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THE MOMENT CURVATURE RELATIONS
FOR
COMPOSITE BEAMS

by
Charles Culver

This work has been carried out as part of an investigation sponsored by the American Institute of Steel Construction.

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

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1. Introduction

Composite beams composed of a concrete slab supported by a steel wide flange section are frequently used in bridge and building construction. In order to compute the moment resistance, deflections, and rotations of the composite section, the moment-curvature relations must be established. This paper presents the results of a study to derive equations for the moment-curvature relations in the elastic plastic region for a concrete steel composite beam.

In the analysis of the composite section, the Bernoulli-Navier hypothesis (bending strain is proportional to the distance from the neutral axis) is assumed to hold. On this basis, the effective width of the concrete slab is divided by the modular ratio \( n \). This reduces the effective width and essentially transforms the two element composite section (concrete-steel) to one material, usually steel, since \( n \) is taken as \( \frac{E_s}{E_C} \). The composite section is then treated as a steel beam unsymmetrical about the axis of bending.

In the elastic range the ordinary methods of analysis used in engineering mechanics are applicable for determining the \( M-\phi \) relations. After the section has yielded, however, the penetration of plastification or the amount of the beam which has
reached the yield stress must be determined before the curvature or the moment resistance for any particular strain distribution can be computed.

For WF shapes the discontinuities of the cross-section make direct computation troublesome. In this report a "point-by-point" method is developed for computing $M-\phi$ curves for composite beams.

2. **Assumptions**

The following assumptions were made in order to develop the moment-curvature relations in this report:

1. The Bernoulli-Navier hypothesis (bending strain is proportional to the distance from the neutral axis) holds.

2. There is complete interaction between the concrete slab and the steel beam, i.e., - There is no slip or relative displacement at the inner face of slab and beam.

3. The stress-strain relations for concrete and steel are those given in Figs. 1 and 2.

4. The effect of strain hardening is neglected.

5. The yield stress is the same for both the web and flange of the steel beam.

6. The tensile strength of concrete is negligible.

7. The beam is subjected to transverse loads only. These loads lie in the plane of symmetry of the cross-section.

8. The influence of vertical shear stress is neglected.
3. **Method of Solution**

The first step in the solution of this problem is to compute the location of the neutral axis or line of zero stress in the elastic range. Since the neutral axis and the centroidal axis coincide for sections with one axis of symmetry, this step involves determination of the location of the centroidal axis. With the location of the neutral axis determined, the yield moment is obtained by use of the elementary bending equation $M_y = \frac{\sigma_y J}{c}$. The curvature or $\phi$ at any section in the elastic range can then be determined by use of the relation $\phi = \frac{M}{EI}$.

With the moment curvature relations in the elastic region known, the next logical step would be to consider the elastic plastic region and determine the $M-\phi$ relations in this range. The analysis will be simplified, however, if the upper limit of the elastic plastic range, the plastic moment, is computed first. The plastic moment and the yield moment are upper and lower bounds for this elastic plastic region and with these bounding values known, the trial and error procedure which will be used in this region and is subsequently outlined will be easier.

In this report a distinction will be made between the plastic moment and the ultimate moment. The plastic moment of a section will be defined as that moment at which the maximum percentage of the given section is stressed to its capacity.

* See Nomenclature pg. 16 for definition of symbols
Computation of the plastic moment involves consideration of the strains which the section must undergo to reach this moment. These strains put limits on the ratios of the cross sectional dimensions of the section. If the section is to develop the plastic moment as defined above the ratios of the cross sectional dimensions must fall within these limits.

To establish these limiting ratios, a stress distribution which is compatible with the definition for the plastic moment is chosen, i.e., a stress distribution such that all or most of the elements of the composite section are working at their maximum capacity. By using the relationship between stress and strain (Fig. 1 and 2) the stress distribution may be converted into a strain distribution. Consideration of equilibrium of horizontal forces ($\int \sigma dA = 0$) and the geometry of the strain distribution leads to various relations or ratios between the cross sectional dimensions of the composite section such as the ratio of the area of the steel section to the area of the concrete slab. This procedure is outlined in Appendix C.

Prior to determining the plastic moment of a particular section, the ratios of the necessary cross sectional dimensions are computed and compared with the ranges for these quantities given in Appendix C. If these ratios fall within the limits given the plastic moment may be determined by applying the formula for $M_p$. If the ratios of the cross sectional dimensions
do not fall within the limits given the section will fail prior to achieving the plastic moment. For this case the section will have failed before the maximum percentage of the cross section is stressed to its capacity. In this case the maximum moment will be termed the ultimate moment. For most commonly used composite beams the ratios of the cross sectional dimensions are such that they fall within the ranges given in Appendix C and the formula given may be used to compute the plastic moment.

After the quantities in the elastic range and at the plastic moment have been determined, it remains to find the $M-\phi$ relations in the elastic plastic region between the yield moment and the plastic moment.

The limit of elastic action is reached when either the outer fibers of the steel beam reach the yield stress and/or the outer fibers of the concrete slab reach the cylinder strength, $f'_c$. After reaching this limit of elastic action, yielding penetrates into the steel beam, the slab, or both. Thus, the ensuing stress distributions are such that a certain proportion of the composite section has reached the yield stress. There are many possible stress distributions for any composite section in this elastic plastic region. In this report, the moment curvature relations for the thirteen stress distributions which are most likely to occur were developed.
Since the composite section is unsymmetrical about the x axis the neutral axis must be located in either the upper half of the web of the steel beam, the top flange, or the concrete slab. The first step in deriving the equations for the elastic plastic region is to assume a location for the neutral axis and a given penetration of yielding into the steel beam and/or slab. These two assumptions completely fix the configuration of the particular stress distribution.

The second step is to reduce the stress distribution to resultant tensile and compressive forces by multiplying the stress by the area over which it acts. In order to satisfy equilibrium at the cross section \( \int A \sigma dA = 0 \) the sum of these tensile and compressive forces must be zero. Reduction of the equilibrium equation leads to an equation for the location of the neutral axis in terms of the proportions of the composite section and the penetration of yielding. The moment resistance for the stress distribution is obtained next by summing the moments produced by the resultant tensile and compressive forces obtained from the stress distribution.

The curvature may be obtained by converting the stress distribution to a strain distribution and then solving for \( \varphi \). The compressive force in the slab or the force which must be resisted by the shear connection is determined by summing the resultant compressive forces in the slab.
The procedure outlined above was used to derive the equations in Appendix A. A sample of the essential steps in this process is given for one particular stress distribution in Appendix A1.

In order to determine the $M-\phi$ relations in the elastic plastic region, one of the thirteen possible stress distributions is chosen and a penetration of yielding into the beam and/or slab assumed and the location of the neutral axis determined by using the equilibrium equation for that particular stress distribution. If the wrong stress distribution has been assumed the location of the neutral axis will not be compatible with the stress distribution assumed and another trial stress distribution must be assumed. Since the neutral axis must move from its position at $M_y$ to that at $M_p$ only those stress distributions which give the location of the neutral axis between these points need be assumed. This will narrow down the possible stress distributions from thirteen to approximately four or five.

4. Example Solution

The method of solution outlined in this report for determining the moment-curvature relations for a composite beam was used for the composite section shown in Fig. 4. The resulting $M-\phi$ curve is given in Fig. 5. This section was tested as part of a research program conducted at Lehigh University on composite beams. The solution for this section is outlined in part B of the Appendix.
The solution in the elastic range for the section in Fig. 4 located the neutral axis in the top flange at the yield moment. It also showed that yielding would occur first in the bottom flange. This indicated that the solution given in Appendix A-5 would hold for the initial stage of the elastic plastic region. Further penetration of yielding into the web of the steel beam caused the neutral axis to move upward. The neutral axis, however, remained in the top flange of the steel beam until yielding had progressed well up into the web. Since the stresses in the concrete were still below $f'_c$, the stress distribution in A-6 of the Appendix would occur next.

Further penetration of yielding into the web of the steel beam caused the neutral axis to move into the slab and the resulting stress distribution was that of A-10 in Appendix B. As yielding progressed through the web the upper fibers of the concrete reached $f'_c$ and the stress distribution in A-12 occurred next. When the yielding of the steel beam penetrated up into the top flange of the steel beam A-12 was no longer applicable and the stress distribution was given by A-13. Finally, with the entire steel section yielded and part of the concrete slab stressed to $f'_c$ the section reached the plastic moment as given in Appendix C. The equations for moment and curvature obtained for this section in the elastic plastic range are given in Table I. After assuming various values for $\eta$ or the penetration of yielding into the steel beam, these equations were solved and the results given in Table II were obtained. These results provided the data for plotting of the $M-\phi$ curve for the test section.
Using the M-Ø relations determined for the section tested, the non-dimensional moment deflection curve shown in Fig. 6 was obtained and compared with test results. The computed points were obtained by using the M-Ø relations and the conjugate beam method to determine the deflections. The solid curve represents values obtained from test results on the particular composite section. It will be noted that the test results are in good agreement with the computed results.

