Power spectrum of total solar radiation on a horizontal surface.

Lynn A. Pohl

Follow this and additional works at: http://preserve.lehigh.edu/etd

Part of the Mechanical Engineering Commons

Recommended Citation
POWER SPECTRUM OF TOTAL SOLAR RADIATION
ON A HORIZONTAL SURFACE

by
Lynn A. Pohl

A Thesis
Presented to the Graduate Committee
of Lehigh University
in Candidacy for the Degree of
Master of Science
in
Mechanical Engineering

Lehigh University
1979
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

May 7, 1979

(date)

Professor in Charge

Chairman of Department
ACKNOWLEDGMENTS

This work was performed as a thesis under Professor R. G. Sarubbi whose guidance and encouragement were invaluable. Special thanks to E. F. Witt and H. J. Luer, my supervisors at Bell Laboratories during this effort, for their patience and support. My deepest appreciation and thanks must go to my wife Nancy and daughter Kristen for their patience and understanding. A great deal of this work was prepared out-of-hours in the time normally allocated to family life. Their sacrifice is tantamount to coauthorship.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>4</td>
</tr>
<tr>
<td>Introduction to Power Spectrum Estimation</td>
<td></td>
</tr>
<tr>
<td>2. Random Processes – Classification</td>
<td>7</td>
</tr>
<tr>
<td>3. Autocovariance Functions</td>
<td>10</td>
</tr>
<tr>
<td>4. Power Spectra</td>
<td>13</td>
</tr>
<tr>
<td>5. Data/Lag Windows – Tapering the Time Series</td>
<td>19</td>
</tr>
<tr>
<td>6. FFT (Fast Fourier Transform) Computing Methods</td>
<td>26</td>
</tr>
<tr>
<td>7. Autocovariance via the FFT</td>
<td>28</td>
</tr>
<tr>
<td>8. Inverse FFT</td>
<td>31</td>
</tr>
<tr>
<td>9. FFT of Two Real Valued Records at One Time</td>
<td>33</td>
</tr>
<tr>
<td>Equipment and Installation</td>
<td></td>
</tr>
<tr>
<td>10. Sensors, Recording Equipment, Station Location and Elevation</td>
<td>35</td>
</tr>
<tr>
<td>11. Types of Recorded Data</td>
<td>36</td>
</tr>
<tr>
<td>Preparation of Data for Analysis</td>
<td></td>
</tr>
<tr>
<td>12. Trend Removal</td>
<td>38</td>
</tr>
<tr>
<td>13. Direct Solar Radiation Spectrum – A Discarded Candidate</td>
<td>41</td>
</tr>
<tr>
<td>14. Total Solar Radiation Spectrum</td>
<td>46</td>
</tr>
<tr>
<td>Inherent Problems in Digital Spectrum Estimation</td>
<td></td>
</tr>
<tr>
<td>15. Aliasing and Averaging of Observations</td>
<td>48</td>
</tr>
<tr>
<td>16. Missing Observations</td>
<td>53</td>
</tr>
<tr>
<td>17. Variability - Confidence Intervals</td>
<td>58</td>
</tr>
</tbody>
</table>
ABSTRACT

POWER SPECTRUM OF TOTAL SOLAR RADIATION ON A HORIZONTAL SURFACE

by

Lynn A. Pohl

Energy systems which use the sun as a source of free fuel have gained widespread popularity over the last decade. With today's exorbitant price for fuel, diminishing fuel resources, and inherent dangers associated with nuclear energy systems, it is no wonder this is so. The aim of this report is to contribute to the technology which will permit solar energy to assume a more significant role in supplying the energy needs of the world.

A power spectrum analysis of total solar radiation on a horizontal surface is presented. The spectrum considered is not a frequency or wavelength decomposition of solar radiation but rather a temporal decomposition by period of the total solar radiation (at all wavelengths which reach the earth) on a horizontal surface. The spectrum is in terms of percent of possible clear day total solar radiation based on observations at 15 minute intervals of 15 minute averages over the period of September 1975, through June 1978, at Lehigh University,
Bethlehem, Pennsylvania. No attempt has been made to test the generality of the spectrum for other locations. The theory and procedures used in this investigation are, of course, independent of location. It is thought that the resultant spectrum may be of general validity and could be applicable to many localities with similar latitude and where the atmospheric contamination is not greatly different from that of Bethlehem. The techniques used in this report can be applied to data obtained at other sites which would give some information on the degree to which spectra are location dependent.

There appear to be statistically significant peaks in the spectrum at several different frequencies indicating that oscillations with these frequencies are more probable than those with neighboring frequencies. The peaks are centered at periods of approximately 18, 7, and 2-1/2 days, and 25, 12, 8, 6, 4-1/2, and 3-1/2 hours.

The significance of this spectrum is that it can be used to extend solar energy technology in other areas. Using standard techniques, it is shown that storage capacity for solar energy systems can be determined with the aid of the spectrum. The spectrum can also be used to determine an optimum weighting function for extrapolation and prediction since it contains probabilistic information about the occurrence of oscillations at differing periods. Finally, for those interested in the physical mechanisms
producing variations in solar radiation, the spectrum can provide valuable insight.
1. Introduction

Panofsky et al. [1,2] compares spectrum analysis with ordinary Fourier analysis. He mentions that in Fourier analysis, a record (time series) is considered to be made up of a finite number of cycles with discrete periods. In spectrum analysis, there are an infinite number of small oscillations with a continuous distribution of periods. The area under the power spectrum curve is a measure of the contribution of these oscillations at various periods to the variance.

In terms of a resultant spectrum curve, given a peak in the spectrum (high point) and gap (low point) at respective periods X and Y, oscillations with a period of X are more probable than those with a period of Y. Note that this has nothing to do with the intensity of solar radiation but merely describes the relative probabilities of different intensities at particular periods.

Power spectrum analysis of meteorological variables is characterized by diurnal and seasonal variations, and a nondeterministic or random variation. The characterization of the nondeterministic variation through power spectrum estimation is the aim of this report. In the process, a considerable amount of effort was expended in removing the trend.

In the mid-fifties Griffith et al. [2] and Van der Hoven [3] presented spectra for vertical and
horizontal wind speed and daily mean temperatures. Their methodology differs from that used in this report partially because the Fast Fourier Transform technique which is used in this report was not yet developed. They were forced to piece together many spectra to avoid excessive computation.

In the late sixties Cumali [4] presented spectra for wind speed, dry bulb temperature, dew point temperature, atmospheric pressure, total solar radiation intensity, and total cloud cover. All of these were based on hourly observations over a ten year span. Cumali emphasizes the first four items and states he is somewhat skeptical of the solar data since it did not compare favorably with values obtained from the standard formula used for solar radiation intensity given in the 1967 ASHRAE GUIDE and DATA BOOK.

Up to and including the late sixties, the amount of solar radiation data gathered in the continental United States was very small. To the author's knowledge, there has not been any attempt to generate a total solar radiation spectrum other than that mentioned above.

The structure of this report is similar to that of Blackman and Tukey [5] which the author found to be extremely useful. For example, details and derivations associated with Section "5" can be found in Appendix "A5."

An introduction to those aspects of power spectrum estimation pertinent to this report is given in
Sections 2-9. These sections are presented for background purposes and more detailed material can be found in the time series analysis texts listed in the references and bibliography. Sections 10 and 11 describe the equipment and installation and may be of particular interest to those contemplating a similar undertaking, or to those who wish to determine if their locality is similar enough to warrant the use of these results. Sections 12-14 indicate the rationale and techniques for pre-conditioning the data before analysis. Alternative methods for presenting the spectrum that were investigated, but later discarded, are also discussed. In addition, curve fits are presented for: total clear day solar radiation on a horizontal surface, the solar equation of time, the maximum value of solar radiation on a horizontal surface on a clear day, and time to sunrise/sunset. All of these equations are for latitude N 40° 36', and should be useful for future efforts in this area.

Sections 15-17 describe the inherent problems in digital spectrum estimation and therefore highlight the limitations embodied in this analysis. Particular attention is given to methods to neutralize or reduce their influence. Section 18 presents a step-by-step recipe used to compute the spectrum. Results, potential use, and recommendations follow in Sections 19-21.
2. Random Processes - Classifications

The subject of random processes is treated extensively in most statistical texts. The aim of this section is to briefly mention those classifications of random processes relevant to this report, and how they form the underlying assumptions in the overall analysis.

Random processes are typically divided into three classes:

1. Stationary
2. Stationary and Ergodic

A random process can be thought of as an (theoretically) infinite collection of sample functions: $X_1(t), X_2(t), \ldots, X_n(t)$, which together make up the random process $X_n(t)$. Stationarity implies averaging over large numbers of sample functions (ensemble averaging) at time $t_1$, gives the same answers as at time $t_2$. Furthermore, all statistical quantities such as mean, variance, and standard deviation are also independent of the time at which they are found.

Most power spectrum analyses implicitly assume that the data is stationary. The theory underlying spectrum analysis is well developed for stationary processes. Nonstationary processes are generally defined
to include all random processes which are not stationary. Adequate spectrum analysis theory for non-stationary processes does not exist yet. Quite simply then, the stationary hypothesis is often invoked for approximation and simplicity.

A general guideline offered by Bendat and Piersol [6] is; "if the basic physical factors which generate the phenomenon are time invariant, then stationarity of the resulting data generally can be accepted without further study." In the case of solar radiation, once the diurnal and seasonal variation has been removed and assuming that atmospheric contamination is statistically time invariant over the time span of interest, then stationarity is a reasonably safe assumption.

A stationary process is called an ergodic process if averages taken along any single sample function are the same as averages over large numbers of sample functions. In practical terms, each sample function is then completely representative of the ensemble. In terms of solar radiation, modified as previously mentioned, ergodicity is also a reasonably safe assumption.

An important type of random process is the Gaussian random process. This means that for any fixed time $t_n$, the joint probability distribution of

$$X_1(t_n), X_2(t_n), \ldots, X_\infty(t_n)$$
follows a multidimensional normal or Gaussian distribution. In the case of an ergodic Gaussian random process, the probability distribution of a single function \( X(t_n) \) will be normally distributed. Here again, this is a reasonably safe assumption for trend free data.

The ergodic hypothesis justifies a single function approach to spectrum analysis though in practice it is impossible to have an infinitely long record as required by this hypothesis. As a result, calculation of a spectrum using a record of finite length introduces variability in spectrum estimates. The effect on variability is discussed by Blackman and Tukey [5] and this report follows their results for estimating the variability. Under the Gaussian hypothesis the estimates are exact and are usually good approximations otherwise [5].

To summarize, the two basic assumptions which are assumed throughout this report are the solar radiation time series after removal of the diurnal and seasonal trend is stationary-ergodic and Gaussian.
3. Autocovariance Functions

In all practical situations, time series are limited to a finite length. They can, however, occur in two forms—continuous and discrete. Spectrum analysis of continuous and discrete series are usually done by analog and digital computations respectively. Discrete time series can be thought of as the result of sampling continuous series at specific instants of time. In this report we will be concerned with the finite-discrete-equally spaced interval time series. The spectrum estimation problems of nonequally spaced interval time series are discussed by Brillinger [7].

This section will look at the infinite continuous series first and then indicate the modifications necessary to obtain results for the finite continuous and then the finite-discrete-equally spaced series.

The autocovariance function for a stationary random process \( X_n(t) \), with mean removed, is defined as the average (over the ensemble) of the product \( X_n(t)X_n(t+\tau) \), or

\[
E[X_n(t),X_n(t+\tau)],
\]

and describes the degree to which data at different times is correlated. If the process is also ergodic [dropping the subscript \( n \)],

\[
E[X(t),X(t+\tau)]
\]
will be independent of time $t$ and will only depend on the time separation $\tau$.

