The effect of combined stresses on the fatigue resistance of longitudinal fillet weldments in "T-1" steel

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THE EFFECT OF COMBINED STRESSES
ON THE FATIGUE RESISTANCE OF LONGITUDINAL
PILLET WELDSMENS IN "T-1" STEEL

by

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ABSTRACT

The purpose of this thesis is to report on the research performed to investigate the effect of combined stresses on the fatigue resistance of longitudinal fillet weldments in "T-1" steel and to determine a basis to correlate the fatigue resistance under combined stresses to that under uniaxial stress.

The specimens tested under a combined stress state were welded built-up beams of "T-1" steel. These specimens were compared to welded tee specimens tested in uniaxial tension and compression.

The correlation between the critical combined stress state and the critical uniaxial stress state was based on two failure theories, the maximum shear stress criterion and the distortion energy criterion.

It was found that the maximum shear stress theory predicted the fatigue resistance under combined stresses reasonably well and that this prediction was conservative.
1. INTRODUCTION

There are many factors that influence the fatigue resistance of a specimen under alternating loads. Some of the more important factors are (1,2)

1. State of stress
2. Range of stress
3. Stress gradient
4. Overstressing and understressing
5. Residual stresses
6. Stress concentrations
7. Sizem effect
8. Frequency of stress repetition
9. Mechanical properties
10. Metallurgical structure

The specific factors investigated in this thesis are the state of stress and the range of stress.

The state of stress of the specimens tested was combined bending and shear. In all but one test the range of stress was one-half the maximum stress. The range of stress for the one beam was three-fourths the maximum stress.
1.1 BACKGROUND

Despite the fact that for almost 50 years the effect of combined stresses on fatigue life has been investigated, few well-established rules\(^{6}\) have been developed for use in design. One reason for this is that the mechanics of failure of materials is not understood, therefore no perfect answer can be given by any one theory which would be applicable to all materials.

Some of the theories presented have been based on three of the failure theories; namely, the principal stress theory, the maximum shear stress theory and the distortion energy theory. These theories, stated analytically, for a combination of bending and shear or bending and torsion are respectively,

\[
\sigma_e = \frac{1}{2} \sigma_x + \sqrt{\sigma_x^2 + 4 \tau_{xy}^2} \quad (1.1)
\]

\[
\sigma_e = \sqrt{\sigma_x^2 + 4 \tau_{xy}^2} \quad (1.2)
\]

\[
\sigma_e = \sqrt{\sigma_x^2 + 3 \tau_{xy}^2} \quad (1.3)
\]

where \(\sigma_e\) is the equivalent unaxial stress, \(\sigma_x\) the normal stress due to bending and \(\tau_{xy}\) the shear stress due to either shear or torsion.

For steel, the maximum shear stress and distortion energy criterions predict the fatigue strength more reasonably than the principal stress theory, which appears to be best suited for cast-iron\(^{6}\), and are conservative.
Gough and Pollard concluded, \(^{(3,4,6\text{ and }7)}\) from tests done mainly from 1935 to 1949, that none of the failure theories either did or could explain fatigue failure. Hence, they proposed the empirical elliptical equation,

\[
\frac{\sigma^2}{\sigma_0^2} + \frac{\tau^2}{\tau_0^2} = 1
\]  \(1.4\)

in which \(\sigma_0\) and \(\tau_0\) represent the fatigue limit of a material in alternating plane bending and alternating torsion respectively, and \(\sigma\) and \(\tau\) the alternating plane bending stress and alternating torsion stress which acting together form a limiting combination.

Findley\(^{(5)}\) proposed that the deviation from the failure theories was in part due to anisotropy and that the maximum shear stress theory, corrected for anisotropy, would form the relation

\[
\sigma_e = \sqrt{\frac{\sigma^2}{\sigma_x^2} + \left(\frac{b}{h}\right)^2 \tau^2_{xy}}
\]  \(1.5\)

in which the term \(b/h\) represents the ratio of fatigue strength in pure bending to that in pure torsion.

1.1.1 Early Work By Reemynqer\(^{(2)}\)

In a study of four beams, which failed due to combined bending and shear, Reemynqer proposed an hypothesis based on the distortion energy failure theory and the static stress relationships of a beam, tested statically, which was of the same length and cross-section as the aforementioned four beams.
This hypothesis stated that,

The change in distortion energy in the
critical combined stress state for fatigue
failure equaled the change in distortion
energy in the critical uniaxial stress state
for fatigue failure.

It was assumed that,

1. The material in the critical region was
   homogeneous and isotropic.
2. The ratios $\sigma_1^{\min.}/\sigma_1^{\max.}$ and $\sigma_2^{\min.}/\sigma_2^{\max.}$
   remained constant during the load cycle.
3. The angle $\theta$ remained unchanged during the
   load cycle ($\theta$ was the angle between
   $\sigma_1$ and $\pi$-axis).
4. The combined stress state was essentially
   biaxial.

Assumption 1 was based on a considerable number of experimental
studies on the material properties. Assumptions 2 and 3 were validated
by the static test of the beam. Assumption 4 was made to simplify the
computations.

This hypothesis stated analytically was,

$$u_1^{\min.} - u_2^{\max.} = u_3^{\max.} - u_4^{\min.}$$  (1.6)
where
\[ u^C = \frac{1 + \mu}{3E} \left( \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \right) \] (1.7)
\[ u^U = \frac{1 + \mu}{3E} s^2 \] (1.8)

where \( \mu \) is Poisson’s Ratio, \( E \) is Young’s Modulus, \( u^C \) and \( u^U \) are the distortion energy for a combined stress state and a uniaxial stress state, \( \sigma_1 \) and \( \sigma_2 \) are the maximum and minimum principal stresses and \( S \) is the fatigue strength for uniaxial tension.

From the beam static test it was found that
\[ \frac{\sigma_1}{P} = \text{constant} \] (1.9)
\[ \frac{\sigma_2}{P} = \text{constant} \] (1.10)

where \( P \) is the total load. Thus
\[ \frac{\sigma_2}{\sigma_1} = \text{constant} \] (1.11)

Substituting Equations 1.7, 1.8 and 1.11 into Equation 1.6 results in

\[ \left( \sigma_{1\text{max.}}^2 - \sigma_{1\text{min.}}^2 \right) \left[ 1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2 \right] = s_\text{max.}^2 - s_\text{min.}^2 \] (1.12)

It had been shown that the four beams yielded in the fillet weld during the first load cycle, therefore

\[ s_{\text{max.}} = F_y \] (1.13)

where \( F_y \) is the uniaxial yield strength of the material.
By the definition of the stress range

$$\sigma_{1r} = \sigma_{1\text{max}} - \sigma_{1\text{min}}.$$  \hfill (1.14)

$$S_r = S_{\text{max}} - S_{\text{min}}.$$  \hfill (1.15)

and from the Hencky-von Mises yield criterion

$$\sigma_{1\text{max}}, \left[ 1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2 \right]^{1/2} = \sigma_{1r}.$$  \hfill (1.16)

