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Ti Huang
Burt Hoffman

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PREDICTION OF PRESTRESS LOSSES IN POST-TENSIONED MEMBERS

by

Ti Huang
Burt Hoffman

Research Project No. 74-3
Prestress Losses
Post-Tensioned Members

LEHIGH UNIVERSITY
Office of Research

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PRESTRESS LOSSES IN POST-TENSIONED MEMBERS

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IN
POST-TENSIONED MEMBERS

by

Ti Huang
Burt Hoffman

Prepared in cooperation with the Pennsylvania Department of Transportation and the U.S. Department of Transportation, Federal Highway Administration. The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Pennsylvania Department of Transportation or the U.S. Department of Transportation, Federal Highway Administration. This report does not constitute a standard, specification or regulation.

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A previous procedure for the estimation of long-term prestress losses in pretensioned concrete bridge members is expanded to apply to post-tensioned members. The procedure is based on the same basic characteristic stress-strain-time relationships for the concrete and steel materials. The time and strain compatibility conditions are modified to reflect the particular conditions in a post-tensioned member. Further modifications are made to allow for multi-stage post-tensioning, as well as application of loads at several stages.

Examples are presented demonstrating the use of these proposed procedures. Results are compared with those obtained from the method contained in the present AASHTO Specifications.
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ABSTRACT

A previous procedure for the estimation of long term prestress losses in pretensioned concrete bridge members is expanded to apply to post-tensioned members. The procedure is based on the same basic characteristic stress-strain-time relationships for the concrete and steel materials. The time and strain compatibility conditions are modified to reflect the particular conditions in a post-tensioned member. Further modifications are made to allow for multi-stage post-tensioning, as well as application of loads at several stages.

A parametric study was made using the computerized general procedure. From this parametric study, a simplified procedure suitable for manual calculations developed.

Examples are presented demonstrating the use of these proposed procedures. Results are compared with those obtained from the method contained in the present AASHTO Specifications. The proposed procedures are shown to produce predictions comparable to the AASHTO method, but allow more flexibility over several parameters.
1. INTRODUCTION

1.1 Background

One of the main problems in the analysis and design of prestressed concrete structural members is the estimation of the loss of prestress over an extended period of time. It has long been recognized that while underestimation of prestress losses may result in premature cracking and excessive deflection of the member, an overestimation could lead to excessive camber, which is less severe but also undesirable. An accurate knowledge of the change of prestress through the member's service life is essential to a satisfactory engineering design.

The study of prestress losses at Lehigh University was started in 1966, under the financial sponsorship of Pennsylvania Department of Transportation and the U. S. Federal Highway Administration. The initial project (PennDOT Research Project 66-17) dealt with pretensioned bridge members only. It included a fundamental study of the characteristics of concrete and steel materials used for these members in the state of Pennsylvania, and culminated in 1973 in the development of a general as well as a simplified procedure for the estimation of prestress losses in pretensioned concrete members. A second project (PennDOT Project 71-9) was started in 1971 and completed in 1976, and dealt with a field study on an experimental bridge. The field observations were used to modify and refine the previous procedure. The research work reported herein is part of a third project in the program. PennDOT Research Project 74-3, entitled "Prestress Losses in Post-Tensioned Members", was started in 1974.
for the purpose of extending the previously developed methods to post-tensioned as well as pre-post-tensioned bridge members.

1.2 Objectives

The objectives covered in this report are listed below.

1. To extend the prediction procedure developed previously for pretensioned members (FL Report 339.9) to apply to post-tensioned members.

2. To modify the computer program PENDOT to perform the above task.

3. To study the various components of prestress losses in post-tensioned structural members, and to develop simple methods for their estimation.


1.3 Definitions

In the rapid development of prestressed concrete, a number of terms have been used rather loosely without precise and universally accepted definitions. As a result, research results in essential agreement may occasionally appear as contradicting one another. For the sake of clarity, a set of consistent definitions are adopted for this report. The authors do not claim authority in pronouncing these definitions, nor do they anticipate quick acceptance by the profession. These definitions are adopted for the sole purpose of enabling a rational discussion.
Prestress: Prestress is defined as the stress introduced into concrete and steel prior to the application of loads. At any time after transfer, prestress is evaluated as the difference between the total stress in the material under load, and the theoretical stress caused by the applied loads. Thus, the prestress may be viewed as the stress remaining in the material if all applied loads, including the self-weight of the member, were temporarily and imaginarily removed. By this definition, the application of load to a member will change the internal stresses, but not the prestress. However, the long-term effect of any sustained loading on prestress is recognized.

Losses: Loss of prestress is evaluated with reference to the tensioning stress in steel just before anchoring. For pretensioned fabrication, the frictional and anchorage losses are generally negligible, and the reference datum may be taken as the steel stress immediately after anchoring to the prestressing bed. The major components of prestress losses are those due to elastic shortening, creep, shrinkage and relaxation.

For post-tensioned members, the frictional and anchorage seating losses are not negligible, and must be properly included. Total prestress loss is therefore directly referenced to the jacking stress. For the sake of convenience, an intermediate prestress value, at the critical location immediately after anchoring, is introduced. It is equal to the jacking stress reduced by the frictional and anchorage losses. The remaining losses can then be calculated based on this intermediate stress value using a procedure similar to the one used for pretensioned members.
1.4 Units

Unless specifically indicated, all quantities in this report are given in consistent kip-inch-day units. Strains are dimensionless, and are expressed in absolute values in calculations. In discussion, it is often convenient to express strains in micro-inches per inch, or micro-strains, which are synonyms and equal to $10^{-6}$.

1.5 Differences between Pre and Post-Tensioned Members

Several fundamental differences between pretensioned and post-tensioned concrete members should be pointed out at the beginning. The most important difference is, of course, the time of tensioning of the strands. In pretensioned members the strands are stretched before the concrete is placed while the post-tensioned tendons are stretched after placement and hardening of concrete. Because the post-tensioned tendons are tensioned after the concrete has cured, they are stressed by jacking against the member itself. On the other hand, the pretensioned strands are tensioned in a prestressing bed, placing no stress on the concrete until transfer time.

Because of the differences in the tensioning procedure, the two types of prestressed concrete members encounter different loss experiences. For pretensioned members, a major loss component takes place at transfer time due to the elastic shortening of concrete. In addition, relaxation of steel in these members starts before creep and shrinkage. On the other hand, a post-tensioned member suffers initial losses due to
anchorage and friction during tensioning, while the elastic loss is relatively small. Shrinkage strain starts before stresses are introduced, and the portion of shrinkage taking place before post-tensioning has no effect on prestress losses\(^7\).

The post-tensioning fabrication procedure also permits various tendons to be stretched at different times. Thus, it is possible to introduce prestress in several increments, enabling a better control on the camber or deflection of the member. This practice will be referred to as "multi-stage post-tensioning" in this report.
2. PREVIOUS RESEARCH

2.1 Previously Developed Theoretical Procedure

In Project 339, Prestress Loss in Pretensioned Concrete Structural Members, and in Project 382, Prestress Losses of In-Service Highway Bridge Members, a basic prediction procedure for prestress losses in pretensioned members was developed. That procedure was based upon the stress-strain-time relationships of concrete and steel materials, four linking relationships and a linear relationship defining concrete stress distribution in the member section. Similar basic relationships will be used in the development of the post-tensioned procedure in Chapters 4 and 5 of this report. For the convenience of discussion of the new procedure, the derivation of the previous basic procedure is shown in this section. The basic relationships are listed below:

(1) Stress-strain-time relationship for steel

\[
 f_s = f_{pu} \left[ A_1 + A_2 S_s + A_3 S_s^2 \right] \\
- \left[ B_1 + B_2 \log(t_s + 1) \right] S_s \\
- \left[ B_3 + B_4 \log(t_s + 1) \right] S_s^2 \]  

(2-1)

where:  
\( f_s \) = steel stress, in ksi  
\( f_{pu} \) = specified ultimate tensile strength of steel, in ksi  
\( S_s \) = steel strain in \( 10^{-2} \)  
\( t_s \) = steel time, starting from tensioning, in days.

The coefficients A and B, which were obtained by a regression analysis of experimental data, are shown in Table 1. Those terms with the A
coefficients represent the instantaneous stress-strain relationship. The time-related relaxation loss of the steel stress is represented by the terms with the B coefficients.

(2) Stress-strain-time relationship for concrete:

After prestressing,

\[
S_c = C_1 f_c + \left[ D_1 + D_2 \log(t_c+1) \right] \\
+ \left\{ \left[ E_1 + E_2 \log(t_c+1) \right] + f_c \left[ E_3 + E_4 \log(t_c+1) \right] \right\} 
\]

After additional load is applied at \( t_c = t_{c5} \):

\[
S_c = C_1 f_c + \left[ D_1 + D_2 \log(t_c+1) \right] \\
+ \left\{ \left[ E_1 + E_2 \log(t_c+1) \right] + f_c \left[ E_3 + E_4 \log(t_c+1) \right] \right\} \\
- E_4 f_{sd} \left[ \log(t_c+1) - \log(t_c+1-t_{c5}) \right] 
\]

where: 
\( S_c \) = concrete strain, in \( 10^{-2} \), contraction positive 
\( f_c \) = concrete stress, in ksi, compression positive 
\( t_c \) = concrete time, in days, starting from the time of transfer, taken as the same as the end of curing period 
\( t_{c5} \) = the age of concrete, in days, when additional load is applied 
\( f_{sd} \) = increment of concrete stress due to the additional load, applied at \( t_c = t_{c5} \), in ksi, compression positive.

In the above expressions, the first term on the right-hand side, \( C_1 f_c \),
represents the elastic component of concrete strain. The shrinkage strain is represented by the D terms, and the creep by the E terms. The experimental coefficients of the concrete materials are shown in Table 2.

If the stress increment \( f_{sd} \) is known, it can be inserted directly into Eq. 2-26. More commonly, the total external applied loads are known instead of \( f_{sd} \). In such cases, \( f_{sd} \) must be properly evaluated to insure continuity in both stress and strain at time \( t_c = t_{c5} \). It is noted that Equation 2-2a applies at this time before the application of loads, and Equation 2-2b applies immediately afterwards. Subtracting these two equations, the relationship between the stress and strain increments is obtained:

\[
\Delta S_c = (C_1 + E_3)f_{sd}
\]

This relationship is combined with the stress compatibility condition, the steel stress-strain relationship and the equilibrium conditions to produce an equation which is solved for \( f_{sd} \). The detailed development is given in Ref. 2.

(3) Time compatibility:

\[
t_s - t_c = k_1
\]

where: \( k_1 \) = time interval from tensioning of steel to transfer of prestress, in days (this includes the time for form setting, casting and curing).

(4) Strain compatibility, at the location of each prestressing strand:
\[ S_s + S_c = k_2 \]  
\hspace{1cm} \text{(2-4)}

where: \( k_2 \) = initial tensioning strain in steel, in \( 10^{-2} \) in/in.

(5) Equilibrium conditions:

\[ \int f_c dA_c - \Sigma f_s a_{ps} = P \]  
\hspace{1cm} \text{(2-5)}

\[ \int f_c xdA_c - \Sigma f_s xa_{ps} = -M \]  
\hspace{1cm} \text{(2-6)}

where: \( A_c \) = area of net concrete section, in \( \text{in}^2 \)
\( a_{ps} \) = area of individual prestressing elements, in \( \text{in}^2 \)
\( x \) = distance to elementary area from the centroidal horizontal axis, in \( \text{in} \)
\( P \) = applied axial load on section, in \( \text{kip} \)
\( M \) = applied bending moment on section, in \( \text{kip-in} \).

The positive direction of \( x \), \( P \) and \( M \) are shown in Fig. 1.

(6) Concrete stress distribution:

\[ f_c = g_1 + g_2 x \]  
\hspace{1cm} \text{(2-7)}

where: \( g_1, g_2 \) = parameters to define concrete stress distribution in member section.