Let

$$E[X(t),X(t+\tau)] = R(\tau), \quad (3.1)$$

where $R(\tau)$ will hereinafter be referred to as the auto-covariance function of $X(t)$. Since $R(\tau)$ depends only on the separation time $\tau$,

$$R(\tau) = E[X(t),X(t+\tau)] = E[X(t),X(t-\tau)] = R(-\tau), \quad (3.2)$$

so that $R(\tau)$ is an even function of $\tau$ and

$$R(0) \geq |R(\tau)| \text{ for all } (\tau). \quad (3.3)$$

The mean ($\mu$) and mean square ($\psi^2$) of $X(t)$ are given by

$$\mu = \sqrt{R(\infty)} \quad (3.4)$$

$$\psi^2 = R(0). \quad (3.5)$$

The variance ($\sigma^2$) is equal to the mean square value ($\psi^2$) minus the square of the mean ($\mu^2$), or

$$\sigma^2 = E[X(t)-\mu]^2 = \psi^2 - \mu^2 = R(0) - R(\infty). \quad (3.6)$$

Under the ergodic hypothesis, Equation 3.1 can be expressed as the sample average

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T X(t)X(t+\tau)dt \quad (3.7)$$
where $T$ is the total observation time and it is assumed that the continuous series begins at time 0 for convenience. If the series is only available for time $T$, then an approximation for Equation 3.7 is

$$
\hat{R}(\tau) = \frac{1}{T-\tau} \int_{0}^{T-\tau} X(t)X(t+\tau)dt
$$

(3.8)

for $0 \leq \tau < T$. Clearly, we can make no estimates for $\tau > T$.

The discrete version of Equation 3.8 where $X(t)$ is sampled at $N$ equally spaced intervals, $h$ apart is,

$$
\hat{R}'(rh) = \frac{1}{N-r} \sum_{n=1}^{N-r} X_n X_{n+r} \quad r = 0,1,2,\ldots,N,
$$

(3.9)

where $r$ is the lag number, $\tau = rh$, and $N$ is the maximum lag.

Many texts refer to $R(\tau)$ as the autocorrelation function. In classical statistics, an autocorrelation function is defined as the normalized ratio

$$
\frac{R(\tau)}{R(0)}
$$
4. Power Spectra

Power spectrum results can appear in two forms: one-sided which are defined for frequencies \((0, \infty)\), and two-sided for frequencies \((-\infty, \infty)\). The distinction is important. Two-sided forms are generally used in the development of the theory while one-sided forms are used in practice to avoid reference to negative frequencies. This can be done because the two-sided spectrum is symmetrical about zero frequency. The two-sided spectrum \([S(f)]\) is defined in terms of the autocovariance function as

\[
S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f \tau} d\tau, \quad -\infty < f < \infty \quad (4.1)
\]

with the condition that

\[
\int_{-\infty}^{\infty} |R(\tau)| d\tau < \infty.
\]

Due to the symmetry properties of \(R(\tau)\) (i.e., \(R(\tau) = R(-\tau)\)),

\[
S(f) = S(-f). \quad (4.2)
\]

The one-sided spectrum \([G(f)]\) is then

\[
G(f) = 2S(f), \quad 0 \leq f < \infty
\]

\[
G(f) = 0 \quad \text{otherwise.}
\]

13
Equation 4.1 states that the power spectrum is the Fourier transform of the autocovariance function. This relation may be inverted to express the autocovariance function as the Fourier transform of the power spectrum, or

\[
R(\tau) = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft} df. \quad (4.4)
\]

To develop the power spectrum in terms of finite Fourier transforms, Equation 3.7 must be redefined over the interval \((-\infty, \infty)\) as

\[
R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} Z(t)Z(t+\tau)dt \quad (4.5)
\]

where

\[
Z(t) = X(t) \quad |t| < T \quad (4.6)
\]

\[
= 0 \quad |t| > T.
\]

Letting

\[
Y(f) = \int_{-\infty}^{\infty} Z(t)e^{-j2\pi ft} dt \quad (4.7)
\]

then,

\[
Z(t) = \int_{-\infty}^{\infty} Y(f)e^{j2\pi ft} df. \quad (4.8)
\]
Now,

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} Z(t) \int_{-\infty}^{\infty} Y(f) e^{j2\pi ft} df \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} Y(f) e^{j2\pi \tau} \int_{-\infty}^{\infty} Z(t) e^{j2\pi ft} dt \, df \quad (4.9)$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} Y(f) Y(-f) e^{j2\pi \tau} df,$$

and from Equation 4.4

$$\int_{-\infty}^{\infty} S(f) e^{j2\pi \tau} df = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |Y(f)|^2 e^{j2\pi \tau} df \quad (4.10)$$

therefore,

$$S(f) = \lim_{T \to \infty} \frac{1}{T} |Y(f)|^2 \quad (4.11)$$

and using Equations 4.3 and 4.6,

$$G(f) = 2 \lim_{T \to \infty} \frac{1}{T} \left| \int_{0}^{T} X(t) e^{-j2\pi ft} \, dt \right|^2 \quad (4.12)$$
Note that the above integral term is the finite Fourier transform of $X(t)$. An estimate of $G(f)$ is given by

$$
\hat{G}(f) = \frac{2}{T} \left| \int_0^T X(t)e^{-j2\pi ft} dt \right|^2.
$$

(4.13)

If $X(t)$ is sampled at $N$ equally spaced intervals, $h$ apart, the discrete form of the Fourier transform of $X(t)$ is

$$
\sum_{n=0}^{N-1} x_n e^{-j2\pi f nh} \quad n = 0, 1, 2, \ldots, N-1
$$

(4.14)

corresponding to harmonics of frequency

$$
f_K = \frac{K}{T} = \frac{K}{Nh} \quad K = 0, 1, 2, \ldots, N-1.
$$

(4.15)

Now, the estimate of the discrete power spectrum $\hat{G}'(f)$ becomes

$$
\hat{G}'(f_K) = \frac{2h}{N} \sum_{N=0}^{N-1} x_n e^{-j2\pi f_k nh} \quad 2
$$

(4.16)

The squared term represents the Fourier components, and as we shall see later [Section 6], a unique method has been developed to evaluate it. Note that Equation 4.16 only requires the original time series and avoids computing the autocovariance function.
Another method for calculating the spectrum known as the Blackman-Tukey or lagged product method computes the spectrum via the autocovariance function. It was popular before the discovery in 1965 of the Fast Fourier Transform (FFT) algorithm which is useful in decreasing computational time in digital computers. There are numerous papers comparing the methods, all of which reach the same general conclusion; there is very little difference between the two. Two papers which cover this very well were written by Rikiishi [8] and by Edge and Liu [9].

This report, due to special considerations for missing data [see Section 16] which require the use of a modified autocovariance function, uses a combination of both methods. The modified autocovariance function is derived in a roundabout way using the FFT method, and then the final spectrum estimates are obtained in a Blackman - Tukey type manner from the autocovariance function but by the direct FFT approach.

To derive the spectrum in terms of the autocovariance function, recall that

\[ G(f) = 2S(f) = 2 \int_{-\infty}^{\infty} R(\tau)e^{-j2\pi ft}d\tau \quad (4.17) \]

or since \( R(\tau) \) is an even function,
\[ G(f) = 2 \int_{-\infty}^{\infty} R(\tau) \cos(2\pi f \tau) d\tau \quad (4.18) \]

where the imaginary part of the exponential drops out due to the symmetry of \( R(\tau) \) and antisymmetry of the imaginary part of the exponential. The discrete form analogous to Equation 4.16 is

\[ \hat{G}'(f_K) = 2h \sum_{r=-N}^{N} \hat{R}'(rh) \cos 2\pi f_K rh \quad (4.19) \]

where \( N \) is the maximum lag, and \( \hat{R}'(rh) \) is the estimate of the autocovariance function at lag \( r \).
5. Data/Lag Windows - Tapering the Time Series

To decrease the effect of the smearing of spectrum estimates because of finite record length, a finite random time series is normally tapered at each end. Tapering is multiplying the time series by a "data window." The analogy in the Blackman and Tukey method is multiplying the autocovariance function by a "lag window." Tapering is equivalent to applying a convolution operation [see Appendix A5] to the Fourier transform of $X(t)$ or $R(\tau)$. Blackman and Tukey [5] state that although tapering the true autocovariance function results in poor autocovariance estimates, their transforms are very good estimates of smoothed values of the true spectrum.

Tapering is well described in the voluminous literature written about this technique. The intent here is to briefly describe those aspects pertinent to this report.

When we analyze data without first tapering it, we are in effect, using a boxcar or rectangular data/lag window. To see this, recall from Equation 4.17 that the true spectrum is

$$G(f) = 2 \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f \tau} d\tau.$$
If the record is of finite length $N$, the value of $R(\tau)$ is only defined for $|\tau|$ less than $|\tau_N|$. The resulting transform is only an estimate of the true spectrum. By defining,

$$U_{\tau N}(\tau) = \begin{cases} 0 & \tau < -\tau_N \\ 1 & -\tau_N \leq \tau \leq \tau_N \\ 0 & \tau > \tau_N \end{cases}$$  \hspace{1cm} (5.1)$$

and multiplying the true autocovariance function $R(\tau)$ by this function, this estimate is,

$$\hat{G}(f) = 2 \int_{-\infty}^{\infty} U_{\tau N}(\tau) R(\tau) e^{-j2\pi f \tau} d\tau$$  \hspace{1cm} (5.2)$$

Equation 5.2 above if written as the convolution of the transforms of $U_{\tau N}(\tau)$ and $R(\tau)$ shows that $\hat{G}(f)$ is a "smeared" estimate of the true spectrum, $G(f)$. The transform of $U_{\tau N}(\tau)$ is given by

$$\frac{\sin(2\pi f \tau_N)}{2\tau_N \frac{2\pi f \tau_N}{}}$$  \hspace{1cm} (5.3)$$

usually referred to as a "spectral window." Figure 5.1 shows the boxcar data/lag window and its associated spectral window. As $\tau_N$ gets larger, the peak at $f=0$ of the spectral window becomes more pronounced and in the limit behaves like a delta function.
The effect of the spectral window (induced by the data/lag window) is to multiply the true value of the spectrum by the window, centered at a particular frequency, and then adding to it contributions of neighboring frequencies. This produces undesirable side effects because of "leakage" through the large main lobe and an infinite number of smaller side lobes (see Figure 5.1). The window then transmits more power than desired. In addition, half of the side lobes are negative. The first as shown in Figure 5.1 is almost one-fifth that of the main lobe.
FIG. 5.1 BOXCAR DATA/LAG AND SPECTRAL WINDOWS
The Blackman-Tukey method may produce theoretically impossible negative spectrum estimates. The effect of the negative side lobes is to increase this possibility at points in the spectrum where the center frequency value is small by comparison with values at neighboring frequencies.

To help alleviate this problem a modification of the boxcar data/lag window is required to decrease the side lobes of the spectral window. Many windows have been proposed. They generally broaden the main lobe but decrease its height, and drastically decrease the size of the sidelobes. The reduction in leakage therefore results in a corresponding widening of the frequency bandwidth of analysis.

The window used in this report is one of several attributable to Parzen [10]. It takes the form

\[ U'_{\tau N}(\tau) = 1 - 6 \left( \frac{|\tau|}{\tau_N} \right)^2 \left( 1 - \frac{|\tau|}{\tau_N} \right) \quad \tau \leq \tau_N/2 \]

\[ = 2 \left( 1 - \frac{|\tau|}{\tau_N} \right)^3 \quad |\tau| > \tau_N/2. \]

(5.4)

Its Fourier transform is

\[ \frac{3}{4} \tau_N^4 \left\{ \frac{\sin(\pi \tau \tau_N/2)}{(\pi \tau \tau_N/2)^4} \right\}. \]

(5.5)
This lag window and corresponding spectral window are shown in Figure 5.2. Notice that the main lobe is broadened but reduced to three-eighths the height of the boxcar spectral window. However, the main feature of interest is that the Parzen spectral window is never negative. This reduces the possibility of computing erroneous negative spectrum estimates.