With the substitution of Equations 1.13, 1.14, 1.15 and 1.16 in Equation 1.12 and rearranging terms

$$\frac{\sigma_{1r}^2 - \frac{2 \sigma_{1r} \sigma_{1r} \sigma_1^2}{1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2}^{1/2} + \frac{2 \sigma_{1r} S_r - S_r^2}{1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2} = 0 \hfill (1.17)$$

The roots of Equation 1.17 are

$$\sigma_{1r} = \frac{S_r}{\left[ 1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2 \right]^{1/2}} \hfill (1.18)$$

and

$$\sigma_{1r} = \frac{\frac{2 \sigma_{1r} S_r - S_r^2}{1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2}^{1/2}} \hfill (1.19)$$

Since in the range of data (for the four beams and nine other beams also studied for combined bending and shear) $S_r = \frac{P_Y}{2}$ in 12 out of 13 beams and $\left[ 1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2 \right]^{1/2} \leq 1.263$ for 11 out of 13 beams (See Tables 8 and 9), Equation 1.19 could result in a value of $\sigma_{1r}$ greater than the yield strength of the material. Thus the valid root is Equation 1.18.
By defining
\[ W_{DB} = \left[ 1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2 \right]^{1/2} \]  
Eq. (1.20)

Equation 1.18 becomes
\[ \sigma_{1r}^{cr} = \frac{S_0}{W_{DB}} \]  
Eq. (1.21)

which states that the critical stress range \( \sigma_{1r}^{cr} \) for a combined stress state equals the critical uniaxial stress range \( S_0 \) divided by a coefficient \( W_{DB} \) which is a function of the beam cross-section and location along the beam and is invariant with the load.

Reemlyder then compared the four beams which failed in the shear span to the regression line determined by the beams which failed in the pure moment region. The equation of this regression line was
\[ \log S_0 = 2.36180 - 0.29536 \log N \]  
Eq. (1.22)

where \( N \) denotes the fatigue life measured in kilocycles. The results are shown in Table 8 (beam numbers 4, 5, 6 and 14) in which two beams are conservative and two unconservative with an average percent error of 6.20%.

It was noted in the web region that there was considerable restraint to lateral strain and a tensile stress \( \sigma_3 \) would develop across the web thickness. Expanding the distortion energy hypothesis for a triaxial stress state, Equation 1.18 becomes
\[ S_0 = \sigma_{1r}^{cr} \left[ 1 - \frac{\sigma_2}{\sigma_1} + \left( \frac{\sigma_2}{\sigma_1} \right)^2 + \left( \frac{\sigma_3}{\sigma_1} \right)^2 - \left( \frac{\sigma_2}{\sigma_1} \right) \left( \frac{\sigma_3}{\sigma_1} \right) - \frac{\sigma_3}{\sigma_1} \right]^{1/2} \]  
Eq. (1.23)
With the substitution of Equation 1.20 and rearranging terms

\[ \sigma_{1r}^{cr} = \frac{S_r}{\left[ K_{DE}^2 - \sigma_3 / \sigma_1 (1 + \sigma_2 / \sigma_1) + (\sigma_3 / \sigma_1)^2 \right]^{1/2}} \]  

(1.24)

With \( \sigma_3 \) a tensile stress the terms in the bracket would be less than \( W_{DE}^2 \). This would increase the value of \( \sigma_{1r}^{cr} \) previously computed, thus causing the results to be more conservative.

1.2 PURPOSE AND SCOPE

The primary purposes of this study are to determine the effect of combined stresses on the fatigue resistance of longitudinal fillet weldments in "T-1" steel and to correlate the fatigue resistance for a combined stress state to the fatigue resistance for a uniaxial stress state.

For this study nine beams, in addition to the four previously mentioned, have been tested. For the correlation between the combined stress state and the uniaxial stress state, the distortion energy hypothesis as proposed by Reemstnyder will be utilized. In addition to this hypothesis, another one based on the maximum shear stress failure theory will be presented for comparison.
2. DESCRIPTION OF TESTS

2.1 TEST PROGRAM

To evaluate the effect of combined stresses, ten beams were tested at various shear stress to tension stress ratios. Four were tested with the loads 12" apart (12-1/2" shear span), four at loads 16" apart (10-1/2" shear span) and two at loads 20" apart (8-1/2" shear span). All ten beams were tested at a stress ratio of 1/2. Added to these ten beams were the four beams tested by Reemnynchter. Three of the four beams were tested at a stress ratio of 1/2 with loads 8" apart (14" shear span). The other beam was tested at a stress ratio of 1/4 with loads 8" apart (14-1/2" shear span).

One of the beams in the program, tested at loads 12" apart (12-1/2" shear span), failed in the pure moment region and data from it was therefore discarded for the purposes of this study. Thus the total number of beams studied for combined stresses was thirteen. The thirteen beams are summarized in Table 1 and Fig. 1.

To determine the stresses in the shear span at the point of failure, a static test was performed on beam 8-7 with seven strain rosettes placed along the tension fillet weld (See Fig. 2). The beam was tested at the four load spans mentioned above.
Tests to determine the stress-strain relationship of both the base metal and weld metal were also performed(2) and the results are tabulated in Tables 12 and 13.

2.2 TEST PROCEDURE

All specimens dynamically loaded were tested in a 220 kip Amieri Alternating Stress Machine at a frequency of 500 cycles per minute. The static tests were performed in the same machine and also in a 300 kip Universal Testing Machine. This equipment has been described in detail elsewhere. (10)

For the static test the beam was loaded at each of the four load spans mentioned previously. For the eight inch load span, readings were recorded at 30, 60, 95, 130, 160 and 190 kips, for the twelve and twenty inch load spans - 50, 100, 150 and 200 kips, and for the sixteen inch load span - 40, 80, 120, 160 and 200 kips.

The various stresses for each load, as determined by the static test were then divided by the corresponding load and an average stress-load ratio was calculated at each rosette for each of the four load spans. From these tests any stress at any point within the range of the rosettes could be calculated. It should be noted that the point of failure for two of the beams (B-13 and B-21) fell outside the range of the rosettes. These beams were loaded dynamically with loads placed 16" apart (10-1/2" shear span). To determine the stress at the point of failure another static test on beam B-7 was performed. For this test it was decided that
the shear span should be kept constant at 10-1/2" thus the center span was reduced from 16" to 8" leaving a 4-1/2" overhang at each end of the beam. This resulted in a small moment, due to the weight of the beam, at each of the supports, but the stress due to this moment was negligible when compared to the other stresses.

On each of the ten beams of the test program a strain gage was attached to the center of the top fillet weld (tension weld) at the mid-span of the beam. Each beam was first loaded statically to the desired strain. Thus the actual maximum stress in the weld at the midspan was determined. The dynamic loads, maximum load being equal to the static load and minimum load depending on the stress ratio, were then applied until failure.