Of primary interest for any composite section, is the shear force developed between the slab and beam. This force must be transmitted by some type of shear connection between slab and beam. With the stress distribution at any section known the C force or compressive force in the slab at that section may be determined. Fig. 7 shows the value of this compressive force plotted along the length of the member.

The shear flow, or the force per unit length which the shear connection must transmit can then be determined from this plot of C force. Since shear flow is the change in C force over a given length \( q = \frac{dC}{dx} \) or the slope of the C force diagram, it may be determined by differentiation. The C force diagram was plotted point by point and there is no equation to represent this curve. Therefore a point by point method must be used to determine the slope of this diagram or the shear flow instead of direct differentiation. The slope was measured graphically at various points in order to determine the values of shear flow plotted in Fig. 8.
There are several points concerning this shear flow diagram which warrant discussion. The shear flow in the elastic range may be determined by using the formula \( q = \frac{V_m}{I} \). In the elastic-plastic region, however, the shear flow does not vary in the same manner as the external shear and therefore the formula \( \frac{V_m}{I} \) cannot be used to determine the shear flow. There has been some discussion\(^1\) concerning the use of this formula in both the elastic and the elastic plastic regions. Since \( q = \frac{V_m}{I} \) was derived from the elastic formula \( \sigma = \frac{M_e}{I} \), it is also an elastic formula and holds only in the elastic range. The error involved in such an approach is pointed out in Fig. 9.

In Fig. 9 a composite beam is subjected to some arbitrary loading. A section of the beam from the location of zero moment to the maximum moment (length "a") is isolated as a free body. Since in this case the maximum moment is equivalent to the plastic moment, the stress distribution and internal forces at this section are the same as those given in Appendix C. If the slab over the length "a" is then isolated as a free body it must be in equilibrium under the forces shown under Part B of Fig. 9. If the assumption that the formula \( q = \frac{V_m}{I} \) may be used in both the elastic and the elastic plastic region is correct, integration of this shear flow over the length "a" must result in a total force equal to the compressive force in the slab (C) at the location of \( M_p \) in order to produce equilibrium. Part C and D of Fig. 9 point out that this is not the case and therefore the assumption that \( q = \frac{V_m}{I} \) may be used in both the

---

elastic and the elastic plastic regions does not satisfy the basic requirement of equilibrium. Furthermore, the error produced by using this formula is not constant for all composite beam sections but is a function of the cross-sectional dimensions. 

The force carried by each shear connector is determined by summing up the shear flow between two adjacent rows of connectors. This is equivalent to integration of the shear flow diagram between connectors. Again, since there is no equation for the shear flow a graphical procedure must be used. The results of this graphical integration are given in Fig. 10 which lists the connector forces.

It will be noted that the connector forces in the elastic plastic region are considerably higher than those in the elastic region. Tests of the strength of individual connectors\(^2\) (push-out tests and double shear tests) indicate that the shear connectors in the elastic plastic region cannot carry these large forces. One possible explanation for this phenomenon is that the highly stressed connectors in the plastic region yield and deform and transmit the excess load (load above the carrying capacity of an individual connector) back to the less highly stressed connectors in the elastic region.

According to this assumption each connector would be carrying the same force at ultimate. If this assumption is valid then a uniform connector spacing could be used even if the shear diagram (not to be confused with the shear flow diagram) varied. The connectors in the regions of high shear would merely yield and transmit the load to the connectors in the regions of low shear.

From the above discussion one would expect that shear connector failure would occur first near the location of the maximum moment where the shear flow is largest since these connectors must deform considerably in order to transmit the load to the less highly stressed connectors. On the contrary, shear connector failure occurred first near the ends of the specimens tested.

The analysis discussed in this report was carried out assuming complete interaction or no slip between the slab and beam. In actuality this case never exists and slip does occur. This slip will alter the distribution of shear flow along the member and consequently the connector forces. Since the shear flow is dependent upon the slip distribution, this slip distribution must first be determined before the true shear flow and connector forces can be evaluated.
Any exact failure theory for composite beams would involve an analysis which considers the properties of the shear connection and the slip or relative displacement between slab and beam. Until such a failure theory is derived, the shear connection may be designed on the assumption that each connector is carrying the same load when the ultimate capacity of the section is reached. This assumption will lead to a satisfactory design if a ductile type shear connector, which will permit a redistribution of forces, is used.

5. **Summary**

The $M-\phi$ relations for a composite beam, in the elastic range, may be determined by the usual methods of engineering mechanics. In the elastic plastic range, however, a point by point method was adopted in this report due to the discontinuities of the cross section. This point by point method involves a trial and error procedure using the equations developed for the possible stress distributions over this elastic plastic range.

The equations derived in this report are only valid for the case of complete interaction or no slip between the elements of the composite section. Since complete interaction is never realized in an actual composite beam, the equations in this
report provide an upper bound to the solution of the problem. The development of a failure theory which would consider the deformations of the concrete slab relative to the steel beam must be developed before an exact solution to the problem of the M-Ø relations may be obtained.
6. ACKNOWLEDGEMENTS

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This report is part of an investigation on Composite Design for Buildings sponsored by the American Institute of Steel Construction. Work on this project is currently in progress at Fritz Laboratory, Lehigh University of which Lynn S. Beedle is the Director and Professor W. J. Eney, Head of the Department of Civil Engineering.
7. NOMENCLATURE

**Section Dimensions**

- \( A_c \) - area of concrete slab \( A_c = b_c d_c \)
- \( A_f \) - flange area of steel beam \( A_f = A_s - w(d_s - 2t_f) \)
- \( A_s \) - area of steel beam
- \( b_c \) - width of concrete slab
- \( C \) - total compressive force at any cross section
- \( c \) - distance from neutral axis of composite section to extreme fiber of steel in tension
- \( d_c \) - depth of concrete slab
- \( d_s \) - depth of steel section
- \( e \) - distance between resultant compressive and tensile forces at \( M_p \)
- \( f'_c \) - cylinder strength of concrete at 28 days
- \( \sigma_y \) - yield stress of steel beam
- \( I \) - moment of inertia of composite section, concrete transformed to equivalent steel area
- \( M_y \) - theoretical yield moment
- \( M_p \) - theoretical plastic moment
- \( m \) - statical moment of transformed compressive concrete area about the neutral axis of the composite section
- \( n = \frac{E_{steel}}{E_{concrete}} \)

**Stress Distributions**
7. NOMENCLATURE (Continued)

Q - Connector force
q - Shear flow per unit length
s - Connector spacing along the longitudinal axis of the beam
T - Total tensile force at any cross section
t_f - Flange thickness
V - External shear force
w - Thickness of Web of steel beam
a - Location of neutral axis in steel beam
\( \eta \) - Percentage penetration of plastification in steel beam
\( \mu \) - Location of neutral axis in slab
\( \xi \) - Percentage penetration of ultimate stress into concrete slab.
\( \varepsilon_{st} \) - Strain hardening strain for steel \( \varepsilon_{st} = 15 \times 10^{-3} \) in/in.
\( \varepsilon_{yc} \) - Yield strain for concrete \( \varepsilon_{yc} = 1.0 \times 10^{-3} \) in/in.
\( \varepsilon_{ys} \) - Yield strain for steel \( \varepsilon_{ys} = \frac{\sigma_y}{E_s} \)
\( \varepsilon_u \) - Ultimate concrete strain \( \varepsilon_u = 3.0 \times 10^{-3} \) in/in.
\( \delta \) - Deflection of beam in inches.
Summary of Equations

Of the many stress distributions for a composite beam in the elastic plastic region the author chose to investigate thirteen possible stress distributions which would be most likely to occur. The equations which determine the location of the neutral axis \((a, \mu)\) were formulated by considering equilibrium of horizontal forces at any cross section, i.e., \(\int \sigma dA = 0\). The equations for the moment resistance provided by a particular stress distribution were determined by resolving the stress distribution into equivalent tensile and compressive forces and summing the moments produced by these forces.