The discrete form of the Parzen window is

\[ U_{Nr} = 1 - 6 \left( \frac{r}{N} \right)^2 \left( 1 - \frac{r}{N} \right) \ 0 \leq r \leq N/2 \]

\[ = 2 \left( 1 - \frac{r}{N} \right)^3 \quad N/2 < r \leq N \]

\[ = 0 \quad \text{elsewhere}, \]

where \( N \) is the maximum lag and \( r = 0, 1, 2, \ldots, N \).

In this report, power spectrum estimates will be computed from the autocovariance functions by combining Equations 4.19 and 5.6 to give

\[ \hat{G}'(f_K) = 2h \sum_{r=-N}^{N} U_{Nr} \hat{R}'(rh)e^{-\frac{12\pi Kr}{N}}. \]  

(5.7)

The imaginary portion of the result can then be ignored.
Fig. 5.2 Parzen data/lag and spectral windows
6. FFT (Fast Fourier Transform) Computing Methods

The Fast Fourier Transform (FFT) method was first described by Cooley and Tukey [11] in 1965. Essentially it is an algorithm for computing Fourier coefficients, such as in Equations 4.14 and 5.7, that requires much less computation time than older methods. The detailed technique is covered well in the literature. The important point here is that the autocovariance function

\[ \hat{R}_n(rh) = \frac{1}{N-R} \sum_{n=1}^{N-r} X_n X_{n+r} \quad r = 0, 1, 2, \ldots, N \]

requires \( N^2 \) real multiply-add operations. Sande [12] has shown it is often faster to calculate autocovariance functions by the double use of the FFT rather than calculate a numerical convolution. This method requires the computation of spectrum estimates which can be inverse transformed [see Section 8] to give the autocovariance function. Bendat and Piersol [6] have shown that a conservative estimate of the speed ratio, if \( N = 2^p \) is

\[ \text{Speed Ratio} = \frac{N^2}{2(4Np)} = \frac{N}{8p} \quad (6.1) \]

If \( N = 8192 \), the FFT approach is approximately 79 times as fast. For Equations 4.14 and 5.7, the number of complex multiply-add operations required for the calculation of the
discrete Fourier transform coefficients with conventional methods is also $N^2$. However, with the FFT, this is reduced to $N \log_2 N$, or $pN$, if $N = 2^p$.

In this report, where we are dealing with fairly large time series, the FFT offers significant savings in computation time.
7. Autocovariance via the FFT

As mentioned in the previous section, autocovariance functions can often be computed by a roundabout method using the FFT rather than by the direct multiply add approach. Since the autocovariance function is just the Fourier transform of the power spectrum, the power spectrum [Equation 4.16] can be computed first, and then inverse transformed. Sande [12] shows that this approach does not yield the usual autocovariance function [Equation 3.9], but rather a circular autocovariance function defined as

\[
\hat{R}_c(rh) = \frac{N-r}{N} \hat{R}'(rh) + \frac{N-(N-r)}{N} \hat{R}'([N-r]h) \quad (7.1)
\]

\[r = 0, 1, 2, \ldots, N.\]

For a detailed explanation and derivation of Equation 7.1 see Appendix A7. The two parts of this equation are biased estimates of \(\hat{R}'(rh)\) and are illustrated in Figure 7.1.

To recover \(\hat{R}'(rh)\) we must eliminate the second term and multiply our result by \(\frac{N}{N-r}\). This can be accomplished by adding \(N\) zeros to the original time series. If we are only concerned with a maximum lag \(r\), only \(r\) zeros need to be added. As you might suspect, this adds considerably to the computation time (and computer memory) required to generate the spectrum. However, as shown below, it is still faster than conventional methods.
FIG. 7.1 CIRCULAR AUTOCOVARIANCE FUNCTION
As we shall see later, the original time series was partitioned into 19 segments each 4096\( (2^{12}) \) points long. The modified autocovariance function [see Section 16] was computed using the FFT of two real valued records at one time [see Section 9]. To avoid circular covariance, each of the 19 segments was extended with zeroes to form segments 8192\( (2^{13}) \) points long. The speed ratio for each segment analogous to Equation 6.1 becomes

\[
\text{Speed Ratio} = \frac{2(4096)^2}{2[(4)(8192)(13)]} \approx 40. 
\] (7.2)

The term \((4096)^2\) in Equation 7.2 represents the number of real multiply-add operations required to compute each of the two numerical convolutions needed to form the modified autocovariance function, if conventional methods are used. The terms \((4)(8192)(13)\) indicate the number of real multiply-add operations (where one complex multiply-add equals 4 real multiply-adds) required to compute the modified autocovariance function, if the roundabout FFT method is used.

To summarize, the roundabout FFT method is approximately 40 times faster than conventional methods, for each segment. For all 19 segments it is approximately 760 times faster.
8. Inverse FFT

To use the roundabout approach to compute autocovariance functions, an inverse discrete Fourier transform is required. Given a discrete Fourier transform as in Equation 4.14, or

\[
\hat{x}'(f_K) = \sum_{r=0}^{N-1} x_r e^{-j2\pi f_K r h} \quad K = 0,1,2,...,N-1 \quad (8.1)
\]

let,

\[
\hat{B}'_n = \frac{1}{N} \sum_{K=0}^{N-1} \hat{x}'(f_K) e^{j2\pi f_K h n} \quad n = 0,1,2,...,N-1 \quad (8.2)
\]

then by substituting \( \hat{x}'(f_K) \) into Equation 8.2 we have,

\[
\hat{B}'_n = \frac{1}{N} \sum_{K=0}^{N-1} \left[ \sum_{r=0}^{N-1} x_r e^{-j2\pi f_K r h} \right] e^{j2\pi f_K h n} \quad (8.3)
\]

\[
= \sum_{r=0}^{N-1} x_r \left[ \frac{1}{N} \sum_{K=0}^{N-1} e^{j2\pi f_K h[n-r]} \right].
\]

For \( n-r \), the term in brackets is equal to 1, and 0 elsewhere, since \( f_K = \frac{K}{Nh} \). Therefore,

\[
\hat{B}'_n = x_n = \frac{1}{N} \sum_{K=0}^{N-1} \hat{x}'(f_K) e^{j2\pi f_K h n} \quad (8.4)
\]
which is equivalent to equation 8.1 except for the sign of the exponential and the constant 1/N. The inverse transform can be computed by the same FFT method.
9. FFT of Two Real Valued Records at One Time

Computing two real valued transforms at one time results in a significant reduction in computer memory and time [6] (for large N, the speed improvement approaches two). This is possible when using an FFT program designed for complex series.

Given two real valued discrete time series $A_n$ and $B_n$, their transforms can be computed by letting

$$C_n = A_n + jB_n. \quad (9.1)$$

The transform of $C_n$ is

$$\hat{C}'(f_K) = \sum_{n=0}^{N-1} [A_n + jB_n] e^{-j2\pi f_K n h} \quad (9.2)$$

where

$$K = 0,1,2,\ldots,N-1$$

and

$$f_K = \frac{K}{Nh}.$$

By noting

$$e^{-j2\pi f_K n h} = e^{-j2\pi kn} = e^{-j2\pi kn/N},$$

it is clear that
\[
e^{-\frac{j2\pi (N-K)}{N}} = e^{-j2\pi n} e^{\frac{j2 \pi n K}{N}} = e^{\frac{j2 \pi n K}{N}}
\]

since
\[
e^{-j2\pi N} = 1.
\]

Therefore (with * indicating complex conjugate),
\[
\hat{C}_K + \hat{C}_{N-K}^* = \sum_{n=0}^{N-1} [A_n + jB_n] e^{-\frac{12 \pi Kn}{N}} + \sum_{n=0}^{N-1} [A_n - jB_n] e^{-\frac{12 \pi Kn}{N}}
\]
\[
= 2 \sum_{n=0}^{N-1} A_n e^{-\frac{12 \pi Kn}{N}}
\]
\[
= 2 \hat{A}_K',
\]

and
\[
\hat{A}_K' = \frac{\hat{C}_K + \hat{C}_{N-K}^*}{2}.
\]

A similar procedure yields
\[
\hat{B}_K' = \frac{\hat{C}_K - \hat{C}_{N-K}^*}{2j}.
\]

These special relations are used to efficiently compute the modified autocovariance function required because of missing data [see Section 16].
10. Sensors, Recording Equipment, Station Location and Elevation

The sensing equipment is located on top of a 9.1 m tower on the roof of Packard Laboratory, Lehigh University, Bethlehem, Pennsylvania. It consists of two Eppley pyranometers for sensing total solar radiation — one on a horizontal plane and the other on a vertical plane facing the equator. These two transducers transmit signals (by cable) to an Esterline-Angus, model D2020, sample data acquisition system and integrator. The resulting digital data is recorded on magnetic tape using a Kennedy incremental tape recorder. Detailed specifications of the equipment are given in Appendix A10.

The station location and elevation are as follows:

Longitude  W78° 23'
Latitude   N40° 36'
Elevation  118 meters (388 feet) above sea level.
11. Types of Recorded Data

The millivolt data output from the D2020 sampler is run through a conversion program which in turn creates another magnetic tape containing the following data:

1. Year
2. Day number
3. Military time at the end of the observation interval in hours and minutes
4. Total solar radiation on the horizontal collector based on an average over the previous 15 minutes in BTU/Ft$^2$
5. Total instantaneous solar radiation on the horizontal collector in BTU/Ft$^2$/Hr
6. Same as #4 but on vertical collector
7. Same as #5 but on vertical collector.

Each day's data is preceded by a header which indicates the following:

1. Year
2. Day number
3. Calculated length of day in hours and minutes [see Appendix A13]
4. Offset of solar noon with respect to clock noon in minutes [see Appendix A13]
5. Total possible solar radiation on a clear day for the horizontal collector in BTU/ft² [see Appendix A13]

6. Same as #5 but for the vertical collector.
Preparation of Data for Analysis

12. Trend Removal

It was noted in Section 1 that solar radiation has a strong diurnal and seasonal variation (now to be referred to as trends) and nondeterministic or random variations. The predictability of the latter is of concern since the trends are predictable from the motion of the earth relative to the sun. The diurnal trend for direct solar radiation on a horizontal surface is a half sine wave whose amplitude and period vary in a predictable way throughout the year. If these trends are not removed, large distortions near the period of the trend (generally low frequency) can appear in the spectrum. In some cases it will tend to completely overshadow the spectrum of the nondeterministic component.

To see that this is true, consider the fictitious spectrum shown in Figure 12.1. The peak in the spectrum represents the distribution of power in the periodic component. Its magnitude indicates that oscillations with a period of $1/b$ are more probable than those at any other period. This is typically what one might expect to see in the case of a strong diurnal trend such as in solar radiation. Actually, if this were the resultant spectrum and the periodic component was contained in a narrow frequency band as shown, there would be no need to
remove the periodic component. The problem arises then, because of the smearing operation implicit in the Fourier transformation process. We have seen in Section 5 that if we do not taper our data, we are, by default, using a boxcar function window. The effect of the window is to smear the "bump" in the spectrum over adjacent frequency bands as shown in Figure 12.2. The power due to the periodic component is spread over the power due to the nondeterministic component [such as at frequency "a" in Figure 12.1] and hence, large distortions can appear in the spectrum. Since trends are deterministic, including them in a spectrum analysis may distort other periodicities which are hidden in the random variations.