Failure for all of the beams was a sudden break in the tension flange. The break was preceded by a crack which started either at the root of the weld or the faying surface between the web and tension flange. The crack first propagated into the web and then into the flange.

2.3 TEST SPECIMENS

The beam specimens used in the study were fabricated from flange quality "T-1" steel. The longitudinal axes of the specimens were parallel to the rolling direction of the plate.

The beams consisted of two 3" x 3/4" flange plates welded to a 5" x 3/8" web plate. The plate material was flame cut to the final
dimensions and then descaled by a Pangborn Roto Blaster with a vane angle of 78° using round steel shot, SAE 170, and an exposure time of ten minutes. The faying surfaces of the web plates were machined to insure good bearing on the flanges. The fillet welds consisted of four 1/4" automatic submerged arc welds over 3/16" x 4" manual tack welds at the midspan of the beam. To insure heat dissipation, the welds were alternated. (lower left, upper right, lower right, upper left) The beam length was 38" with a net span of 37". The cross-section of the beam is shown in Fig. 1.

The chemical composition, mechanical properties and metallurgical treatment of the plate material and weld material is summarized in Tables 12 and 13.
3. THEORETICAL ANALYSIS

By using the measured static stress relationships of beam B-7 and a rigorous application of the maximum shear stress failure theory, another hypothesis for fatigue failure in a combined stress state will now be presented. As in the hypothesis proposed by Reemanyder (Section 1.1.1) the same four assumptions will be made. That is

1. The material in the critical region is homogeneous and isotropic.
2. The ratios $\sigma_1^{\text{min}}/\sigma_1^{\text{max}}$ and $\sigma_2^{\text{min}}/\sigma_2^{\text{max}}$ remain unchanged during the load cycles,
3. The angle $\theta$ remains unchanged during the load cycle. ($\theta$ is the angle between $\sigma_1$ and the $x$-axis.)
4. The combined stress state is essentially biaxial.

This hypothesis is as follows:

The change in the maximum shear stress in the critical combined stress state for fatigue failure equals the change in the maximum shear stress in the critical uniaxial stress state for fatigue failure.

The hypothesis, written analytically, is

$$\tau_{\text{max.} c} - \tau_{\text{min.} c} = \tau_{\text{max.} u} - \tau_{\text{min.} u} \quad (3.1)$$
for which the maximum shear stress for combined stresses is

\[ \tau^c = \frac{1}{2} (\sigma_1 - \sigma_2) \]  \hspace{1cm} (3.2)

and the maximum shear for the uniaxial stress state is

\[ \tau^u = \frac{s}{2} \]  \hspace{1cm} (3.3)

all terms are as defined in Section 1.1.1. With the substitution of
Equations 3.2 and 3.3 into Equation 3.1 and noting that \( \sigma_2/\sigma_1 = \text{constant} \)
(Equation 1.11)

\[ \left( \sigma_{1_{\text{max}}}, \sigma_{1_{\text{min}}} \right) \left( 1 - \frac{\sigma_2}{\sigma_1} \right) = s_{\text{max}} - s_{\text{min}}. \]  \hspace{1cm} (3.4)

From the definition of the stress range

\[ \sigma_{1r} = \sigma_{1_{\text{max}}} - \sigma_{1_{\text{min}}}. \]  \hspace{1cm} (1.14)

\[ s_r = s_{\text{max}} - s_{\text{min}}. \]  \hspace{1cm} (1.15)

Equation 3.4 may be written as

\[ \sigma_{1r} = \frac{s_r}{(1 - \sigma_2/\sigma_1)} \]  \hspace{1cm} (3.5)

Letting

\[ W_{\text{MS}} = (1 - \sigma_2/\sigma_1) \]  \hspace{1cm} (3.6)

Equation 3.5 becomes

\[ \sigma_{1r} = \frac{s_r}{W_{\text{MS}}} \]  \hspace{1cm} (3.7)
This Equation is similar to Equation 1.21

\[ \sigma_{cr} = \frac{S_x}{W_{BS}} \]  

(1.21)

and also states that the critical stress range \( \sigma_{cr} \) for a combined stress state equals the critical uniaxial stress range \( S_x \) divided by a coefficient \( W_{BS} \) which is a function of the beam cross-section and location along the beam and is invariant with the load. The only difference between Equations 1.21 and 3.7 is in the coefficient \( W \).

Expanding the maximum shear stress hypothesis to a triaxial stress state it is noted that the failure theory states that half the difference between the maximum and minimum principal stress is equal to the shear stress. Since \( \sigma_3 \) is a tensile stress, and \( \sigma_2 \) is compressive, this theory is

\[ \frac{1}{2} (\sigma_1 - \sigma_2) = \tau \]  

(3.2)

which is the same as Equation 3.2. Thus with the same reasoning as in the biaxial stress state the critical stress range for a triaxial stress state is

\[ \sigma_{cr} = \frac{S_x}{W_{BS}} \]  

(3.7)

which is the same as the critical stress range for a biaxial stress state.

Another method of presenting the maximum shear stress and distortion energy hypotheses would be in terms of the normal stresses, \( \sigma_x \) and \( \sigma_y \), and the shear stress, \( \tau_{xy} \), instead of the principal stresses, \( \sigma_1 \) and \( \sigma_2 \).
From the maximum shear stress failure theory and Equation 3.3

\[ \sigma_1 - \sigma_2 = s \quad (3.8) \]

Knowing that

\[ \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \nu \frac{\sigma_{xy}}{\sigma_y}} \]

\[ \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \nu \frac{\sigma_{xy}}{\sigma_y}} \]

Substituting for the principal stresses, Equation 3.8 becomes

\[ s = \left(\sigma_x - \sigma_y\right) \sqrt{1 + 4 \left(\frac{\sigma_{xy}}{\sigma_x - \sigma_y}\right)^2} \quad (3.9) \]

Rearranging terms results in the non-dimensionalized equation

\[ \frac{\sigma_x - \sigma_y}{s} = \frac{1}{\sqrt{1 + 4 \left(\frac{\sigma_{xy}}{\sigma_x - \sigma_y}\right)^2}} \quad (3.10) \]

From the Hencky-von Mises Failure Criterion and Equation 1.13

\[ \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = s^2 \quad (3.11) \]

Once again substituting for the principal stresses

\[ \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3 \sigma_{xy}^2 = s^2 \quad (3.12) \]

