terms with the A coefficients represent the instantaneous stress-strain relationship. The time-related relaxation loss of the steel stress is represented by the terms with the B coefficients. It is noted that different steels may be used in the same member.

The stress-strain-time relationship for concrete is

\[ S_c = -C_1 f_c + \left[ D_1 + D_2 \log(t_c + 1) \right] + \left[ E_1 + E_2 \log(t_c - t_{s1} + 1) \right] \]

\[ - f_c E_3 - E_4 (f_c - \sum f_{sdi}) \log(t_c - t_{s1} + 1) \]

\[ - E_4 \sum f_{sdi} \log(t_c - t_{s1} + 1) \]  \hspace{1cm} (2-2)

where \( S_c \) = concrete strain, in \( 10^{-2} \), contraction positive

\( f_c \) = concrete stress, tension positive

\( t_c \) = concrete time, starting from the time when curing is terminated.

\( t_{s1} \) = age of concrete when the i-th stage stress increment is applied

\( f_{sdi} \) = increment of concrete stress at the i-th stage.

In Eq. 2-2, the first term on the right hand side, \( -C_1 f_c \), represents the elastic strain, the second term, with D coefficients, represents shrinkage, and the remaining terms with E coefficients represent the creep strain. The summation operations cover all stress increments which have
already taken place at the time in question \( t_{s_i} \leq t_c \). The empirical coefficients \( C, D \) and \( E \) for concretes commonly used for prestressed concrete bridge members in Pennsylvania are shown in Table 2.

The stress increments \( f_{s_{di}} \) are evaluated by an iterative procedure considering the conditions before and after the application of load or post-tensioning.

The compatibility conditions are

\[
(t_s)_i = t_c - t_{s_i} \quad (2-3)
\]

and

\[
S_{s_i} + S_{c_i} = k_{4i} \quad (2-4)
\]

Here \( (t_s)_i \) = length of relaxation time for steel elements of the i-th stage

\[
k_{4i} = \text{strain compatibility constants, in } 10^{-2}
\]

\( S_{s_i} \) and \( S_{c_i} \) are concurrent steel and concrete strains.

At the post-tensioning time \( t_{s_i} \), \( S_{s_i} \) is the tensioning strain in steel of the i-th stage, which is directly under control and known. The corresponding \( S_{c_i} \), however, is not a controlled input. It includes the effects of all prestressing and loading up to and including the i-th stage, and is calculated by an iterative process. The constant \( k_{4i} \) is determined by adding \( S_{s_i} \) and \( S_{c_i} \) at time \( t_{s_i} \), and is maintained constant for all subsequent time.

The equilibrium equations are

\[
\int f_c \, dA + \Sigma(f_{s_i} \, a_{s_i}) = -P \quad (2-5)
\]

7
\[ \int f_c x \, dA_c + \sum (f_{si} a_{si} x_i) = M \]  

(2-6)

where:  
- \( A \) = area of net concrete section  
- \( a_{si} \) = area of prestressing steel of \( i \)-th stage  
- \( x \) = distance to elementary area from the centroidal horizontal axis  
- \( P \) = applied axial load on section  
- \( M \) = applied bending moment on section.

The positive directions of \( x \), \( P \) and \( M \) are shown in Fig. 1. The integrations are over the net concrete area and the summations are over all steel elements which have already been tensioned and anchored to the member.

A linear distribution of concrete stresses (corresponding to linear strain distribution) is considered

\[ f_c = g_1 + g_2 x \]  

(2-7)

where \( g_1, g_2 \) = parameters to define concrete stress distribution.

Combination of equations (2-1), (2-2), (2-3) and (2-4) shows that the steel stress in any given element, \( f_{si} \), can be expressed as a quadratic function of the concurrent concrete stress, \( f_{ci} \).

\[ f_{si} = R_{li} + R_{2i} f_{ci} + R_{3i} f_{ci}^2 \]  

(2-8)

where the \( R \) coefficients are functions of the material properties, the compatibility constants and the time parameter \( t_c \). Substituting Equation 2-7 into the equilibrium conditions and integrating,

\[ A g_1 + \sum (f_{si} a_{si}) = -P \]  

(2-9)
\[ \sum_{i} (f_{s_i} \times s_i x_i) = M \]  
\[ (2-10) \]

Substitution of Equations (2-7) and (2-8) into (2-9) and (2-10) results in two quadratic equations of \( g_1 \) and \( g_2 \), which can be solved simultaneously by a numerical method. The stresses and strains in steel and concrete are then determined by back substitution.

2.2 **Friction and Anchorage Seating Losses**

For post-tensioned members, the losses due to friction and anchorage seating are calculated at each stage of tensioning to determine the initial applied post-tensioning force before invoking the general procedure described in the preceding section. The theoretical relationship is found in standard references.\(^{(4,10)}\) Expressed in terms of stresses, this relationship can be written as follows:

\[ f_x = f_{sj} e^{-(Kx+\mu\alpha)} \]  
\[ (2-11a) \]

where \( f_x \) = steel stress at distance \( x \) from the jacking end  
\( f_{sj} \) = jacking stress at end of member  
\( x \) = distance from jacking end, in feet  
\( K \) = wobbling coefficient, in ft\(^{-1}\)  
\( \mu \) = friction coefficient  
\( \alpha \) = change of tendon direction in distance \( x \), in radians.

Typical values for \( K \) and \( \mu \) are found in standard references.\(^{(4)}\)

In Reference 1, a procedure was presented for determining the steel stress at a section \( x \) ft from the jacking end after anchorage seating. The equations were based on a profile of uniform curvature, when Equation 2-11a
can be simplified into

\[ f_x = f_{sj} e^{-kx} \]  

(2-11b)

where \( k = K + \omega \phi \)

\( \phi \) = curvature of tendon profile, in radians per foot.

The anchorage length \( l_a \), over which the anchorage seating loss is distributed, is first calculated

\[ l_a = \frac{1}{K} \ln \left[ 1 - \sqrt{\frac{E_s \Delta_a}{f_{sj}}} \right] \]  

(2-12)

where \( E_s \) = modulus of elasticity of post-tensioning steel

\( \Delta_a \) = anchorage seating distance, in feet.

The steel stress after anchorage seating is

\[ f_x = f_{sj} e^{-k(2l_a - x)} \]  

for \( x \leq l_a \)  

(2-13a)

or

\[ f_x = f_{sj} e^{-kx} \]  

for \( x > l_a \)  

(2-13b)

If the anchorage length calculated by Equation 2-12 exceeds the maximum available length, then the anchorage seating loss penetrates the entire member, \( l_a \) assumes the maximum value, and the steel stress after anchorage seating is

\[ f_x = \left[ f_{sj} - \frac{E_s \Delta_a k}{1 - e^{-k l_a}} \right] e^{-k(l_a - x)} \]  

(2-14)
The steel stress after anchorage seating, $f_x$ in Equations 2-13 and 2-14, are used as the "initial" prestress at tensioning time in the general procedure of Section 2.1. (This stress $f_x$ corresponds to the initial strain $S_{s1}$ in Equation 2-4 at time $t_{s1}$.) Obviously, the friction and anchorage seating loss is

\[ \text{ACT} = f_{sj} - f_{pi} \]  

(2-15)

2.3 Practical Procedure for Pretensioned Members

The simplified procedure for pretensioned members was based on a linear growth of prestress loss with respect to the logarithm of concrete age. The initial time at transfer is taken as the same as end of curing. The end of service life is taken at a concrete age of 100 years. All pretensioning strands are taken as concentrated at the center of gravity of the strands. Detailed description of this procedure is in References 3 and 2.