In these equations, \( f_c, f_s, S_c \) and \( S_s \) are functions of \( x \), and in Eqs. 2-5 and 2-6, the integrations are over the net concrete area and the summations cover all prestressing steel elements. Substituting Eq. 2-7 into 2-5 and 2-6, and performing the integrations,

\[ A \cdot g_1 - \Sigma (f_s + f_{cs})a_{ps} = P \]  
\hspace{1cm} \text{(2-8)}
\[ I g_2 - \Sigma (f_s + f_{cs})x_{ps} = -M \]  
\hspace{1cm} (2-9)

where: \( f_{cs} = \text{concrete fiber stress at the level of prestress steel} \)

\( x_{ps} = \text{x distance for an individual prestressing element} \)

Therefore

\[ f_{cs} = g_1 + g_2 x_s \]  
\hspace{1cm} (2-10)

To simplify further derivation, a group of parameters are introduced

\[ P_1 = A_1 f_{pu} \]

\[ P_2 = [A_2 - B_1 - B_2 \log(t_c + k_1 + 1)] f_{pu} \]

\[ P_3 = [A_3 - B_3 - B_4 \log(t_c + k_1 + 1)] f_{pu} \]

for \( t_c \leq t_{c5} \)

\[ Q_1 = D_1 + E_1 + (D_2 + E_2) \log(t_c + 1) \]

for \( t_c \geq t_{c5} \)

\[ Q_1 = D_1 + E_1 + (D_2 + E_2) \log(t_c + 1) - E_4 f_{sd} \log(t_c + 1 - t_{c5}) \]

\[ Q_2 = C_1 + E_3 + E_4 \log(t_c + 1) \]

Then

\[ f_s = P_1 + P_2 S + P_3 S^2 \]  
\hspace{1cm} (2-11)

\[ S_c = Q_1 + Q_2 f_c \]  
\hspace{1cm} (2-12)

Substituting into Eq. 2-4:

\[ S_s = k_2 + Q_1 - Q_2 f_{cs} \]  
\hspace{1cm} (2-13)

Substituting into Eq. 2-11:

\[ f_s = P_1 + P_2 (k_2 - Q_1 - Q_2 f_{cs}) + P_3 (k_2 - Q_1 - Q_2 f_{cs}) \]

\[ = R_1 + R_2 f_{cs} + R_3 f_{cs}^2 \]  
\hspace{1cm} (2-14)
where: 

\[
R_1 = P_1 + P_2(k_2-Q_1) + P_3(k_2-Q_1)^2
\]

\[
R_2 = -Q_2\left[P_2 + 2P_3(k_2-Q_1)\right]
\]

\[
R_3 = P_3 Q_2^2
\]

Substituting Eqs. 2-10 and 2-14 into the equilibrium conditions 2-8 and 2-9

\[
A_g \ g_1 - \Sigma\left[R_1 + (R_2+1)(g_1+g_2 \times s) + R_3(g_1+g_2 \times s)^2\right] a_{ps} = P \tag{2-15}
\]

\[
I_g \ g_2 - \Sigma\left[R_1 + (R_2+1)(g_1+g_2 \times s) + R_3(g_1+g_2 \times s)^2\right] x_s a_{ps} = -M \tag{2-16}
\]

These equations are simultaneous quadratic equations in \(g_1\) and \(g_2\), and can be written in the form of Eq. 2-18 by introducing the following parameters

\[
U_1 = R_1 \ a_{ps} + p
\]

\[
V_1 = R_1 \ \Sigma \ x \ a_{ps} - M
\]

\[
U_2 = (R_2+1)A_{ps} - A_g
\]

\[
V_2 = (R_2+1)\Sigma \ x \ a_{ps} = U_3
\]

\[
U_3 = (R_2+1)\Sigma \ x \ a_{ps}
\]

\[
V_3 = (R_2+1)\Sigma \ x^2 \ a_{ps} = I_g
\]

\[
U_4 = R_3 \ a_{ps}
\]

\[
V_4 = R_3 \ \Sigma \ x \ a_{ps} = \frac{1}{2} U_5
\]

\[
U_5 = 2R_3 \ \Sigma \ x \ a_{ps}
\]

\[
V_5 = 2R_3 \ \Sigma \ x^2 \ a_{ps} = 2U_6
\]

\[
U_6 = R_3 \ \Sigma \ x^2 \ a_{ps}
\]

\[
V_6 = R_3 \ \Sigma \ x^3 \ a_{ps}
\]

Then

\[
U_1 + U_2 \ g_1 + U_3 \ g_2 + U_4 \ g_1^2 + U_5 \ g_1 \ g_2 + U_6 \ g_2^2 = 0 \tag{2-18}
\]

\[
V_1 + V_2 \ g_1 + V_3 \ g_2 + V_4 \ g_1^2 + V_5 \ g_1 \ g_2 + V_6 \ g_2^2 = 0
\]

If prestressing steel is concentrated at one level, then \(x_s\) becomes a constant for all elements, and is equal to \(e_g\) by definition. Replacing \(x_s\) by \(e_g\) and performing all summation in Eqs. 2-17, the parameters \(U\) and
V become simplified as follows:

\[
\begin{align*}
U_1 &= R_1 A_{ps} + P \\
V_1 &= R_1 e_g A_{ps} - M \\
U_2 &= (R_2+1)A_{ps} - A_g \\
V_2 &= (R_2+1)e_g A_{ps} - I_g \\
U_3 &= (R_2+1)e_g A_{ps} = V_2 \\
V_3 &= (R_2+1)e_g A_{ps} - I_g \\
U_4 &= R_3 A_{ps} \\
U_5 &= 2R_3 A_{ps} e_g = 2V_4 \\
U_6 &= R_3 A_{ps} e_g^2 = \frac{1}{2} V_5 \\
\end{align*}
\]

Substituting these equations into Eqs. 2-18, the quadratic terms can be eliminated by multiplying the first equation by \(e_g\) and subtracting the second

\[
(P e_g + M) - (A_g e_g)g_1 + I_g g_2 = 0
\]

Therefore

\[
g_2 = \frac{A_g e_g}{I_g} g_1 - \frac{P e_g + M}{I_g}
\]

Substituting into Eq. 2-7

\[
f_c = (1 + \frac{A_g e_g}{I_g} x) g_1 - \frac{P e_g + M}{I_g} x
\]

It is clear that Eqs. 2-18 can be transformed into a quadratic equation in terms of \(g_1\) by means of Eq. 2-19. However, a more useful form of the equation is obtained by eliminating \(g_1\) and \(g_2\) from Eqs. 2-15, 2-16, and 2-13. Replacing \(x\) by \(e_g\), these equations become

\[
A_g g_1 - \left[ R_1 + (R_2+1)(g_1+g_2 e_g) + R_3 (g_1+g_2 e_g) \right] A_{ps} = P
\]
Multiply Eq. 2-20 by \( I_g \), Eq. 2-21 by \( (\frac{Ag}{g} e) \), add these two equations, and substitute Eq. 2-22

\[
A \ g \ f_{cs} = \left[ R_1 + (R_2+1)f_{cs} + R_3 f_{cs}^2 \right] A_{ps} \ \left( \frac{1}{A_g} + \frac{e_g}{I_g} \right) = \frac{P}{A_g} - \frac{Me_g}{I_g}
\]

Therefore

\[
f_{cs} = \left[ R_1 + (R_2+1)f_{cs} + R_3 f_{cs}^2 \right] A_{ps} \ \left( \frac{1}{A_g} + \frac{e_g}{I_g} \right) = \frac{P}{A_g} - \frac{Me_g}{I_g}
\]

Two parameters are introduced

\[
\beta = \frac{1}{A_{ps} \left( \frac{1}{A_g} + \frac{e_g}{I_g} \right)}
\]

\[
f_{cl}' = \frac{P}{A_g} + \frac{Me_g}{I_g}
\]

Equation 2-23 is then transformed into Eq. 2-24

\[
(R_1 - 3f_{cl}') + (R_2 - \beta + 1)f_{cs} + R_3 f_{cs}^2 = 0
\]

It is important to note that \( f_{cl}' \) is the nominal concrete stress at c.g.s. caused by the applied loads, based on gross section properties, and using a tension positive sign convention. The dimensionless geometrical parameters \( \beta \) is closely associated with the ratio of steel prestress to concrete prestress.
The equilibrium equations, 2-5 and 2-6, can also be simplified to yield the value of steel stress at any arbitrary time:

\[ f_s = (\beta-1)f_{cs} + \beta f_{cl} \]  \hspace{1cm} (2-25)

By definition, the steel prestress and prestress loss can be evaluated by the following equations:

\[ f_p = f_s - f_{sl} \]  \hspace{1cm} (2-26)

\[ \Delta f_p = f_{si} - f_p = f_{si} - f_s + f_{sl} \]  \hspace{1cm} (2-27)

where:  
- \( f_p \) = steel prestress, in ksi
- \( f_{sl} \) = steel stress caused by applied loads including member weight and all permanent loads, in ksi
- \( \Delta f_p \) = loss of prestress, in ksi
- \( f_{si} \) = initial steel stress immediately upon anchorage, in ksi

2.2 Previously Developed Practical Procedure

A simple procedure for the hand calculation of prestress losses in a pretensioned concrete member has also been developed in the previous projects. The procedure entails first the estimation of losses at the beginning and end of service life of the member, taken at transfer time and 100 years afterwards, respectively. Prestress losses at intermediate times are calculated by linear interpolation with respect to the logarithm of time. For a short time period after the application of external loading, the prestress is taken as remaining constant. This procedure was previously given in Ref. 8, as follows:
Input data needed:

Concrete material characteristics

Initial tensioning stress $f_{si}$

Transfer time $k_1$

Geometrical design parameter $\beta$

Nominal concrete stress at c.g.s. caused by full long term load $f_{ct}'$

Nominal concrete stress at c.g.s. caused by loads, applied later than transfer of prestress $\Delta f_{ct}'$

Age of concrete when $\Delta f_{ct}'$ is applied $t_{cs}$

Step 1: Initial prestress loss, at transfer time:

$$IL = REL_1 + EL$$  \hspace{1cm} (2-28)$$

The two parts of initial loss $IL$ are:

$$REL_1 = \text{pretransfer relaxation loss, dependent upon } f_{si} \text{ and } k_1, \text{ and calculated from the steel stress-strain-time relationship, Eq. 4-1}$$

$$EL = \text{Elastic loss of prestress}$$

$$= \frac{n_i}{n_i + \beta - 1} (f_{si} - REL_1)$$  \hspace{1cm} (2-29)$$

where $n_i = \text{Initial modulus ratio}$

Step 2: Final prestress loss, taken at the end of 100 years

$$TL = SRL + ECR - LD$$  \hspace{1cm} (2-30)$$
The three components of the final loss TL are:

**SRL** = Component independent of concrete stress, dependent upon concrete characteristics and $f_{si}$

**ECR** = Component directly dependent upon concrete stress

= 2.2 EL

**LD** = Effect of applied load, including weight of members

$$LD = \frac{n_i \beta}{\beta + n_i - 1} f_{ci} \quad (2-31)$$

where $\gamma = 2.9$ for lower bound losses

3.3 for upper bound losses

Step 3: Auxiliary final prestress loss: If the load creating $\Delta f_{ci}'$ were eliminated, the member would be under a lighter load over its entire life, and the final loss would be higher

$$T_{LD} = TL + \Delta LD \quad (2-32)$$

$$\Delta LD = (\gamma-1) \frac{n_i \beta}{n_i + \beta - 1} (\Delta f_{ci}') \quad (2-33)$$

Step 4: Loss of prestress at intermediate time $t_c$

(a) Before the application of load, $t_c < t_{c5}$

$$PL = IL + 0.22(TL - IL) \log t_c \quad (2-34)$$

(b) Duration of prestress excursion period, $\Delta t$:

$$\frac{\log(t_{c5} + \Delta t)}{\log t_{c5}} = \frac{TL_{D} - IL}{TL - IL}$$
(c) During the prestress excursion period,

\[ t_{c5} < t_c < t_{c5} + \Delta t \]

\[ PL = IL + 0.22(TL - IL) \log t_{c5} \]  \hspace{1cm} (2-35)

(d) After the prestress excursion period, \( t_c > t_{c5} + \Delta t \)

\[ PL = IL + 0.22(TL - IL) \log t_c \]  \hspace{1cm} (2-36)
3. FRICITION AND ANCHORAGE LOSSES

3.1 Introduction

At the time of tensioning of a post-tensioned member, two components of prestress loss take place, namely, those due to friction and anchorage seating. These losses are unique to post-tensioned tendons. They are both location-dependent because of friction, but their distribution along the length of the member are quite different. Given the frictional characteristics, these losses can be accurately calculated for any specified location in the member. The steel prestress at an interior location, after the friction and anchorage losses, controls the subsequent losses in manners very similar to the steel stress before transfer in a pretensioned member. This steel prestress value is used as the basis for calculations of long-term stress conditions, to be described in Chapter 4.