Again non-dimensionalizing and taking the square root of Equation 3.12 results in

\[ \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2} \sqrt{\frac{1}{\sqrt{1 + 3 \left(\frac{\sigma_{xy}}{\sigma_x - \sigma_y}\right)^2}}} = \frac{1}{\sqrt{1 + 3 \left(\frac{\sigma_{xy}}{\sigma_x - \sigma_y}\right)^2}} \quad (3.13) \]
In both Equations 3.10 and 3.13 $\sigma_y$ is due to bending, $\tau_{xy}$ due to shear and $\sigma_y$ is due to the effect of the load application. If the quantity $\sigma_y$ could be reduced to a minimum and neglected the two Equations would respectively become

$$\frac{\sigma_y}{S} = \frac{1}{\sqrt{1 + 4 \left(\frac{\tau_{xy}}{\sigma_y}\right)^2}} \quad (3.16)$$

$$\frac{\sigma_x}{S} = \frac{1}{\sqrt{1 + 3 \left(\frac{\tau_{xy}}{\sigma_y}\right)^2}} \quad (3.15)$$

Then the two hypotheses could be compared at the same shear stress to normal stress ratio. But since $\sigma_y$ is a significant factor for the beams of this study, the shear stress to normal stress ratio of any beam in the program will be different for each hypothesis.
4. ANALYSIS OF RESULTS

The results of beams which failed in the shear span were compared in Fig. 3 to the results of welded tee specimens tested under the supervision of Reemsnyder at stress ratios of 1/2, 0, and -1. The tee specimen regression line for a stress ratio of 1/4 was determined from the regression lines for stress ratios of 1/2, 0, and -1.

The regression lines of the tee specimens estimated the fatigue lives of the beams reasonably well when based on the stress in the pure moment region. However, the stress at the point of failure, computed by simple bending theory, was considerably less than that in the pure moment region.

It has been pointed out(2) that a combined stress state may be more critical in fatigue than a uniaxial stress state. It was indicated(2) that the superposition of a shearing stress on a uniaxial tensile stress lowered the fatigue life. The most critical region in a cyclically loaded beam should then be directly over the load where the bending stress and the shear stress, each computed by simple bending theory, are each maximum and therefore produce the most critical combined stress state. However, after inspection of the beams, it was found that no cracks occurred over the load and that most of the cracks occurred in the shear span two to five inches outside the load (see Table 1). In the simple bending theory it is assumed that no normal stresses exist transverse to the longitudinal axis. The static test of beam B-7
however indicated a perturbation of the simple bending theory due to the
local affect of load application and showed that normal stresses \( \sigma_y \),
transverse to the longitudinal axis, existed in the fillet weld.

The distortion energy criterion for fatigue failure under com-
bined stresses is

\[
\sigma_{cr} = \left( \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 \right)^{1/2} 
\]

(4.1)

whereas the maximum shear stress criterion for fatigue failure in a
combined stress state is

\[
\tau_{cr} = 1/2 \left( \tau_1 - \tau_2 \right) 
\]

(4.2)

Rearranging Equations 4.1 and 4.2 and dividing by the total load results
in

\[
\frac{\sigma_{cr}}{P} = \frac{\sigma_1}{P} \left[ 1 - \sigma_2/\sigma_1 + (\sigma_2/\sigma_1)^2 \right]^{1/2} 
\]

(4.3)

\[
\frac{\tau_{cr}}{P} = \frac{\sigma_1}{2P} (1 - \sigma_2/\sigma_1) 
\]

(4.4)

Using the measured values of \( \sigma_1/P \) and \( \sigma_2/P \) (Tables 2, 3, 4 and 5) values
of \( \sigma_{cr}/P \) and \( \tau_{cr}/P \) were computed at two inch intervals along the fillet
weld for each of the four loads spans and are listed in Tables 6 and 7.
Figs. 4 and 5 are graphical representations of the two tables.

From Table 6 and Fig. 4 it can be seen that \( \tau_{cr}/P \) is critical
either in the shear span or at the load point in every case but the one
in which the shear span is largest. From the results of the static test
on beam B-7 it would seem that the minimum principal stress at two inches
from midspan is in error when compared with other points in the center section, particularly at zero and four inches from midspan. If this is true then \( \sigma_{cr}/P \) (and \( \tau_{cr}/P \)) would be less at this point. Thus for the largest shear span loading the critical region would then be either in the shear span or pure moment region which agrees with the test results in which about half of the beams failed in the shear span and the other half in the pure moment region. Thus from the maximum shear stress failure theory, it can be concluded that the shear span is critical.

However, the distortion energy criterion, Table 7 and Fig. 5, results in just the opposite where in all cases the critical \( \sigma_{cr}/P \) occurs in the center section. However, it was noted in Section 1.1.1 that a tensile stress \( \sigma_3 \), which was not measurable, existed in the weld region. Inclusion of this stress may or may not make the shear span the critical region, but due to this stress further investigation of the distortion energy theory would be advisable.

4.1 CORRELATION WITH THEORY

To test the hypotheses for fatigue failure in a combined stress state, the static stress relationships of beam B-7 were assumed to be representative for all beams failing in the shear span. Then Equations 1.21 and 3.7 could be applied with values of \( \sigma_{cr}^{\text{meas}} \), \( \tau_{cr} \), \( W_{D3} \) and \( W_{DE} \) being interpolated from the principal stress values listed in Tables 2, 3, 4 and 5.

First the beams failing in the shear span were compared to the beams which failed in the pure moment region. The critical uniaxial stress
for the beams was computed from Equation 1.22 which is the regression line determined from the beams which failed in the pure moment region

\[ \log S_y = 2.36130 - 0.29536 \log N \tag{1.22} \]

The predicted critical stress range from each hypothesis was then computed from Equations 1.21 and 3.7.

The comparison between \( \sigma_{1r}^{\text{calc.}} \) and \( \sigma_{1r}^{\text{meas.}} \) for the two hypotheses is shown in Table 8 and Fig. 6 where it can be seen that for the maximum shear stress hypothesis the predicted critical stress range is conservative for seven of the specimens and unconservative for six with an average percent error of 7.73\%. This type of scatter is common in fatigue testing.

From the same Table and Figure it is noted that the distortion energy hypothesis predicts a critical stress range greater than the measured critical stress for eleven of the thirteen specimens. The average percent error for the distortion energy hypothesis is 11.72\% which is about one and a half times as great as the average percent error of the maximum shear stress hypothesis.

The beam specimens were then compared to the welded tee specimens. The critical uniaxial stress range for the welded tee specimens was determined from the Equation (2)

\[ \log S_y = 2.425 - 0.3184 \log N \tag{4.5} \]
It should be noted that this regression line was computed from the welded tee specimens tested at stress ratios of 1/2, 0 and -1 and also from the beams that failed in the pure moment region.

The predicted critical stress range and the measured critical stress range for both hypotheses are compared in Table 9 and Fig. 7. Again the average percent error for the maximum shear stress hypothesis is less (7.87% versus 10.81%).

However, in this correlation, the maximum shear stress hypothesis is conservative for only five of the thirteen specimens whereas the distortion energy hypothesis is conservative for nine of the thirteen specimens.

Expanding the results to a triaxial stress state it is noted that for the maximum shear stress hypothesis the results would be the same. But for the distortion energy hypothesis the results would become even more conservative.