Step 1: Initial prestress loss, at transfer time:

\[ \text{IL} = \text{REL}_1 + \text{EL} \]  

(2-16)

The two parts of initial loss IL are:

\[ \text{REL}_1 = \text{pretransfer relaxation loss, dependent upon } f_{pi} \]  

and $k_1$, and calculated from the steel stress-strain-time relationship, or from Fig. 3.

\[ \text{EL} = \text{elastic loss of prestress} \]

\[ \text{EL} = \frac{n_i}{n_i + 2 - 1} (f_{pi} - \text{REL}_1) \]  

(2-17)
where \( n_i \) = initial modulus ratio
\[ \vartheta = \text{geometrical design parameter,} \]
\[ = \frac{1}{A_p s \left( \frac{1}{A} + \frac{e^2}{I} \right)} \]
\[ A_p s = \text{total area of pretensioning steel} \]
\[ e = \text{eccentricity of pretensioning steel} \]
\[ k_1 = \text{transfer time, or the length of relaxation time} \]
\[ \text{for steel at the time of transfer} \]
\[ f_{pi} = \text{initial tensioning stress.} \]

Step 2: Final prestress loss, taken at the end of 100 years

\[ TL = SRL + ECR - LD \quad (2-18) \]

The three components of the final loss \( TL \) are:

\[ SRL = \text{component independent of concrete stress, dependent} \]
\[ \text{upon concrete characteristics and } f_{pi} \text{ from Fig. 4} \]
\[ ECR = \text{component directly dependent upon concrete stress} \]
\[ = 2.2 EL \]
\[ LD = \text{effect of applied load, including weight of member} \]
\[ = 2n f_{ct} \quad (2-19) \]

when \( f_{ct} \) = concrete fiber stress caused by applied loads,
\[ \text{including weight of member.} \]

Step 3: Loss of prestress at intermediate time \( t_c \)

\[ PL = IL + 0.22(TL-IL) \log t_c \quad (2-20) \]
In Reference 2 a slightly expanded version was given, allowing for the addition of permanent load at some time after transfer. It should be pointed out that while all geometric quantities refer to the net concrete section elsewhere in this report, in this section, they refer to the gross section. It is not difficult to remove this "double" standard. In fact, the only modifications needed are to replace $\delta$ (for gross section) by $\delta+1$ (for net section) in Equations 2-17 and 2-18. However, this "unification" is not recommended since in practice it is more convenient to work with gross section properties for pretensioned members. But for post-tensioned members, the use of net section properties cannot be avoided because there is no direct relationship among the gross section, the net section, and the amount of prestressing steel. Excessive error could result if proper section properties are not used, particularly for heavily prestressed members.

2.4 Practical Procedure for Post-Tensioned Members

The simplified procedure for post-tensioned members is in a format very similar to that for pretensioned members given in the preceding section. (1) It also involves the estimation of an initial loss at the time of anchorage and a final loss at the end of service life (taken as 100 years).

The initial loss involves only the effects of friction and anchorage seating, and is calculated by the methods of Section 2.2.

$$IL = ACF$$ (2-21)
The final loss is estimated by a combination of several terms

\[ TL = ACF + EL + BLL - S - CRA - LD \]  \hspace{1cm} (2-22)

where ACF = loss due to anchorage seating and friction
EL = elastic shortening loss
BLL = "basic" long-term loss = SRL + CR
S = correction to shrinkage loss
CRA = correction for multistage post-tensioning
LD = effect of applied load.

Estimation of intermediate losses is based on a linear growth with respect to the logarithm of time.

\[ PL = IL + 0.22(TL-IL) \log(t - t_{s1}) \]  \hspace{1cm} (2-23)

The estimation of ACF has been discussed in Section 2.2. The remaining components in Equation 2-22 can be estimated for individual elements (of k-th stages) as follows:

\[ EL = -n \sum_{i=k+1}^{L} f_{sdi} \]  \hspace{1cm} (2-24)

where \( f_{sdi} \) = concrete prestress at level of k-th stage steel, caused by the i-th stage prestressing

\[ = -a_{si} f_{pi} \left( \frac{1}{A} + \frac{\epsilon_{i,k}}{I} \right) \]

BLL = SRL + CR

where SRL = component independent of concrete stress, from Fig. 4
CR = component dependent of concrete stress
\[ S = C_s \log t_{sk} \]  
where \( C_s \) = empirical constant

\[ = 4.0, 2.2 \text{ and } 3.0 \text{ for upper bound, lower bound and average loss estimate, respectively.} \]

\[ \text{CRA} = -0.26 n \sum f_{sdi} \log (t_{sk} - t_{si} + 1) \]
\[ - 0.44 n f_{cg} \log (t_{sk} - t_{si} + 1) \]  
\[ \text{LD} = 2 n f_{cl} + n f_{cg} \]  
where \( f_{cl} \) = concrete stress caused by applied permanent loads, except the member's own weight.
\( f_{cg} \) = concrete stress caused by member's own weight.

In Reference 1, formulas were also presented for estimating average losses in all tendons in a post-tensioned member.
3. EXTENSION OF THE PROCEDURES

3.1 Pretensioned Members

The general procedure described in Section 2.1 applies directly to post-tensioned members. For pretensioned members, a simpler version has been presented in earlier reports. It is noted that the post-tensioned member procedure actually can be used to analyze pretensioned members as well, if a few special adjustments are made. The advantages of so doing is that the same procedure can apply to both types of fabrication, which then leads to the handling of pre-post-tensioned members.

The primary difference between the pre- and post-tensioning methods of fabrication lies in the time compatibility constants. Post-tensioning tendons are stretched after concrete has hardened, and concrete is compressed at the same time as the steel elements are tensioned. In contrast, pretensioning strands are stretched prior to the casting of concrete, and concrete is not compressed until transfer. In the general procedure, all time is referenced to the same origin, at the end of curing. Consequently, the time constant \( t_{si} \) in Equation 2-3 must be negative for pretensioning operations. (Note that Equation 2-3 is used with Equation 2-1 to describe the steel relaxation behavior.) On the other hand, since transfer is assumed to take place at the end of curing, the time parameter controlling creep in concrete is \( t_c \), and \( t_{si} \) in Equation 2-2 must be taken as zero. Thus, \( t_{si} \) are assigned different values in Equations 2-2 and 2-3, in order that creep and relaxation behaviors are properly represented.
Equation 2-4 is also affected by pretensioning. At tensioning time \( t_{si} \), concrete is not stressed, and \( S_{ci} \) is zero. Therefore, the constant \( k_{4i} \) reduces to the initial tensioning strain at the critical section, corresponding to the initial tensioning stress \( f_{pi} \), and is a directly controlled input quantity.

The use of the general procedure for pretensioned concrete members is accomplished by making the special assignments on \( t_{si} \) and \( k_{4i} \) as described above.

3.2 Application of Permanent Loads

By virtue of the definition of prestress in Section 1.3, the addition of external loads (thrust and bending moment) causes no immediate change in prestress. But if the loads remain permanently, the increment stresses cause changes in the creep and relaxation behaviors of the materials, hence there is a long term effect on prestress loss. In terms of the general procedure described in Section 2.1, the application of external loads causes three changes in the formulation. Obviously, equilibrium conditions 2-5 and 2-6 are affected, by the changes in \( P \) and \( M \). The concrete characteristic Equation 2-2 is also changes by additional terms involving the new stress increment \( f_{sd1} \). These effects are similar to that of a post-tensioning operation, which imparts an eccentric compression on the member section, creating a new stress increment. This similarity was fully utilized in the computer program.