3.2 Development of Equations

The first loss component to occur is the one due to friction. As a tendon is jacked, it slides against the wall of the conduit which contains the tendon, and frictional force is developed to resist the sliding motion, causing the tendon stress to decrease from the jacking end(s). Figure 6 shows the typical distribution of frictional loss.

Frictional loss is calculated in terms of intentional and unintentional curvatures in the tendon profile. The basic equation for friction loss is:

\[ f_x = f_{sj} e^{-[Kx + \mu \alpha]} \]  

(3-1a)
where: \( x \) = distance from jacking end to point \( x \) (in feet)

\[
f_{sJ} = \text{jacking stress at the end}
\]

\[
f_x = \text{steel stress at a location } x
\]

\[
K = \text{wobble coefficient, in units per ft}
\]

\[
\mu = \text{coefficient of friction between the post-tensioned tendon and the conduit.}
\]

For small values of friction, Eq. 3-la can be closely approximated by \(^{9}\)

\[
f_x = f_{sJ} \left[ 1 - (Kx + \mu \alpha) \right]
\]

(3-lb)

In most practical cases, the tendon profile may be treated as having constant curvature. In such cases, Eq. 3-la can be shortened by introducing a new coefficient

\[
k = K + \mu \frac{\alpha_t}{l}
\]

where: \( l \) = length over which the curvature is constant

\[
\alpha_t = \text{total angle change over the distance } l
\]

Then

\[
f_x = f_{sJ} e^{-kx}
\]

(3-lc)

Anchorage loss, the second component, is the result of slippage and/or deformation in the anchoring device when the tendon is anchored after tensioning. Because of friction, the loss due to anchorage seating is not uniformly distributed over the full length of the member, but rather heavily concentrated near the end(s) being anchored. In this case, the tendon slides inward, in the direction opposite to that during the tensioning, so the friction force is also reversed. A higher stress loss near the end of the member results as shown in Fig. 7. This loss
will be distributed over a length sufficient to make up for the change in length of the tendon. The shortening of tendon during anchorage is obtained by integrating over the anchorage length

$$\Delta a = \int_{0}^{l_a} \frac{\Delta f_s}{E_s} \, dx$$ \hspace{1cm} (3-2)

where:  
\( \Delta a = \) slippage or deformation distance. 
\( l_a = \) anchorage length, length over which the anchorage seating loss of prestress is distributed, See Fig. 7. 
\( E_s = \) modulus of elasticity of steel 
\( \Delta f_s = \) change in steel stress due to anchorage loss.

Using Eq. 3-1a to evaluate steel stresses before and after anchoring, and considering the case of uniform curvature, Eq. 3-2 becomes

$$\Delta a \ E_s = \int_{0}^{l_a} \left( f_{sj} \ e^{-kx} - f_{sj} \ e^{-k(2l_a-x)} \right) dx$$

Performing the integrations

$$\Delta a \ E_s = \frac{f_{sj}}{k} (1 - e^{-k\ell_a}) + \frac{f_{sj}}{k} (e^{-2k\ell_a} - e^{-k\ell_a})$$

$$\frac{\Delta a \ E_s}{f_{sj}} \ k = 1 - 2e^{-k\ell_a} + e^{-2k\ell_a}$$

$$= (1 - e^{k\ell_a})^3$$

$$1 - e^{-k\ell_a} = \sqrt[3]{\frac{\Delta a \ E_s}{f_{sj}}} \ k$$

solving for the anchorage length
For a location outside the anchorage length \((x > l_a)\), the steel stress is not affected by the anchorage losses and is defined by Eq. 3-1. For a location inside the anchorage length \((x < l_a)\), steel stress is affected by both friction and anchorage seating, and:

\[
\sigma_x = \sigma_{sj} e^{-k(2l_a-x)}
\]  

(3-4)

It is clear that the preceding derivation is valid only if the tendon slipback does not penetrate the entire length of the member, i.e., the anchorage length is less than the effective beam length.

\[ l_a \leq L_e \]

where: \(L_e\) = "effective beam length" or the maximum length available for distribution of anchorage seating losses; one half of member length if tensioning is done from both ends simultaneously; length of member if post-tensioning is done from one end only.

For short members, or members with low friction coefficients, \(l_a\) calculated by Eq. 3-3 may exceed the maximum available length \(L_e\), which is clearly not acceptable. This case is shown in Fig. 8. Once again integrating the change in stress over the anchorage length and setting it equal to the anchorage loss, results in

\[
\Delta_a = \int_0^{L_e} \frac{\Delta f_s}{E_s} dx
\]  

(3-5)
If the tendon stress at the jacking end after anchorage is designated by $f_e$,

$$
\Delta f_s = f_{sj} e^{-kx} - f_e e^{kx}
$$

Therefore,

$$
\Delta_a E_s = \int_0^{L_e} (f_{sj} e^{-kx} - f_e e^{kx}) dx
$$

Performing the integration

$$
\Delta_a E_s = \frac{f_{sj}(1 - e^{-kL_e}) - f_e (-1 + e^{kL_e})}{k} (1 - \frac{e^{kL_e}}{kL_e})
$$

$$
f_e = \frac{-k \Delta_a E_s + f_{sj} (-e^{-kL_e} + 1)}{1 + e^{kL_e}}
$$

$$
f_e = f_{sj} e^{-kL_e} - \frac{k \Delta_a E_s}{e^{kL_e} - 1}
$$

For a section at a distance $x$ from the end, the steel stress after anchorage is:

$$
f_x = f_e e^{kx} = \left[ \frac{k \Delta_a E_s}{1 - e^{kL_e}} + f_{sj} e^{-kL_e} \right] e^{kx}
$$

$$
f_x = \left[ f_{sj} - \frac{k \Delta_a E_s}{1 - e^{-kL_e}} \right] e^{-k(L_e-x)} \tag{3-6}
$$

The above derivation deals with tendon profiles in a single uniform curvature only. For profiles with multiple curvatures, somewhat more complicated formulas will be needed. Derivation of these formulas will not be presented here, but can be found in available literature.\(^{(3)}\)
In summary the procedure for calculating friction and anchorage losses includes the following steps:

1. Compute the anchorage length using Eq. 3-3 and determine if the full anchorage length is acceptable within the maximum available length ($l_a \leq L_e$).

2. If $l_a > L_e$, anchorage loss penetrates the entire beam, and steel stress at any location is calculated by Eq. 3-5.

3. If $l_a \leq L_e$, anchorage loss does not penetrate the entire beam.

4. At a section outside of the anchorage length ($x \geq l_a$), there is no prestress loss due to anchorage slippage. Steel stress is calculated for friction loss only, use Eq. 3-1.

5. For sections inside the anchorage length ($x < l_a$), steel stress is influenced by both friction and anchorage seating, and is determined by Eq. 3-4.

Computer subroutine ANCFR follows the aforementioned procedure to determine the initial tensioning stress at the center of the beam.

3.3 Selection of the Critical Section

Structural design of any member is usually controlled by the conditions at a certain cross section. For a simply supported beam, the midspan can generally be taken as this critical location. Here the moments due to dead end live loads tend to be at the maximum. Also, the anchorage and friction losses tend to reduce the prestress to a relatively low value, particular if the post-tensioning is performed from both ends.
Because of these considerations, the computer program calculates the stresses for the midspan section only.
4. GENERAL PROCEDURE

4.1 Introduction

In many aspects, the procedure for long-term losses in post-tensioned beams is similar to the one for pretensioned members. Both procedures share the same material stress-strain-time relationships (Eq. 2-1, 2-2a, 2-2b), and the same assumption of a linear concrete stress distribution (Eq. 2-7). Each procedure also contains four linking conditions on compatibility and equilibrium. Changes were made in these linking relationships to accommodate the sequence of events in the post-tensioning method of fabrication.

In the previous procedure, a mixed sign convention was used for the stresses (and prestresses). A positive stress signifies tension in steel, but compression in concrete. In the new procedure, a uniform and consistent sign convention is adopted so that all tensile stresses are positive. This change caused several sign changes in the formulation, and enables an easier interpretation of the computed results. Additional modifications were made in the formulation to allow for multi-stage post-tensioning and loading. These modifications greatly improved the flexibility of the procedure, so that it can easily be adapted to handle pre-post-tensioning or segmental method of construction.\(^1\)

4.2 The Time Parameters

In the pretensioning fabrication procedure, all prestressing strands are tensioned at practically the same time, and also released from the bed
at practically the same time. Therefore, only one steel time parameter \( t_s \) is needed. The transfer of prestress is done immediately upon the termination of curing, hence the same time parameter \( t_c \) applies to both the creep and the shrinkage components of the concrete strain. The linking relationship (2-3)

\[
t_s = t_c + k_1
\]

shows that the tensioning of steel precedes the prestressing of concrete by the "transfer time" \( k_1 \). The same constant \( k_1 \) applies to all pretensioned steel elements.

In the post-tensioning procedure, tensioning of steel follows the hardening of concrete. The shrinkage and creep of concrete are controlled by different time parameters. In addition, multi-stage post-tensioning requires different time values for each steel element. To afford the flexibility needed, all times are referred to a single starting point at the termination of curing. This single time variable is designated as \( t_c \). An array of time values \( (t_{s1}, t_{s2}, \ldots) \) are used to designate the time when successive post-tensioning operations are performed. Thus, the time parameter controlling the relaxation of the \( i \)th stage post-tensioned steel would be

\[
(t_s)_i = t_c - t_{si}
\]  

(4-1)

Notice that \( t_{si} \) are constants similar to the \( k_1 \) constant in Eq. 2-3 for pretensioned members. The steel time variables \( (t_s)_i \) will be used only temporarily for derivation purposes.
4.3 Modifications in Material Characteristic Equations

4.3.1 Steel Equation

The characteristic equation for prestressing steel has previously been developed as follows:

\[ f_s = f_{pu} \left( A_1 + A_2 S_s + A_3 S_s^2 - \left[ B_1 + B_2 \log (t_s + 1) \right] S_s - \left[ B_3 + B_4 \log (t_s + 1) \right] S_s^2 \right) \]  \hspace{1cm} (2-1)

As pointed out before, the terms with the B coefficients represent the loss of steel stress due to relaxation. This expression for relaxation loss was developed empirically from long-term relaxation loss data. An inaccuracy at the initial times is tolerated to achieve long term stability. As can be easily seen, the inaccuracy amounts to an overestimation of relaxation loss by \( f_{pu} \left( B_1 S_s + B_3 S_s^2 \right) \) at time \( t_s = 0 \). This inaccuracy decreases rapidly with time, and becomes negligible when \( t_s \) exceeds approximately ten days.

In the previous procedure for pretensioned members, this initial inaccuracy in relaxation estimation was not apparent, since the full expression (2-1) was used only after the transfer of prestress, which took place a day or more after tensioning. For the immediate stress-strain response at tensioning time, a shortened expression including only the terms with the A coefficients was used.

\[ f_s = f_{pu} \left( A_1 + A_2 S_s + A_3 S_s^2 \right) \]  \hspace{1cm} (4-2)
For post-tensioned members, a similar approach was used. For each prestressing steel element, Eq. 4-2 is used at the time of its tensioning \((t_s = 0)\), and the complete Eq. 2-1 is used for all subsequent times. When several steel tendons are tensioned one after another, this approach causes the calculated stress in each tendon to change rather drastically when the next tendon is stretched. The amount of the stress change would appear to be inconsistent with the elastic behavior of the cross-section, but actually only reflects the discontinuity caused by the initial inaccuracy in relaxation, mentioned earlier, which is included in the calculation for the first time.

To avoid any misunderstanding of these apparent discontinuities in steel stresses, it is decided that the shortened Eq. 4-2 will be used for each steel element, not only at the time of its tensioning, but also for subsequent times until \((t_s)_{1} = 1\) day, after which time the full Eq. 2-1 will be used. Figure 9 illustrates the variation of steel stress by this technique. It should be emphasized that for long term analyses, the full amount of relaxation is accounted for, and no inaccuracy is induced. The use of the full equation 2-1 is delayed merely to remove the appearance of inconsistency when several post-tensioning stages occur within the same day.