A-1 Stress distribution One - Neutral axis in web of steel beam; part of bottom flange of steel beam plastic, slab elastic

A-1.1 Stresses - using the stress distribution diagram the following stresses were determined by means of similar triangles. Since the concrete slab was transformed to an
equivalent steel area by use of the modular ratio "n", the stresses $\sigma_1$ and $\sigma_2$ given below are stresses in the transformed section. In order to obtain the actual stresses in the concrete of the composite section, the stresses in the transformed section ($\sigma_1$ and $\sigma_2$) must be divided by the modular ratio "n".

$$\sigma_1 = \sigma_y \left[ \frac{d_c}{d_s} \frac{2 + \frac{d_s}{d_c} - 2\alpha \frac{d_s}{d_c}}{1 + 2\alpha - 2\eta \frac{t_f}{d_s}} \right]$$

$$\sigma_2 = \sigma_y \left[ \frac{1-2\alpha}{1 + 2\alpha - 2\eta \frac{t_f}{d_s}} \right]$$

$$\sigma_3 = \sigma_y \left[ \frac{1-2\alpha - 2 \frac{t_f}{d_s}}{1+2\alpha - 2\eta \frac{t_f}{d_s}} \right]$$

$$\sigma_4 = \sigma_y \left[ \frac{1+2\alpha - 2 \frac{t_f}{d_s}}{1+2\alpha - 2\eta \frac{t_f}{d_s}} \right]$$

Al.2 Forces - The forces shown in the force diagram are computed by multiplying the stress by the area over which it acts. For example, the compressive force $C_1$ is obtained by multiplying the transformed area of the slab by the triangular stress distribution,
the compressive force $C_2$ is obtained by multiplying the transformed area of the slab by the rectangular stress distribution, $C_3$ and $C_4$ are the compressive forces in the top flange resulting from the triangular and rectangular stress distributions, $C_5$ is the compressive force resulting from the stress triangle in the web above the neutral axis. The force $T_1$ is the tensile force caused by the triangle of stress below the neutral axis in the web, $T_2$ and $T_3$ are tensile forces resulting from the rectangle and triangle of stress in the bottom flange, and $T_4$ is the tensile force due to the rectangle of yield stress in the plastic portion of the bottom flange.

\[
C_1 = \frac{\sigma_1 - \sigma_2}{2} \frac{A_c}{n}
\]
\[
C_2 = \frac{\sigma_2}{n} \frac{A_c}{n}
\]
\[
C_3 = \frac{\sigma_2 - \sigma_3}{2} \frac{A_r}{2}
\]
\[
C_4 = \sigma_3 \frac{A_r}{2}
\]
\[
C_5 = \frac{\sigma_3}{2} w \left( \frac{ds}{2} - ads - t_f \right)
\]
\[
T_1 = \frac{\sigma_4}{2} w \left( \frac{ds}{2} + ads - t_f \right)
\]
\[
T_2 = \frac{\sigma_4}{2} \frac{A_r}{2} (1-\eta)
\]
\[
T_3 = \frac{\sigma_y - \sigma_4}{2} \frac{A_r}{2} (1-\eta)
\]
\[
T_4 = \frac{\sigma_y}{2} \frac{A_r}{2} \eta
\]
Al.3 Equilibrium - In order to satisfy equilibrium at the section, the equation $\int \sigma dA = 0$ must be satisfied. By equating the sum of the horizontal forces to zero this equation is satisfied. Summing horizontal forces, considering compressive forces positive and tensile forces negative, and reducing leads to the following:

$$C_1 + C_2 + C_3 + C_4 + C_5 - T_1 - T_2 - T_3 - T_4 = 0$$

$$\frac{A_c}{n} \left[ 1 + \frac{d_c}{d_s} - 2\alpha \right] + \frac{A_f}{2} \left[ \eta^2 \frac{t_f}{d_s} - 4\alpha \right] + \frac{wds}{2} \left[ \beta \frac{t_f}{d_s} - 4\alpha \right] = 0$$

Al.4 Location of the Neutral Axis -

Using the equilibrium equation and solving for $\alpha$ the location of the neutral axis is determined in terms of $\eta$ or the percentage of the bottom flange which has reached the yield stress. For each composite beam there is a unique value for $\alpha$ for each value of $\eta$ for this stress distribution. In order for the equation to be valid for a particular section, i.e., for the particular section to assume this stress distribution the value of $\alpha$ determined from this equation must fall within the prescribed limits and the
value of the stress at the extreme fiber of the slab must be less than $n f'_c$. If after assuming value for $\eta$, the computed value for $\alpha$ does not fall within the prescribed limits or the concrete stress exceeds $n f'_c$, this means that the section cannot develop the assumed stress distribution.

$$\alpha = \frac{1}{2} \left[ \frac{A_c (1 + \frac{d_c}{d_s}) + A_f}{2A_s d_s} \frac{t_f}{d_s} \eta^2 \right]$$

The neutral axis for a composite section must always to above mid depth of the steel section (assuming symmetrical rolled steel beams) and since this equation only holds when the neutral axis is in the web the following limits may be applied:

$$0 < \alpha \leq \frac{1}{2} - \frac{t_f}{d_s} \quad 0 < \eta \leq 1.0$$

**Al.5 Moment Resistance for Given Stress Distribution** -

Taking moments of the forces shown in the force diagram about the bottom flange of the steel beam and simplifying the equation leads to:
\[
M_1 = \left[ \frac{\sigma_y A_s d_s}{1+2a-2\eta t_f d_s} \right] \left\{ \frac{A_c}{nA_s} \left( 1+\frac{3}{2} \frac{d_c}{d_s} - 2a - \alpha \frac{d_c}{d_s} + \frac{2}{3} \left[ \frac{d_c}{d_s} \right]^2 \right) + \frac{A_f}{A_s} \right\}
\]

\[
\left( \frac{1}{2} - \frac{t_f}{d_s} - a + \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^2 + \frac{n^3}{6} \left[ \frac{t_f}{d_s} \right]^2 \right) + \frac{w d_s}{A_s} \left( \frac{1}{6} + 2\alpha \frac{t_f}{d_s} - a - \frac{t_f}{d_s} + 2 \left[ \frac{t_f}{d_s} \right]^2 \right)
\]

\[
\frac{1}{3} \left[ \frac{t_f}{d_s} \right]^3
\]

It will be noted that only the first term contains any dimensional quantities, all the other terms are non-dimensional ratios or parameters. Taking the yield stress in kips per square inch, the area of the steel beam in square inches, and the depth of steel beam in inches will result in units of kip inches for the moment.

Al. 6 Determination of Curvature (\(\phi\))

\[\tan \phi = \frac{\varepsilon_y}{\gamma}\]

Since \(\phi\) is small the tangent of the angle and the angle itself are approximately equal

\[\phi = \frac{\varepsilon_y}{\gamma} = \frac{\varepsilon_y}{\frac{d_s}{2} + \alpha d_s - \eta t_f \frac{t_f}{d_s}} = \frac{2 \varepsilon_y}{d_s + 2\alpha d_s - 2\eta t_f \frac{t_f}{d_s}}\]

Al. 7 Determination of Compressive Force in the Slab

\[C = C_1 + C_2 = \frac{\sigma_y A_c}{n} \left[ \frac{1-2\alpha + \frac{d_c}{d_s}}{1+2\alpha - 2\eta \frac{t_f}{d_s}} \right]\]
APPENDIX A

Comment on Method used - The moment equation and the equilibrium equation contain the unknown parameters $a$ and $\eta$. Using the expression for $a$ given in A-1.4 the moment equation could be reduced to a function of $\eta$ only. This would lead to a more complicated expression. It is much easier to assume a given penetration of yielding ($\eta$), determine $a$ from the equilibrium equation and then compute the moment using these values of $a$ and $\eta$.