Each post-tensioning stage is expanded to include also an additional applied moment. As both operations affect the same three parameters, \( P, M \)
and \( f_{sd1} \), the expansion does not increase the calculation effort significantly. The simple application of permanent load can now be regarded as a fictitious post-tensioning stage with zero steel area. In each case, the stress increments \( f_{sd1} \) are evaluated by an iterative process starting from an estimate based on linearly elastic behavior.

The casting of bridge deck on top of a precast, prestressed beam causes the weight of the deck slab to be supported by the beam, and any subsequently applied load to be supported by the composite section. In the general procedure, any load supported by composite section is replaced by an equivalent load system (axial load \( P \) and bending moment \( M \)) which, when applied to the precast section, produces the same stresses. By this load conversion strategy, the general solution subroutine uses the properties of the precast section only. The composite section properties are used only in a preliminary subroutine for load conversion. As a consequence, the procedure developed here cannot accept post-tensioning of steel after the deck slab has been placed, since the losses in these post-tensioned steel cannot be properly evaluated. This restriction is justifiable by the understanding that post-tensioning of the composite section is rarely done in practice.

3.3 Pre-Post-Tensioning

Pre-post-tensioning signifies a fabrication procedure in which a pretensioned member is further prestressed by post-tensioning. This procedure is used when the design initial prestressing force exceeds the capacity of the pretensioning bed. Part of the prestressing force, compatible with the
initial allowable stress requirements, is used in the pretensioning stage. The remainder of the required force is introduced later, after the member has been removed from the bed, by post-tensioning. PennDOT design practice places the lower limit for pre-post-tensioning consideration at 1500 kips.\(^{(11)}\)

It is easy to see that the general procedure for prestress loss estimation can be adapted to pre-post-tensioned members. Such a member is obviously prestressed in several stages. The first stage will be a pretensioning one. Hence \(t_{sl}\) is negative (steel tensioning before placing concrete) for equation 2-3, and taken as 0.0 for equation 2-2. The remaining stages will be those of post-tensioning.

3.4 Segmental Construction

The segmental construction technique has gained popularity in recent years for long span bridges. Segments of the bridge member are precast, often in a wide but short configuration. These precast segments are then assembled in the field, and held together by post-tensioning. Details of the construction procedure vary, and falsework for erection may or may not be involved. This technique may be viewed as an extension of the post-tensioning procedure, with the added feature that the member length increases as fabrication progresses. The segments are usually not prestressed, and the number of segments is usually quite large, necessitating a large number of post-tensioning stages.

The change of effective member length causes an increase of member weight bending moment as segments are added. The length controlling friction and anchorage seating losses also increases. To account for
these effects, the new member length is included in the input for each stage. The increment of dead load moment is calculated and added to the previous moment and used in the equilibrium equation (2-10). In the general computer program, a provision is made that a missing input for the "new member length" will be interpreted to indicate no change in length. This provision takes care of the situation where several tensioning stages are used at the same length, as well as regular post-tensioning and pre-post-tensioning procedures.

3.5 Generalized Computer Program

The generalized computer program is based on the general procedure described in Section 2.1 (for more details, see Reference 1), and include all the modifications discussed in the several preceding sections. It can be used to analyze pretensioned, post-tensioned, pre-post-tensioned as well as segmental members. A brief flow-chart of this program is given in Appendix B, including the requirement on input data cards, and a sample data set.

The input data for each problem includes the properties of the net concrete cross section and descriptions of each prestressing stage. All pretensioned strands are released from bed at the end of curing ($t_c = 0$). Each of the post-tensioning stages may include simultaneously an applied bending moment and a change of member length. Simple addition of loads is treated as a fictitious stage of post-tensioning, with a steel area of zero. A cast-in-place deck slab may be added after all post-tensioning. The program calculates the stresses and prestresses at each stage and at a preselected set of concrete ages up to 100 years, which is assumed to be the service life of the member.
3.6 Modification of the Simplified Procedures

The manual procedures for pre- and post-tensioned members (Sections 2.3 and 2.4) were examined to facilitate their combination and application to pre-post-tensioning and segmental structures.

It is noted that in post-tensioned members the jacking stress, \( f_{sj} \), is measured while the member is under the influence of its own weight. In line with the definition of prestress, given in Section 1.3, the measured quantity is not the steel prestress, but includes the gravity load stress as well. If the member weight were imaginarily removed, the steel stress would obviously decrease. In this sense, the initial prestress loss should include the elastic effect of the member weight at the time of tensioning.

To reflect this effect, a term is added to Equation 2-21

\[
IL = ACF + n f_{cg}
\]

where \( f_{cg} \) = concrete fiber stress, at level of steel, caused by member's own weight at time of tensioning.

Previously, this "elastic loss" effect of member weight was dealt with indirectly, by Equation 2-27, where a smaller coefficient was used for \( f_{cg} \). The new format for \( IL \) allows Equation 2-27 to be modified and unified with Equation 2-19, by redefining \( f_{cl} \) to include the effects of member weight as well as all other permanent loads. Thus,

\[
LD = 2 n f_{cl}
\]

and

\[
TL = IL + SRL + EL + CR - S - LD - CRA
\]

It is seen that Equation 2-22a yields the same result as Equation 2-22, the two equations differ in appearance only.
For multi-stage post-tensioning, the linear semi-logarithmic rule for
growth of prestress losses (Eq. 2-23) is applied piecewise for each time
interval between two consecutive stages. The application of each additional
stage of post-tensioning (and/or loading) causes an immediate elastic loss
and also changes in TL by increments in EL, CR and LD. Since the EL
increment occurs at the beginning of the interval, it is reasonable to be
included in IL, and removed from Equation 2-22a as a separate term.

The IL formulation can be further supplemented by an REL₁ term
signifying the relaxation term before transfer. This term is obviously
zero for post-tensioning. The purpose of including this term is to unify
the pretension and post-tension procedures. Thus, Equations 2-16 and 2-21a
are combined into

\[ IL = REL₁ + EL + ACF + nf_{cg} \] (3-1)

\[ EL = -n_i [f_{sdp} + \Sigma f_{sdi}] \] (3-2)

where \( f_{sdp} \) = initial concrete prestress caused by pretensioned steel

\[ = -(f_p - REL₁)_i (n_i + β - 1), \text{ for pretensioned steel only.} \]

\( f_{sdi} \) = concrete prestress increment caused by the \( i \)-th post-
tension stage

\( f_{cg} \) = concrete stress caused by member weight at time of post-
tensioning.

In Equation 3-2 the summation covers only subsequently applied stages.
Correspondingly, the equations of TL (Eq. 2-18 and 2-22a) are also combined:

\[ TL = IL + (SRL - REL₁ - S) + (CR - CRA) - LD \] (3-3)
In Equation 3-4, the summation covers all stages.

The intermediate loss is

\[ PL = IL + 0.22(TL - IL) \log(t_c - t_{sl}) \]  

where \( t_{sl} \) = concrete age when \( i \)-th stage prestress is introduced.

The IL and TL values are calculated for each of the time intervals. Each stage causes changes by increments in EL, CR and LD.