For the correlation based on the normal stresses and the shear stress, values for the uniaxial stress were computed from the critical uniaxial stress range and the critical uniaxial maximum stress. The stress range values were calculated from Equation 1.22

\[ \log S_r = 2.36180 - 0.29536 \log N \] (1.22)

whereas the maximum stress values were computed from

\[ S_{\text{max.}} = 200.2 - 46.62 \log N, \ R = 1/2 \] (4.6)

and \[ S_{\text{max.}} = 151.4 - 35.19 \log N, \ R = 1/4 \] (4.7)
which are the regression lines for welded tee specimens. The symbol \( R \) denotes the stress ratio. Values of \( \sigma_x, \sigma_y \) and \( \gamma_{xy} \) were interpolated from Tables 2, 3, 4 and 5.

The maximum shear stress hypothesis is plotted in Fig. 8 which is taken from Table 10. Fig. 9 and Table 11 correspond to the distortion energy hypothesis.

By making \( \gamma_{xy}/f(\sigma) \) the independent variable the predicted \( f(\sigma)/S \) falls above and below the measured \( f(\sigma)/S \) for both hypotheses. Again the distortion energy hypothesis is more conservative but still has an average percent error greater than the maximum shear stress hypothesis when based on stress range values. However, when based on the maximum stress values the average percent errors are about the same.

It should be noted that the results of Equations 3.10 and 3.13 should not be taken at face value since the quantity \( \sigma_y \) is included but would not be taken into consideration in designing for combined stresses.

In the correlations of these hypotheses, scatter would be due to the statistical nature of fatigue, incomplete representation of the dynamically tested beams by the statically tested beam B-7 and the interpolation of the stress values from the Tables.
5. CONCLUSIONS

The results as presented, though limited by the fact that most specimens were tested at a stress ratio equal to half the maximum stress, indicates that the hypothesis based on the maximum shear stress failure theory correlates the fatigue resistance under combined stresses to that of a uniaxially stress state more reasonably than the distortion energy hypothesis when based on principal stresses.

The correlation based on the normal stresses and shear stress indicates that there is not much of a distinction between the two hypotheses. If more work is done in this field the normal stress due to load application should be eliminated so that the two hypotheses could be compared at the same shear stress to bending stress ratio.

Also the results should not be applied to steels other than "T-1" steel or welds other than the Lincoln L-70 fillet weld. Though the maximum shear stress hypothesis predicted the fatigue resistance of the beams of this study more reasonably than the distortion energy hypothesis, the author believes that both hypotheses should be investigated if any work is done on other types of steels and welds.

If it is found in future work that the distortion energy hypothesis predicts the fatigue resistance more reasonably in other steels than the triaxial stress state should be investigated not only for lateral strain but also for residual stresses.
6. Nomenclature

\( b \) fatigue strength in pure bending
\( P_y \) uniaxial yield strength
\( N \) fatigue life, cycles or kilocycles
\( P \) load, kips
\( S \) uniaxial stress, ksi
\( t \) fatigue strength in pure torsion
\( U \) distortion energy
\( W \) function of beam cross-section and location along beam
\( \sigma \) normal stress, ksi
\( \tau \) shear stress, ksi
\( \theta \) angle between maximum principal stress and longitudinal axis
7. TABLES AND FIGURES
<table>
<thead>
<tr>
<th>Beam</th>
<th>R</th>
<th>Midspan</th>
<th>$S_{\text{max}}$, ksi</th>
<th>Point of Failure</th>
<th>Life, kilocycles</th>
<th>Load Span, inches</th>
<th>Shear Span, inches</th>
<th>Crack Location Measured from Midspan</th>
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<tbody>
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<td>47.0</td>
<td>36.9</td>
<td>2155.4</td>
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<td>74.1</td>
<td>61.78</td>
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<td>14</td>
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<td>37.1</td>
<td>29.43</td>
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<td>1937.6</td>
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<td>10-1/2</td>
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<td>11.20*</td>
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* Crack in Weld Only
** Crack in Weld and Web Only
**TABLE 1 SUMMARY OF BEAMS (continued)**

<table>
<thead>
<tr>
<th>Beam</th>
<th>R</th>
<th>$S_{\text{max}}$</th>
<th>Point of Failure</th>
<th>Life span kilocycles</th>
<th>Load Span inches</th>
<th>Shear Span inches</th>
<th>Crack Location Measured from Midspan</th>
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</thead>
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<td>B-18</td>
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<td>57.2</td>
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<td>11.00, 9.50*</td>
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<td>50.0</td>
<td>32.19</td>
<td>1720.6</td>
<td>20</td>
<td>8-1/2</td>
<td>12.50, 12.00*</td>
</tr>
<tr>
<td>B-21</td>
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<td>59.5</td>
<td>0.113</td>
<td>623.4</td>
<td>16</td>
<td>10-1/2</td>
<td>14.50*, 14.50*</td>
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<td>B-22</td>
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<td>28.62</td>
<td>1809.5</td>
<td>20</td>
<td>8-1/2</td>
<td>13.00, 15.00*, 26.00*, 26.50*, 26.75*, 27.125*, 27.50*</td>
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<td>37.01</td>
<td>1988.2</td>
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<td>11.00, 1.00*, 8.25*, 11.25*, 21.75*</td>
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<tr>
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<td>2705.0</td>
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<td>12-1/2</td>
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* Crack in Weld Only
** Crack in Weld and Web Only
**TABLE 2  STATIC TEST - BEAM B-7 - FIRST CYCLE**