A-2 Stress Distribution Two - Neutral axis in web of steel beam; bottom flange and part of web of steel beam plastic, slab elastic

A2.1 Location of Neutral Axis

\[
\alpha' = \frac{1}{2} \left[ \frac{A_c}{nA_s} (1 + \frac{d_c}{d_s}) + \frac{A_f}{A_s} (\eta - \frac{1}{2} \frac{t_f}{d_s}) + \frac{wds}{A_s} (\eta^2 + \left[ \frac{t_f}{d_s} \right]^2 - 2\eta \frac{t_f}{d_s}) \right]
\]

\[
= \frac{A_c}{nA_s} + \frac{A_f}{A_s} + \frac{wds}{A_s} (1 - 2 \frac{t_f}{d_s})
\]

\[
0 < \alpha \leq \frac{1}{2} - \frac{t_f}{d_s}, \quad \frac{t_f}{d_s} \leq \eta
\]

A2.2 Moment Resistance
\[ M_2 = \left[ \frac{\sigma_y A_s d_s}{1 + 2\alpha - 2\eta} \right] \left\{ \frac{A_c}{n A_s} \left( 1 + \frac{3}{2} \frac{d_c}{d_s} - 2\alpha - \alpha \frac{d_c}{d_s} + \frac{2}{3} \left[ \frac{d_c}{d_s} \right]^2 \right) + \frac{A_f}{A_s} \left( \frac{1}{2} - \alpha - \frac{t_f}{d_s} + \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^2 + \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^3 \right) \right\} \]

\[ \left( 1 - \frac{3}{2} \frac{d_c}{d_s} - 2\alpha \right) d_s - 2t_f \] 

\[ \frac{1}{6} \alpha + \frac{2}{3} \frac{t_f}{d_s} - \frac{t_f}{d_s} + 2 \left[ \frac{t_f}{d_s} \right]^2 - \eta \left[ \frac{t_f}{d_s} \right]^3 \]

A2.3 Curvature \( \phi \)

\[ \phi = \frac{2 \varepsilon_y}{d_s + 2 \alpha d_s - 2\eta d_s} \]

A2.4 Compressive Force in the Slab

\[ C = \frac{\sigma_y A_c}{n} \left[ \frac{1 - 2\alpha + \frac{d_c}{d_s}}{1 + 2\alpha - 2\eta} \right] \]

A3 Stress Distribution Three - Neutral axis in web of steel beam; part of bottom flange of steel beam plastic, part of concrete slab at ultimate strength.

A3.1 Location of Neutral Axis

\[ \alpha = \left[ \frac{A_f}{2A_s} - \eta \frac{t_f}{d_s} + \frac{A_c}{A_s} \frac{f'_{ct}}{\sigma_y} \left( \frac{1}{2} + \frac{1}{2n} \frac{\sigma_y}{f'_{ct}} \right) \frac{-\xi}{2n} \frac{\sigma_y}{f'_{ct}} - \eta \frac{t_f}{d_s} + \frac{\xi}{2} \right] \]

\[ -n \xi \frac{t_f}{d_s} \left[ \frac{2 \frac{A_f}{A_s} + \frac{wds}{A_s} \left( 2 - 4 \frac{t_f}{d_s} \right) + \frac{A_c}{A_s} \frac{f'_{ct}}{\sigma_y} \left( \frac{1}{n} \frac{\sigma_y}{f'_{ct}} - \frac{\xi}{n} \frac{\sigma_y}{f'_{ct}} - 1 - \xi \right)}{2 \frac{A_f}{A_s} + \frac{wds}{A_s} \left( 2 - 4 \frac{t_f}{d_s} \right) + \frac{A_c}{A_s} \frac{f'_{ct}}{\sigma_y} \left( \frac{1}{n} \frac{\sigma_y}{f'_{ct}} - \frac{\xi}{n} \frac{\sigma_y}{f'_{ct}} - 1 - \xi \right)} \right] \]
A second equation for $a$ is obtained from the stress distribution by means of similar triangles.

$$\frac{\sigma_y}{\frac{d_s}{2} + a \cdot d_s - \eta t_f} = \frac{nf'_c}{\frac{d_s}{2} - a \cdot d_s - \xi d_c + d_c}$$

$$a = \frac{\left[ \frac{1}{2} - \frac{\xi d_c}{ds} + \frac{d_c}{ds} - \frac{n}{2} \cdot \frac{f'_c}{\sigma_y} + \frac{n \cdot t_f}{\sigma_y} \cdot \frac{f'_c}{ds} \right]}{1 + n \cdot \frac{f'_c}{\sigma_y}}$$

The two equations are solved simultaneously for $a$ and $\xi$ by assuming values for $\eta$. In order for this stress distribution to hold the values of $a$, $\eta$, and $\xi$ must be within the following limits:

$$0 < a \leq \frac{1}{2} - \frac{t_f}{ds} \quad 0 < \eta \leq 1.0 \quad 0 < \xi$$

A3.2 Moment Resistance

$$M_3 = \left[ \frac{\sigma_y A_s d_s}{1+2a-2\eta \cdot \frac{t_f}{ds}} \right] \left\{ \frac{A_c \left( \frac{\varepsilon}{3} \cdot \frac{f'_c}{\sigma_y} + \frac{a \cdot f'_c}{\sigma_y} \cdot \frac{d_c}{ds} - \frac{\xi \cdot f'_c}{\sigma_y} \cdot \frac{d_c}{ds} + a \cdot \frac{f'_c}{\sigma_y} \cdot \frac{t_f}{ds} - \frac{2a \cdot \varepsilon}{3} \cdot \frac{f'_c}{\sigma_y} \cdot \frac{d_c}{ds} - \frac{a \cdot \varepsilon}{3} \cdot \frac{f'_c}{\sigma_y} \cdot \frac{d_c}{ds} - \eta \cdot \frac{f'_c}{\sigma_y} \cdot \frac{t_f}{ds}}{\frac{2 \eta \varepsilon}{3} \cdot \frac{f'_c}{\sigma_y} \cdot \frac{d_c}{ds} + \frac{\eta \varepsilon}{3} \cdot \frac{f'_c}{\sigma_y} \cdot \frac{d_c}{ds} + \frac{1}{2n} \cdot \frac{f'_c}{\sigma_y} \cdot \frac{d_c}{ds}} \right\}$$

$$\frac{a}{n} - \frac{\varepsilon}{2n} + \frac{a \varepsilon}{n} + \frac{1}{6n} \cdot \frac{d_c}{ds} - \frac{a}{3n} \cdot \frac{d_c}{ds} - \frac{\varepsilon}{3n} \cdot \frac{d_c}{ds}$$
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\[ + \frac{2a\xi}{3n} \frac{dc}{ds} - \frac{\xi^2}{6n} \frac{dc}{ds} - \frac{\xi^2}{3n} \frac{dc}{ds} + \frac{1}{2} \frac{f_c'}{\sigma_y} + \frac{1}{3} \frac{c}{\sigma_y} \frac{dc}{ds} + \]

\[ a \frac{f_c'}{\sigma_y} + \frac{2a}{3} \frac{f_c'}{\sigma_y} \frac{dc}{ds} - \eta \frac{f_c'}{\sigma_y} \frac{tf}{ds} - \frac{2\eta}{3} \frac{f_c'}{\sigma_y} \frac{dc}{ds} \frac{tf}{ds} + \frac{Ar}{As} \]

\[ (\frac{1}{2} - a - \frac{tf}{ds} + \frac{2}{3} \left[ \frac{tf}{ds} \right]^2 + \frac{n^3}{6} \left[ \frac{tf}{ds} \right]^2 ) + \frac{wds}{As} (\frac{1}{6} - a \]

\[ - \frac{tf}{ds} + 2 \alpha \frac{tf}{ds} + 2 \left[ \frac{tf}{ds} \right]^2 - \frac{1}{3} \left[ \frac{tf}{ds} \right]^3 \} \]

A3.3 Curvature \( \phi \)

\[ \phi = \frac{2\varepsilon_y}{d_s + 2a d_s - 2\eta tf} \]

A3.4 Compressive Force in the Slab

\[ C = \sigma_y A_c \left[ \frac{n \xi f_c'}{2\sigma_y} + \frac{nf_c'}{2\sigma_y} + \frac{1 - \frac{\xi}{2} - a + \alpha \xi}{1 + 2a - 2\eta \frac{tf}{ds}} \right] \]

A4. Stress Distribution Four - Neutral Axis in web of steel beam; bottom flange and part of steel beam in plastic, part of concrete slab at ultimate strength.