A summary of the combined manual procedure is given in Appendix C. Demonstrative examples are given in Chapter 4.
4. DEMONSTRATIVE EXAMPLES

4.1 Introduction

Two examples are presented in this chapter to demonstrate the use of the simplified procedure described in Section 3.6. A pre-post-tensioned member is analyzed in Section 4.2, and a segmental bridge structure is analyzed in Section 4.3. Both structures are also analyzed by the general computer program. The results of these computations show that these procedures produce compatible estimates of prestress losses.

4.2 Example of Pre-Post-Tensioned Member

The pre-post-tensioned beam member used in this demonstrative example is similar to a pretensioned member which has been used as an example before. (3) The member has an AASHTO Type V I-beam cross section, a length of 103 ft, and is prestressed with a total of seventy-three 7/16 in. stress-relieved 270 K strands. The total initial prestress force is 1584 kips, with the centroid of steel 12.20 in from the bottom of the beam. (12) Forty-nine of the strands are pretensioned, and the remaining 24 strands are post-tensioned at a concrete age of 30 days. A superimposed deck slab 7 1/2 in. thick and 84 in. wide is added at concrete age of 40 days. Initial tensioning in all prestressing strands is 0.7 of the ultimate. Friction and anchorage seating losses are neglected. Additional information is given in Fig. 5.

Calculations by the simplified procedure are done separately for the pretensioned and post-tensioned strands.
For the pretensioned strands (stage 1)

Initial loss (at transfer, $t_c = 0$)

\[ \text{REL}_1: \quad k_1 = 1 \text{ day} \quad (t_{g1} = -1 \text{ day}) \]

\[ f_{pi} = 0.7 \quad f_{pu} = 185.5 \text{ ksi} \]

\[ \text{REL}_1 = 0.02 \quad f_{pu} = 5.3 \text{ ksi} \]

\[ \text{EL:} \quad 3 = \frac{1}{5.73 \left( \frac{1}{1013} + \frac{19.76^2}{521163} \right)} = 101 \]

\[ f_{sd1} = \frac{185.5 - 5.3}{101 + 5 - 1} = -1.72 \text{ ksi} \]

\[ \text{EL} = -5 \times (-1.72) = 8.6 \text{ ksi} \]

\[ \therefore \text{IL} = 5.3 + 8.6 = 13.9 \text{ ksi} \]

Final total loss (at 100 years)

\[ \text{SRL} = 39.1 \text{ ksi} \]

\[ \text{CR} = -1.2(5)(-1.72) = 10.3 \text{ ksi} \]

\[ f_{cl} = 16790 \times \frac{19.76}{521163} = 0.64 \text{ ksi} \]

\[ \text{LD} = 2(5)(0.64) = 6.4 \text{ ksi} \]

\[ \text{TL} = 13.9 + (39.1 - 5.3) + 10.3 - 6.4 = 51.6 \text{ ksi} \]

Modification by stage 2 prestressing (after $t_c = 30$)

\[ f_{sd2} = 2.41(185.5) \left( \frac{1}{1013} + \frac{19.76^2}{521163} \right) = -0.78 \]

\[ \Delta \text{EL} = -5(-0.78) = 3.9 \text{ ksi} \]

\[ \Delta \text{CR} = 1.2(3.9) = 4.7 \text{ ksi} \]

\[ \therefore \text{IL} = 13.9 + 3.9 = 17.8 \text{ ksi} \]

\[ \text{TL} = 51.6 + 3.9 + 4.7 = 60.2 \text{ ksi} \]
Modification by casting of deck slab (after $t_c = 40$)

$$\Delta f_{cl} = 10440 \times \frac{19.76}{521163} = 0.40 \text{ ksi}$$

$$\Delta LD = 2(5)(0.40) = 4.0 \text{ ksi}$$

$$\therefore TL = 60.2 - 4.0 = 56.2 \text{ ksi}$$

For the post-tensioned strands (stage 2):

Initial loss (at post-tensioning time, $t_c = 30$)

$$ACF = 0$$

$$f_{cg} = 0.64 \text{ ksi}$$

$$\therefore TL = 0 + 5(0.64) = 3.2 \text{ ksi}$$

Final total loss (at 100 years)

$$SRL = 39.1 \text{ ksi}$$

$$S = 2.2 \log(30) = 3.3 \text{ ksi}$$

$$\Sigma f_{sdi} = -1.72 - 0.78 = -2.50 \text{ ksi}$$

$$CR = -1.2(5)(-2.50) = 15.0 \text{ ksi}$$

$$f_{cl} = 0.64 \text{ ksi}$$

$$LD = 2(5)(0.64) = 6.4 \text{ ksi}$$

$$CRA = -(0.26)(5)(-1.72) \log(30)$$

$$-(0.44)(5)(0.64) \log(30)$$

$$= 1.2 \text{ ksi}$$

$$\therefore TL = 3.2 + (39.1-3.3) + (15.0-1.2) - 6.4$$

$$= 46.4 \text{ ksi}$$

Modification by casting of deck slab (after $t_c = 40$)

$$\Delta f_{cl} = 0.40 \text{ ksi}$$

$$\Delta LD = 4.0 \text{ ksi}$$

$$\therefore TL = 46.4 - 4.0 = 42.4 \text{ ksi}$$
These results are shown in Fig. 6 along with results from the computerized general procedure. The comparisons are seen to be very good.

4.3 Example of Segmental Bridge

The segmental bridge structure used as the second illustrative example in this report is the experimental bridge at the Test Track Facilities near University Park, Pennsylvania. This bridge was constructed in two halves, each consisted of seventeen segments, with a total span length of 121.1 ft. The design of this bridge has been given elsewhere, (6) and will not be presented in this report. Figure 7 shows the general plan of this bridge. Some of the relevant data are given in Table 3.

Each of the half-bridges are post-tensioned in five stages. The first four are by $\frac{1}{4}$ in Stressteel bars of the 160 K grade, over increasing effective member length as segments are added. The final stage involves eleven multistrand tendons through the entire member, but following several different profiles. In the analysis, this final stage is further separated, and prestress losses are evaluated for each tendon separately. The horizontal curvature of the bridge was calculated manually and treated as additional vertical curvature in the calculation. (9)

Analysis by the simplified procedure was done for each stage of prestressing steel separately. The frictional and anchorage seating losses were calculated first. The concrete stress increments caused by prestress as well as additional load (due to additional member length) was then determined. Computations of the loss components in Equations 3-1 and 3-3 were then carried out. The detailed calculations for Stage 5, consisting
of one 09/5 multistrand tendon, are presented here. Similar calculations were made on all other tendons.