**STRESSES AT BOTTOM OF FLANGE**

<table>
<thead>
<tr>
<th>Rosette*</th>
<th>Measured from Midspan</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>22</th>
<th>25</th>
<th>37</th>
<th>40</th>
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<tbody>
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<td>x</td>
<td>inches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\frac{\sigma_1}{F}$</td>
<td>Meas.</td>
<td>0.204</td>
<td>0.271</td>
<td>0.329</td>
<td>0.358</td>
<td>0.413</td>
<td>0.409</td>
<td>0.424</td>
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<tr>
<td></td>
<td>Calc.</td>
<td>0.200</td>
<td>0.249</td>
<td>0.295</td>
<td>0.340</td>
<td>0.399</td>
<td>0.388</td>
<td>0.388</td>
</tr>
<tr>
<td>$\frac{\sigma_2}{F}$</td>
<td>Meas.</td>
<td>-0.0832</td>
<td>-0.0628</td>
<td>-0.0443</td>
<td>-0.0347</td>
<td>-0.0405</td>
<td>-0.0101</td>
<td>0.0333</td>
</tr>
<tr>
<td></td>
<td>Calc.</td>
<td>-0.0240</td>
<td>-0.0193</td>
<td>-0.0143</td>
<td>-0.0138</td>
<td>-0.0106</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>Meas.</td>
<td>-0.410</td>
<td>-0.232</td>
<td>-0.1346</td>
<td>-0.0970</td>
<td>0.0982</td>
<td>-0.0247</td>
<td>0.0786</td>
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<tr>
<td></td>
<td>Calc.</td>
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<td>-0.0775</td>
<td>-0.0485</td>
<td>-0.0406</td>
<td>-0.0246</td>
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<td>0</td>
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<tr>
<td>$\Theta$</td>
<td>Meas.</td>
<td>27.0°</td>
<td>23.3°</td>
<td>22.8°</td>
<td>19.0°</td>
<td>9.58°</td>
<td>3.37°</td>
<td>1.34°</td>
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<td>$\frac{\sigma_{44}}{F}$</td>
<td>Meas.</td>
<td>0.172</td>
<td>0.245</td>
<td>0.308</td>
<td>0.344</td>
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<td>0.176</td>
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<td>0.388</td>
<td>0.388</td>
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<tr>
<td>$\frac{\sigma_{45}}{F}$</td>
<td>Meas.</td>
<td>-0.0516</td>
<td>-0.0381</td>
<td>-0.0333</td>
<td>-0.0208</td>
<td>0.0448</td>
<td>-0.00973</td>
<td>0.0334</td>
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<tr>
<td></td>
<td>Calc.</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{\tau_{45}}{F}$</td>
<td>Meas.</td>
<td>0.0698</td>
<td>0.0674</td>
<td>0.0667</td>
<td>0.0729</td>
<td>0.0371</td>
<td>0.0151</td>
<td>0.0044</td>
</tr>
<tr>
<td></td>
<td>Calc.</td>
<td>0.0691</td>
<td>0.0691</td>
<td>0.0651</td>
<td>0.0690</td>
<td>0.0651</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Location of Rosettes is shown in Fig. 2*


<table>
<thead>
<tr>
<th>Rosette</th>
<th>Measured from Midspan</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>22</th>
<th>25</th>
<th>37</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>x inches</td>
<td></td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

| $\sigma_1$ | Meas. | .202 | .269 | .306 | .324 | .359 | .364 | .363 |
| $\sigma_1$ | Calc. | .200 | .249 | .295 | .340 | .335 | .335 | .335 |
| $\sigma_2$ | Meas. | -.0920 | -.0661 | -.0524 | -.0219 | .0222 | .0150 | .0248 |
| $\sigma_2$ | Calc. | -.0240 | -.0193 | -.0143 | -.0138 | 0 | 0 | 0 |
| $\sigma_2$ | Meas. | -.459 | -.246 | -.1715 | -.0677 | .0618 | .0412 | .0663 |
| $\sigma_2$ | Calc. | -.120 | -.0775 | -.0485 | -.0406 | 0 | 0 | 0 |
| $\theta$ | Meas. | 29.0° | 23.4° | 21.4° | 13.4° | 3.26° | 1.38° | 1.55° |
| $\theta$ | Calc. | 19.1° | 15.6° | 12.4° | 11.1° | 0 | 0 | 0 |
| $\sigma_n$ | Meas. | .163 | .240 | .286 | .318 | .358 | .364 | .363 |
| $\sigma_n$ | Calc. | .176 | .230 | .281 | .339 | .335 | .335 | .335 |
| $\sigma_{xy}$ | Meas. | -.0535 | -.0347 | -.0328 | -.0165 | .0226 | .0150 | .0251 |
| $\sigma_{xy}$ | Calc. | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{xy}$ | Meas. | .1005 | .0955 | .0816 | .044 | .0113 | .00503 | .00565 |
| $\sigma_{xy}$ | Calc. | .0691 | .0691 | .0651 | .0690 | 0 | 0 | 0 |

*Location of Rossette is shown in Fig. 2*
TABLE 4  STATIC TEST - BEAM D-7 - THIRD CYCLE
STRESSES AT BOTTOM OF FLANGE

$p =$ Total Load
$P/2$ was applied below Rosette 7
Midspan at Rosette 40

<table>
<thead>
<tr>
<th>Rosette*</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>22</th>
<th>25</th>
<th>37</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured from Midspan $x$ inches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Meas.</td>
<td>.201</td>
<td>.243</td>
<td>.279</td>
<td>.2915</td>
<td>.314</td>
<td>.326</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Calc.</td>
<td>.200</td>
<td>.249</td>
<td>.295</td>
<td>.284</td>
<td>.281</td>
<td>.281</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Meas.</td>
<td>-.0768</td>
<td>-.0663</td>
<td>-.0365</td>
<td>.00410</td>
<td>.0300</td>
<td>.0186</td>
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<tr>
<td>$\sigma_2$</td>
<td>Calc.</td>
<td>-.0246</td>
<td>-.0193</td>
<td>-.0143</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$\sigma_3$</td>
<td>Meas.</td>
<td>-.382</td>
<td>-.199</td>
<td>-.131</td>
<td>.0141</td>
<td>.0954</td>
<td>.0570</td>
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<tr>
<td>$\sigma_3$</td>
<td>Calc.</td>
<td>-.120</td>
<td>-.0775</td>
<td>-.0485</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>$\theta$</td>
<td>Meas.</td>
<td>$27.0^\circ$</td>
<td>$16.9^\circ$</td>
<td>$13.2^\circ$</td>
<td>$3.34^\circ$</td>
<td>$4.35^\circ$</td>
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<td>$\theta$</td>
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<td>$15.6^\circ$</td>
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<tr>
<td>$\sigma_n$</td>
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<td>.281</td>
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<td>.281</td>
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<td>.0691</td>
<td>.0651</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Location of Rosettes is shown in Fig. 2
<table>
<thead>
<tr>
<th>Rosette*</th>
<th>Measured from Midspan</th>
<th>1</th>
<th>6</th>
<th>7</th>
<th>22</th>
<th>25</th>
<th>37</th>
<th>40</th>
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<tbody>
<tr>
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<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>$\sigma_1$</td>
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<td>.218</td>
<td>.237</td>
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<td>.249</td>
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<tr>
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<td>Calc.</td>
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<td>.249</td>
<td>.228</td>
<td>.230</td>
<td>.228</td>
<td>.228</td>
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<td>.0094</td>
<td>.0042</td>
<td>.0112</td>
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<td>Calc.</td>
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<td>-.0193</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>.0639</td>
<td>.0378</td>
<td>.0170</td>
<td>.0655</td>
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<td>$\sigma_1$</td>
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<td>-.0775</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>15.3°</td>
<td>3.54°</td>
<td>1.38°</td>
<td>1.59°</td>
<td>2.08°</td>
<td>1.39°</td>
</tr>
<tr>
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<td>Calc.</td>
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<td>15.6°</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>$\sigma_3$</td>
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<td>.255</td>
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<td>.247</td>
<td>.246</td>
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<tr>
<td>P</td>
<td>Calc.</td>
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<td>-.0157</td>
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<td>$\tau_{xy}$</td>
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<td>.0383</td>
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<td>Calc.</td>
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<td>4&quot;</td>
<td>6&quot;</td>
<td>8&quot;</td>
<td>10&quot;</td>
<td>12&quot;</td>
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$\sigma_1/P$ = .424, .408, .413, .358, .329, .271, .204