One other modification adopted in the procedure for post-tensioned member is the permissible variability of steel characteristics for multi-stage post-tensioning. The steel characteristic equation is specified for each post-tensioning stage separately, allowing for a design which utilizes several types of prestressing steel in the same member.
4.3.2 Concrete Equation

The first modification to the concrete stress-strain-time relationship involves the starting times of the creep and the shrinkage. For pretensioned members both the shrinkage and the creep were taken as starting at the termination of curing, \( t_c = 0 \). In contrast, for post-tensioned members, the creep starts at the time of first tensioning which is significantly later than the end of curing. Rewriting Eq. 2-2a, after adjustments to conform with the new sign convention for concrete stresses, the basic form of the relationship is

\[
S_c = -C f_c + [D_1 + D_2 \log (t_c + 1)] + [E_1 + E_2 \log (t_c - t_{s1} + 1)]
- f_c [E_3 + E_4 \log (t_c - t_{s1} + 1)]
\]

(4-3a)

where \( t_{s1} \) = concrete time at which the first tendon is tensioned, in days.

Likewise, after multistage post-tensioning or additional loading, the relationship is

\[
S_c = -C f_c + [D_1 + D_2 \log (t_c + 1)] + [E_1 + E_2 \log (t_c - t_{s1} + 1)]
- f_c E_3 - E_4 (f_c - \sum f_{sdi}) \log (t_c - t_{s1} + 1)
- E_4 \sum f_{sdi} \log (t_c - t_{si} + 1)
\]

(4-3b)

where \( f_{sdi} \) = increment of concrete stress caused by the post-tensioning, or loading, of the \( i \)th stage,

and the summation operations cover all stress increments which have already taken place \( (t_{si} \leq t_c) \)
Immediately after tensioning the first strand, $t_c = t_{sl}$, Eq. 4-3a reduces to

$$S_c = -C_1 f_c + [D_1 + D_2 \log (t_c + 1)] + E_1 - E_3 f_c$$

This reduced equation contains shrinkage strain $[D_1 + D_2 \log (t_c + 1)]$ and creep strain $(E_1 - E_3 f_c)$. While $[D_1 + E_2 \log (t_c + 1)]$ represents correctly the shrinkage strain at this time, the $[E_1 - E_3 f_c]$ term for creep strain is an initial inaccuracy similar to that in the steel relaxation expression, discussed in Section 4.3.1. In order to remove the appearance of inconsistency, the same treatment as for the steel characteristic relationship is adopted. The creep component is omitted until one day after the first stage post-tensioning. In other words, for $t_c \leq t_{sl} + 1$, a shortened concrete relationship is used:

$$S_c = -C_1 f_c + [D_1 + D_2 \log (t_c + 1)]$$

(4-4)

Similar to the treatment of the steel relationship, this arbitrary delay for the inclusion of creep strain permits all steel elements tensioned one after another to show conformal loss behavior. For long term analysis, the full amount of creep strain is included, and no inaccuracy is induced. (See Fig. 10).

An additional complication in the concrete expression, Eq. 4-3b, involves the evaluation of the stress increments $f_{sdi}$. The direct solution method mentioned earlier (Section 2.1) is impractical for the generalized case under consideration, and an iterative method is employed instead. The concrete stresses immediately before loading are directly calculated. An initial approximation of $f_{sdi}$ is calculated based on linear elasticity,
where $\Delta P_i$ and $\Delta M_i$ are the loads being applied.

The concrete stresses immediately after loading are then calculated based on these approximate values of $f_{sd'1}$. The difference between the two sets of concrete stresses yields new estimates of $f_{sd'1}$. These new estimates of $f_{sd'1}$ enable the recalculation of the concrete stress after loading, which in turn yields improved estimates of $f_{sd'1}$. The iteration procedure is repeated until $f_{sd'1}$ values are essentially stabilized. This is usually achieved in four or five cycles. More details are given in Appendix C.

4.4 Post-Tensioning

When a steel element is post-tensioned the stress in the element is being controlled and known. The stresses (strains) in the elements which have previously been tensioned and anchored to the member are not controlled, but are free to respond to the additional axial load and moment imposed by the new tensioning force. In the developed procedure, the post-tensioning is initially treated as an application of an equivalent eccentric load on the "previous" section which includes the concrete cross-section plus all previously anchored steel. The equivalent axial load is the tensile force in the element being stretched. The equivalent applied moment is the product of the tensile force and the distance between the steel being post-tensioned and the centroid of the net concrete section. Figure 13 helps to clarify this equivalent force system for post-tensioning.
For the determination of stress and strain conditions immediately after the \( i \)th stage of post-tensioning, at time \( t_c = t_{si} \), the equivalent load system for the \( i \)th post-tensioning is added to the actual external loads, and the "previous" section is analyzed for the total loads. Notice that each post-tensioning creates \( f_{sd} \) increments in all concrete fibers, which cause changes in the concrete characteristic equation. Hence, this analysis involves the iterative procedure described in Section 4.3. For times \( t_c > t_{si} \), the \( i \)th stage steel \( a_{si} \) is added to the effective section, and the equivalent load system for post-tensioning is removed.

Another modification which has been incorporated into the post-tensioned prediction procedure involves the strain compatibility constant. In the pretensioned member the concrete is not stressed at tensioning, and \( k_2 \) is equal to the initial steel tensioning strain. In the post-tensioned case, concrete is stressed simultaneously with steel. Therefore, the strain constants are not so directly known. For each post-tensioned steel element

\[
S_{si} + S_{ci} = k_{4i}
\]  

(4-6)

where \( S_{si} \) and \( S_{ci} \) are concurrent steel and concrete strains. Notice that at time \( t_{si} \), \( S_{si} \) is the tensioning strain in steel, directly under control but \( S_{ci} \) is the total concrete strain (in compression) caused by all prestressing and loading including the stage under consideration. \( k_{4i} \) can only be determined by summing \( S_{si} \) and \( S_{ci} \) at time \( t_{si} \), and is kept constant afterwards.
4.5 Applied Loads

Loads which have influence on the prestress losses in a post-tensioned bridge member include the weight of the precast member, that of the added deck slab, and any other sustaining load. The live and impact loads are transient in nature, and have negligible effect on the loss of prestress, hence are not included in this analysis. Most of the applied loads are supported by the precast member itself. Only a small portion of long-term loads is carried by the composite section of beam and deck slab. Weight of a future wearing surface is the most common example of these loads.

In the prestress loss estimation procedure presented herein, the loads supported by composite section are replaced by equivalent load systems (axial load $P$ and bending moment $M$) which, when applied to the precast section, produce the same stresses. By this load conversion strategy, the general solution subroutine uses the properties of the precast section only. The composite section properties are used only in a preliminary subroutine for load conversion. As a consequence, the program developed here cannot accept post-tensioning of steel after the deck slab has been placed, since post-tensioning of the composite sections cannot be properly converted to desired equivalent load system. This restriction is justified by the understanding that post-tensioning of the composite section is rarely done in practice.

As noted previously with regard to the losses in pretensioned members, the time when an external load is applied to a member has negligible
influence on the long-term prestress losses. The same holds for post-
tensioned members. Therefore, for the sake of convenience, all externally
applied loads, with one exception, are taken as activated at the same
time when the deck slab is placed.

The lone exception to the above is the weight of the precast member
itself. This load becomes active in the member as soon as post-tensioning
prestress is introduced to cause negative bending. In fact, the jacking
force is measured with the member weight in action. In the proposed pro-
cedure, the member weight is assumed to be fully active upon the first
stage post-tensioning ($t_c = t_{sl}$). For members which do not camber up
until after several stages of post-tensioning, this assumption induces
slight errors in the steel elements of these early stages, but its effect
on the total prestress is negligible.

4.6 Effective Cross-Section

In the previous procedure for prestress loss estimation in preten-
sioned members, gross concrete section properties were used in the compu-
tations. The use of gross section properties was chosen for practical
convenience, and is justifiable because the gross section can be directly
related to the concrete and steel areas. That is

$$A_g = A_c + A_{ps}$$

For post-tensioned members, such a simple relationship does not exist.
Post-tensioning tendons are generally placed in conduits whose areas
are several times larger than the areas of the tendons contained. In addition, the amount of steel effective in resisting an applied load changes with each stage of post-tensioning. In the new procedure, all calculations of cross-section properties refer to the net concrete section. The effective steel area is accounted for separately.

4.7 Summary of Procedure for Post-Tensioned Members

With the basic relationships modified as described in the preceding sections, the general procedure for post-tensioned members is derived in a manner very similar to that for pretensioned members. As shown in Appendix C, the basic relationships are combined into two simultaneous quadratic equations in $g_1$ and $g_2$ which defines the distribution of concrete stresses (Eq. 2-7)

$$U_1 + U_2g_1 + U_3g_2 + U_4g_1^2 + U_5g_1g_2 + U_6g_2^2 = 0 \quad (4-7)$$

$$V_1 + V_2g_1 + V_3g_2 + V_4g_1^2 + V_5g_1g_2 + V_6g_2^2 = 0 \quad (4-8)$$

These equations have exactly the same form as Eqs. 2-15. However, the meaning of the coefficients $U$ and $V$ are somewhat different as can be seen from Appendix C.

For the analysis of a post-tensioned member, the necessary input information include:

The net concrete section properties: Area, centroidal axis location, moment of inertia.
The characteristics of concrete material

For each post-tensioning steel element:

The characteristics of steel material

The areas and location of the element, \( a_{si} \) and \( x_i \)

The initial tensioning stress at the critical section, after friction and anchorage losses

The time of post-tensioning, \( t_{si} \)

For applied external loads:

The moment caused by the weight of the precast member is taken as applied at the time of first stage post-tensioning (\( t_{sl} \))

All other loads are taken as applied at the time of casting of deck slab

To determine the condition at a specified time \( t_c \), the coefficients \( Q_1 \) and \( Q_2 \) are first calculated, the calculation of \( Q_1 \) includes contributions from all available \( f_{sdi} \) terms (for which \( t_{si} < t_c \)). Next, the \( R_{1i} \), \( R_{2i} \), and \( R_{3i} \) coefficients are determined for steel elements of each preceding stages (for which \( t_{si} < t_c \)). Summations in Eqs. C-7 are then performed, and Eqs. 4-7 and 4-8 solved. The concrete stress at any location is determined by Eq. 2-7. The steel stress \( f_s \) is calculated by using Eqs. 4-3b, 4-6 and 2-1.

At each stage of post-tensioning and loading, additional stress increments \( f_{sdi} \) are introduced, which requires the analysis of the member both before and after the event, while maintaining the time parameter unchanged (at \( t_c = t_{si} \)). The iteration procedure is described in Appendix C.
On account of the dependency of the solution procedure on the $f_{sd1}$, it is clear that the analysis of a given member must follow the time sequence of its stages of post-tensioning and loading. Between two post-tensioning (or loading) stages, the solution is direct and does not require cumulation of short time intervals.

Computer programs BEAM has been developed to carry out the analysis of post-tensioned members as described above. It analyzes a given post-tensioned member over its entire service life, assumed to be 100 years after casting of the precast beam member. Concrete and steel stress distributions are determined before and after each stage of post-tensioning, before and after application of deck load, and at a series of preselected concrete ages from 1 day to 36500 days. A brief flow chart of this program is given in Appendix B.
5. PRACTICAL PROCEDURE

5.1 Introduction

The practical procedure is a simplification of the general procedure described in Chapter 4. It is designed for manual computation. As in the case of pretensioned members, the practical procedure for post-tensioned members involves the estimation of an initial loss at the time of tensioning, a final loss at the end of service life, (taken at the concrete age of 100 years), and the loss at any intermediate time.