Highpass digital filters are usually of little value in removing these types of trends [13]. The best method appears to be to fit the trend using least-squares techniques [14]. This is the approach taken in this analysis. Section 14 and Appendix A14 describe the detailed approach used to remove the double trend.
FIG. 12.1  FICTITIOUS POWER SPECTRUM

FIG. 12.2  WINDOW EFFECT ON SPECTRUM IN FIG 12.1
13. Direct Solar Radiation Spectrum - A Discarded Candidate

The solar energy falling on collectors or sensors on the earth's surface can be split up into three components [15]: direct or beam radiation, diffuse radiation, and solar radiation reflected from the ground. Beam radiation is energy which is not interfered with or absorbed by atmospheric gases, water vapor, or dust, and arrives at the earth's surface without change in direction. Diffuse radiation is energy received after its direction has been changed by reflection or scattering by the atmosphere. Reflected radiation is energy reflected from the earth's surface or from objects on the earth's surface.

The most obvious way to remove the diurnal and seasonal trends seemed to be to generate a spectrum in terms of beam radiation. For a horizontal collector, reflection from the ground is not a variable since the collector cannot see the ground. The beam component of radiation falling on a horizontal collector ($R_{bh}$) is

$$R_{bh} = R_b \cos \theta_z,$$  \[(13.1)\]

where $R_b$ is the direct beam from the sun and $\theta_z$ is the zenith angle (i.e., the angle between $R_b$ and the vertical). The zenith angle as shown in Appendix A13 accounts for the diurnal and seasonal variations and therefore
\[
\frac{R_{bh}}{\cos \theta_z}
\]

is a trend-free random time series. Two problems arise. Only total (direct + diffuse) solar radiation \(R_{th}\) is available for data analysis in this study and the diffuse component does not follow a cosine trend, Also as \(\theta_z\) approaches 90°,

\[
R_b = \frac{R_{bh}}{\cos \theta_z} + \infty,
\]

which drastically magnifies errors in the magnitude of the time series data near sunrise and sunset.

The second problem can be eliminated by removing points near sunrise and sunset and treating them as missing observations [see Section 16].

To isolate the direct beam portion of solar energy, the diffuse component must be known. Using both the horizontal and vertical collector data, it is shown in Appendix A13 that the diffuse and direct components can be expressed as a function of the available total horizontal and vertical components both of which are known from the available data, and one other component the reflectance \(\rho\), which is not known.
\[ R_{th} = R_{bh} + R_{dh} \]

and

\[ R_{tv} = R_{bh}[C+\rho/2] + [R_{dh}/2][1+\rho] \quad (13.3) \]

give

\[ R_{bh} \text{ and } R_{dh}. \]

This assumes that the diffuse component is uniformly distributed over the sky and the ground is of infinite horizontal extent whose surface reflects solar radiation diffusely. The variables in Equation 13.3 are defined as:

- \( R_{th} \) = Total radiation on a horizontal surface
- \( R_{bh} \) = Beam radiation on a horizontal surface
- \( R_{dh} \) = Diffuse radiation on a horizontal surface
- \( R_{tv} \) = Total radiation on a vertical surface
- \( C \) = Ratio of beam radiation on the vertical surface to that on the horizontal surface (known).
- \( \rho \) = Ground reflection coefficient.

If we assume that on clear days the diffuse component comes mostly from an apparent origin near the sun (i.e., forward scattering), the second part of Equation 13.3 becomes

\[ R_{tv} = (R_{bh}+R_{dh})(C+\rho/2) \quad (13.4) \]

The reflection coefficient \( \rho \), can vary substantially with the conditions of the terrain. Liu and Jordan [15] suggest using \( \rho = 0.2 \) when the ground
is covered with less than one inch of snow or no snow, and $\rho = 0.7$ otherwise. They make no attempt to justify these values. They also assume that $\rho$ is isotropically distributed (i.e., not dependent on direction) and Dave [16] has found that this is not entirely satisfactory. Hunn and Calafell [17] have shown that $\rho$ is considerably variable.

Numerous attempts were made to eliminate $R_{dh}$ in Equations 13.3 and 13.4 with the expectation of solving for $\rho$ and noting how it varied with terrain conditions. Weather Bureau records were checked to find various conditions of snow cover. Days with low humidity, strong breezes, and low pollution were used to minimize the effect of water vapor, dust, and other contaminants. Noon values were used to minimize the air mass effect. All of these attempts proved less than satisfactory because it is difficult to separate the reflected and diffuse radiation. Even on a clear day, as reported by Liu and Jordan [18], diffuse radiation on a horizontal surface can be as much as 12% of the total.

There are many methods published concerning estimation of the diffuse component of total solar radiation. Most are limited to one location and exhibit significant variation when comparisons are made. Despite its obvious utility, very few locations measure diffuse radiation.
With the above in mind, the idea of generating a spectrum of normalized beam radiation was discarded. Since most available solar radiation data is in the form of total radiation and collectors that are feasible for practical use do not distinguish between radiation components, the decision was made to use total radiation and eliminate the trends by other means. This method is covered in the next Section (14) and the general equations shown in Appendix A13 will be useful in understanding its content. A facility which would measure diffuse as well as total radiation would be desirable.
14. Total Solar Radiation Spectrum

If the diurnal and seasonal trends can be removed from total solar radiation on a horizontal surface, then the resultant time series is random. However, there still remains the periodic discontinuous effect due to sunrise and sunset.

It may seem that the logical approach is to ignore the evening data points and create a new time series composed of each day from sunrise to sunset laid end-to-end. This results in a spectrum that would have no meaning in terms of actual clock time, since the length of the daylight period is also seasonal. There does not appear to be an obvious method to normalize the data in the time domain while preserving the equally spaced intervals so important to simplifying power spectrum analysis.

The approach used in this report is to treat evening points as zero. They were ignored when computing and removing the mean from the time series. The standard deviation about the mean of each of the means of the 19 segments analyzed was approximately 12%. This supports to some extent the assumption in Section 2 of stationarity.

In terms of the autocovariance function of Equation 3.9 (repeated here)

\[ \hat{R}'(r) = \frac{1}{N-r} \sum_{n=1}^{N-r} x_n x_{n+r}, \quad r = 0, 1, 2, \ldots, N-1, \]
treating evening points as zero implies there is no correlation between daytime and evening data. This built-in periodicity should appear in the spectrum and at a period roughly equal to 8 to 12 hours.

This report uses a least-squares curve fit of instantaneous clear day total solar radiation, encompassing both the diurnal and seasonal trends, as a basis for computing a time series representing the percent of possible clear day total solar radiation.

Appendix A14 derives and discusses the above curve fit as well as presenting others that were tried and found unsatisfactory. It also covers the rationale for removing points near sunrise and sunset to avoid spurious results when taking percentages of two small values.
Inherent Problems in Digital Spectrum Estimation

15. Aliasing and Averaging of Observations

The relationship between sampling a continuous time series at equal spacing and the original time series is important, since estimates of the spectrum of the sampled series must be used to estimate the spectrum of the original. Sampling leads to sinusoids of high frequency appearing at low frequencies and the two frequencies will be indistinguishable or aliased with one another. The classical example of aliasing is the phenomenon in movies of stagecoach wheels seeming to go backwards. This effect is caused by the position of the spokes when the movie frame is shot. The period is not one complete revolution of the wheel, but rather a function of the spoke separation. If there are "n" spokes, one period is 1/n of a revolution.

To clarify aliasing, observe the two sinusoids A and B shown in Figure 15.1. If B is the real continuous series of period 2, then harmonic A, generated by sampling B at intervals of 3, in a sense has the combined power of B and all other harmonics that are indistinguishable from it by sampling. For example, A has as period of 6, whereas the original series B contained no such cycle with this period. Note, if we had sampled at the points X, Y, and Z, no fictitious oscillation would have resulted. Aliasing then is greatly influenced by sample spacing.
FIG. 19.1 ALIASING SCHEME
When data are sampled at intervals \( h \) apart, the highest frequency which can be defined by sampling at a rate of \( 1/h \) is \( 1/2h \). This frequency \( (f_N) \)

\[
f_N = 1/2h
\]  

is often referred to as the Nyquist folding frequency in honor of Nyquist [19]. The use of the term folding is appropriate since for any frequency \( f_1 \) in the range \( 0 \leq f_1 < f_N \), \( f_2 \) in the range \( f_N < f_2 < 2f_N \) is folded back on \( f_1 \). The higher frequencies aliased with \( f_1 \) are

\[
(2f_N \pm f_1), (4f_N \pm f_1), ...
\]

Note that in Equations 4.16 and 5.7, the results are only unique out to \( K = N/2 \) corresponding to \( f_K = 1/2h \), the Nyquist frequency.

It follows that spectrum estimates with frequencies \( 2f_N \pm f \), \( 4f_N \pm f \), \( ... \) will be superimposed on the estimates at \( f \). One way to avoid this is to choose the sampling interval \( h \) small enough so that there is no appreciable power above the Nyquist frequency. This may result in excessive computing time, especially if this is done by running the computations over and over again with succeedingly smaller values of \( h \) until no changes in the resulting spectrum are noticeable.
This report uses $h = 15$ minutes, corresponding to a sampling rate of 4 samples per hour and a Nyquist frequency of 2 cycles per hour. Although very little contribution to the spectrum is expected to exist above this range, to help offset the aliasing problem, 15 minute averages obtained with an analog data averager were used. Averaging acts like a filter which eliminates the high frequencies from the time series.

Appendix A15 derives and discusses the effect of averaging on the spectrum. The result indicates averaging will cause the spectrum $\hat{G}'(f_K)$ to be underestimated by a factor of

$$\left[ \frac{\sin(\pi f_K h)}{\pi f_K h} \right]^2$$

or in Equation form,

$$\overline{G}'(f_K) = \hat{G}'(f_K) \left[ \frac{\sin(\pi f_K h)}{\pi f_K h} \right]^2$$ (15.3)

where $\overline{G}'(f_K)$ is the spectrum obtained from averaging.

As $f_K \to 0$ or as $h \to 0$,

$$\left[ \frac{\sin(\pi f_K h)}{\pi f_K h} \right]^2 \to 1$$
and falls to $4/\pi^2$ as $f_K \rightarrow 1/2h$ (the Nyquist frequency). Thus, our estimates based on averages are approximately true at low frequencies but are attenuated to around 40% near the Nyquist frequency. At frequencies which are even multiples of the Nyquist frequency, any contribution to the spectrum is nullified, thereby decreasing any unknown aliasing effects. For practical applications of solar energy, one is not as concerned with variations corresponding to periods of a few hours as with those on the order of days.
16. Missing Observations

As is the case with most data collected over a long period of time, the data used for this analysis contained a fairly substantial number of missing observations. Magnitude errors also existed, which were treated as missing observations. In addition, the first four and last four daylight points in each day were treated as missing observations [see Appendix A14] to avoid spurious results when taking percentages of small values near sunrise and sunset.

The data also exhibited extended periods of missing observations due to building repairs and inoperable equipment. In particular, the data from 5/19/76 to 8/15/76 and from 8/20/77 to 9/30/77, corresponding to 89 and 42 days respectively, was missing.

As is shown in Section 17, in order to reduce the variability of the spectrum estimates, and to reduce the amount of computer memory required, the time series must be sectioned into equal length segments so the estimates can be averaged. This enabled the three intervals of data, separated by the two long stretches of missing data, to be individually sectioned and the remaining data in the interval discarded. In all, 7% of the data was lost. There were 77824 observations used or approximately 2-1/4 years of data. Of these, (8673)
were missing points (11%), and (41641) were evening points (54%).

Normally, if only a few points in a row are missing, they can be replaced by a linear combination of their neighbors with very little biasing of the spectrum. However, in addition to the two large gaps there were a number of cases where data was missing for a few days in a row. This can lead to an unacceptable bias in the spectrum estimates. Jones (20) and Parzen (21) estimate this bias for the case when missing points are replaced by zeroes (the mean). Both assume the data is missing in some periodic way. Bloomfield (22) and Scheinok (23) use the zero replacement scheme and assume the data is missing in some general random manner.