A4.1 Location of Neutral axis

\[ a = \begin{bmatrix} \frac{Ar(\eta - \frac{1}{2} \frac{tf}{ds}) + wds}{As} (\eta^2 + \left[ \frac{tf}{ds} \right]^2 - 2\eta \frac{tf}{ds}) + \frac{Ac(\xi f_c'}{2\sigma_y} - \eta \xi f_c'}{\sigma_y} + \\
\frac{1}{2n} + \frac{1}{3} \frac{f_c'}{\sigma_y} - \eta \frac{f_c'}{\sigma_y} \left[ \frac{2A_r}{A_s} + \frac{wds}{A_s} (2 - 4 \frac{tf}{ds}) + \frac{Ac}{As} \right. \left. \left( \frac{1}{n} - \frac{\xi f_c'}{\sigma_y} - \frac{\xi}{n} - \frac{f_c'}{\sigma_y} \right) \right] \end{bmatrix} \]
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\[ a = \left[ \frac{1}{2} + \eta n \frac{f_c^l}{\sigma_y} - \frac{n}{2} \frac{f_c^l}{\sigma_y} + \frac{d_c}{\sigma_y} - \xi \frac{d_c}{\sigma_y} \right] \]

\[ \frac{1 + n \frac{f_c^l}{\sigma_y}}{1 + n \frac{f_c^l}{\sigma_y}} \]

\[ 0 < a \leq \frac{1}{2} - \frac{t_f}{d_s} \quad \frac{t_f}{d_s} \leq \eta \quad 0 < \xi \]

A4.2 Moment Resistance

\[ M_4 = \left( \frac{\sigma_y A_s d_s}{1 + 2\alpha - 2\eta} \right) \left\{ \frac{A_c}{A_s} \left( \frac{\xi}{2} \frac{f_c^l}{\sigma_y} + \frac{\xi}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} - \frac{\xi}{6} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} + \alpha \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} \right) \right\} \]

\[ + \frac{2\alpha}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} - \frac{\alpha \xi}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} - \eta \xi \frac{f_c^l}{\sigma_y} + \frac{2\eta \xi}{3} \]

\[ \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} + \frac{\eta \xi}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} + \frac{1}{2} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} + \alpha \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} - \eta \xi \frac{f_c^l}{\sigma_y} + \frac{2\eta \xi}{3} \]

\[ \frac{1}{2n} - \frac{\alpha}{n} + \frac{1}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} + \frac{2\alpha}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} - \frac{2\eta}{3} \frac{f_c^l}{\sigma_y} \frac{d_c}{d_s} + \frac{1}{6n} \frac{d_c}{d_s} - \frac{\alpha}{3n} \frac{d_c}{d_s} - \frac{\xi}{3n} \frac{d_c}{d_s} + \frac{2\alpha \xi}{3n} \frac{d_c}{d_s} - \frac{\xi}{2n} + \frac{\alpha \xi}{n} + \frac{\xi^2}{6n} \frac{d_c}{d_s} - \frac{\alpha \xi^2}{3n} \frac{d_c}{d_s} \]

\[ + \frac{A_F}{A_s} \left( \frac{1}{2} - \frac{t_f}{d_s} + \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^2 \right) \frac{4wds}{A_s} \left( \frac{1}{6} + 2 \left[ \frac{t_f}{d_s} \right]^2 \right) \]

\[ + \frac{\eta^3}{3} - \eta \left[ \frac{t_f}{d_s} \right]^2 - \alpha - \frac{t_f}{d_s} + 2 \alpha \frac{t_f}{d_s} - \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^3 \right\} \]
A4.3 Curvature \( \phi \)

\[
\phi = \frac{2 \varepsilon_y}{d_s + 2a d_s - 2\eta d_s}
\]

A4.4 Compressive Force in the Slab

\[
c = \frac{\sigma_y A_c}{n} \left[ \frac{n s f'_c}{2 \sigma_y} + \frac{n t f'_c}{2 \sigma_y} + \frac{1}{2} - a - \frac{\xi}{2} + \alpha \zeta \right] \frac{1 + 2a - 2\eta}{1 + 2a - 2\eta}
\]

A5. Stress Distribution Five - Neutral Axis in Top Flange of steel beam; part of bottom flange of steel beam plastic, slab elastic.

A5.1 Location of Neutral Axis

\[
a = \frac{1}{2} \left[ \frac{A_c}{n A_s} (1 + \frac{d_c}{d_s}) + \frac{A_f}{2 A_s} \left( \eta^2 \frac{t_f}{d_s} \right) \right]
\]

\[
\left[ \frac{A_c}{n A_s} + \frac{A_f}{A_s} + \frac{w d_s}{A_s} (1 - 2 \frac{t_f}{d_s}) \right]
\]

\[
\frac{1}{2} - \frac{t_f}{d_s} \leq a \leq \frac{1}{2}
\]

0 \leq \eta \leq 1.0

A5.2 Moment Resistance

\[
M_5 = \frac{\sigma_y A_s d_s}{1 + 2a - 2\eta} \frac{t_f}{d_s} \left\{ \frac{A_c}{A_s} \left( \frac{3}{2n} \frac{d_c}{d_s} + \frac{2}{3n} \left[ \frac{d_c}{d_s} \right]^2 + \frac{1}{n} - \frac{2a}{n} - \frac{a}{n} \frac{d_c}{d_s} \right) \right.
\]

\[
+ \frac{A_f}{A_s} \left( \frac{1}{2} - a + \frac{n^3}{6} \left[ \frac{t_f}{d_s} \right]^2 - \frac{t_f}{d_s} + \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^2 \right) \frac{w d_s}{A_s}
\]

\[
\left. \left( \frac{1}{6} + 2 \left[ \frac{t_f}{d_s} \right]^2 - a - \frac{t_f}{d_s} + 2a \frac{t_f}{d_s} - \frac{4}{3} \left[ \frac{t_f}{d_s} \right]^3 \right) \right\}
\]
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A5.3 Curvature $\psi$

$$ \psi = \frac{2 \varepsilon_y}{d_s + 2a d_s - 2\eta_t f} $$

A5.4 Compressive Force in the Slab

$$ C = \frac{\sigma_y A_c}{n} \left[ 1 - 2 \alpha + \frac{d_c}{d_s} \right] \left[ 1 + 2 \alpha - 2\eta \frac{t_f}{d_s} \right] $$

A6. Stress Distribution Six - Neutral axis in top flange of steel beam; bottom flange and part of the web of steel beam plastic, slab elastic

A6.1 Location of Neutral Axis

$$ \alpha = \frac{1}{2} \left[ \frac{A_c}{n A_s} (1 + \frac{d_c}{d_s}) + \frac{A_f}{A_s} (\eta - \frac{1}{2} \frac{t_f}{d_s}) + \frac{w_d s}{A_s} (\eta^2 - 2\eta \frac{t_f}{d_s} + \frac{[\frac{t_f}{d_s}]^2}{d_s^2}) \right] $$

$$ \left[ \frac{A_c}{n A_s} + \frac{A_f}{A_s} + \frac{w_d s}{A_s} (1 - 2 \frac{t_f}{d_s}) \right] $$

$$ \frac{1}{2} - \frac{t_f}{d_s} \leq \alpha \leq \frac{1}{2} \quad \frac{t_f}{d_s} \leq \eta $$
A6.2 Moment Resistance

\[ M_6 = \left[ \frac{\sigma_y A_s d_s}{1+2\alpha - 2\eta} \right] \left\{ \frac{A_c}{A_s} \left[ \frac{3}{2n} \frac{d_c}{d_s} + \frac{2}{3n} \left[ \frac{d_c}{d_s} \right]^2 + \frac{1}{n} \right] - 2\alpha - \frac{a}{n} \frac{d_c}{d_s} \right\} \]

\[ + \frac{A_f}{A_s} \left[ \frac{1}{2} - \alpha - \frac{t_f}{d_s} + \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^2 + \frac{\eta}{2} \frac{t_f}{d_s} \right] \frac{w d_s}{A_s} \]

\[ \left\{ \frac{1}{6} \frac{n^3}{3} + 2 \left[ \frac{t_f}{d_s} \right]^2 - \frac{t_f}{d_s} - \alpha + 2\alpha \frac{t_f}{d_s} - \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^3 - \eta \left[ \frac{t_f}{d_s} \right]^2 \right\} \]

A6.3 Curvature \( \varphi \)

\[ \varphi = \frac{2} {d_s + 2\alpha} \frac{\sigma_y}{d_s - 2\eta} \]

A6.4 Compressive Force in the Slab

\[ C = \sigma_y A \frac{1 - 2\alpha + \frac{d_c}{d_s}} {n} \left[ \frac{1 + 2\alpha - 2\eta} {1 + 2\alpha - 2\eta} \right] \]

A7. Stress Distribution Seven - Neutral axis in the top flange of the steel beam; part of the bottom flange of the steel beam plastic, part of the concrete slab at ultimate strength.