A format similar to that for the pretensioned members is used, and whenever possible, calculations are made similar. The initial loss involves only that caused by friction and anchorage seating. For the final loss, an estimate of the long-term losses is first made on the basis of pretensioning, and several correcting terms are added. In symbolic form, the formula is

\[ TL = ACF + ES + BLL - S - CRA - LD \]  

(5-1)

when

- \( ACF \) = loss due to anchorage seating and friction
- \( ES \) = Elastic shortening loss
- \( BLL \) = "Basic" long-term loss, including shrinkage, creep and relaxation effects
- \( S \) = Correction to shrinkage loss
- \( CRA \) = Correction for multistage post-tensioning
- \( LD \) = Effect of applied load

Each of the component terms is discussed in detail in the following sections.
The estimation of intermediate losses is based on a linear growth with respect to the logarithm of time. The calculation is similar to Eq. 2-28 for pretensioned members, except that the beginning time is that of first tensioning, \( t_{sl} \). Therefore

\[
PL = ACF + (TL - ACF) \frac{\log (t - t_{sl})}{\log (36500 - t_{sl})}
\]

For practical purpose, the \( t_{sl} \) term in the denominator can be ignored, and

\[
PL = ACF + 0.22 (TL - ACF) \log (t - t_{sl}) \tag{5-2}
\]

5.2 Friction and Anchorage Losses

Friction and anchorage seating losses, ACF in Eq. 5-1, are directly calculated by Eqs. 3-1, 3-4 or 3-5. In most practical cases, the frictional coefficients \( \mu \) and \( K \) are not large and simpler formulas based on the linear approximation (Eq. 3-1b) may be used.

Friction and anchorage losses are the only components occurring at the "initial" time. Relaxation of steel has not yet begun, and elastic shortening loss does not occur until a later stage of post-tensioning takes place. Therefore, the "initial loss" in a post-tensioning steel element is the same as the quantity ACF. As mentioned earlier, the steel stress after the ACF losses is taken as the "initial" stress value in the estimation of long-term losses.
5.3 Elastic Loss

Prestress loss due to elastic shortening of a post-tensioned member is dependent upon the sequence of post-tensioning. This loss component can be calculated accurately, using the well-known linear elastic formula, by a step-by-step cumulation. At the \( i \) step of tensioning, a previously tensioned \( j \)th step (\( j < i \)) steel element suffers a stress loss of

\[
(ES)_{ji} = n a_{si} f p_i (\frac{1}{A} + \frac{e_i e_j}{I})_{i-1}
\]  

(5-3)

where \( A \), \( e \), and \( I \) refer to the area, eccentricity, and moment of inertia, respectively, of the composite section containing all steel up to the \( (i-1) \) step. The total elastic loss in the \( j \)th steel is

\[
(ES)_j = \sum_{i=j+1}^{n} (ES)_{ji}
\]

The "average" elastic loss is

\[
ES = \frac{1}{\sum_{j} (ES)_j} \sum_{j} (ES)_j (a_{sj})
\]

\[
= \frac{n}{\sum_{j} (a_{sj})} \sum_{j} a_{sj} f p_i (\frac{1}{A} + \frac{e_i e_j}{I})_{i-1}
\]  

(5-4)

In most practical cases, the scattering of the prestressing steel is not large, and the successive composite sections differ only slightly from one another. Equations 5-3 and 5-4 can then be simplified considerably by referring to the net concrete section and the average eccentricity.

\[
(ES)_{ji} \approx n a_{si} f p_i (\frac{1}{A} + \frac{e^2}{I})
\]

\[
= \frac{n a_{si} f p_i}{a_{ps}}
\]
\[ ES = C_{es} \left( \frac{f_{pi}}{\beta} \right) \]  \hspace{1cm} (5-5)

where
\[ \beta = \frac{1}{A_{ps} \left( \frac{1}{A} + \frac{e^2}{A} \right)} \]  \hspace{1cm} (5-6)
\[ C_{es} = \frac{1}{2} \left[ 1 - \frac{\sum(a_{si}^2)}{A_{ps}^2} \right] \]  \hspace{1cm} (5-7)

Notice that \( \beta \) is the same dimensionless parameter previously used in connection with Equation 2-18, but refers to the net concrete section for post-tensioned members. The coefficient \( C_{es} \) clearly has values within the limits 0 and 0.5, depending upon the separation of \( A_{ps} \) into \( a_{si} \). For single stage tensioning, \( C_{es} \) obviously equals zero. For practical purposes, it is recommended that \( C_{es} \) be taken as 0.25 for two stage post-tensioning and 0.37 where more than two stages are involved.

5.4 Basic Long Term Loss

The basic long term loss represents the total prestress loss caused by shrinkage, creep and relaxation, assuming all prestressing to be applied at concrete age of zero. It resembles the total loss of a pretensioned member, except that the elastic loss is not included. The estimation of BLL is made based on the previous procedure, including two parts:

\[ BLL = SRL + CR \]

The SRL part is controlled by the initial steel stress and the characteristics of concrete, and independent of the concrete stress. It is estimated by the aid of Figure 3. The CR part is primarily controlled by the initial concrete stress at the level of the steel element and also affected by the initial steel stress. It is similar to the ECR term used in the pretensioning
procedure, but does not include the initial elastic shortening effect. It is estimated by Figs. 11 and 12. More simplified, the CR term may be estimated as \( 1.2 n f_{cs} \), where \( f_{cs} \) is the initial concrete stress at the level of steel.

The components ACF, ES and BLL combine to give the anticipated prestress loss, at the concrete age of 100 years, in a post-tensioned member, prestressed at time zero, and carrying no external load.

5.5 Correction to Shrinkage Loss

The first correction term \( S \) reflects the fact that post-tensioned members are typically prestressed some length of time after the end of curing. A significant portion of the shrinkage strain takes place before the application of prestress, and has no effect on the loss. The concrete characteristic equation (4-3b) indicates the amount of shrinkage strain in question is

\[
D_1 + D_2 \log(t_{s1} + 1)
\]

Because of the interlocking influence of shrinkage, creep and relaxation, it is not convenient to derive the stress change directly from this strain value. Instead, empirical coefficients were developed based on parametric calculations using a slightly simpler mathematical form

\[
S = C_s \log t_{s1}
\]  \hspace{1cm} (5-8)

where the recommended value for the coefficient \( C_s \) is 2.2, 4.0 and 3.0 for lower bound, upper bound and average concrete, respectively.

If several stages of post-tensioning are involved, each steel element will experience a different amount of shrinkage correction, and the average
correction is

\[ S = C_s \frac{1}{A_{ps}} \sum a_{si} \log t_{si} \quad (5-9) \]

5.6 Effect of Applied Load

By the definition of prestress adopted in Section 1.3, the application of an external load has no immediate effect on the prestress. However, because the material stresses are changed, a long-term effect exists for a sustaining load. This effect is reflected by the term LD in the general formulation, Eq. 5-1.

Evaluation of the term LD is done in the same manner as in the pretensioned members method. All sustained external loads are taken as applied at the same time, coincident with the casting of deck slab concrete. The long term effect is a decrease in prestress loss by an amount proportional to the elastic stress caused by the applied loads.

\[ LD = 2n_1 f_{c,t} \]

where \( f_{c,t} \) = elastic concrete stress at level of prestressing steel caused by applied loads.

The effect of the weight of the member itself is somewhat different from the above. As discussed in Section 4.5, the post-tensioning jacking force is measured with the member cambered up, and the weight active in the member. Therefore, the measured jacking force includes the tension caused by gravity load. The initial prestress, as defined in Section 3.1, is lower by this same elastic stress value. To compensate for this effect,
the coefficient for the dead load stress is reduced.

\[ LD = 2n_1 f_{cL} + n_1 f_{cg} \]  \hspace{1cm} (5-10)

Here \( f_{cg} \) is the concrete stress, at the level of steel due to members' own weight, and \( f_{cL} \) represents the concrete stress caused by all other long-term loads.

5.7 Modification for Multi-Stage Post-Tensioning

The multi-stage tensioning of steel elements at different times affects the prestress losses in several aspects. Each post-tensioning stage causes elastic shortening and subsequent creep losses in each previously anchored element. On the other hand, the newly tensioned steel element is obviously not affected by shrinkage and creep strains in concrete which occurred before its tensioning. These effects can be separately considered and systematically estimated for each element. Although such a procedure will be lengthy and tedious, a discussion is necessary before a practical approximation can be developed.

Consider the \( k \)th stage post-tensioning of a member, at time \( t_{sk} \), when steel elements with area \( a_{sk} \) are tensioned, producing an increment of concrete stress, \( f_{sdk} \), at c.g.s. The additional final loss in any previously anchored steel include the elastic and the creep components. From Sections 5.3 and 5.4

\[ (ES)_k = -n f_{sdk} \]

\[ (CR)_k = -1.2 n f_{sdk} \]

The final prestress loss in \( a_{sk} \) can be estimated by the general formula, Eq. 5-1. The elastic term \( ES \) is obviously zero. The terms ACF, SRL and
LD are estimated as discussed in Sections 5.2, 5.4 and 5.6. The remaining terms are

\[ CR = -1.2 n \sum_{i=1}^{k} f_{sd} \]
\[ S = -C_s \log t_{sk} \]

The correction for early creep is made on the basis of linear development with respect to the logarithm of time.

\[ (CRA)_k = -1.2 n \sum_{i=1}^{k-1} \left[ f_{sd} \log(t_{sk} - t_{si} + 1) / \log(36500 + 1) \right] \]
\[ - 2n f_{cg} \log(t_{sk} - t_{si} + 1) / \log(36500 + 1) \]
\[ = -0.26 n \sum_{i=1}^{k} \left[ f_{sd} \log(t_{sk} - t_{si} + 1) \right] \]
\[ - 0.44 n f_{cg} \log(t_{sk} - t_{si} + 1) \]

Notice that the effects of gravity load and the previously applied pre-stress must be considered separately. The prestress decreases gradually with time, while the concrete stress caused by gravity load remains practically unchanged.

It should be kept in mind that the loss in element \(a_{sk}\) estimated above, will be modified again by any subsequent tensioning. With loss in each element estimated in the above manner, the "average" loss in all steel can be developed.

The "average" loss due to shortening, creep and shrinkage correction have already been shown in Sections 5.3, 5.4 and 5.5. The average correction for creep is
\[ \text{CRA} = \frac{1}{A_{ps}} \sum a_{sk}(\text{CRA})_k \]

\[ = -0.44 f_{cg} n_i \frac{1}{A_{ps}} \sum a_{sk} \log(t_{sk} - t_{si} + 1) \]

\[ - 0.26 n \frac{1}{A_{ps}} \sum a_{sk} \left[ \sum_{i=1}^{k-1} f_{sdi} \log(t_{sk} - t_{si} + 1) \right] \]
6. EXAMPLES AND COMPARISONS

6.1 An Idealized Example

To demonstrate the practical method described in Chapter 5, an idealized post-tensioned bridge member is analyzed as an example. A more realistic example is given in Section 6.2. In the idealized example, the area of prestressing steel was assigned rounded values to simplify computation, and the concrete section is only defined in terms of the several properties. However, it must be emphasized at the outset that these simplifications do not change the nature of the problem, only the numerical values. The demonstration remains perfectly valid.

The example beam is 50 ft long, fabricated with the concrete exhibiting lower bound loss characteristics. The properties of the net concrete section are

\[
\begin{align*}
\text{Area } A &= 426 \text{ sq in} \\
\text{Depth } h &= 36 \text{ in} \\
\text{Centroidal distance from top } y_t &= 17.83 \text{ in} \\
\text{Moment of inertia } I &= 95726 \text{ in}^4
\end{align*}
\]

Post-tensioning is done in two stages, at concrete ages of 1 and 45 days, respectively. Each stage involves a steel area of 1 sq in. The first group is located at 30 in from the top \( (p_1 = 12.17 \text{ in}) \), while the second stage steel is located 6 inches higher \( (p_2 = 6.17 \text{ in}) \). After frictional and anchorage losses, the "initial tensioning stress" for all steel is \( 0.7 f_{pu} \), or 189 ksi, at the midspan section.

A cast-in-place deck 7.5 inches thick and 60 inches wide is placed at a concrete age of 180 days.
The applied moments at midspan section are 1664 kip-in for girder weight, and 1755 kip-in for deck slab.