Jones (24) developed a method using the zero replacement scheme where the pattern in which points are missing does not matter. This is the theory that is used in this report. Jones shows you can get reasonable results with only 50% of the data. A synopsis of his theory is presented below.

Let $I_n$ be an indicator sequence

\[
I_n = 1 \text{ if } X(nh) \text{ is observed}
\]

\[
I_n = 0 \text{ if } X(nh) \text{ is missing}, \quad (16.1)
\]
then an unbiased estimate of the autocovariance function \( \hat{R}'(rh) \) is

\[
\hat{R}'(rh) = \frac{1}{C_r} \sum_{n=0}^{N-1} X_{n+r}I_{n+r}I_n \quad r = 0,1,2,\ldots,N-1
\]  \hspace{1cm} (16.2)

where

\[
C_r = \sum_{n=0}^{N-1} I_{n+r}I_n \quad (16.3)
\]

and assuming that

\[ C_r > 0. \]

Equations 16.2 and 16.3 can be computed using the methods described in Section 7 except there is no need to multiply the result by \( \frac{N}{N-r} \). Note also that the two series \( X_n \) and \( C_r \) can be transformed simultaneously as shown in Section 9. Since \( C_r \) must be greater than 0, this implies at least one pair of observations, separated by each lag \( r \) needed, must exist. This can limit the maximum lag \( r \) that can be used depending on the position of the missing observations (i.e., 0's) in the data. It is easy to see that the probability of \( C_r \) equaling 0 is much greater for large lags. In fact, for a series with a number of leading or trailing zeroes, \( C_r \) will be zero for large lags. This is another reason for removing the two large
gaps of data. Of the 19 segments (each 4096 points long) analyzed, several resulted in $C_r = 0$ for large lags.

An estimate of the spectrum using Jones' theory is given by

$$\hat{G}'(f_K) = 2h \sum_{r=-N}^{N} \hat{R}'(rh) U_{Nr} e^{-j2\pi f_K rh}$$  \hspace{1cm} (16.4)

where $N$ is the maximum lag, and $U_{Nr}$ is the Parzen lag window (Figure 5.2).

The variance is given by

$$\text{Var}[\hat{G}'(f_K)] = G^2(f_K) \sum_{r=-N}^{N} U_{Nr}^2 / C_r,$$  \hspace{1cm} (16.5)

and the equivalent degrees of freedom (edf) is

$$\text{edf} = 2 \left( \sum_{r=-N}^{N} U_{Nr}^2 / C_r \right).$$  \hspace{1cm} (16.6)

Note that missing observations tend to increase the variance and therefore decrease the edf. Note also that if the maximum lag ($N$) is decreased, the variance also decreases and the edf is increased. We have seen that $C_r = 0$ for large lags. We can now eliminate at least all lags ($r$) greater than that where $C_r = 0$, and decrease the variance as well. In this report all lags above
r = 3600 were discarded. Although this results in loss of frequency resolution it does, as we shall see in the next Section (17), result in a reasonable value for the edf.

The difficulty with this theory is it is desirable to have some prior knowledge of $C_n$ to predict the edf. A great deal of effort is required to obtain this knowledge of $C_n$ [see Equation 16.3]. This involves either $N^2$ real multiply-add operations or $8N\log_2 N$ operations if the FFT roundabout method is used. There is also the possibility of several iterations of either varying the lag or the number of segments. It is interesting to note that both varying the lag and varying the segments affect the variability of the estimates [see Section 17] but segmenting involves the transformation of sequences which are much shorter. When computations are to be performed on computers with limited core storage, this is a distinct advantage.

The strategy used in this report was to section the original time series into segments whose length was compatible with "prime time" computer memory allocation. The maximum lag was then varied to achieve a reasonable value for edf.
17. Variability - Confidence Intervals

In spectrum analysis one must trade frequency resolution for stability or vice versa. Blackman and Tukey [5] have shown assuming the original time series is Gaussian, the spectrum estimates given by Equation 16.4 follow a chi-square distribution and hence, each is a chi-square variable with

\[ 2 \sum_{r=-N}^{N} U_{r}^{2}/C_{r} \]  

(17.1)

equivalent degrees of freedom. Each estimate of \( \hat{G'(f_{K})} \) will then have a sampling distribution \([25]\) given by,

\[ \frac{\hat{G'(f_{K})}}{G(f_{K})} = \frac{X_{\text{edf}}^{2}}{\text{edf}} \]  

(17.2)

where \( X_{\text{edf}}^{2} \) is the chi-square variable with edf equivalent degrees of freedom.

A \((1-\alpha)\) confidence interval can then be established by standard statistical methods as

\[ \frac{(\text{edf})\hat{G'(f_{K})}}{X_{\text{edf};\alpha/2}^{2}} \leq G(f_{K}) < \frac{(\text{edf})\hat{G'(f_{K})}}{X_{\text{edf};1-\alpha/2}^{2}} \]  

(17.3)

Given \( \alpha \), the confidence interval increases as the edf decreases. There are two methods for increasing the edf.
The first, involves limiting the lag \( r \) in Equation 16.3 which in turn decreases

\[
\sum u_{Nr}^2 / c_r
\]

and therefore increases the edf. The second, involves sectioning the time series into equal length segments, averaging the estimates at each frequency, and then by the \( X^2 \) addition theorem for independent variables, summing the edf's. Both these methods result in loss of frequency resolution.

The approach of this report, as was previously mentioned [Section 16] is to use a combination of both methods. In particular, the data was sectioned into 19 segments, each containing 4096 points. The modified autocovariance function [Equation 16.2] was computed for lags \( r = 0,1,2,...3600 \). This resulted in 105 equivalent degrees of freedom.
Recipe for Computations

18. Descriptive Approach

This section is intended as a guide for those contemplating a similar undertaking or for those who wish to verify these results. It is a compilation of the theories and techniques discussed in Sections 1-17. Although the recipe for computations is tailored to the data from this analysis, it can easily be modified for use with other time series.

For simplicity, it will be assumed that the raw data as in Section 11, and the theoretical expression for clear day solar radiation intensity at any time \( t \) [Equation A14.2 using Equations A14.4] are available. The following is a chronological account of the procedure used to generate the spectrum.

1. Split the data into 3 segments eliminating the two large gaps of data.
2. Flag the missing and evening points.
3. Compute the ratio of the observed (averaged) radiation to the theoretical (Equation A14.2), with \( t = (\text{real solar time} - 7 \, 1/2 \, \text{minutes}) \). Multiply by 100 to get the above in terms of percentages.
4. Section each of the 3 segments into segments 4096\( (2^{12}) \) points long. Discard the data left over
at the end of each of the 3 segments. This produces 19 segments.

5. Remove the mean from each segment individually. Ignore missing and evening points when computing the mean.

6. Form the 19 indicator time series of Equation 16.1, and insert zeroes for the missing and evening points in the 19 standard time series. This produces 38 series, each 4096 points long.

7. Extend each series to double its length (i.e., 8192) with zeroes.

8. Compute the FFT (two at a time) as shown in Section 9. This produces 38 complex valued series, each 8192 points long.

9. Square the real and imaginary part of each complex value and add them together.

10. Inverse transform (See Section 8) each series (two at a time) and divide by 8192. Discard the last half of the results. If the result of the indicator series \( C_r \) is zero at any lag \( r \), note the lag number. In this analysis \( C_r = 0 \) at lag \( r \geq 4093 \) for several segments. The above produces 19 numerically convolved series, each 4096 points long, for both the indicator and standard series.
11. Compute the equivalent degrees of freedom (edf per Equation 16.6) for various lags up to the lag at which $C_r = 0$ (if any). Note that this produces 19 independent edf's which are then added together to form the edf for the final spectrum.

12. Select a lag that produces reasonable results for the edf. In this analysis, edf = 105, corresponding to a lag of 3600, was assumed reasonable.

13. Compute the modified autocovariance functions (Equation 16.2) for all lags up to the one selected in #12. This produces 19 modified autocovariance functions of equal length.

14. Use the FFT to compute the 19 spectra (Equation 16.4) using the Parzen lag window (Equation 5.6). Discard the last half of the results since they overlap the first half. In this analysis, this produces 1800 spectrum estimates equally spaced from $f = 0$ to $f = 2$ cyc/hr (the Nyquist frequency).

15. Average the 19 estimates at each frequency to form the final spectrum.

16. Compute confidence intervals (Equation 17.3) for as many estimates as desired.
19. Analysis of Spectrum

Figures 19.1 through 19.4 show the spectrum in four sections: 0.0 to 0.5, 0.5 to 1.0, 1.0 to 1.5, and 1.5 to 2.0 cycles/hour. The units for the ordinate are [% possible clear day solar radiation KJ/m²] per cycle per hour. All four figures are plotted on the same scale to preserve the property that the variance contributed within a frequency range is given by the area under the spectral curve.

The spectrum is relatively flat and small in magnitude beyond 0.3 cycles/hr (i.e., periods less than 3.3 hours) indicating that oscillations with these frequencies are less probable than those with frequencies below 0.3 cycles/hr.

Figures 19.5 through 19.7 show the spectrum (all with the same scale) for frequency intervals of: 0.0 to 0.1, 0.1 to 0.2, and 0.2 to 0.3 cycles/hr. There appear to be statistically significant peaks in the spectrum centered at periods of approximately 18, 7, and 2 1/2 days, and 25, 12, 8, 6, 4 1/2, and 3 1/2 hours. This indicates that oscillations with these periods are more probable than those with neighboring periods. The statistical significance of the major peaks is shown by a plot of a 95% confidence interval.
Peaks at 18, 7, and 2 1/2 days, and 24 hours are most significant since their magnitude greatly exceeds those of peaks at other periods. They could perhaps reflect the average time between frontal passages. The 7 day cycle seems to agree with the intuitive feeling of weekly periodicities in weather.

The peak at 12 hours is thought mainly to be due to the built in periodicity introduced by treating evening points as zeroes. It is not clear what effect, if any, this has on oscillations at other periods. The peaks at periods less than 12 hours are thought to be due to the position of the sun and to changes in the atmosphere such as water vapor content, pollution, and wind speed. All of these factors affect absorption and scattering of solar radiation.
FIG. 19.2 POWER SPECTRUM OF PERCENT OF POSSIBLE CLEAR DAY SOLAR RADIATION ON A HORIZONTAL SURFACE AT BETHLEHEM, PA. (9/75-6/78).

SPECTRAL ESTIMATES
FIG. 19.3 POWER SPECTRUM OF PERCENT OF POSSIBLE CLEAR DAY SOLAR RADIATION ON A HORIZONTAL SURFACE AT BETHLEHEM, PA. (9/75-6/78).
FIG. 19.4 POWER SPECTRUM OF PERCENT OF POSSIBLE CLEAR DAY SOLAR RADIATION ON A HORIZONTAL SURFACE AT BETHLEHEM, PA. (9/75-6/78).
FIG. 19.5  POWER SPECTRUM OF PERCENT OF POSSIBLE CLEAR DAY SOLAR RADIATION ON A HORIZONTAL SURFACE AT BETHLEHEM, PA (9/75-6/78).
FIG. 19.6 POWER SPECTRUM OF PERCENT OF POSSIBLE CLEAR DAY SOLAR RADIATION ON A HORIZONTAL SURFACE AT BETHLEHEM, PA. (9/75-6/78)
Fig. 19.7 Power spectrum of percent of possible clear day solar radiation on a horizontal surface at Bethlehem, PA. (9/75-6/78).
20. Use of Results

This section is intended to identify a few of the many applications of the solar radiation spectrum. The intent is not to present rigorous descriptions, but rather to stimulate interest in the utility of the spectrum.