A7.1 Location of the Neutral Axis
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\[
\alpha = \left[ \frac{A_c}{A_s} \left( \frac{\xi}{2} \frac{f'_c}{\sigma_y} - \eta \xi \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} + \frac{1}{2} \frac{f'_c}{d_s} \frac{t_f}{\sigma_y} + \frac{1}{2n} - \frac{\xi}{2n} \right) \right.
\]

\[
+ \frac{A_f}{A_s} \left( \frac{n}{2} \frac{t_f}{d_s} \right)
\]

\[
\left. \frac{A_c}{A_s} \left( \frac{1}{n} - \xi \frac{f'_c}{\sigma_y} - \eta \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} \right) + 2 \frac{A_f}{A_s} + \frac{w_d s}{A_s} \left( 2 - 4 \frac{t_f}{d_s} \right) \right]
\]

\[
\alpha = \left[ \frac{1}{2} + \frac{d_c}{d_s} - \xi \frac{d_c}{d_s} - n \frac{f'_c}{2 \sigma_y} + n \eta \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} \right]
\]

\[
\left. \frac{1}{2} + \frac{d_c}{d_s} - \xi \frac{d_c}{d_s} - n \frac{f'_c}{2 \sigma_y} + n \eta \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} \right]
\]

\[
1 + n \frac{f'_c}{\sigma_y}
\]

\[
\frac{1}{2} - \frac{t_f}{d_s} \leq \alpha \leq \frac{1}{2} \quad 0 < \eta \leq 1.0 \quad 0 < \xi
\]

A7.2 Moment Resistance

\[
M_7 = \left[ \frac{\sigma_y A_s d_s}{1 + 2n - 2n t_f} \right]
\]

\[
\left\{ \frac{A_c}{A_s} \left( \frac{1}{2n} - \frac{\alpha}{8n} + \frac{\alpha \xi}{n} + \frac{1}{6n} \frac{d_c}{d_s} - \frac{\alpha}{3n} \frac{d_c}{d_s} \right) \right.
\]

\[
- \frac{\xi}{3n} \frac{d_c}{d_s} - \xi^2 \frac{d_c}{6n} + \frac{\alpha \xi^2}{3n} \frac{d_c}{d_s} + \frac{\xi}{2} \frac{f'_c}{\sigma_y} +
\]

\[
\left. \frac{\xi}{2} \frac{f'_c}{\sigma_y} - \eta \xi \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} + \frac{\xi}{3} \frac{f'_c}{\sigma_y} \frac{d_c}{d_s} + \frac{2a \xi}{3} \frac{f'_c}{\sigma_y} \frac{d_c}{d_s} \right]
\]

\[
- \frac{2n \xi}{3} \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} - \xi^2 \frac{f'_c}{\sigma_y} \frac{d_c}{d_s}
\]

\[
\frac{\alpha \xi^2}{3} \frac{f'_c}{\sigma_y} \frac{d_c}{d_s} + \frac{\eta \xi^2}{3} \frac{f'_c}{\sigma_y} \frac{d_c}{d_s} \left( \frac{1}{3} \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} + \frac{1}{2} \frac{f'_c}{\sigma_y} + \frac{\alpha \xi}{3n} \right)
\]

\[
\eta \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} + \frac{2a \xi}{3} \frac{f'_c}{\sigma_y} \frac{d_c}{d_s} -
\]

\[
\frac{2n}{3} \frac{f'_c}{\sigma_y} \frac{d_c}{d_s} \left( \frac{1}{3} \frac{f'_c}{\sigma_y} \frac{t_f}{d_s} + \frac{2a \xi}{3n} \frac{d_c}{d_s} \right) \]
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\[ + \frac{A_f}{A_s} \left( \frac{1}{2} + \frac{\eta}{6} \left[ \frac{t_f}{d_s} \right]^2 + \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^2 - \alpha - \frac{t_f}{d_s} \right) \]

\[ + \frac{wds}{A_s} \left( \frac{1}{6} + 2 \alpha \frac{t_f}{d_s} - \frac{t_f}{d_s} - \alpha + 2 \left[ \frac{t_f}{d_s} \right]^2 - \frac{4}{3} \left[ \frac{t_f}{d_s} \right]^2 \right) \]

A7.3 Curvature \( \varphi \)

\[ \varphi = \frac{2 \varepsilon_y}{d_s + 2ad_s - 2\eta t_f} \]

A7.4 Compressive Force in the Slab

\[ C = \sigma_y A_c \left[ \frac{\xi n f^l_c}{2 \sigma_y} + \frac{n f^l_c}{2 \sigma_y} + \frac{1 - \xi - \alpha + \alpha \xi}{1 + 2\alpha - 2 \eta \frac{t_f}{d_s}} \right] \]

A8. Stress Distribution Eight - Neutral axis in the top flange of the steel beam; bottom flange and part of the web of the steel beam plastic, part of the concrete slab at ultimate strength.

A8.1 Location of the Neutral Axis

\[ a = \begin{bmatrix} \frac{A_c}{A_s} \left( \frac{\xi}{2} \frac{f^l_c}{\sigma_y} - \frac{\eta f^l_c}{\sigma_y} + \frac{1}{2n} - \xi - \frac{f^l_c}{2\sigma_y} - \frac{A_f}{A_s} (\eta - \frac{1}{2} \frac{t_f}{d_s}) \right) \\
+ \frac{wds}{A_s} \left( \frac{\eta}{2} + \left[ \frac{t_f}{d_s} \right]^2 - 2\eta \frac{t_f}{d_s} \right) \\
\frac{A_c}{A_s} \left( \frac{1}{n} \right) - \xi \frac{d_c}{d_s} - \frac{\xi}{n} \frac{f^l_c}{\sigma_y} + 2 \frac{A_f}{A_s} + \frac{wds}{A_s} \left( 2 - 4 \frac{t_f}{d_s} \right) \\
\frac{1}{2} \frac{d_c}{d_s} - \xi \frac{d_c}{d_s} - \eta \frac{f^l_c}{2 \sigma_y} + n \eta \frac{f^l_c}{\sigma_y} \right] \]

\[ a = \begin{bmatrix} \frac{1 + \frac{d_c}{d_s} - \xi \frac{d_c}{d_s} - \frac{n f^l_c}{2 \sigma_y} + n \eta \frac{f^l_c}{\sigma_y}}{1 + n \frac{f^l_c}{\sigma_y}} \end{bmatrix} \]
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\[
\frac{1}{2} \leq \frac{t_f}{d_s} \leq a \leq \frac{1}{2}, \quad \frac{t_f}{d_s} \leq \eta, \quad 0 < \xi
\]

A8.2 Moment Resistance

\[
M_8 = \left[ \frac{\sigma_y A_s d_s}{1 + 2a - 2\eta} \right] \left\{ \frac{A_c}{A_s} \left( \frac{\xi}{2} \frac{f_c^1}{\sigma_y} + \alpha \frac{f_c^1}{\sigma_y} - \eta \frac{f_c^1}{\sigma_y} + \frac{\xi}{3} \frac{f_c^1}{d_s} \right) + \frac{2a \xi}{3} \frac{f_c^1}{\sigma_y} \frac{d_s}{d_s} - \frac{a}{3n} \frac{d_c}{d_s} - \frac{2\eta}{3} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} - \frac{\xi}{6} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} - \frac{\xi}{3n} \frac{d_c}{d_s} - \frac{\xi}{2n} + \frac{\xi^2}{6n} \frac{d_c}{d_s} - \frac{\alpha + 2a \xi}{3n} \frac{d_c}{d_s} + \frac{\alpha}{n} - \frac{\alpha^2}{n} \frac{d_c}{d_s} + \frac{2a \xi}{3n} \frac{d_c}{d_s} + \frac{\alpha}{2} \frac{f_c^1}{\sigma_y} - \eta \frac{f_c^1}{\sigma_y} + \frac{1}{3} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} + \frac{2a \xi}{3} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} \right. \\
+ \frac{2\eta}{3} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} + \frac{\alpha}{2} \frac{f_c^1}{\sigma_y} - \eta \frac{f_c^1}{\sigma_y} + \frac{1}{3} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} + \frac{2a \xi}{3} \frac{f_c^1}{\sigma_y} \frac{d_c}{d_s} \right. \\
+ \frac{A_f}{A_s} \left( \frac{1}{2} - \frac{t_f}{d_s} - a + \frac{n}{2} \frac{t_f}{d_s} + \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^2 \right) \\
+ \frac{w d_s}{A_s} \left( \frac{1}{6} - a - \frac{1}{2} \frac{t_f}{d_s} + a \frac{t_f}{d_s} + \eta \frac{t_f}{d_s} - \frac{n}{2} \right. \\
\left. + \alpha \eta + \frac{\eta^2}{2} - \eta^2 \frac{t_f}{d_s} + \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^3 + \frac{1}{2} \left[ \frac{t_f}{d_s} \right]^2 - \alpha \eta^2 \right. \\
+ \alpha \left[ \frac{t_f}{d_s} \right]^2 + \frac{\eta^3}{3} - \eta \left[ \frac{t_f}{d_s} \right]^2 \right\}
\]

A8.3 Curvature \( \phi \)

\[
\phi = \frac{-2 \frac{\varepsilon_y}{d_s}}{d_s + 2a d_s - 2\eta d_s}
\]

A8.4 Compressive Force in the Slab

\[
c = \frac{\sigma_y A_c}{n} \left[ \frac{n \xi}{2} \frac{f_c^1}{\sigma_y} + \frac{n}{2} \frac{f_c^1}{\sigma_y} + \frac{1}{2} - \frac{\xi}{2} - a + \alpha \xi \right]
\]

\[
1 + 2a - 2\eta
\]
APPENDIX A

A9. Stress Distribution None - Neutral axis in the concrete slab; part of the bottom flange of the steel plastic, slab elastic.