The losses in the first stage steel are:

SRL: From Fig. 3, for \( f_{pi} = 0.7 f_{pu} \)

\[
SRL = 39.1 \text{ ksi}
\]

CR: \( f_{cs} = f_{sd1} = -189(1) \left( \frac{1}{426} + \frac{12.17^2}{45726} \right) = -1.06 \text{ ksi} \)

\( n = 5 \)

\[
CR = -1.2(5)(-1.06) = 6.4 \text{ ksi}
\]

LD: \( f_{cg} = 1664 \times \frac{12.17}{45726} = 0.44 \text{ ksi} \)

\( f_{ct} = 1755 \times \frac{12.17}{95726} = 0.47 \text{ ksi} \)

\[
LD = 2(5)(0.47) + 5(0.44) = 6.9 \text{ ksi}
\]

S: 

\[
t_{s1} = 1
\]

\[
S = 2.2 \log 1 = 0
\]

Corrections due to later tensioning:

\[
f_{sd2} = -189(1) \left[ \frac{1}{426} + \frac{6.17(12.17)}{45726} \right] = -0.75 \text{ ksi}
\]

\[
ES = -5(-0.75) = 3.8 \text{ ksi}
\]

\[
CR = -1.2(5)(-0.75) = 4.5 \text{ ksi}
\]

\[
TL_1 = 39.1 + 6.4 - 6.9 - 0 + 3.8 + 4.5 = 46.9 \text{ ksi}
\]

Final prestress \( = 189 - 46.9 = 142.1 \text{ ksi} \)

Final steel stress \( = 142.1 + 5(0.44+0.47) = 146.7 \text{ ksi} \)

Similarly, the losses in the second stage steel are:
SRL: 39.1 ksi

CR: 

\[ f_{cs} = f_{sd1} + f_{sd2} = -189(1) \left( \frac{1}{426} + \frac{12.17(6.17)}{45726} \right) \]

\[ -189(1) \left( \frac{1}{426} + \frac{6.17^2}{45726} \right) = -0.75 - 0.60 = -1.35 \text{ ksi} \]

CR = -1.2(5)(-1.35) = 8.1 ksi

LD: 

\[ f_{cg} = 0.25 \text{ ksi} \quad f_{cl} = 0.25 \text{ ksi} \]

LD = 2(5)(0.24) + 5(0.23) = 3.5 ksi

S: 

\[ t_s = 45 \]

S = 2.2 log 45 = 3.6 ksi

CRA: 

\[ -0.26(5)(-0.75) \log(45-1+1) \]

\[ -0.44(5)(+0.23) \log(45-1+1) \]

= 1.62 - 0.82 = 0.80 ksi

\[ TL_2 = 39.1 + 8.1 - 3.5 - 3.6 - 0.80 = 39.3 \text{ ksi} \]

Final prestress = 189 - 39.3 = 149.7 ksi

Final steel stress = 149.7 + 5(0.23+0.24) = 152.1 ksi

Alternately, the "average" prestress loss can be calculated approximately by treating all steel as concentrated at one level, then

\[ \rho = \frac{1}{2}(12.17 + 6.17) = 9.17 \]

The loss components are:

\[ \beta = \frac{1}{2} \left( \frac{1}{426} + \frac{9.17^2}{45726} \right) = 119.4 \]

\[ C_{es} = \frac{1}{2} \left( 1 - \frac{t^2 + t^2}{2^2} \right) = 0.25 \]

\[ ES = 0.25(5) \frac{189}{119.4} = 2.0 \text{ ksi} \]

SRL: 

SRL = 39.1 ksi
CR:

\[ f_{cs} = -\frac{189}{119.4} = -1.6 \text{ ksi} \]

\[ CR = -1.2(5)(-1.6) = 9.6 \text{ ksi} \]

LD:

\[ f_{cg} = 1664 \times \frac{9.17}{45726} = 0.33 \text{ ksi} \]

\[ f_{cl} = 1755 \times \frac{9.17}{45726} = 0.35 \text{ ksi} \]

\[ LD = 2(5)(0.35) + 5(0.33) = 5.2 \text{ ksi} \]

S:

\[ 2.2 \left(\frac{1}{2}\right)(1 \times \log 1 + 1 \times \log 45) = 1.8 \text{ ksi} \]

CRA:

\[ -0.26(5)\left(\frac{1}{2}\right)(1)(\frac{1}{2})(-1.6) \log(45-1+1) \]

\[ -0.44(0.33)(5)\left(\frac{1}{2}\right)(\log 1 + \log 45) \]

\[ = 1.07 - 0.61 = 0.5 \text{ ksi} \]

\[ TL = 39.1 + 2.0 + 9.6 - 5.2 - 1.8 - 0.5 = 43.2 \text{ ksi} \]

Final prestress = 189 - 43.2 = 146.8 ksi

Final steel stress = 146.8 + 5(0.33+0.35) = 150.2 ksi

In comparison, the computerized procedure, described in Chapter 4, yielded the following results:

For the first stage steel: \( TL_1 = 47.8 \text{ ksi} \)

Final prestress = 140.9 ksi

Final steel stress = 145.8 ksi

For the second stage steel: \( TL_2 = 42.5 \text{ ksi} \)

Final prestress = 146.2 ksi

Final steel stress = 148.7 ksi

This beam was also analyzed by the method specified in the current AASHTO Specifications, (9) assuming an average relative humidity of 70%. The
calculated total loss is 37.4 ksi, corresponding to a final average steel stress of 151.6 ksi.

Figure 14 shows the results from the general as well as the simplified procedures. Also shown is the result obtained by the AASHTO method. Discussion of these results is given in Section 6.3.

6.2 A Realistic Example

The second demonstrative example for the proposed methods was taken from the 1975 AASHTO Interim Bridge Specifications. (9) That document contains a number of examples illustrating the estimation of prestress losses by the newly adopted AASHTO method. Example Problem No. IV deals with a post-tensioned member, and is chosen for this report. The member in question has a triple-box cross-section, and has a simple span of 162 ft. The section properties (of the net concrete area) are as follows:

\[
A = 55.7 \text{ ft}^2 = 8,021 \text{ in}^2 \\
I = 346 \text{ ft}^4 = 7,175,000 \text{ in}^4 \\
y_c = 2.89 \text{ ft} = 34.7 \text{ in} \\
y_b = 3.61 \text{ ft} = 43.3 \text{ in}
\]

Concrete compressive strengths are 4500 psi at 28 days, and 3500 psi at post-tensioning, with an initial steel-to-concrete modulus ratio of 7.8. Prestressing steel consists of 290 stress-relieved 1/2 in diameter strands of the 270 K grade. The centroid of prestressing steel is placed on a parabolic profile, with zero eccentricity at ends and an eccentricity of 2.61 ft (31.3 in) at midspan. Tensioning is done from one end only, to a jacking stress of \( \frac{0.726 f_{pu}}{\mu} \) or 195.9 ksi. The frictional coefficients are \( \mu = 0.25 \) and \( K = 0.0002 \). The anchorage seating distance is 0.25 in.
The calculations for the losses due to friction and anchorage seating are identical for the method proposed in the report as for the method demonstrated by AASHTO. By Eqs. 3-3 and 3-4, it is found that the anchorage length is 87.8 ft., exceeding half span length, and that the steel stress at midspan upon anchoring, \( f_{pi} \), is 188.7 ksi (approximately 0.70 \( f_{pu} \)). These results are nearly identical to those given by AASHTO.

Detailed information on the shrinkage and creep characteristics of concrete and the sequence of post-tensioning were not given in the AASHTO document. For the application of the proposed method, it is assumed that all strands are tensioned within a relatively short time, at a concrete age of 14 days, and only the average loss is calculated. The following calculations are based on the concrete having the upper bound loss characteristics.

\[
A_{ps} = 290(0.153) = 44.37 \text{ sq in}
\]

\[
ES = 0.37(7.8)(\frac{189}{86.3}) = 6.3 \text{ ksi}
\]

\[
SRL = 53 \text{ ksi}
\]

\[
f_{cs} = -\frac{189}{86.3} = -2.19 \text{ ksi}
\]

\[
CR = -1.2(7.8)(2.19) = 20.5 \text{ ksi}
\]

\[
S = 4.0 \log(14) = 4.6 \text{ ksi}
\]

52
LD: The moment due to girder weight at midspan is 329000 k-in

\[ f_{cg} = \frac{329000 \times 31.3}{7175000} = 1.44 \text{ ksi} \]

\[ LD = 7.8 \times 1.44 = 11.2 \text{ ksi} \]

The total "average" prestress loss is:

\[ TL = 6.3 + 53 + 20.5 - 4.6 - 11.2 = 64.0 \text{ ksi} \]

\[ \therefore \text{Final prestress} = 188.7 - 64.0 = 124.7 \text{ ksi} \]

\[ \text{Final steel stress} = 124.7 + 7.8(1.44) = 135.9 \text{ ksi}. \]

Similar calculations, using the characteristics of lower bound concrete, resulted in a total loss of 52.1 ksi, a final prestress of 136.6 ksi, and a final steel stress of 147.8 ksi.

The computer program was also used to analyze the member assuming the steel elements to be tensioned in three stages, involving 90, 100 and 100 strands respectively. For upper bound concrete, the computerized calculation resulted in a final "average" steel prestress of 128.4 ksi and a final steel stress of 137.3 ksi. For lower bound concrete, the resulting values are 141.2 ksi and 148.8 ksi, respectively.

In comparison to the above estimations by the methods proposed here, the AASHTO document shows an estimated total loss of 33.2 ksi, and a final steel stress of 155.8 ksi.

6.3 Comparison of Results

The example calculations in Sections 6.1 and 6.2 show clearly that the practical procedure, described in Chapter 5, and the computerized general procedure, described in Chapter 4, give very nearly the same estimate for
prestress losses. These close agreements are hardly surprising, since the practical procedure was empirically developed to approximate the general procedure.

In comparison with the present AASHTO method, as described in detail in the 1975 Interim Specifications, the results predicted by the procedure developed in this report appear to be significantly higher, particularly when the concrete material with upper bound loss characteristics is involved. Here it must be pointed out that the comparison may have been distorted by two apparent fundamental differences between the methods. First, the definitions of "prestress" are probably different. For the proposed procedures, the terms "prestress" and "prestress losses" are clearly and explicitly defined in Chapter 1. For the AASHTO method, these terms are not so clearly defined. Note that in the estimation of both the elastic (ES) and the creep (CR) components, the dead load stresses are included (in $f_{c_{ir}}$ and $f_{c_{ds}}$). It is reasonable to assume that in the AASHTO method, the dead load stress is not separated from the prestress, and the "losses" measure the difference between the initial and final steel stresses under full dead load condition. Secondly, the meaning of "final time" is probably different in the two methods. For the proposed procedure, the "end of service life" was arbitrarily placed at a concrete age of 100 years. For the AASHTO method, this time is again not clearly defined. However, indications exist that a much shorter time is assumed, probably in the range of 20 years. Considering that prestress losses "grow" approximately linearly with the logarithm of concrete age in days, the total loss at 20 years can be expected to be approximately 85% of that at 100 years. When both of these two differences are taken into consideration, the predictions made by the proposed procedures compare quite well with the present AASHTO method.
Further comparison of the proposed procedures with the AASHTO method reveals that the proposed procedures offer much more flexibility over a large number of parameters. The most important of these is the characteristics of the concrete material, which influences the elastic, shrinkage as well as creep components of the losses. In the AASHTO method, only the effect on elastic shortening is considered, the shrinkage and creep components are both taken as independent of the concrete material. In the proposed procedures, the effects of concrete properties are fully incorporated. It is the authors' opinion that the concrete properties influence the shrinkage and creep behaviors of concrete significantly, and these effects must be adequately included to provide accurate estimates of prestress losses. The consideration of concrete material properties is particularly important when dealing with post-tensioned members, since the range of variation is larger in comparison to pretensioned members.

Additional flexibilities allowed by the proposed procedures but not by the AASHTO method include the multi-layer placement of steel, the multi-stage tensioning and the application of long-term loads at several stages. The AASHTO method yields only one solution, for the "average" loss in all steel. The proposed procedures allow the loss in each element to be evaluated separately. They also allow the estimation of losses at various times over the entire lifetime of the member. In short, the proposed procedures give much more detailed estimation than the present AASHTO method.

The only parameter considered by the AASHTO method but not the procedures proposed herein is the relative humidity of the environment. The basic concrete characteristic equations were developed from strain measurements on concrete specimens stored in the Fritz Engineering Laboratory, with
a moderate environment. Subsequent field study indicated that the characteristic equations apply reasonably well to the prevailing condition at the Pennsylvania State Test Track, with an average relative humidity of approximately 70%\(^9\). For areas substantially drier or wetter than Pennsylvania, an adjustment to the shrinkage coefficients (D\(_1\) and D\(_2\)) in the concrete characteristic equations would appear appropriate. Without field data from locations with significantly different environmental conditions, these adjustments cannot be incorporated properly. However, using the procedures as they are probably will not induce excessive error.
7. SUMMARY AND CONCLUSIONS

This report contains a computerized general procedure for the estimation of prestress losses in post-tensioned concrete bridge members, and a simplified procedure which is suitable for manual calculations. These procedures are proposed for the use by the Pennsylvania Department of Transportation, as an alternative to the method described in the AASHTO Specifications.