$(1 - \sigma_2/\sigma_1)$ = .9214, 1.0247, .9018, 1.097, 1.1346, 1.232, 1.410

$\tau_{cr}/P$ = .195, .209, .186, .196, .186, .167, .143

$\sigma_1/P$ = .363, .364, .359, .324, .306, .269, .202

$(1 - \sigma_2/\sigma_1)$ = .9317, .9588, .9382, 1.0677, 1.1719, 1.246, 1.459

$\tau_{cr}/P$ = .169, .174, .168, .173, .179, .168, .148

$\sigma_1/P$ = .303, .326, .314, .2915, .279, .249, .201

$(1 - \sigma_2/\sigma_1)$ = .9082, .9430, .9046, .9259, 1.131, 1.199, 1.382

$\tau_{cr}/P$ = .138, .154, .142, .144, .158, .140, .139

$\sigma_1/P$ = .246, .247, .249, .238, .257, .218, .189

$(1 - \sigma_2/\sigma_1)$ = .9545, .9830, .9622, .9361, .9151, 1.0995, 1.354

$\tau_{cr}/P$ = .117, .122, .120, .111, .118, .120, .128

$x = \text{distance along fillet weld from midspan}$

$\tau_{cr} = \text{equivalent uniaxial fatigue strength from maximum shear stress criteria}$

$\tau_{cr}/P = \sigma_1/2P \times (1 - \sigma_2/\sigma_1)$

$\sigma_1/P$ & $\sigma_2/\sigma_1$ from Tables 2, 3, 4, and 5
### TABLE 7  EQUIVALENT UNIAXIAL FATIGUE STRENGTH

DISTORTION ENERGY THEORY - BEAM B-7

<table>
<thead>
<tr>
<th>Distance From Load to Midspan (in.)</th>
<th>α</th>
<th>0</th>
<th>2'</th>
<th>4'</th>
<th>6'</th>
<th>8'</th>
<th>10'</th>
<th>12'</th>
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<td></td>
<td></td>
<td>α</td>
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<td>σ₁/Ρ</td>
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<td>.408</td>
<td>.413</td>
<td>.358</td>
<td>.329</td>
<td>.271</td>
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<td>W₀</td>
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<td>1.074</td>
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<td>σₑ/P</td>
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<td>.414</td>
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<td>σ₁/Ρ</td>
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<td>.364</td>
<td>.359</td>
<td>.324</td>
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<td>.269</td>
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<tr>
<td></td>
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<td>σₑ/P</td>
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<td>.956</td>
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<td>.317</td>
<td>.301</td>
<td>.289</td>
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<td>.247</td>
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<td>.993</td>
<td>.982</td>
<td>.970</td>
<td>.955</td>
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<td>σₑ/P</td>
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<td>.245</td>
<td>.244</td>
<td>.231</td>
<td>.245</td>
<td>.230</td>
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</tbody>
</table>

- α = distance along fillet weld from midspan
- σₑ = equivalent uniaxial fatigue strength from distortion energy criteria

$$σ_{cr} = \sigma_1/\rho \left[ 1 - \sigma_2/\sigma_1 + (\sigma_2/\sigma_1)^2 \right]^{1/2}$$

$$\sigma_1/\rho & \sigma_2/\sigma_1 \text{ (From Tables 2, 3, 4, and 5)}$$

$$W_{de} = \left[ 1 - \sigma_2/\sigma_1 + (\sigma_2/\sigma_1)^2 \right]^{1/2}$$
### TABLE 8  CORRELATION OF HYPOTHESES

<table>
<thead>
<tr>
<th>Beam</th>
<th>$\sigma_{\text{meas.}}$</th>
<th>$S_T$</th>
<th>$\sigma_{\text{calc.}} = \frac{S_F}{W}$</th>
<th>% Error</th>
<th>$\sigma_{\text{calc.}} = \frac{S_F}{W}$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-4</td>
<td>19.6</td>
<td>23.83</td>
<td>1.153</td>
<td>20.66</td>
<td>- 5.56</td>
<td>1.085</td>
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<tr>
<td>B-5</td>
<td>30.5</td>
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<td>1.1216</td>
<td>28.4</td>
<td>6.89</td>
<td>1.066</td>
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<td>1.1348</td>
<td>29.84</td>
<td>9.22</td>
<td>1.074</td>
</tr>
<tr>
<td>B-14</td>
<td>23.43</td>
<td>26.55</td>
<td>1.1119</td>
<td>23.82</td>
<td>- 1.66</td>
<td>1.060</td>
</tr>
<tr>
<td>B-16</td>
<td>18.93</td>
<td>27.16</td>
<td>1.252</td>
<td>21.64</td>
<td>- 14.31</td>
<td>1.147</td>
</tr>
<tr>
<td>B-17</td>
<td>22.20</td>
<td>24.60</td>
<td>1.1746</td>
<td>20.96</td>
<td>5.59</td>
<td>1.098</td>
</tr>
<tr>
<td>B-18</td>
<td>19.71</td>
<td>27.56</td>
<td>1.323</td>
<td>20.80</td>
<td>5.53</td>
<td>1.194</td>
</tr>
<tr>
<td>B-20</td>
<td>19.25</td>
<td>25.47</td>
<td>1.388</td>
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<td>4.41</td>
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<td>1.263</td>
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<td>22.29</td>
<td>1.1984</td>
<td>18.62</td>
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<td>1.113</td>
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</table>

**Average Error**
- Maximum Shear Stress  
  ave. error = 7.73%
- Distortion Energy  
  ave. error = 11.72%

**Note:**  
\[ \text{% Error} = \frac{\text{Meas.} - \text{Calc.}}{\text{Meas.}} \times 100 \]

\[ \log S_x = 2.36180 - 0.29536 \log N \]
<table>
<thead>
<tr>
<th>Beam</th>
<th>$\sigma_{\text{meas.}}$</th>
<th>$S_r$</th>
<th>$\sigma_{\text{cr}}$</th>
<th>$\frac{S_r}{\sigma_{\text{cr}}}$</th>
<th>$%$ Error</th>
<th>$\sigma_{\text{cr}}$</th>
<th>$%$ Error</th>
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</table>

Average Error - Maximum Shear Stress
ave. error = 7.57%

Distortion Energy
ave. error = 10.61%

Note:

$$\% \text{ error} = \frac{\text{Meas.} - \text{Calc.}}{\text{Meas.}} \times 100$$

Log $S_r = 2.425 - 0.3184 \log N$
<table>
<thead>
<tr>
<th>Beam</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
<th>$\frac{\sigma_x - \sigma_y}{S}$</th>
<th>$\frac{\sigma_x - \sigma_y}{2\tau}$</th>
<th>$%$ Error</th>
<th>$\frac{S}{S_{\text{max.}}}$</th>
<th>$%$ Error</th>
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</table>