Recorded jacking stress = 276/1.38 = 200 ksi

Effective member length = 121.1 ft

Anchorage seating distance = 1/8 in

Total curvature (vertical and horizontal) = 0.00197 rad/ft

The anchorage length was found to be less than half-span length

The initial prestress at midspan was found by Equation 2-13b

\[ f_{pi} = 188.2 \text{ ksi} \]

The concrete stresses before stage 5 are

\[ \sum_{i=1}^{4} f_{sd_i} = -1.385 \text{ ksi} \]

\[ \sum_{i=1}^{4} f_{cg_i} = 0.686 \text{ ksi} \]

The stress increments are

\[ f_{sd5} = -(1.38)(188.2) \left[ \frac{1}{4314} + \frac{37.5^2}{2386000} \right] = -0.215 \text{ ksi} \]

\[ f_{cg5} = 55240 \frac{37.5}{2386000} = 0.869 \text{ ksi} \]

Initial losses (at time of post-tensioning, \( t_c = 213 \))

\[ ACF = 200.0 - 188.2 = 11.8 \text{ ksi} \]

\[ IL = 11.8 + 5(0.686+0.869) = 19.6 \text{ ksi} \]
Final total loss (at 100 years)

\[ SRL = 39.1 \text{ ksi} \]

\[ S = 2.2 \log(213) = 5.1 \text{ ksi} \]

\[ CR = -1.2(5)(-1.385 - 0.215) = 9.6 \text{ ksi} \]

\[ LD = 2(5)(0.686+0.869) = 15.6 \text{ ksi} \]

CRA is calculated for \( f_{c_{gi}} \) and \( f_{s_{di}} \) for the first four steps

\[ = 0.26 \text{ ksi} \]

\[ TL = 19.6 + (39.1-5.1) + (9.6-0.3) - 15.6 \]

\[ = 47.3 \text{ ksi} \]

Modifications for the later stages (after \( t_c = 221 \))

\[ \Sigma \Delta f_{s_{di}} = -2.287 \text{ ksi} \]

\[ \Delta EL = -5(-2.287) = 11.4 \text{ ksi} \]

\[ \Delta CR = 1.2(11.4) = 13.7 \text{ ksi} \]

\[ TL = 19.6 + 11.4 = 31.0 \text{ ksi} \]

\[ TL = 47.3 + 11.4 + 13.7 = 72.4 \text{ ksi} \]

Comparison of these results with those from the computerized general procedure is shown in Fig. 8. The linear approximations are seen to agree very closely to the computer output.
5. PRESTRESS RECOVERY AFTER STEAM CURING

5.1 Statement of the Problem

During the investigation on prestress losses in pretensioned members in an earlier project (PennDOT project 66-17), a question was raised concerning the effect of the temperature excursion during steam curing. Obviously, stress in pretensioned strands decreases during the curing period because of the elevated temperature. After curing has been terminated, the strands contract and regain some of the lost stress. A question has been raised whether there remains any lasting effect on steel stress after the initial high temperature curing. There was concern that after concrete has set and adhered to the steel, the strands may not be able to regain the thermal loss.

Results from a previous project (PennDOT project 71-9) appear to have answered the foregoing question in the negative. However, question persisted because force measurements were made on prestressing bed only, and it was feared that strands embedded in concrete may not experience the same stress recovery. Therefore, as part of this present project, an experimental study was carried out to specifically determine this effect.

5.2 Experimental Study

Three sets of two specimens each were tested immediately after prestress transfer in an effort to determine the actual prestress force in the member.
The specimens were 4 in by 6 in in cross section, and 10 ft-6 in long. Prestressing was done by one ½-in diameter stress-relieved 270 K strand, placed at the center of cross section (Fig. 9). A sheet metal plate was cast at mid-length, effectively separating each specimen into two halves between which no tensile stress can be transmitted. Force in each strand was monitored continuously during the fabrication period by a load cell attached at the dead end. Simultaneously, concrete temperature was monitored by embedded thermal devices.

Steam curing was started approximately 5 hours after the pretensioning of strands. A temperature of 130°F to 140°F was maintained for about three days. Transfer of prestress was carried out after the specimens had cooled off. A typical record of prestress force variation is shown in Fig. 10. This curing cycle is similar to those used by the producing plants for the basic specimens in the previous project (PennDOT Research Project 66-17). (13)

Immediately after detensioning the specimens were tested under flexural load to determine the cracking moment. It should be noted that the central location of the prestressing strand resulted in uniform precompression in the specimen. In addition, the bending moment caused no stress in the strand since it was on the neutral axis. The "cracking" moment, which caused the preformed crack to re-open, produced a tensile stress in the bottom concrete fiber, of the same magnitude as the uniform precompression. In this manner, the cracking moment is directly related to the prestressing forces in the specimen at the time of testing.

During the testing, each specimen was loaded and unloaded several times, and the opening of the preformed crack was detected by strain gages adjacent to the crack as well as a straddling clip gage. Both gages responded linearly up to the cracking moment. After the crack opening,
the strain gage readings became stabilized, indicating complete elimination of concrete precompression. At the same time, the clip gage readings began to increase more rapidly, reflecting the reduced stiffness of the cracked section. Typical results of these tests are shown in Figures 11 and 12. Similar results were obtained from all specimens.

5.3 Discussion of Test Results

The recorded strand force variations throughout the curing period are shown in Fig. 10. The variation is essentially the same as that of temperature, measured by an embedded thermocouple near the mid-length section of the specimen. The strands were initially stretched to stresses of 183.1 and 183.7 ksi, respectively. The pre-curing relaxation losses were 3.2 and 3.5 ksi, approximately 1.8%, over a five hour period. Under steam curing, with a temperature of over 130°F, the maximum stress loss was nearly 14 ksi, or about 8%. However, after steam cutoff the stress increased as temperature dropped. The last readings just before transfer were 177.0 and 176.5 ksi, respectively. Thus, the net stress loss over the curing period was 3.5 and 3.1 ksi, respectively, less than 2% of the initial stress. These values compared very well with the predictions by Equation 2-1, indicating no residual effect of the 60-hour excursion into high temperatures.

Figures 11 and 12 show the strain gage as well as the clip gage responses to the applied bending load. Both gages measured the same strains with different scale factors until the cracking load was reached. Afterwards each type of gage gave different information. The SR-4 strain gages, being near the "free" crack surface recorded no further change in strain.
However, the clip gage was mounted across the crack and the opening of the crack was directly measured. Therefore, readings from the clip gage increased faster after cracking.

The SR-4 strain gages give a clear indication of the cracking load by the abrupt change in slope of the load versus strain curve. For specimen No. 1 the apparent cracking load is in the vicinity of 1600 lb, while that for specimen No. 2 is approximately 1200 lb.

While the strain gages measured the changes of concrete fiber strain directly, the clip gage readings were those of the bending strain in the legs. Calibration of the clip gage had yielded the scale factor that the clip gage readings are 0.138 times the actual concrete strain. It should be noted that for the purpose of this experiment, the precise value of this conversion factor is not important. It was only necessary to note the "break" in the plotted curves in order to determine the cracking load.

The slope of the clip gage reading vs. load curves did not show a distinctive break, but instead changed gradually and smoothly. This is because the crack grew under increasing bending moment, continuously changing the effective section resisting the load. As a consequence, it was not practical to determine the precise moment when the crack began to open. Nevertheless, there can be no doubt that the softening of the section coincided with the stabilizing of the strain gage reading.

Originally it was planned that the actual prestress force in the specimens will be calculated from the observed cracking moments value, and then compared with the load cell readings prior to transfer in order to
detect any discrepancy between the steel strand force inside and outside of concrete. The difficulties in obtaining a precise value for the cracking moment made such an approach impractical. Instead, the calculations were reversed. The measured strand force before transfer was used to calculate the cracking load. Any excess of this calculated cracking moment in comparison with the observed value would then indicate probable thermal loss of prestress.