A9.1 Location of the Neutral Axis

\[ \mu^2 + \mu \left[ \frac{w ds}{A_c} \left( \frac{4n}{d_s} \frac{t_f}{d_s} - 2n \right) - 2n \frac{A_f}{A_c} \right] + \left[ 1 + n \frac{A_f}{A_c} \right] \]

\[ \left( \frac{\eta^2}{2} \frac{t_f}{d_c} - \frac{d_s}{d_c} \right) + 2n \frac{w ds}{A_c} \left( \frac{t_f}{d_c} - \frac{1}{2} \frac{d_s}{d_c} \right) = 0 \]

\[ 0 < \mu, \quad 0 < \eta \leq 1.0 \]

A9.2 Moment Resistance

\[ M_q = \frac{\sigma_y A_s d_s}{1 + \mu \frac{d_c}{d_s} - \eta \frac{t_f}{d_s}} \left\{ \frac{A_c}{A_s} \left( \frac{1}{2n} \frac{d_c}{d_s} \right) \left( \frac{\mu}{2n} \frac{d_c}{d_s} + \frac{\mu^2}{2n} \frac{d_c}{d_s} + \frac{1}{3n} \left[ \frac{d_c}{d_s} \right]^2 \right) - \right. \]

\[ \frac{\mu}{2n} \left[ \frac{d_c}{d_s} \right]^2 + \frac{\mu}{6n} \left[ \frac{d_c}{d_s} \right]^2 \right) + \frac{A_f}{A_s} \left( \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^2 \right) - \]

\[ \frac{\mu}{2} \frac{d_c}{d_s} - \frac{1}{2} \frac{t_f}{d_s} + \frac{\eta^3}{12} \left[ \frac{t_f}{d_s} \right]^2 \right) + \frac{w ds}{A_s} \left( \mu \frac{t_f}{d_s} \frac{d_c}{d_s} \right) \]

\[ + \left[ \frac{t_f}{d_s} \right]^2 - \frac{\mu}{2} \frac{d_c}{d_s} - \frac{1}{6} - \frac{2}{3} \left[ \frac{t_f}{d_s} \right]^3 \right) \}

A9.3 Curvature \( \phi \)

\[ \phi = \frac{\sigma_y}{d_s + \mu d_c - \eta t_f} \]

A9.4 Compressive Force in the Slab

\[ C = \frac{\sigma_y A_c}{n} \left( \frac{d_c}{d_s} \right) \left( \frac{1 - \mu}{2} \right) \frac{1}{1 + \mu \frac{d_c}{d_s} - \eta \frac{t_f}{d_s}} \]
A10. Stress Distribution Ten - Neutral axis in the concrete slab; bottom flange and part of the web of the steel beam plastic, slab elastic.

A10.1 Location of the Neutral Axis

\[ \mu^2 + \mu \left[ \frac{wds}{A_c} \left( 4n \frac{t_f}{d_s} - 2n \right) - 2 - 2n \frac{A_f}{A_c} \right] + 1 + \]

\[ + \frac{A_f}{A_c} \left( m \frac{d_s}{d_c} - n \frac{d_s}{d_c} - \frac{n t_f}{2 d_c} \right) \]

\[ + \frac{wds}{A_c} \left( n \frac{t_f}{d_s} \frac{t_f}{d_c} - 2n \frac{d_s}{d_c} - n \frac{d_s}{d_c} + m \right) \frac{d_s}{d_c} + \]

\[ 2n \frac{t_f}{d_c} \]

\[ 0 < \mu \leq \frac{t_f}{d_s} \leq \eta \]

A10.2 Moment Resistance

\[ M_{10} = \left[ \frac{\sigma_y A_s d_s}{1 + \mu d_c - \eta d_s} \right] \left\{ \frac{A_c}{A_s} \left( \frac{1}{2n} \frac{d_c}{d_s} + \frac{1}{3n} \left[ \frac{d_c}{d_s} \right]^2 - \mu \frac{d_c}{d_s} - \frac{\mu}{2n} \left[ \frac{d_c}{d_s} \right]^2 \right) + \frac{A_f}{A_s} \left( \frac{1}{6} \frac{t_f}{d_s} \right)^2 - \left( \frac{1}{2} \frac{t_f}{d_s} - \frac{\mu}{2} \frac{d_c}{d_s} + \frac{n}{4} \frac{t_f}{d_s} \right) + \frac{wds}{A_s} \left( \frac{n^3}{6} - \frac{1}{6} + \right) \right\} \]

\[ \left[ \frac{t_f}{d_s} \right]^2 - \frac{n}{2} \left[ \frac{t_f}{d_s} \right]^2 + \mu \frac{d_c}{d_s} \frac{t_f}{d_s} - \frac{\mu}{2} \frac{d_c}{d_s} - \]

\[ - \frac{1}{3} \left[ \frac{t_f}{d_s} \right]^3 \]
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Al9.3 Curvature $\phi$

$$\phi = \frac{\varepsilon_y}{d_s + \mu d_c - \eta d_s}$$

Al9.4 Compressive Force in the Slab

$$C = \frac{\sigma_y A_c}{n} \left[ \frac{\frac{d_c}{d_s}}{(1 - \frac{H_o}{H})^{\frac{1}{2}}} \right] \left[ \frac{1}{1 + \mu d_c - \eta} \right]$$

All. Stress Distribution

Eleven - Neutral axis in the concrete slab; part of the bottom flange of the steel beam plastic, part of the concrete slab at ultimate strength.

All.1 Location of the Neutral Axis

$$\mu^2 + \mu \left[ 2 \frac{A_r}{A_c} \frac{\sigma_y}{f_c^*} \xi - \frac{d_s}{d_c} - \eta \frac{t_f}{d_c} - 1 - \frac{w d_s}{A_c} \left( 4 \frac{\sigma_y}{f_c} \frac{t_f}{d_s} - 2 \frac{\sigma_y}{f_c} \right) \right]$$

$$+ \frac{w d_s}{A_c} \left( \frac{\sigma_y}{f_c} \frac{d_s}{d_c} - 2 \frac{\sigma_y}{f_c} \frac{t_f}{d_c} \right) + \frac{A_r}{A_c} \left( \frac{\sigma_y}{f_c^*} \frac{d_s}{d_c} - \frac{\eta^2}{2} \frac{\sigma_y}{f_c} \frac{t_f}{d_c} - \frac{\xi d_s}{d_c} + \eta \frac{d_s}{d_c} - \eta \frac{t_f}{d_c} \right) = 0$$

$$\mu = \left[ \begin{array}{c} 1 - \xi - n \frac{f_c^*}{\sigma_y} \left( \frac{d_s}{d_c} - \eta \frac{t_f}{d_c} \right) \\ \frac{1 + n \frac{f_c^*}{\sigma_y}}{\sigma_y} \end{array} \right]$$

$$0 < \mu \quad 0 < \eta \leq 1.0 \quad 0 < \xi$$
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All.2 Moment Resistance