Conclusions derived from the studies leading to the development of these prediction procedures include the following:

1. Prestress losses in post-tensioned members are reasonably estimated by either the general or the simplified procedure.

2. Using the simplified manual procedure, prestress losses may be estimated either for each steel element separately, or for the entire steel in average.

3. The proposed procedures represent improvements of the present AASHTO method, as many more parameters are incorporated into these procedures.

4. The multi-staged post-tensioning causes a significant change in the elastic component of prestress losses. However, unless a substantial period of time elapses between stages, the effect on shrinkage and creep components is negligible.

5. The proposed procedures produce predictions of prestress losses which appear higher than the present AASHTO method. However, the difference may be caused by different time and stress parameters.
8. ACKNOWLEDGMENTS

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Special thanks are due the supporting personnel at Fritz Engineering Laboratory for their contributions in producing this report.
9. **TABLES**
TABLE 1 COEFFICIENTS FOR STEEL SURFACES

Instantaneous Stress-Strain Relationship

\[ A_1 = -0.4229 \]
\[ A_2 = 1.21952 \]
\[ A_3 = -0.17827 \]

Relaxation Coefficients - Stress Relieved Strands

<table>
<thead>
<tr>
<th>Size</th>
<th>Manufacturers</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/16 in.</td>
<td>B</td>
<td>-0.05243</td>
<td>0.00113</td>
<td>0.11502</td>
<td>0.05228</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>-0.04697</td>
<td>-0.01173</td>
<td>0.10015</td>
<td>0.05943</td>
</tr>
<tr>
<td></td>
<td>U</td>
<td>-0.06036</td>
<td>0.00891</td>
<td>0.12068</td>
<td>0.02660</td>
</tr>
<tr>
<td></td>
<td>ALL</td>
<td>-0.05321</td>
<td>0.00291</td>
<td>0.11294</td>
<td>0.03763</td>
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<tr>
<td>1/2 in.</td>
<td>B</td>
<td>-0.06380</td>
<td>0.00359</td>
<td>0.12037</td>
<td>0.05673</td>
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<tr>
<td></td>
<td>C</td>
<td>-0.07880</td>
<td>-0.00762</td>
<td>0.14598</td>
<td>0.05920</td>
</tr>
<tr>
<td></td>
<td>U</td>
<td>-0.06922</td>
<td>0.00844</td>
<td>0.13645</td>
<td>0.04394</td>
</tr>
<tr>
<td></td>
<td>ALL</td>
<td>-0.07346</td>
<td>0.00620</td>
<td>0.13847</td>
<td>0.04608</td>
</tr>
<tr>
<td>ALL</td>
<td></td>
<td>-0.05867</td>
<td>0.00023</td>
<td>0.11860</td>
<td>0.04858</td>
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</table>

Low-Relaxation Strands

<table>
<thead>
<tr>
<th>Size</th>
<th>Manufactures</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/16 in.</td>
<td>L</td>
<td>-0.00412</td>
<td>0.00142</td>
<td>0.02203</td>
<td>0.01605</td>
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<tr>
<td>1/2 in.</td>
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<td>0.01399</td>
<td>0.04435</td>
<td>0.00923</td>
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<tr>
<td>ALL</td>
<td></td>
<td>-0.01403</td>
<td>0.00609</td>
<td>0.03245</td>
<td>0.01395</td>
</tr>
</tbody>
</table>
TABLE 2 COEFFICIENTS FOR CONCRETE SURFACES

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Upper Bound</th>
<th>Lower Bound</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Strain C₁*</td>
<td>0.02500</td>
<td>0.02105</td>
<td>0.02299</td>
</tr>
<tr>
<td>D₁</td>
<td>-0.00668</td>
<td>-0.00066</td>
<td>-0.00289</td>
</tr>
<tr>
<td>Shrinkage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D₂</td>
<td>0.02454</td>
<td>0.01500</td>
<td>0.02031</td>
</tr>
<tr>
<td>E₁</td>
<td>-0.01280</td>
<td>-0.00664</td>
<td>-0.01592</td>
</tr>
<tr>
<td>E₂</td>
<td>0.00675</td>
<td>-0.00331</td>
<td>0.00649</td>
</tr>
<tr>
<td>Creep</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E₃</td>
<td>-0.00600</td>
<td>-0.00371</td>
<td>0.00256</td>
</tr>
<tr>
<td>E₄</td>
<td>0.01609</td>
<td>0.01409</td>
<td>0.01153</td>
</tr>
</tbody>
</table>

*Note: C₁ = 100/E where E is modulus of elasticity for concrete, in ksi
10. FIGURES
Fig. 1 Sign Convention for Applied Loads
Fig. 2 Initial Relaxation Loss
Average Steel
\( k_1 = 3 \) days

Upper Bound

Lower Bound

Shrinkage Loss

Fig. 3 SRL Part of Final Prestress Loss

65
Fig. 4 ECR Part of Final Prestress Loss - Upper Bound
Fig. 5  ECR Part of Final Prestress Loss – Lower Bound
Fig. 6 Typical Steel Stress Variation Due to Frictional Effects
Fig. 7 Steel Stress After Anchorage Seating - Partial Penetration

Fig. 8 Steel Stress After Anchorage Seating - Full Penetration
Based on Eq. (4-2)

Fig. 9 Steel Stress vs. Time Variation
Elastic Effect = $C_1 f_C$

Discontinuity at $t_{s1} + 1$

No creep discontinuity

Eq. (2-2)

Fig. 10 Concrete Strain vs. Time Variation
Fig. 11 CR Component of Prestress Loss - Lower Bound

\[ CR/f_{pu} = 0.6, 0.7, 0.8 \]

\[ CR = 1.2 \cdot n_i f_c \]
Fig. 12 CR Component of Prestress Loss - Upper Bound
$a_{si}$ being post-tensioned

Section Resisting Post-Tensioning

Equivalent Force System

$P = a_{si} f_{si}$

$M = -Px_i$

Fig. 13 Equivalent System for Post-Tensioning Stage $i$
General Procedure

First Stage Steel

AASHTO

First Stage Steel

Second Stage Steel

Simplified Procedure

Deck cast at $t_c = 180$

$ts1 = 1$

$ts2 = 45$

Fig. 14 Prestress Losses vs. Time, Idealized Example
11. REFERENCES

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8. Huang, Ti
   PRESTRESS LOSSES IN IN-SERVICE BRIDGE BEAMS AND REFINEMENT OF
   PREDICTION METHOD, Fritz Engineering Laboratory Report No. 382.5,
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9. American Association of State Highway Officials
   STANDARD SPECIFICATIONS FOR HIGHWAY BRIDGES, Interim Specifications,
   1975.
The notations used in this report were defined upon their first appearance in the text. Those of a general importance are assembled in this Appendix for easy reference. Several notations are used only once and are not included in the following listing. An effort has been made to make the units of the symbols consistent. Unless specifically indicated otherwise, all quantities are expressed in consistent kip-inch-day units.

\[ A_{si} = \text{Area of an individual steel element} \]
\[ A, A_c = \text{Area of net concrete section} \]
\[ A_g = \text{Area of the gross cross-section} \]
\[ A_{ps} = \text{Total area of all prestressing steel elements} \]
\[ ACF = \text{Prestress loss due to friction and anchorage seating} \]
\[ BLL = \text{Basic long-term loss, prestress loss due to shrinkage, creep and relaxation} \]
\[ C_{es} = \text{Coefficient for estimation of elastic loss, see Eq. 507} \]
\[ C_s = \text{Coefficient for estimation of shrinkage correction, see Eq. 5-8} \]
\[ CR = \text{Prestress loss due to creep of concrete} \]
\[ CRA = \text{Correction to prestress loss for multistage post-tensioning} \]
\[ e = \text{Eccentricity of prestress, referring to the net cross-section} \]
\( e_g \) = Eccentricity of prestress, referring to the gross cross-section

\( E_s \) = Modulus of elasticity of post-tensioning steel

\( E_L, E_S \) = Prestress loss due to elastic shortening

\( f_c \) = Fiber stress in concrete

\( f_{cg} \) = Concrete fiber stress, at level of steel, caused by member's own weight

\( f_{cl} \) = Concrete fiber stress, at level of steel, caused by the applied long term loads other than the member's own weight

\( f_{cs} \) = Concrete fiber stress at c.g.s. at an arbitrary time

\( f_p \) = Prestress in steel

\( f_{pu} \) = Specified ultimate tensile strength of prestressing steel

\( f_s \) = Stress in prestressing steel

\( f_{sd} \) = Increment of concrete stress, due to application of load at time \( t_c = t_{c5} \)

\( f_{sdi} \) = Increment of concrete stress, at level of steel, due to the post-tensioning of the \( i \)th stage

\( f_{si} \) = Initial steel stress immediately upon anchorage

\( f_{sj} \) = Jacking steel stress at the end

\( f_{sl} \) = Steel stress caused by applied loads, at end of service life

\( f_x \) = Steel stress after anchorage seating, at a point \( x \) distance from the jacking end

-78-
$g_1$, $g_2$ = Parameters to define concrete stress distribution in member section, see Eq. 2-7

$I, I_g$ = Moment of inertia of gross cross-section

$IL$ = Initial total prestress loss, immediately after transfer

$k$ = Combined friction coefficient, defined in Section 3.2

$k_1$ = Time interval from tensioning of steel to transfer

$k_2$ = Initial tensioning strain in steel, in $10^{-2}$

$k_{4i}$ = Strain compatibility constant for the $i^{th}$ stage prestressing steel, defined by Eq. 4-6, in $10^{-2}$

$K$ = Wobble coefficient of post-tensioning system, in ft.$^{-1}$

$L_a$ = Length over which the anchorage seating loss is distributed, in ft.

$L_e$ = "Effective" member length, maximum length over which the anchorage seating loss may be distributed, in ft.

$M$ = Bending moment on section caused by applied load

$n_i$ = Modular ratio at transfer time

$P$ = Axial load on section caused by applied load

$PL$ = Total prestress loss $t_c$ days after transfer

$REL$ = Prestress loss due to relaxation

$REL_1$ = Relaxation loss occurring before transfer

$S$ = Correction to prestress loss, accounting for the shrinkage occurring before post-tensioning
$S_s$ = Strain in concrete, in $10^{-2}$

$S_s$ = Steel strain, in $10^{-2}$

SRL = One part of the final prestress loss, independent of section geometry

t$_c$ = Time from transfer

t$_{c5}$ = Age of concrete when long term load is applied to prestressed concrete member

t$_s$ = Time from initial tensioning of steel

t$_{s1}$ = Age of concrete when first stage of post-tensioning steel is stretched

t$_{si}$ = Age of concrete when the $i^{th}$ stage post-tensioning takes place

$(t_s)_i$ = Steel "age" after post-tensioning, for the $i^{th}$ stage steel

TL = Total prestress loss at end of service life

x = Distance of elementary area from the centroid axis of gross cross-section

= Distance of a given section from the jacking end in ft., see Section 3.1

$\alpha$ = Total angle change in the steel profile, from the jacking end to a point $x$ distance away, in radians

$\alpha_x$ = Total angle change over a distance $\lambda$, in which the curvature of prestressing steel is constant, in radians
$\beta = \frac{1}{A_{ps} \left( \frac{1}{A} + \frac{e^2}{I} \right)}$, (Eq. 5-6)

(\beta \text{ refers to gross section properties in Chapter 2})

$\gamma = \text{Magnification factor for } n_1 \text{ to reflect effects of shrinkage, creep and relaxation, dimensionless}$

$\Delta_a = \text{Anchorage seating distance, in inches}$

$\mu = \text{Frictional coefficient between prestressing tendon and its conduit}$
APPENDIX B

FLOW DIAGRAM FOR PROGRAM BEAM

Program BEAM

Input Section Properties (Read Card 1)

Initialize Parameters
Subroutine INITI

Set Stage Counter

Input Next Event
(Read Card 2 or 4)

Card 2
Post-Tensioning

Analyze for
Intermediate Times
Subroutine INTERM

Analyze for
Post-Tensioning
Subroutine POST

Card 2 or 4

Card 4
No More Tensioning

Analyze for
Remaining Time
Subroutine LASTS

END

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Subroutine INITI

Define the Concrete Characteristic Coefficients

Calculate Moment Due to Member's Own Weight

Set Initial Values for Control Variables

Return
Subroutine INTERM

Analyze Section for Specified Time
Subroutine PREDI

Output Results
Subroutine PROUT

Return
Subroutine POST

Input Additional Data
Read Card 3

Analyze Before Post-Tensioning
Subroutines PREDI, PROUT

Calculate Post-Tensioning Force
Subroutine ANCFR

Calculate Equivalent Load System
Estimate $h_{1i}, h_{2i}$

Repeat 5 Times

Analyze After Post-Tensioning
Subroutine PREDI

Calculate $h_{1i}, h_{2i}$

Calculate $k_{4i}$

Calculate Steel Stresses and Prestresses

Output Results
Subroutine PROUT

Return
Subroutine
LASTS

Input Data on Deck Casting (Read Card 5)

Analyze for Times Before Deck Casting
Subroutine PREDI, PROUT

Calculate Loading Due to Deck
Estimate $h_1$, $h_2$

Repeat 5 times.