From Fig. 10, the steel stress just before transfer was 177.0 and 176.5 ksi, respectively for the two specimens. The calculated cracking moments were 25.9 and 25.8 kip-in, corresponding to applied loads of 1229 and 1225 lb, respectively. These values agree almost perfectly with the observed result for specimen 2. For specimen 1, the observed cracking moment was actually higher, opposite the consequence of reduced prestress due to thermal loss. From these comparisons, it was concluded that the thermal recovery of strand tension must be taken as real, and no permanent prestress loss need to be considered for the excursion into high temperature during the curing period.
6. **SUMMARY AND CONCLUSIONS**

The primary purpose of this report is to expand the previously developed procedures for use with pre-post-tensioned and segmental structures. The modifications of the procedures are described in Chapter 3. In Chapter 4 demonstrative examples are presented for both the computerized general procedure and the simplified manual procedure. The two procedures are seen to produce compatible results. Chapter 5 presents the results of a separate study on the effect of steam curing.

In the following are listed the specific conclusions derived from the studies presented herein.

1. The general procedure based on the linking of material characteristics equations is applicable to all types of prestressed concrete members, including pretensioned, post-tensioned, pre-post-tensioned beams, as well as segmental structures.

2. The simplified procedures, presently previously in several reports, can be modified and unified, so that the same procedure can be used to predict prestress losses for all types of prestressed concrete members.

3. Steam curing causes the steel pretension to decrease significantly under elevated temperature. However, this thermal loss is nearly completely recovered upon cooling after the termination of curing. There is no need to consider any long-term loss due to the excursion into high temperature.
4. Adherence of concrete to steel in a pretensioned member does not prevent the steel from regaining its thermal loss of stress upon cooling.

5. The growth of prestress loss in any steel element may be taken as linearly related to the logarithm of time since the prestressing of concrete.
7. ACKNOWLEDGMENTS

The research work reported herein was conducted at the Fritz Engineering Laboratory of Lehigh University. Financial Sponsorship was provided by the Pennsylvania Department of Transportation, the United States Federal Highway Administration and the Reinforced Concrete Research Council. The interest and support of these agencies are gratefully acknowledged.

Development of the general computerized procedure was started by Messrs. C. S. Hsieh, Peter Rimbos and Burt Hoffman, who served as research assistants on the project at different times. Their contributions on the reported work were invaluable. Mr. Hoffman also contributed on the simplified procedure.

Special thanks are due the supporting personnel at Fritz Engineering Laboratory for their contributions in producing this report.
8. TABLES
TABLE 1  CHARACTERISTIC COEFFICIENTS FOR PRESTRESSING STEEL

(Equation 2-1)

<table>
<thead>
<tr>
<th>Instantaneous Stress-Strain Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of all Steel</td>
</tr>
<tr>
<td>$A_1 = -0.04229$</td>
</tr>
<tr>
<td>$A_2 = 1.21952$</td>
</tr>
<tr>
<td>$A_3 = -0.17827$</td>
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<table>
<thead>
<tr>
<th>Relaxation Coefficients  -  Stress Relieved Strands</th>
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</thead>
<tbody>
<tr>
<td>Size</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>7/16 in.</td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1/2 in.</td>
</tr>
<tr>
<td></td>
</tr>
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<td></td>
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<table>
<thead>
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<th>Low-Relaxation Strands</th>
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<td>7/16 in.</td>
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<tr>
<td>1/2 in.</td>
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<tr>
<td>Average</td>
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TABLE 2 CHARACTERISTIC COEFFICIENTS FOR CONCRETE

(Equation 2-2)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Upper Bound Losses</th>
<th>Lower Bound Losses</th>
<th>Average Losses</th>
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<tbody>
<tr>
<td>Elastic Strain $C_1^*$</td>
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<td>Shrinkage</td>
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<td>$E_4$</td>
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<td>0.01409</td>
<td>0.01153</td>
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</table>

*Note: $C_j = 100/E_c$ where $E_c$ is modulus of elasticity for concrete, in ksi
TABLE 3 POST-TENSIONING OF SEGMENTAL BRIDGE

<table>
<thead>
<tr>
<th>Stage</th>
<th>$a_{si}$ (in$^2$)</th>
<th>Member Length (ft)</th>
<th>Drape (in)</th>
<th>$t_{si}$ (days)</th>
<th>Jacking Force (kips)</th>
<th>Seating Distance (1/32 in)</th>
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<td>0.5</td>
<td>12.0</td>
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<td>80.44</td>
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<td>121.1</td>
<td>3.0</td>
<td>40.4</td>
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<td>3.0</td>
<td>40.4</td>
<td>221</td>
<td>548</td>
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<td>121.1</td>
<td>16.25</td>
<td>40.4</td>
<td>221</td>
<td>592</td>
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<tr>
<td>5-6</td>
<td>3.68</td>
<td>121.1</td>
<td>53.0</td>
<td>40.4</td>
<td>221</td>
<td>732</td>
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9. FIGURES
Fig. 1 Sign Convention for Applied Loads
Fig. 2 Friction and Anchorage Seating Losses
Initial Tensioning Stress, $f_{si} = 0.8$ fpu

Fig. 3 Initial Relaxation Loss
Figure 4: SRL Part of Final Prestress Loss
Concrete Characteristics: Lower Bound Losses, $n_l = 5$

Steel Characteristics: Average for 270 K stress-relieved strands

Strand Properties: Area per strand = 0.117 in$^2$
\[ f_{pu} = 265 \text{ ksi} \]

Precast Concrete Section: AASHTO Type V
\[ A = 1013 \text{ in}^2 \]
\[ I = 521163 \text{ in}^4 \]
\[ f'_c = 6250 \text{ psi} \]

Pretensioning: Stage 1
\[ \text{Amount of steel: } 49 \text{ strands, } a_{s1} = 5.73 \text{ in}^2 \]
\[ \text{Tensioning stress: } f_{pi} = 0.70 f_{pu} = 185.5 \text{ ksi} \]
\[ \text{Time of tensioning: } t_{s1} = -1.0 \text{ day} \]
\[ \text{Member weight moment} = 16790 \text{ kip-in} \]

Post-tensioning: Stage 2
\[ \text{Amount of steel: } 24 \text{ strands, } a_{s2} = 2.81 \text{ in}^2 \]
\[ \text{Tensioning stress: } f_{pi} = 0.70 f_{pu} = 185.5 \text{ ksi} \]
\[ \text{Time of tensioning: } t_{s2} = 30 \text{ days} \]

Deck Slab:
\[ \text{Time of casting: } t_{s3} = 40 \text{ days} \]
\[ \text{Slab weight moment} = 10440 \text{ kip-in} \]
\[ f'_c = 3500 \text{ psi} \]
\[ \text{Structural thickness} = 7 \text{ in.} \]

Fig. 5 Example Pre-Post-Tensioned Beam
Fig. 6  Prestress Losses in Pre-Post-Tensioned Beam
Fig. 7 Experimental Segmental Bridge
Figure 8: Prestress Loss in Segmental Bridge Stage 5-1
Fig. 9 Specimen for Thermal Effect Test
Fig. 10 Steel Stress versus Time
Fig. 11 Load versus Strain, Specimen No. 1
Fig. 12 Load versus Strain, Specimen No. 2
1. Huang, Ti and Hoffman, Burt  

2. Huang, Ti  
PRESTRESS LOSSES IN IN-SERVICE BRIDGE BEAMS AND REFINEMENT OF PREDICTION METHOD, Fritz Engineering Laboratory Report No. 382.5, Lehigh University, October 1976.

3. Huang, Ti  
PRESTRESS LOSSES IN PRETENSIONED CONCRETE STRUCTURAL MEMBERS, Fritz Engineering Laboratory Report No. 339.9, Lehigh University, August 1973.