Meteorologists have always been concerned with extrapolation and prediction of weather variables. For solar radiation, the spectrum generated in this report presents a quantitative method to accomplish this. It can be used to predict periodicities in solar radiation since the spectrum contains probabilistic information about oscillations at differing periods.

For some, it may be sufficient to recognize that solar radiation exhibits definite periodicities, but for others the ultimate question is "Why?" The spectrum can provide valuable insight for a better understanding of the large scale processes operating in the atmosphere.

Another valuable application is for comparison purposes with spectra of other variables such as wind speed and temperature. Comparisons are usually made using an extension of the basic formulations outlined in this report. Instead of dealing with statistical functions to describe the properties of data from one
individual random process, joint properties of different data from two or more random processes are considered. When analyzing joint properties of two random processes, the statistical function similar to the autocovariance function is referred to as the cross-autocovariance function, and the spectrum becomes the cross-spectrum.

The actual formulations of these new functions are similar to the single series approach. Given two random processes \( X(t) \) and \( Y(t) \), then the cross-autocovariance function is given by

\[
R_{XY}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} X(t)Y(t+\tau)dt \quad (20.1)
\]

and the cross-spectrum is just the Fourier transformation of \( R_{XY}(\tau) \). The cross-autocovariance function describes the general dependence of one set of data on the other. It differs from the single series function in that \( R_{XY}(\tau) \) is not necessarily maximum at \( \tau = 0 \), and it is not an even function. However, it is symmetrical about \( \tau = 0 \) when \( X \) and \( Y \) are interchanged, or

\[
R_{XY}(-\tau) = R_{YX}(\tau). \quad (20.2)
\]

The cross-spectrum is a complex valued quantity such that

\[
G_{XY}(f) = Re_{XY}(f) - jIm_{XY}(f) \quad (20.3)
\]

73
where the magnitude $|G_{XY}(f)|$ and phase angle $\theta_{XY}(f)$ are given by

$$|G_{XY}(f)| = \sqrt{\text{Re}^2_{XY}(f) + \text{Im}^2_{XY}(f)} \quad (20.4)$$

and

$$\theta_{XY}(f) = \tan^{-1} \left[ \frac{\text{Im}_{XY}(f)}{\text{Re}_{XY}(f)} \right] \quad (20.5)$$

A coherence function $C_{XY}(f)$ indicating the amplitude correlation between the two functions is given by

$$C_{XY}(f) = \frac{|G_{XY}(f)|^2}{G_x(f)G_y(f)} \quad (20.6)$$

which ranges in value between 0 and 1. If $C_{XY}(f)=1$, the series are fully correlated in amplitude and if it is 0, they are uncorrelated.

The phase angle and coherence function are extremely valuable for comparison purposes. The phase lag or lead of frequency components of the series can now be determined as well as amplitude correlations.

It should be noted that one of the requirements for this method is the two time series must be sampled identically. Although it was not mentioned in Section 11,
wind speed and temperature were simultaneously recorded with the solar radiation data, at the Bethlehem site. Cross-spectra should be available in the near future.

The next application of the solar radiation spectrum to be discussed is its use in determining storage capacity for a solar energy system.

Let $P(t)$ be the percent of clear day solar power supplied and $D(t)$ be the percent of clear day solar power demand. If there are no losses in transfer, conversion, and storage of power, then the excess energy $E(t)$ at any time is given by

$$E(t) = \int_0^t P(\tau) d\tau - \int_0^t D(\tau) d\tau. \quad (20.7)$$

As an example, consider the case of random supply and constant demand equal to the average value of the supply power (i.e., average value of percent of clear day solar power). Then,

$$E(t) = \int_0^t [P(\tau) - \bar{P}] d\tau \quad (20.8)$$

where $\bar{P}$ is the average power given by

$$\bar{P} = \lim_{T \to \infty} \frac{1}{T} \int_0^T P(t) dt. \quad (20.9)$$
E(t) then is similar to the input time series in this report. Because P(t) fluctuates randomly, E(t) is a random variable with a mean value of 0, which can be negative. This means that even with a storage system that accepts all excess power during those periods when supply exceeds demand, there will be periods when the energy stored will be depleted and the supply power will not be able to meet demand. In addition, a storage system which is large enough to capture all excess energy will probably not use a large portion of its capability a large portion of the time and therefore will not be economical. Design decisions on storage have to be made on a statistical basis.

One design consideration for storage is when E(t) goes through zero, since at this point either the demand is not being met or there is excess supply. Power must either be drawn from auxiliary sources or, (when there is excess power) it must be dumped into storage. A useful statistic would be the average number of times per unit time either depletion or repletion occurs which is equal to half the average number of times E(t) becomes zero.

In a similar manner, given a design storage capacity S_t, a useful statistic would be the number of times per unit time that S_t is exceeded which indicates how often energy must be dumped.
The average number of times per unit time that \( E(t) \) exceeds the value \( \alpha \) is given by

\[
\bar{N}_\alpha = \left[ \frac{\int_0^\infty f^2 G_E(f) df}{\int_0^\infty G_E(f) df} \right]^{1/2} e^{-\alpha^2/\int_0^\infty G_E(f) df} \tag{20.10}
\]

where \( G_E(f) \) is the power spectrum of \( E(t) \).

If \( \alpha = 0 \), the average number of times per unit time that either depletion or repletion of storage occurs is given by

\[
\bar{N}_0 = \left[ \frac{\int_0^\infty f^2 G_E(f) df}{\int_0^\infty G_E(f) df} \right]^{1/2} \tag{20.11}
\]

The above is only one simple example where the spectrum is useful in solving statistical problems associated with storage capacity. The distribution of peaks of \( E(t) \) is also helpful in determining storage capacity. The distribution of times between crossings is also of interest since it gives the probability of time span for which demand will not be met. In addition, the problem becomes much more complex when power-transfer losses and periodic demand are considered.
21. Recommendations for Future Research

The following recommendations are offered, not as detailed plans, but as suggestions of ways in which this analysis can be improved and extended. They are based on the thought that this report is only a "first step" to understanding variations in solar radiation.

The spectrum resulting from this analysis is limited in that it can only give information about oscillations at periods up to one month. While this may be adequate for short term forecasts of insolation, it does not ascertain long term trends in solar radiation which may affect solar energy systems. To extend the range of analysis, more data is required.

Further investigation is required to determine the degree to which the clear day solar radiation curve fit, and substitution of zeroes for evening observations, bias the spectrum. Although the curve fit has been shown to be reasonably accurate [See Appendix A14], it is quite possible that it may introduce subtle diurnal and seasonal periodicities. The zero replacement scheme for missing observations introduces a built-in periodicity in the spectrum at a period roughly equal to 8-12 hours, and may contribute to periodicities at other periods.
One way to avoid curve fits is to measure diffuse as well as total solar radiation. A spectrum of trend-free normalized beam radiation (i.e., total minus diffuse) would then be possible.

To determine the degree to which spectra are location dependent, data should be obtained from other sites for analysis. This is an important point. The instrumentation required for measuring solar radiation is expensive and observations must be made over long periods of time. If relationships can be developed for differing locations based on sites that routinely record solar radiation, then the above time and expense can be avoided.

A comparison of solar radiation with other meteorological variables is another area in need of more detailed research. For combined energy systems such as a solar-wind system, the extent to which these energy sources complement each other is important. Magnitude and phase relationships need to be developed through the use of cross-spectra.
REFERENCES


BIBLIOGRAPHY

In addition to the articles and books cited in the text, those listed below have been valuable references for the ideas and techniques described in this report.


Appendix A5
Convolution

Given

\[ \hat{G}(f) = 2 \int_{-\infty}^{\infty} U_{TN}(\tau) R(\tau) e^{-j2\pi f \tau} d\tau \]

and noting that

\[ U_{TN}(\tau) = \int_{-\infty}^{\infty} U_{TN}(f) e^{j2\pi f \tau} df \]

and

\[ U_{TN}(f) = \int_{-\infty}^{\infty} U_{TN}(\tau) e^{-j2\pi f \tau} d\tau \]

and letting

\[ R(\tau) = \int_{-\infty}^{\infty} S(f_1) e^{j2\pi f_1 \tau} df_1 \]

then
\[ \hat{G}(f) = 2 \int_{-\infty}^{\infty} U_{TN}(\tau) \left[ \int_{-\infty}^{\infty} S(f_1) e^{j2\pi f_1 \tau} df_1 \right] e^{-j2\pi f \tau} d\tau \]

\[ = 2 \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} U_{TN}(\tau) e^{-j2\pi (f-f_1) \tau} d\tau \right] S(f_1) df_1 \]

\[ = 2 \int_{-\infty}^{\infty} U_{TN}(f-f_1) S(f_1) df_1. \]

Note that \( U_{TN}(f) \) and \( S(f) \) are interchangeable. This relation is referred to as convolution. Multiplication in the time domain is therefore equivalent to convolution in the frequency domain and vice versa. \( \hat{G}(f) \) is the Fourier transform of the product of \( U_{TN}(\tau) \) and \( R(\tau) \) so the result is the convolution of their transforms.
Appendix A7
Circular Autocovariance Function

Since only a finite range Fourier series or transform can be computed with discrete data, this finite range establishes the fundamental period of the series. In effect, deterministic periodic records in which \(X(t)\) is a single cycle are being analyzed. Hence, an autocovariance function obtained from \(X(t)\) of the form

\[
\hat{R}_c(rh) = \frac{1}{N} \sum_{n=1}^{N} X_n X_{n+r} \quad r = 0, 1, 2, \ldots, N \quad (A7.1)
\]

will be periodic or circular since \(X(t)\) because of the finite length of the record is also periodic (i.e., \(X_{n+r} = X_{n+r-N}\) when \(n+r\geq N\)).

It therefore does not truly represent the actual or consistent estimate of the autocovariance function given by Equation 3.9 as

\[
\hat{R}_c^{\prime}(rh) = \frac{1}{N-r} \sum_{n=1}^{N-r} X_n X_{n+r} \quad r = 0, 1, 2, \ldots, N.
\]

On expansion, it is clear that Equation A7.1 can be written in two parts, namely

87
\[
\hat{R}_C'(rh) = \frac{1}{N} \sum_{n=1}^{N-r} x_n x_{n+r} + \frac{1}{N} \sum_{n=1}^{r} x_n x_{n+N-r} \tag{A7.2}
\]

The first term is an estimate for \( \hat{R}'(rh) \) but the second term has nothing to do with \( \hat{R}'(rh) \) and is only present due to the periodic assumption. Substituting \( r = N - r \) in Equation 3.9 results in

\[
\hat{R}'((N-r)h) = \frac{1}{N-(N-r)} \sum_{n=1}^{r} x_n x_{n+N-r} \tag{A7.3}
\]

therefore Equation A7.2 written in terms of Equation 3.9 is

\[
\hat{R}_C'(rh) = \frac{N-r}{N} \hat{R}'(rh) + \frac{N-(N-r)}{N} \hat{R}'([N-r]h) \tag{A7.4}
\]

corresponding to Equation 7.1 in Section 7.
Appendix A10

Equipment Specifications

Sensor


Characteristics

- Sensitivity: 7.5 mv. per cal/cm$^2$-min (approx.)
- Impedance: 300 ohms
- Temperature Dependence: ±1.5% from -20°C to 40°C
- Linearity: ±1% from 0 to 2.0 cal/cm$^2$-min.
- Response Time: 3 to 4 seconds (1/e signal)
- Cosine Response: ±2% from normalization, 10-90°
- Shipping Weight: 5.4 kg (12 lbs)
- Mechanical Vibration: up to 20 g's
- Spectral Range*: 0.28 to 2.80 microns (10$^{-6}$M)

* Only radiation of wavelengths between 0.29 and 2.5 microns reach the earth's surface [26].