Analyze After Deck Casting
Subroutine PREDI

Revise $h_1$, $h_2$

Calculate Stresses After Deck Casting

Output Results
Subroutine PROUT

Analyze for remaining Life of Member
Subroutines PREDI, PROUT

Return
Subroutine PREDI

1. Initialize Values for U and V Coefficients

2. For Each Stage of Post-Tensioning Steel, \( a_{si} \)
   - Select \( P_1, P_2 \) and \( P_3 \) for \( a_{si} \)
   - Get \( Q_1 \) and \( Q_2 \) for the Location of \( a_{si} \) (CONCS)
   - Calculate \( R_{1i}, R_{2i}, R_{3i} \)
   - Increment U and V Coefficients

3. Solve Simultaneous Equations For \( g_1 \) and \( g_2 \)

4. Return
Subroutine PROUT

Write out Concrete Stress at the Top of the Beam

Write out Concrete and Steel Stresses, and Steel Prestress for Steel Elements of each Stage

Write out Concrete Stress at the Bottom of the Beam

Calculate and Output the Total Steel Force and its Eccentricity

Return
Subroutine ANCFR

Calculate Anchorage Length

Compare Anchorage Length with Effective Length, Also compare Location of Critical Sections

Calculate Steel Stress at Critical Section After Anchorage and Friction Losses

Return
Subroutine CONCS

\[
Q_1 = D_1 + D_2 \log (t_c + 1)
\]
\[
Q_2 = 0
\]

If \( t_c > t_{s1} + 1 \)
Add Creep Terms to \( Q_1, Q_2 \)

If \( t_c > t_{s2} \)
Add \( f_{sdi} \) terms to \( Q_1 \)

Return
Input Data Required for Program BEAM

Card No. 1: Properties of the Concrete Member: Area, total depth, centroidal location and moment of inertia of the net section; span length; and characteristics of concrete material.

Card No. 2: Partial data on post-tensioning, to be followed by card No. 3. These may be any number of sets of cards 2 and 3 as needed.

Concrete age at post-tensioning, size, manufacturer and characteristics of steel, number of strands tensioned, location of steel and initial jacking stress.

Card No. 3: Additional data on post-tensioning, supplementing these on card No. 2.

Jacking from one or two ends, friction and wobble coefficients, profile of steel tendon, and anchorage seating distance.

Card No. 4: A blank card signaling the completion of all post-tensioning stages.

Card No. 5: Data on deck slab: Concrete age when deck is cast, compressive strength of concrete in slab and in beam, size of deck slab and any additional moment to be carried by the precast beam or the composite sections.
APPENDIX C
DERIVATION OF EQUATIONS

The computerized general procedure is based on the material stress-strain-time relationships, time and strain compatibility relationships, equilibrium conditions and the linear distribution of concrete stresses, as listed below.

The steel material relationship

\[
f_s = f_{pu} \left[ A_1 + A_2 S_s + A_3 S_s^2 - [B_1 + B_2 \log (t_s + 1)] S_s^2 \right]
\]

(2-1)

The concrete material relationship

\[
S_c = -C_1 f_c + [D_1 + D_2 \log (t_c + 1)] + [E_1 + E_2 \log (t_c - t_{s1} + 1)]
- E_3 f_c - E_4 (f_c - \sum f_{sdi}) \log (t_c - t_{s1} + 1)
- E_4 \sum [f_{sdi} \log (t_c - t_{s1} + 1)]
\]

(4-3b)

The time compatibility linkage for the steel element post-tensioned at the \(i^{th}\) stage.

\[
(t_{s'i}) = t_c - t_{s1}
\]

(4-1)

The strain compatibility linkage for the \(i^{th}\) stage steel

\[
S_{s1} + S_{ci} = k_{4i}
\]

(4-6)
The equilibrium conditions

\[ \int f_c dA_c + \sum (f_{si} a_{si}) = -P \]  \hspace{2cm} (C-1)

\[ \int f_c x dA_c + \sum (f_{si} a_{si} x_i) = M \]  \hspace{2cm} (C-2)

The linear distribution of concrete stresses

\[ f_c = g_1 + g_2 x \]  \hspace{2cm} (2-7)

All notations in these equations have been defined when they appear in the main body of this report, and will not be repeated here. It is emphasized that each steel element, \( a_{si} \), has its own characteristic equation (2-1), and its own time parameter \( (t_s) \) and that \( f_s, S_s, f_c, \) and \( f_{sdi} \) are all functions of \( x \). It is also noted that the \( x \) distances refer to the centroidal axis of the net concrete section, the integrations are over the net concrete section, and that at any given time \( t_c \), the summations in Eqs. 4-3b, C-1 and C-2 cover all steel elements which have been post-tensioned, i.e., \( t_{si} < t_c \). Figure 1 shows the positive directions of \( P, M \) and \( x \).

For the sake of brevity in the following derivations, Eqs. 2-1 and 4-3b are rewritten in shorter forms:

\[ f_s = P_1 + P_2 S_s + P_3 S_s^2 \]  \hspace{2cm} (C-3)

\[ S_c = Q_1 - Q_2 f_c \]  \hspace{2cm} (C-4)
where \( P_1 = A_1 f_{pu} \)

\[
P_2 = [A_2 - B_1 - B_2 \log (t_s + 1)] f_{pu}
\]

\[
P_3 = [A_3 - B_3 - B_4 \log (t_s + 1)] f_{pu}
\]

\[
Q_1 = [D_1 + D_2 \log (t_c + 1)] + [E_1 + E_2 \log (t_c + 1)]
- E_4 \sum f_{sdi} [\log (t_c - t_{si} + 1) - \log (t_c - t_{s1} + 1)]
\]

\[
Q_2 = C_1 + E_3 + E_4 \log (t_c - t_{s1} + 1)
\]

Substitute (C-4) into (4-6)

\[
S_{si} = k_{4i} - Q_1 + Q_2 f_{ci}
\]  
(C-5)

Substitute (C-5) and (4-1) into (C-3) for the steel element of the \( i^{th} \) stage,

\[
f_{si} = P_{li} + P_{2i} (k_{4i} - Q_1 + Q_2 f_{ci}) + P_{3i} (k_{4i} - Q_1 + Q_2 f_{ci})^2
= R_{li} + R_{2i} f_{ci} + R_{3i} f_{ci}^2
\]

(C-6)

where

\[
R_{li} = P_{li} + P_{2i} (k_{4i} - Q_1) + P_{3i} (k_{4i} - Q_1)^2
\]

\[
R_{2i} = Q_2 [P_{2i} + 2 P_{3i} (k_{4i} - Q_1)]
\]

\[
R_{3i} = P_{3i} Q_2^2
\]

Substituting (2-7) into the equilibrium equations (C-1) and (C-2), and performing the integrations,

\[
A_{ci} g_i + \sum (f_{si} a_{si}) = -p
\]  
(C-1a)
\[ I_n g_2 + \sum (f_{si} a_{si} x_i) = M \] (C-2a)

Substitute (C-6) and expand, two quadratic equations in \( g_1 \) and \( g_2 \) are obtained.

\[ U_1 + U_2 g_1 + U_3 g_2 + U_4 g_1^2 + U_5 g_1 g_2 + U_6 g_2^2 = 0 \] (4-7)

\[ V_1 + V_2 g_1 + V_3 g_2 + V_4 g_1^2 + V_5 g_1 g_2 + V_6 g_2^2 = 0 \] (4-8)

in which

\[
\begin{align*}
U_1 &= P + \sum (R_{li} a_{si}) \\
U_2 &= A_c + \sum (R_{2i} a_{si}) \\
U_3 &= \sum (R_{3i} a_{si} x_i) \\
U_4 &= \sum (R_{4i} a_{si} x_i) \\
U_5 &= 2 \sum (R_{3i} a_{si} x_i) \\
U_6 &= \sum (R_{3i} a_{si} x_i^2) \\
V_1 &= -M + \sum (R_{li} a_{si} x_i) \\
V_2 &= \sum (R_{2i} a_{si} x_i) = U_3 \\
V_3 &= I_n + \sum (R_{2i} a_{si} x_i^2) \\
V_4 &= \sum (R_{3i} a_{si} x_i) = \frac{1}{2} U_5 \\
V_5 &= 2 \sum (R_{3i} a_{si} x_i^2) = 2U_6 \\
V_6 &= \sum (R_{3i} a_{si} x_i^3)
\end{align*}
\] (C-7)

It should be pointed out once again that in the above equations, all summations are over the steel elements which have already been tensioned, i.e., \( t_{si} < t_c \).

Each stage of post-tensioning, or the application of external load, causes changes of concrete stresses, and introduces new \( f_{stdi} \) terms into the concrete characteristic equation (4-3b). The evaluation of these stress increments is done by an iterative procedure, starting from approximate values based on linearly elasticity. It is noted that the increments
in question are the differences of concrete stresses before and after the "loading."

\[ f_{sdi} = f_{ca} - f_{cb} \]  \hspace{1cm} (C-8)

where the subscripts a and b stand for "after" and "before," respectively.

From Eq. 2-7

\[ f_{ca} = g_{1a} + g_{2a}x \]
\[ f_{cb} = g_{1b} + g_{2b}x \]

Therefore

\[ f_{sdi} = (g_{1a} - g_{1b}) + (g_{2a} - g_{2b})x \]
\[ = h_{1i} + h_{2i}x \]  \hspace{1cm} (C-9)

where

\[ h_{1i} = g_{1a} - g_{1b} \]
\[ h_{2i} = g_{2a} - g_{2b} \] \hspace{1cm} (C-10)

At the time of the \(i\)th stage post-tensioning (or loading), \(t_c = t_{si}\), the stress condition just before is directly determined by solving the simultaneous equations (4-7) and (4-8). For the "after" conditions, the initial \(f_{sdi}\) values are obtained by linear elastic approximations

\[ f_{sdi} \approx (a_{si} f_{si}) \left( -\frac{1}{A} - \frac{x_i}{I} x \right) \] \hspace{1cm} (C-11)

or, more generally,

\[ f_{sdi} \approx -\frac{\Delta P}{A} + \frac{\Delta M}{I} x \] \hspace{1cm} (4-5)

Terms corresponding to this approximate \(f_{sdi}\) are added to the concrete characteristic equation. Similarly, the external load terms in the
equilibrium equations, P and M, are incremented by the newly applied loads at this time. The "previous" section, is then analyzed again using the general solution procedure, yielding the first approximate solutions for \( g_{1a} \) and \( g_{2a} \). It should be pointed out that for this solution, the stress increment \( f_{sdi} \) for the stage under question is included in the concrete characteristic equation, but the steel area for the current stage is not included in the equilibrium equations.

The approximate solution for \( g_{1a} \) and \( g_{2a} \), as described above, enables the calculation of improved estimate \( f_{sdi} \) by using equations (C-10) and (C-9). A new solution for the "after" stress condition can now be made, based on the new value of \( f_{sdi} \). This iterative process is continued until no significant changes in \( h_{1i} \) and \( h_{2i} \) are made. The computer program subroutine provides for five iterations. Usually, only two or three cycles are needed to reach stabilized solutions.