4. American Association of State Highway and Transportation Officials  

5. Hoffman, Burt  

AN EXPERIMENTAL POST-TENSIONED SEGMENTAL CONCRETE BOX GIRDER BRIDGE, PennDOT Publication #118, September 1974.

7. Huang, Ti and Tansu, John  
SOME OBSERVATIONS ON THE PRESTRESS LOSS BEHAVIOR OF BEAMS IN AN EXPERIMENTAL BRIDGE, Fritz Engineering Laboratory Report No. 382.2, Lehigh University, April 1974.

8. Rimbos, Peter G.  

9. American Concrete Institute  

10. Nilson, Arthur H.  

11. Pennsylvania Department of Transportation  

12. Pennsylvania Department of Transportation  
STANDARDS FOR BRIDGE DESIGN (Prestressed Concrete Structures), Form BD-201, 1973.

13. Huang, Ti and Frederickson, Donald C.  
APPENDIX A

NOTATIONS

The notations used in this report were defined upon their first appearance in the text. Those of a general importance are assembled in this Appendix for easy reference. Several notations are used only once and are not included in the following listing. An effort has been made to make the units of the symbols consistent. Unless specifically indicated otherwise, all quantities are expressed in consistent kip-inch-day units. All stresses are positive in tension.

\[ a_{si} \] = Area of an individual steel element

\[ A \] = Area of net concrete section

\[ A_{ps} \] = Total area of all prestressing steel elements

\[ ACF \] = Prestress loss due to friction and anchorage seating

\[ BLL \] = Basic long-term loss, including effects of shrinkage, creep and relaxation

\[ C_s \] = Coefficient for estimation of shrinkage correction, see Eq. 2-25

\[ CR \] = Prestress loss due to creep of concrete

\[ CRA \] = Correction to prestress loss for multistage post-tensioning

\[ e \] = Eccentricity of prestress, referring to the net cross section

\[ E_s \] = Modulus of elasticity of post-tensioning steel

\[ ECR \] = Component of final prestress loss, dependent on concrete stress

\[ EL \] = Prestress loss due to elastic shortening

\[ f_c \] = Fiber stress in concrete

\[ f_{cg} \] = Concrete fiber stress, at level of steel, caused by member's own weight at the time of post-tensioning
Concrete fiber stress, at level of steel, caused by the applied long-term loads, including member's own weight except in Section 2.4 (see p. 21)

$f_c^{i}$ = Prestress in steel

$f_{pi}^{i}$ = Initial prestress in steel, at tensioning time, after ACF losses

$f_{pu}$ = Specified ultimate tensile strength of prestressing steel

$f_s$ = Stress in prestressing steel

$f_{sdi}^{i}$ = Increment of concrete stress, at level of steel, due to the post-tensioning of the i-th stage

$f_{sj}$ = Jacking steel stress at the end

$f_x$ = Steel stress after anchorage seating, at a point x distance from the jacking end

$g_1, g_2$ = Parameters to define concrete stress distribution in member section, see Eq. 2-7

$I$ = Moment of inertia of net cross section

$IL$ = Initial total prestress loss, immediately after introduction of concrete prestress

$k$ = Combined friction coefficient, defined in Section 2.2

$k_1$ = Time interval from tensioning of steel to transfer, for pretensioned members

$k_{4i}$ = Strain compatibility constant for the i-th stage prestressing steel, defined by Eq. 2-4 in $10^{-2}$

$K$ = Wobble coefficient of post-tensioning system, in $ft^{-1}$

$L_a$ = Anchorage length, in $ft$, defined in Eq. 2-12

$LD$ = Effect of applied permanent load on final prestress loss

$M$ = Bending moment on section caused by applied load, see Fig. 1

$n$ = Modular ratio of steel to concrete
\( P \) = Axial load on section caused by applied, see Fig. 1

\( PL \) = Total prestress loss at concrete age of \( t_c \)

\( REL \) = Relaxation loss occurring before transfer

\( S \) = Correction to prestress loss, accounting for the shrinkage occurring before post-tensioning

\( S_c \) = Strain in concrete, in \( 10^{-2} \), contraction positive

\( S_s \) = Steel strain, in \( 10^{-2} \)

\( SRL \) = One part of the final prestress loss, independent of concrete stress

\( t_c \) = Age of concrete starting from end of curing

\( t_{sl} \) = Age of concrete when first stage of prestress is introduced

\( t_{si} \) = Age of concrete when the \( i \)-th stage prestress is introduced

\( (t_s)_i \) = Steel "age" after tensioning, for the \( i \)-th stage steel

\( TL \) = Total prestress loss at end of service life

\( x \) = Distance of elementary area from the centroid axis of net cross section, see Fig. 1

\( = \) Distance of a given section from the jacking end in ft, see Section 2.2

\( \alpha \) = Total angle change in the steel profile, from the jacking end to a point \( x \) distance away, in radians

\( \beta \) = A dimensionless parameter of the section geometry

\[ \beta = \frac{1}{A_p s \left( \frac{1}{A} + \frac{e^2}{I} \right)} \]

\( \Delta_a \) = Anchorage seating distance, in feet

\( \mu \) = Frictional coefficient between prestressing tendon and its conduit

\( \omega \) = Curvature of tendon profile, in radians per foot.

58
Input Number of Pretensioning Stages (Card 1)

Input Section Properties (Card 2)

Initialize Parameters
Subroutine INITI

Set Stage Counter

Analyze for Pretensioning
Subroutine PRE

Input Next Event
(Card 4, 6, 8 or EOF)

Identify Card

EOF

No more stages

Card 4

Analyze for Intermediate
time Subroutine
INTERM

Card 6

Add deck slab
Subroutine LASTS

Card 8

Add external moment
Subroutine ADDM

Post-tension-
ing
Subroutine
POST

End

Analyze for Remaining Life
to 100 years
Subroutines
PREDI, PROUT
Subroutine
ADDM

Analyze for Times
up to \( t_i \) before Loading
Subroutines PREDI, PROUT

Calculate Equivalent Load System

\[ a_{i_1} = 0 \]

Calculate Stress Increments
Subroutine PREDI

Output Conditions after Loading
Subroutine PROUT

Return
Subroutine ANCFR

Calculate Anchorage Length

Compare Anchorage Length with Effective Length, also Compare Location of Critical Sections

Calculate Steel Stress at Critical Section after Anchorage and Friction Losses

Return
Subroutine

CONCS

\[ Q_1 = D_1 + D_2 \log(t_c+1) \]
\[ Q_2 = 0 \]

If \( t_c > t_{s1} + 1 \)
Add Creep Terms to \( Q_1, Q_2 \)

If \( t_c > t_{s2} \)
Add \( f_{sdi} \) terms to \( Q_1 \)

Return
Subroutine INITI

Define the Concrete Characteristic Coefficients

Calculate Moment Due to Member's Own Weight

Set Initial Values for Control Variables

Return
Subroutine

INTERM

Analyze Section for Specified Time
Subroutine PREDI

Output Results
Subroutine PROUT

Return
Subroutine LASTS

Input Data on Deck Slab
   (Card 7)

   Analyze for Times before Deck Casting
   (Subroutines PREDI, PROUT)

   Calculate Loading Due to Deck
   Estimate \( h_1, h_2 \)

   Repeat 5 times

   Analyze after Deck Casting
   Subroutine PREDI

   Revise \( h_1, h_2 \)

   Calculate Stresses after Deck Casting

   Output Results
   Subroutine PROUT

   Return
Subroutine POST

Input Additional Data (Read Card 5)