Description

The pyranometer is a differential thermocouple made of 48 plated, copper on constantan junctions located radially; hot-junction receivers are covered with a stable black coating, cold-junction receivers, are whitened with non-hygroscopic sulphate.
Data Acquisition and Recording System

Digital Data Acquisition System and Integrator

Model D2020, Esterline Angus Corporation, P. O. Box 24000, Indianapolis, Indiana 46224.

Number of Channels:  20
Input Ranges:       1, 10, 100 MV
Data Rate:          1 hr, 20, 10, 5, 2, 1 min, 20, 10 sec. continuous
Channel Scan Rate:  2.5 channels/sec
Appendix A13
General Solar Equations

Throughout this Appendix and the next, the seasonal variation of total clear day solar radiation on a horizontal surface is expressed in the form

\[ y = a - b \cos \left( \frac{2\pi(d+10)}{365} \right) \]

(where \( d \) = the day of the year) as suggested by Hirschmann [27].

A fairly accurate representation of the total clear day solar radiation on a horizontal surface at latitude N 40° as reported by Duffie and Beckman [26] is given by

\[ R_{th\text{-}day} = 19585 - 10502 \cos \left( \frac{2\pi(d+10)}{365} \right) \text{kJ/m}^2\text{-day} \quad (A13.1) \]

where the argument of \( \cos \) is in radians. All other arguments of trigonometric functions hereinafter are in degrees except as noted.

The zenith angle \( \theta_z \) is given by

\[ \cos \theta_z = \sin \delta \sin \phi + \cos \delta \cos \phi \cos 15t \quad (A13.2) \]

where \( t \) is the time elapsed from solar noon in hours with mornings positive and afternoons negative, and \( \phi \) is the latitude with north positive and,
is the sun's declination as approximated by Cooper [28].

Solar time can be found by taking

Standard time + Equation of time +
4 (standard meridian - longitude in degrees west).

The equation of time accounts for the variation in the speed of the sun with respect to the earth because of the earth's elliptical orbit. The last two terms give the result in minutes and a fairly good approximation to the Equation of time (E) is

\[ E = 7.25 \sin \left( \frac{(d-183) \cdot 2\pi}{365} \right) + 9.75 \sin \left( \frac{(d-84) \cdot 4\pi}{365} \right) \]  

where the sin arguments are in radians. To obtain \( t \), solar time is subtracted from 12. The time to sunrise (\( t_{sr} \)) can be found by letting \( \theta_Z = 90^\circ \) in Equation A13.2 giving,

\[ \cos 15t_{sr} = -\tan \phi \tan \delta. \]  

Assuming that diffuse radiation is minimal on clear days, the total clear day radiation on a horizontal surface (\( R_{th-day} \)) in terms of direct beam radiation \( R_b \) (assumed constant throughout the day) is given by

\[ R_{th-day} = 2 \int_{0}^{t_{sr}} R_b \cos \theta_Z dt \quad (KJ/m^2-day). \]  

(A13.6)
This equation may be integrated giving,

\[ R_{th-day} = \frac{2}{15} R_b \left[ 15t \sin \delta \sin \phi + \frac{\cos \delta \cos \phi \sin 15t}{\pi/180} \right]. \]  \hspace{1cm} (A13.7)

The radiation intensity at any time \( t \) on a horizontal surface \( (R_{th}) \) is related to \( R_b \) by,

\[ R_{th} = R_b \cos \theta_Z = R_b [\sin \delta \sin \phi + \cos \delta \cos \phi \cos 15t]. \]  \hspace{1cm} (A13.8)

If \( R_b \) from Equation A13.7 is substituted into Equation A13.8, then

\[ R_{th} = \frac{15}{2} R_{th-day} \frac{[\sin \delta \sin \phi + \cos \delta \cos \phi \sin 15t]}{15t \sin \delta \sin \phi + \frac{\cos \delta \cos \phi \sin 15t}{\pi/180}}. \]  \hspace{1cm} (A13.9)

The total clear day radiation on a vertical surface \( (R_{tv-day}) \) in terms of direct beam radiation \( R_b \) (assumed constant throughout the day) is given by

\[ R_{tv-day} = 2 \int_0^t R_b \cos \theta_v \, dt \hspace{1cm} (KJ/m^2) \]  \hspace{1cm} (A13.10)

where \( \theta_v \) is the angle between the direct beam and the normal to a vertical surface given by

\[ \cos \theta_v = -\sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos 15t + \cos \delta \sin \gamma \sin 15t, \]  \hspace{1cm} (A13.11)
and \( \gamma \) is the deviation of the normal to the surface from the local meridian, the zero point being due south, east positive, and west negative.

Equation A13.10 can be integrated giving

\[
R_{\text{tv-day}} = \frac{2R_b}{15} \left[ -15t \sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \sin 15t \right] \frac{\pi}{180} - \frac{\cos \delta \sin \gamma \cos 15t}{\pi/180}. \tag{A13.12}
\]

These equations will be useful in understanding the curve fits in Appendix A14.

Derivation of Equations 13.3 relating total horizontal and vertical radiation to beam, diffuse and reflected radiation.

For vertical surfaces pointing south the angle between the direct beam from the sun and the horizontal is given by

\[
\cos \theta_v = \cos(\phi-90^\circ) \cos \delta \cos 15t + \sin(\phi-90^\circ) \sin \delta
\]

(\(A13.13\))

\[
= \sin \phi \cos \delta \cos 15t - \cos \phi \sin \delta
\]

The ratio of beam radiation on a vertical surface \((R_{bv})\) to that on a horizontal \((R_{bh})\) is given by,

\[
C = \frac{R_{bv}}{R_{bh}} = \frac{R_b \cos \theta_v}{R_b \cos \theta_z} = \frac{\cos \theta_v}{\cos \theta_z}. \tag{A13.14}
\]
Under the assumptions that the diffuse component of total solar radiation is uniformly distributed over the sky dome and the ground is of infinite horizontal extent whose surface reflects solar radiation diffusely,

\[ R_{th} = R_{bh} + R_{dh} \]  \hspace{1cm} (A13.15)

The vertical surface sees 1/2 of the ground and the sky dome, therefore,

\[ R_{tv} = R_{bh} + R_{dh}/2 + (R_{bh} + R_{dh})(\rho/2) \]  \hspace{1cm} (A13.16)

\[ = R_{bh}[C+\rho/2] + (R_{dh}/2)[1+\rho] \]

where

\[ \rho = \text{ground reflection coefficient.} \]
Appendix A14

Curve Fits for Trend Removal

In the following discussion, all curve fits were based on what appeared to be the best 45 clear days over the range of the data. Best, implies radiation data for those days exhibiting a near perfect cosine form. Table A14.1 indicates the days used, which were selected from data plots.

Several attempts to curve fit the horizontal surface clear day data were made. The first obvious choice was to compare the radiation intensity at any time \( t \), \( (R_{th}) \) given in Equation A13.9, to actual data by using the approximations for \( R_{th} \)-day, \( \delta \), and \( E \) given in Appendix A13. The approximations for \( R_{th} \)-day and \( E \) were visual curve fits to data presented by Duffie and Beckman [26]. Without going into detail, the difference was substantial but not surprising since the above estimation for \( R_{th} \)-day assumes there is no diffuse radiation on clear days. A check of \( E \) with published charts indicated very reasonable agreement. Equation A13.9 was then used as a basis for a least-squares curve fit to \( R_{th} \)-day, resulting in

\[
R_{th} \text{-day} = 19529-10639 \cos \left[ \frac{2\pi(d+10)}{365} \right]. \quad (A14.1)
\]
<table>
<thead>
<tr>
<th>Clear Days Used for Curve Fits</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table A14.1</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1975</th>
<th>1977</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/6</td>
<td>1/18</td>
</tr>
<tr>
<td>11/17</td>
<td>1/27</td>
</tr>
<tr>
<td>1976</td>
<td></td>
</tr>
<tr>
<td>1/19</td>
<td></td>
</tr>
<tr>
<td>2/14</td>
<td></td>
</tr>
<tr>
<td>2/23</td>
<td></td>
</tr>
<tr>
<td>3/23</td>
<td></td>
</tr>
<tr>
<td>3/28</td>
<td></td>
</tr>
<tr>
<td>4/5</td>
<td></td>
</tr>
<tr>
<td>4/12</td>
<td></td>
</tr>
<tr>
<td>4/23</td>
<td></td>
</tr>
<tr>
<td>4/30</td>
<td></td>
</tr>
<tr>
<td>8/18</td>
<td></td>
</tr>
<tr>
<td>8/21</td>
<td></td>
</tr>
<tr>
<td>8/22</td>
<td></td>
</tr>
<tr>
<td>8/31</td>
<td></td>
</tr>
<tr>
<td>9/8</td>
<td></td>
</tr>
<tr>
<td>9/13</td>
<td></td>
</tr>
<tr>
<td>10/4</td>
<td></td>
</tr>
<tr>
<td>10/23</td>
<td></td>
</tr>
<tr>
<td>11/2</td>
<td></td>
</tr>
<tr>
<td>12/5</td>
<td></td>
</tr>
<tr>
<td>12/24</td>
<td></td>
</tr>
<tr>
<td>12/31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1978</td>
</tr>
<tr>
<td></td>
<td>2/10</td>
</tr>
<tr>
<td></td>
<td>2/19</td>
</tr>
<tr>
<td></td>
<td>2/23</td>
</tr>
<tr>
<td></td>
<td>4/8</td>
</tr>
<tr>
<td></td>
<td>4/12</td>
</tr>
</tbody>
</table>
This led to a better result but also unsatisfactory because of the large residual error. The next attempt involved varying the 10 in the term \((d+10)\). This term corresponds to the winter and summer solstice when the sun is at its lowest and highest positions respectively. It was thought, perhaps due to some terrain or environment dependent phenomenon, this would make a positive difference. In fact, \((d+16)\) produced better but not satisfactory results. Apparently maximum/minimum radiation does not depend on astronomical considerations alone. Absorption and scattering by water vapor and particles also control the receipt of solar radiation on the ground, and may account for the forward shift of the maximum/minimum. The search for reasons for this result will be left for future study. The above variation was used on all subsequent refinements.

Improvements in curve fitting were judged by the degree to which the residual squared error could be reduced. To further improve the fit a new strategy was developed expressing \(R_{th}\) as a function of \(R_{th}\) maximum per day or,

\[
R_{th} = R_{th,\text{max}} \cos\left(\frac{t\pi}{2t_{sr}}\right) \tag{A14.2}
\]

where the argument of \(\cos\) is in radians. Attempts to curve fit \(R_{th,\text{max}}\) per day using this new equation led to
errors of the same order, but it suggested the possibility of getting better values of time to sunrise ($t_{sr}$) by curve fitting since it became obvious that a good deal of the error occurred around sunset and sunrise. It's possible that this is due to increased diffuse radiation at these times which would depart from the cosine trend.

Equation A14.2 can be solved for $t_{sr}$ as

$$t_{sr} = \frac{\pi}{2 \cos^{-1} \left( \frac{R_{th}}{R_{th_{max}}} \right)} \quad \text{(A14.3)}$$

A curve fit of the above $t_{sr}$ using the results of the previous curve fit of $R_{th}$ led to a 30% reduction in the standard deviation over the last method which used $t_{sr}$ from Equation A13.5. A check to see how time from solar noon affected the total error indicated the first and last four points in the day were major contributors. They were eliminated from consideration and the curve fits were redone. A very reasonable result ensued, and once again, $(d+16)$ produced the best results.