**Average Error - Stress Range**

ave. error = 7.59%

**Maximum Stress**

ave. error = 10.06%

**Note:**

\[
\% \text{ Error} = \frac{\text{Meas.} - \text{Calc.}}{\text{Meas.}} \times 100
\]
### TABLE 12  CALCULATIONS FOR $f(\sigma)/S$ vs. $\tau_{xy}/f(\sigma)$ DIAGRAM

#### DISTORTION ENERGY HYPOTHESIS

<table>
<thead>
<tr>
<th>Beam</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
<th>$\frac{\sqrt{\sigma_x^2 - \sigma_y^2 + \tau_{xy}^2}}{S}$ calc.</th>
<th>$\frac{\sqrt{\sigma_x^2 - \sigma_y^2 + \tau_{xy}^2}}{S}$ meas.</th>
<th>% Error</th>
<th>$S_{\text{max.}}$</th>
<th>$S_{\text{max.}}$: meas.</th>
<th>% Error</th>
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<tr>
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<td>10.77</td>
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<td>.799</td>
<td>-12.64</td>
<td>44.79</td>
<td>.851</td>
<td>-5.76</td>
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<td>56.80</td>
<td>3.98</td>
<td>15.32</td>
<td>.912</td>
<td>.924</td>
<td>1.30</td>
<td>64.18</td>
<td>.910</td>
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<td>B-6</td>
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<td>4.70</td>
<td>17.08</td>
<td>.908</td>
<td>.949</td>
<td>4.21</td>
<td>68.80</td>
<td>.935</td>
<td>2.89</td>
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<tr>
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<td>29.43</td>
<td>1.90</td>
<td>7.85</td>
<td>.913</td>
<td>.861</td>
<td>-6.06</td>
<td>39.67</td>
<td>.768</td>
<td>-18.22</td>
</tr>
<tr>
<td>B-16</td>
<td>33.73</td>
<td>-5.44</td>
<td>13.84</td>
<td>.837</td>
<td>.678</td>
<td>-23.44</td>
<td>53.75</td>
<td>.685</td>
<td>-22.19</td>
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<td>B-17</td>
<td>42.36</td>
<td>-5.80</td>
<td>9.45</td>
<td>.939</td>
<td>.925</td>
<td>-1.51</td>
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<td>.970</td>
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<td>B-18</td>
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<td>-6.90</td>
<td>16.64</td>
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<td>.679</td>
<td>-17.39</td>
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<tr>
<td>B-20</td>
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<td>-8.63</td>
<td>18.97</td>
<td>.742</td>
<td>.732</td>
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<td>B-21</td>
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<td>23.53</td>
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<td>B-22</td>
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<td>.673</td>
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<td>-5.50</td>
<td>40.19</td>
<td>.924</td>
<td>4.65</td>
</tr>
</tbody>
</table>

**Average Error - Stress Range**  
\[ \text{ave. error} = 10.05\% \]

**Maximum Stress**  
\[ \text{ave. error} = 10.23\% \]

**Note:**  
\[ \% \text{Error} = \frac{\text{Meas.} - \text{Calc.}}{\text{Meas.}} \times 100 \]
### Table 12. Properties of Plate Material

<table>
<thead>
<tr>
<th>Plate Size</th>
<th>Yield Strength ksi</th>
<th>Ultimate Strength ksi</th>
<th>% Elong. in 2&quot;</th>
<th>% Red. of Area</th>
<th>% C</th>
<th>% Hg</th>
<th>% P</th>
<th>% S</th>
<th>% Si</th>
<th>% Ni</th>
<th>% Cr</th>
<th>% Mo</th>
<th>% Cu</th>
<th>% B</th>
<th>% Va</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8</td>
<td>114.0</td>
<td>122.7</td>
<td>19.5</td>
<td>53.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/4</td>
<td>117.2</td>
<td>126.7</td>
<td>21.2</td>
<td>59.2</td>
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<td></td>
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</tbody>
</table>

#### Heat Treating Cycle

- **Austenitizing Temp.**: 1700°F
- **Tempering Temp.**: 1250°F

<table>
<thead>
<tr>
<th>Time</th>
<th>Heat Treatment</th>
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</thead>
<tbody>
<tr>
<td>3/4&quot; P_L</td>
<td>70 min.</td>
</tr>
<tr>
<td>3/8&quot; P_L</td>
<td>43 min.</td>
</tr>
</tbody>
</table>

*0.2% Offset

Modulus of Elasticity = 29.5 x 10^3 ksi

Poisson's Ratio = 0.323
<table>
<thead>
<tr>
<th>WELD</th>
<th>PROCESS</th>
<th>ELECTRODE</th>
<th>MECHANICAL PROPERTIES</th>
<th>CHEMICAL COMPOSITION</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Yield Strength ksi</td>
<td>Tensile Strength ksi</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>3/16&quot; Tack</td>
<td>Manual</td>
<td>1/8&quot; diameter E11018</td>
<td>101</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>Shield</td>
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<tr>
<td></td>
<td>Metal Arc</td>
<td>Atom Arc T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final 1/4&quot; Fillet</td>
<td>Automatic</td>
<td>5/64&quot; diameter Lincoln Electric No. L-70</td>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>
Fig. 1 GEOMETRY OF BEAMS
Fig. 2 STATIC TEST - BEAM B-7 - INSTRUMENTATION
Fig. 3 RESULTS OF BEAMS FAILING IN SHEAR SPAN
Fig. 4 EQUIVALENT CRITICAL STRESS - MAXIMUM SHEAR STRESS CRITERION
Fig. 5  EQUIVALENT CRITICAL STRESS - DISTORTION ENERGY CRITERION
Note: Predicted Critical Stress Range Based on Eq.1.22

LEGEND

○ - Maximum Shear Stress Theory
△ - Distortion Energy Theory

Fig. 6 CORRELATION OF HYPOTHESES
Note: Predicted Critical Stress Range Based on Eq. 4.5

LEGEND
- Maximum Shear Stress Theory
- Distortion Energy Theory

Fig. 7 CORRELATION OF HYPOTHESES
LEGEND

○ - Stress Range
△ - Maximum Stress

Fig. 8 NORMAL STRESS/UNIAXIAL STRESS vs. SHEAR STRESS/NORMAL STRESS
Fig. 9 NORMAL STRESS/UNIAXIAL STRESS vs. SHEAR STRESS/NORMAL STRESS
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9. V I T A

The author was born in Philadelphia, Pennsylvania on October 25, 1939, the third child of Mr. and Mrs. Warren E. Feeman.

He graduated from Edison High School in Philadelphia in June 1957. He then attended Drexel Institute of Technology in Philadelphia from September 1957 to June 1962. There he completed the requirements of the Department of Civil Engineering and was awarded the degree of Bachelor of Science in Civil Engineering in June 1962.

He was appointed to a research assistantship at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania in September 1962 and has been associated with the project on fatigue of fillet weldments in "7-1" steel and with industrial testing.