Adjust Member Weight Moment if Needed

Analyze before Post-Tensioning
Subroutines PREDI, PROUT

Calculate Post-Tensioning Force
Subroutine ANCFR

Calculate Equivalent Load System
Estimate $h_{1i}, h_{2i}$

Repeat 5 Times

Analyze after Post-Tensioning
Subroutine PREDI

Revise $h_{1i}, h_{2i}$

Calculate $k_{4i}$

Calculate Steel Stresses and Prestresses after Post-Tensioning

Output Results
Subroutine PROUT

Return
Subroutine

PRE

Input Pretensioning Data (Card 3)
Increment Stage Center
Calculate Initial Steel Stresses and Strains

Calculate Steel Stresses before Transfer

Analyze for Conditions after Transfer
Subroutines PREDI, PROUT

Return
Subroutine PREDI

Initialize Values for U and V Coefficients

For Each Stage of Prestressing Steel, $a_{si}$

Select $P_1$, $P_2$, and $P_3$ for $a_{si}$

Get $Q_1$ and $Q_2$ for the Location of $a_{si}$
(Subroutine CONCS)

Calculate $R_{1i}$, $R_{2i}$, $R_{3i}$

Increment U and V Coefficients

Solve Simultaneous Equations for $g_1$ and $g_2$

Return
Subroutine PROUT

Output Concrete Stress at the Top of the Beam

Output Concrete Stress, Steel Stress, and Steel Prestress for Steel Elements of Each Stage

Output Concrete Stress at the Bottom of the Beam

Calculate and Output the Total Steel Force and its Eccentricity

Return
### INPUT DATA REQUIRED FOR GENERAL PROGRAM

<table>
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<th>Symbol</th>
<th>Description</th>
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<td>Card No. 1</td>
<td>(one card)</td>
</tr>
<tr>
<td>IPRE</td>
<td>Number of pretensioned stages</td>
</tr>
<tr>
<td>Card No. 2</td>
<td>(one card, properties of concrete member)</td>
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<tr>
<td>AGR.</td>
<td>Net area of concrete in the beam (in²)</td>
</tr>
<tr>
<td>CMI</td>
<td>Moment of inertia of the net area of concrete (in⁴)</td>
</tr>
<tr>
<td>YNET</td>
<td>Distance from the top of the beam to the centroid of the net section (in)</td>
</tr>
<tr>
<td>SPANL</td>
<td>Span length (ft)</td>
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<tr>
<td>NSUCO</td>
<td>Code for concrete characteristics</td>
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<td>DEPTH</td>
<td>Depth of beam (in)</td>
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<th>Card No. 3</th>
<th>(one card for each pretensioning stage) (I-th stage typical)</th>
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<td>TIME(I)</td>
<td>Time of pre-tensioning (days after end of curing) (must be negative)</td>
</tr>
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<td>NTYST</td>
<td>Code for strand manufacturer and size</td>
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<td>Code for steel characteristics</td>
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<td>STRANDS</td>
<td>Number of strands in the stage</td>
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<tr>
<td>YDIST(I)</td>
<td>Distance from top of beam to level of steel (in)</td>
</tr>
<tr>
<td>FSP(I)</td>
<td>Initial tensioning stress (in fraction of guaranteed ultimate strength)</td>
</tr>
</tbody>
</table>
Card No. 4 (one card for each post-tensioning stage, each to be followed by a card No. 5)

TIME(IN) Time of post-tensioning (days after end of curing)
NTYST Code for manufacturer and size
NSUST Code for steel characteristics
STRANDS Number of elements in the stage
YDIST(I) Distance from top of beam to level of steel (in)
FSP(I) Initial jacking stress (in fraction of guaranteed ultimate strength)

Card No. 5 (one card for each post-tensioning stage, following card No. 4)

Y Number of ends the tendon is jacked from
XKK Wobble coefficient (ft⁻¹)
DELT Anchorage seating distance (in)
XMU Coefficient of friction
EE Distance from top of beam to steel at the end of the beam (in)
CMBM Extra moment added to the beam (k-in)
ALENGTH New length of member (ft) (if no change leave blank)

Card No. 6 (A blank card after last set of cards 4 and 5)

Card No. 7 (one card immediately following card 6)

STIME Time deck is cast (days after end of curing)
FCSL Strength of deck concrete (psi)
FCBM Strength of beam concrete (psi)
CMBM Extra moment added to the beam (k-in)
TSL Structural thickness of deck (in)
TSLW Total thickness of the deck (in)
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<th>WSL</th>
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<td>CMCO</td>
<td>Extra moment added to the composite section (k-in)</td>
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</table>

(one card for each application of moment, may appear anywhere after a No. 5 card)

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<tr>
<th>STIME</th>
<th>Time moment is added (days after end of curing)</th>
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</thead>
<tbody>
<tr>
<td>CMCO</td>
<td>Moment added to the section (k-in)</td>
</tr>
</tbody>
</table>

**Sample Input for Pre-Post-Tensioned Example Beam**

Card 1: 1.0

Card 2: 1013.0, 521163.0, 31.04, 103.0, 2.0, 63.0

Card 3: -1.0, 2.0, 7.0, 49.0, 50.80, 0.70

Card 4: 30.0, 2.0, 7.0, 24.0, 50.8, 0.70

Card 5: 2, 0.000001, 0.0, 0.0, 50.8, 0.0

Card 6: Blank

Card 7: 40.0, 3500., 6250., 0.0, 7.0, 7.5, 84.0, 0.0

No No. 8 card is needed.
APPENDIX C

SUMMARY OF SIMPLIFIED PROCEDURE

Loss of prestress is estimated for each prestressing element separately and is based on a linear semi-logarithmic growth for each time interval between two consecutive stages. The formulations given below are for the steel element of k-th stage. As each subsequent post-tensioning stage is applied, additional terms $f_{sdi}$ and $f_{cgi}$ are introduced, as well as increment in $f_{ct}$. Therefore, IL and TL are both changed.

1. Initial loss at the time when concrete is prestressed

$$
IL = REL_1 + EL + ACF + n \sum_{i=1}^{k} f_{cgi}
$$

$REL_1$ = initial relaxation in pretensioned steel before transfer, from Fig. 3

EL = elastic loss

$$
= -n \sum f_{sdi}
$$

$f_{sdi}$ = concrete prestress introduced at the i-th stage post-tensioning. The summation covers only the stages subsequent to stage k for post-tensioning steel, but include all stages for pretensioning steel.

ACF = anchorage and friction loss.

$f_{cgi}$ = concrete stress caused by gravity load activated at time of i-th stage prestressing. This term is ignored for pretensioning steel.
2. Final total loss at concrete age of 36500 days

\[ TL = IL + (SRL - REL_1 - S) + (CR - CRA) - LD \]

\[ SRL = \text{from Fig. 4} \]
\[ S = c_s \log t_{sk} \]
\[ CR = -1.2 n \sum_{i=1}^{k-1} f_{sdi} \]
\[ LD = 2n f_{cl} \]
\[ CRA = n \sum_{i=1}^{k-1} (-0.26 f_{sdi} - 0.44 f_{cgi}) \log(t_{sk} - t_{si} + 1) \]

\( f_{cl} \) = concrete stress due to applied permanent loads, member weight included.

\( t_{sk} \) = concrete age when k-th stage prestress is introduced.

3. Intermediate loss

\[ PL = IL + 0.22(TL - IL) \log(t_c - t_{sk}) \]