The final curve fits were,

$$t_{sr} = 5.57 - 1.21 \cos \left[ \frac{(d+16)2\pi}{365} \right]$$

and

$$R_{th_{max}} = 2565 - 968 \cos \left[ \frac{(d+16)2\pi}{365} \right] \quad \text{(per day)} \quad \text{(A14.4)}$$
corresponding to a standard deviation of 80 KJ/m\(^2\)-Hr, and an average error of less than 5%.

Using these two equations to determine \( R_{th} \), now expressed as the total theoretically possible clear day radiation on a horizontal surface at any time \( t \), the input time series to the spectrum was put in terms of percent of possible \( R_{th} \) with evening and missing points handled as previously mentioned in Sections 14 and 16.

These curve fits used the instantaneous rather than the 15 minute average data. The reason for this was to maintain realistic values for \( t_{sr} \) and \( R_{th} \). Since the spectrum uses average values, they were assumed to occur at the recorded time minus 7-1/2 minutes, a reasonable assumption.

The following is a derivation of the equations for the least-squares curve fits of Equations A14.4. All of the previously mentioned curve fits, although somewhat different, follow the same basic pattern.

Given that,

\[
R_{th} = R_{th_{\text{max}}} \cos \left( \frac{t\pi}{2t_{sr}} \right) \tag{A14.5}
\]

then \( R_{th_{\text{max}}} \) (per day) and \( t_{sr} \) can be expressed

\[
R_{th_{\text{max}}} = A - B \cos \left[ \frac{(d+16)2\pi}{365} \right] \tag{A14.6}
\]
and

\[ t_{sr} = C - D \cos \left[ \frac{(d+16)2\pi}{365} \right]. \]  \hspace{1cm} (A14.7)

Since we know \( R_{th_{\text{max}}} \) (now to be referred to as \( R_{th_{\text{max}}}^* \)) for each of the 45 days, letting

\( \frac{(d+16)2\pi}{365} = p_n \)

then

\[ \Sigma \left( R_{th_{\text{max}}} - R_{th_{\text{max}}}^* \right)^2 = \Sigma (A-B \cos p_n - R_{th_{\text{max}}}^*)^2. \]  \hspace{1cm} (A14.8)

So,

\[ \frac{3}{3A} = 0 = 45A - B \Sigma \cos p_n - \Sigma R_{th_{\text{max}}}^* \cos p_n \]  \hspace{1cm} (A14.9)

and

\[ \frac{3}{3B} = 0 = -A \Sigma \cos p_n + B \Sigma \cos^2 p_n + \Sigma R_{th_{\text{max}}}^* \cos p_n \]  \hspace{1cm} (A14.10)

therefore,

\[ A = \frac{\Sigma R_{th_{\text{max}}}^* + B \Sigma \cos p_n}{45} \]  \hspace{1cm} (A14.11)

and substituting \( A \) into Equation A14.10 after some simplification results in

\[ B = \frac{45 \left[ \Sigma R_{th_{\text{max}}}^* \cos p_n \right] - \left[ \Sigma R_{th_{\text{max}}}^* \right] \left[ \Sigma \cos p_n \right]}{\left[ \Sigma \cos p_n \right]^2 - 45 \Sigma \cos^2 p_n}. \]  \hspace{1cm} (A14.12)
To find C and D, let

\[ t_{sr} = \frac{\pi}{2 \cos^{-1} \left( \frac{R_{th}}{R_{th_{max}}} \right)} \]  

(A14.13)

so that

\[ \Sigma \left( t_{sr} - t_{sr}' \right)^2 = \Sigma \left( C - D \cos p_n - t_{sr}' \right)^2 \]  

(A14.14)

and the procedure is the same except the 45 in Equations A14.11 and A14.12 is replaced by the number of points (constant for all days) at which values of \( R_{th} \) and \( R_{th_{max}} \) are used.
Appendix A15

Effect of Averaging

For convenience, this section will develop the effect of averaging in terms of the Fourier transform of the original time series rather than the Blackman-Tukey approach using the autocovariance function. It can be shown that the result is the same.

The Fourier transform (FT) of a real or complex valued function $X(t)$ is given by

$$X(f) = \int_{-\infty}^{\infty} X(t)e^{-j2\pi ft}dt \quad (-\infty < f < \infty). \quad (A15.1)$$

Let

$$w(t) = \frac{1}{h} \int_{t-h}^{t} X(u)du \quad (A15.2)$$

be the result of averaging $X(t)$ over the interval $h$. If $w(t)$ is sampled at points $nh$ where $n = 0, 1, 2, \ldots, N-1$, Equation A15.1 can be written as

$$\bar{X}(f) = h \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} w(t)\delta(t-nh)e^{-j2\pi ft}dt \quad (A15.3)$$

where $\bar{X}(f)$ is the FT based on averaging, and $\delta(t-nh)$ samples $w(t)$ as given above. Since $\delta(t-nh)$ samples $(w(t)$ at a finite number of points, a data window $D_i(t)$ must be
introduced where \( D_1(t) \) is defined over the range \( 0 \leq t \leq T \) \((T = Nh)\) and zero elsewhere. Equation A15.3 now becomes

\[
\overline{X}(f) = h \int_{-\infty}^{\infty} \left[ D_1(t)w(t) \sum_{n=-\infty}^{\infty} \delta(t-nh)e^{-j2\pi ft} \right] dt \quad (A15.4)
\]

or alternatively

\[
\overline{X}(f) = h \int_{-\infty}^{\infty} \left[ \frac{D_1(t)w(t)}{h} \sum_{n=-\infty}^{\infty} \delta(t-nh)e^{-j2\pi ft} \right] dt \quad (A15.5)
\]

where \( w(t) \) is redefined as \( w(t) = \int_{t-h}^{t} X(u)du \).

Equation A15.5 is now in terms of an infinite range FT. The FT of the product of the two terms is just the convolution of their transforms. The second term is the well known transform \([29]\)

\[
\int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(t-nh)e^{-j2\pi ft} dt = \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{h}). \quad (A15.6)
\]

The first term is of primary concern since it indicates the effect of averaging. This term

\[
\int_{-\infty}^{\infty} \frac{D_1(t)w(t)}{h} e^{-j2\pi ft} dt \quad (A15.7)
\]

can be evaluated using integration by parts, or letting

\[
u = \frac{D_1(t)w(t)}{h} \quad \text{and} \quad dv = e^{-j2\pi ft} dt
\]
then,

\[ du = \frac{1}{h} \left\{ D_1(t) \frac{d[w(t)]}{dt} dt + \frac{d[D_1(t)]}{dt} w(t) dt \right\} \]

and

\[ v = \frac{e^{-j2\pi ft}}{-j2\pi f} \]

The evaluation of the term

\[ \frac{d[w(t)]}{dt} = \frac{d}{dt} \left[ \int_{t-h}^{t} X(u) du \right] \]

requires the generalized Leibnitz formula,

\[ \frac{dF}{da} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial f(X,a)}{\partial a} dX - \frac{f(a,\alpha)}{dx} f(a,\alpha) \frac{da}{dx} + f(b,a) \frac{db}{da} \]

where

\[ F(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(X,a) dX \]

resulting in

\[ \int_{t-h}^{t} \frac{\partial f(X,u)}{\partial t} du - X(t-h) \frac{d(t-h)}{dt} + X(t) \frac{dt}{dt} \]

which is equal to

\[ X(t) - X(t-h). \]
Therefore,

\[
du = \frac{1}{h} \left\{ D_1(t)[X(t)-X(t-h)]dt + \frac{d[D_1(t)]}{dt} w(t) dt \right\}.
\]

Equation A15.7 now becomes

\[
\left[D_1(t)w(t)\right]\frac{e^{-j2\pi ft}}{h}\left[\frac{e^{-j2\pi ft}}{-j2\pi f}\right]^{\infty}_{-\infty}
\]

\[
- \int_{-\infty}^{\infty} \left[D_1(t)[X(t)-X(t-h)] + \frac{d[D_1(t)]}{dt} w(t) \right] dt.
\]

(A15.8)

The first term is zero since

\[
D_1(\pm \infty) = 0,
\]

and Equation A15.8 can be written as

\[
\frac{1}{j2\pi fh} \left\{ \int_{-\infty}^{\infty} D_1(t)X(t)e^{-j2\pi ft} dt - \int_{-\infty}^{\infty} D_1(t)X(t-h)e^{-j2\pi ft} dt \right\}
\]

\[
+ \int_{-\infty}^{\infty} \frac{d[D_1(t)]}{dt} w(t)e^{-j2\pi ft} dt \right\}.
\]

(A15.9)

The first two terms of this equation simplify to

\[
D_1(f) * X(f) - D_1(f) * X(f)e^{-j2\pi fh}
\]
where * indicates convolution. Now letting the third term equal B, we have

\[
\frac{1}{2\pi fh} \left[ D_1(f) * X(f) \right] \left[ \frac{1 - e^{-j2\pi fh}}{j} \right] + \frac{1}{j2\pi fh} [B]. \tag{A15.10}
\]

The term

\[
\frac{1 - e^{-j2\pi fh}}{j}
\]

reduces to

\[
e^{-j\pi fh} [2 \sin(\pi fh)],
\]

so Equation A15.10 becomes

\[
\left[ D_1(f) * X(f) \right] e^{-j\pi fh} \left[ \frac{\sin(\pi fh)}{\pi fh} \right] + \frac{1}{j2\pi fh} [B]. \tag{A15.11}
\]

The second term above,

\[
\frac{1}{j2\pi fh} [B] = \frac{1}{j2\pi fh} \int_{-\infty}^{\infty} \frac{d[D_1(t)]}{dt} w(t) e^{-j2\pi ft} dt
\]

\[
= \frac{1}{j2\pi fh} \int_{0}^{T} \frac{d[D_1(t)]}{dt} w(t) e^{-j2\pi ft} dt \tag{A15.12}
\]

since the data window is 0 outside this range.
For symmetrical data windows such as the Parzen window [Section 5], note that

\[
\frac{d[D_1(t)]}{dt} = - \frac{d[D_1(T-t)]}{dt}
\]

and

\[e^{-j2\pi f t} = e^{-j2\pi f(T-t)}\]

where

\[f = 1/T.\]

Due to the Gaussian nature of \(w(t)\), it can be argued that as \(T \to \infty\), the results of integrating Equation A15.12 between \((0,T/2)\) and \((T/2,T)\) will cancel and therefore, for finite \(T\) the result of Equation A15.12 is small and can be ignored.

The final result of averaging is then

\[
\bar{X}(f) = [D_1(f) * Y(f)] e^{-j\pi f h} \frac{\sin(\pi f h)}{\pi f h} \quad (A15.13)
\]

The factor \(e^{-j\pi f h}\) only indicates a linear phase shift, so the effect of averaging on the power spectrum given by

\[
S(f) = \frac{1}{T} |X(f)|^2,
\]
is to underestimate the true values by

\[ \left[ \frac{\sin(\pi fh)}{\pi fh} \right]^2. \]

At frequencies \( f = n/h(n = 1, 2, 3, \ldots) \), all information of \( \bar{X}(f) \) is lost. This corresponds to frequencies which are multiples of the averaging period. Since the Nyquist frequency is \( 1/2h \), the effect of averaging is to filter out power existing between frequencies \( n/2h \) above the Nyquist frequency at odd multiples of the Nyquist frequency. At frequencies less than the Nyquist a smearing effect similar to that produced by the finite record, is produced.
VITA

The author was born in Oxford, New Jersey on September 26, 1948. He graduated from Lafayette College, Easton, Pennsylvania, in 1976 with the Degree of Bachelor of Science (Mechanical Engineering). He is a member of Pi Tau Sigma, Tau Beta Pi, and Phi Beta Kappa. He has been employed by Bell Laboratories, Whippany, New Jersey since 1968. The work described in this report was performed between September, 1977, and May, 1979, while attending Lehigh University.