\[ M_{ll} = \frac{\sigma_y A_s d_s}{1 + \frac{\mu}{d_s} \frac{d_y}{d_s} - \eta \frac{f_r}{d_s}} \left\{ \frac{A_c}{A_s} \frac{f_c'}{\sigma_y} \left( \frac{\xi}{2} + \frac{\xi}{3} d_c - \frac{\xi^2}{6} d_c + \frac{2 \mu \xi}{3} \frac{d_c}{d_s} + \right. \right. \\
\left. \left. \frac{\mu \xi}{3} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu \xi^2}{6} \left[ \frac{d_c}{d_s} \right]^2 - \eta \frac{f_r}{d_s} - \frac{\eta}{3} \frac{d_c}{d_s} \frac{d_r}{d_s} \\
+ \frac{\eta \xi^2}{6} \frac{d_c}{d_s} \frac{d_r}{d_s} + \frac{1}{2} - \frac{\mu}{3} \frac{d_c}{d_s} \frac{d_r}{d_s} + \frac{\mu}{3} \frac{d_c}{d_s} \right. \right. \\
- \frac{2 \mu^2}{3} \frac{d_c}{d_s} \frac{d_r}{d_s} + \frac{\mu}{3} \left[ \frac{d_c}{d_s} \right]^3 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 \left( \frac{d_c}{d_s} \right)^2 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 \right. \right. \\
\left. \left. \frac{\mu^3}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu}{2} \frac{d_r}{d_s} + \frac{\mu}{6} \frac{d_c}{d_s} \frac{d_r}{d_s} - \frac{\eta}{3} \frac{d_c}{d_s} \frac{d_r}{d_s} + \frac{\mu}{6} \frac{d_c}{d_s} \frac{d_r}{d_s} \right. \right. \\
\left. \left. \frac{\mu}{6} \frac{d_c}{d_s} \frac{d_r}{d_s} - \frac{\mu \eta}{6} \frac{d_c}{d_s} \frac{d_r}{d_s} + \frac{\mu}{6} \frac{d_c}{d_s} \frac{d_r}{d_s} \right) \\
\frac{\mu}{6} \frac{d_r}{d_s} \frac{d_c}{d_s} \left( \frac{\frac{f_r}{d_s}}{d_s} \frac{d_c}{d_s} + \frac{\frac{f_r}{d_s}}{d_s} \frac{d_c}{d_s} \right) - \frac{\mu^3}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 \right. \right. \\
\left. \left. \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^3}{6} \left[ \frac{d_c}{d_s} \right]^2 \right) \\
\frac{\mu}{6} \frac{d_r}{d_s} \frac{d_c}{d_s} \left( \frac{\frac{f_r}{d_s}}{d_s} \frac{d_c}{d_s} + \frac{\frac{f_r}{d_s}}{d_s} \frac{d_c}{d_s} \right) - \frac{\mu^3}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 \right. \right. \\
\left. \left. \frac{\mu^2}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu}{6} \left[ \frac{d_c}{d_s} \right]^2 - \frac{\mu^3}{6} \left[ \frac{d_c}{d_s} \right]^2 \right) \right\} \right. \right. \\
\left. \left. \frac{2}{3} \left[ \frac{f_r}{d_s} \right]^3 \right. \right. \right. \]

All.3 Curvature \( \phi \)

\[ \phi = \frac{\xi_y}{d_s + \mu d_c - \eta f_r} \]

All.4 Compressive Force in the Slab

\[ C = f_c' A_c \left[ \frac{1}{2} + \frac{\xi}{2} - \frac{\mu}{2} \right] \]
A12. Stress Distribution

Twelve - Neutral axis in the concrete slab; bottom flange and part of the web of the steel beam plastic, part of the slab at ultimate strength.

A12.1 Location of the Neutral Axis

\[
\begin{align*}
\mu^2 - &\mu \left[ \frac{ds}{dc} - \xi - 1 - \eta \frac{ds}{dc} + 2 \frac{Af}{Ac} \frac{\sigma_y}{f_c} - \frac{4}{Ac} \frac{\sigma_y}{f_c} \frac{tf}{ds} \right] \\
&+ 2 \frac{ws}{Ac} \frac{\sigma_y}{f_c} \frac{tf}{ds} + \left[ \eta \frac{ds}{dc} - \frac{ds}{dc} - \xi \frac{ds}{dc} + \eta \xi \frac{ds}{dc} - \eta \frac{Af}{Ac} \frac{\sigma_y}{f_c} \frac{tf}{ds} \right] \\
&+ \frac{1}{2} \frac{Af}{Ac} \frac{\sigma_y}{f_c} \frac{tf}{ds} + \frac{Af}{Ac} \frac{\sigma_y}{f_c} \frac{ds}{dc} + \frac{ws}{Ac} \frac{\sigma_y}{f_c} \frac{ds}{dc} - \frac{ws}{Ac} \frac{\sigma_y}{f_c} \frac{tf}{ds} = 0
\end{align*}
\]

\[
\mu = \left[ \frac{1 - \xi - n \frac{f_c}{\sigma_y} \frac{ds}{dc} + \eta n \frac{f_c}{\sigma_y} \frac{ds}{dc}}{1 + n \frac{f_c}{\sigma_y}} \right]
\]

\(0 < u \quad \frac{tf}{ds} \leq \eta \quad 0 < \xi\)

A12.2 Moment Resistance

\[
M_{12} = \left[ \frac{\sigma_y A_s ds}{1 + \mu \frac{dc}{ds} - \eta} \right] \left\{ \frac{Ac}{As} \frac{f_c}{\sigma_y} \left( \frac{1 + \mu \frac{dc}{ds} - \eta}{2} - \frac{2}{3} \frac{dc}{ds} - \frac{2}{3} \frac{dc}{ds} \right) + \frac{\xi}{2} + \frac{2}{3} \mu \xi \frac{dc}{ds} - \frac{\eta \xi}{2} + \frac{1}{3} \frac{dc}{ds} + \frac{\mu}{3} \left[ \frac{dc}{ds} \right]^2 - \frac{\eta}{3} \frac{dc}{ds} - \frac{\mu}{3} \left[ \frac{dc}{ds} \right]^2 + \frac{\mu}{6} \frac{dc}{ds} + \frac{\xi}{3} \frac{dc}{ds} \right.
\]

\[
+ \frac{\mu \xi}{3} \frac{dc}{ds} - \frac{\eta \xi}{3} \frac{dc}{ds}
\]
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$$\frac{\mu^2}{6} \frac{d_c}{ds} + \frac{\mu}{6} \left[ \frac{d_c}{ds} \right]^2 - \frac{\mu}{6} \frac{d_c}{ds} - \frac{\xi}{6} \frac{d_c}{ds} - \frac{2}{6} \frac{d_c}{ds} -$$

$$\frac{\mu^2}{6} \left[ \frac{d_c}{ds} \right]^2 - \frac{\mu}{6} \left[ \frac{d_c}{ds} \right]^2 + \frac{\mu}{6} \frac{d_c}{ds} - \frac{\eta}{6} \frac{d_c}{ds} + \frac{Af}{As} \left( \frac{1}{6} \left[ \frac{tf}{ds} \right]^2 -$$

$$\frac{\mu}{2} \frac{d_c}{ds} - \frac{1}{2} \frac{tf}{ds} + \frac{n}{4} \frac{tf}{ds} \right) + \frac{wds}{As} \left( \mu \frac{d_c}{ds} \frac{tf}{ds} -$$

$$- \frac{\mu}{2} \frac{d_c}{ds} \right) + \left[ \frac{tf}{ds} \right]^2 - \frac{1}{3} \left[ \frac{tf}{ds} \right]^3 - \frac{1}{6} + \frac{n^3}{6} - \frac{n}{2} \left[ \frac{tf}{ds} \right]^2 \right)$$

A12.3 Curvature $\phi$

$$\phi = \frac{\varepsilon_y}{d_s + \mu \frac{d_c}{ds} - \eta d_s}$$

A12.4 Compressive Force in the Slab

$$C = f \int A_c \left[ \frac{1}{2} + \frac{\xi}{2} - \frac{\mu}{2} \right]$$

A13. Stress Distribution Thirteen - Neutral axis in the concrete slab; bottom flange, web and part of top flange plastic, part of concrete slab at ultimate strength.

A13.1 Location of the Neutral Axis

$$\mu^2 + \mu \left[ \frac{d_s}{d_c} - 1 - \xi - \eta \frac{d_s}{d_c} + 2 \frac{Af}{A_c} \frac{\sigma_y}{f_y} + \frac{wds}{A_c} \frac{\sigma_y}{f_y} \left( 2 - 4 \frac{tf}{d_s} \right) \right]$$

$$+ \left[ \eta \frac{d_s}{d_c} - \frac{d_s}{d_c} - \xi \frac{d_s}{d_c} + \eta \xi \frac{d_s}{d_c} + \frac{Af}{A_c} \frac{\sigma_y}{f_y} \left( 2 \frac{d_s}{d_c} - \frac{d_s}{d_c} \right) \right.$$

$$\left. - \frac{1}{2} \frac{d_s}{d_c} \frac{d_s}{d_c} - 2 \eta \frac{d_s}{d_c} + \eta \frac{d_s}{d_c} \frac{d_s}{d_c} \right) \right] + \frac{wds}{A_c} \frac{\sigma_y}{f_y} \left( 2 \frac{d_s}{d_c} - 2 \eta \frac{d_s}{d_c} \right.$$}

$$4 \frac{tf}{d_c} + 4 \eta \frac{tf}{d_c} \right] = 0$$
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\[
\mu = \left[\frac{1 - \xi - n \frac{f_c}{\sigma_y} \frac{d_s}{d_c} + \eta n \frac{f_c}{\sigma_y} \frac{d_s}{d_c}}{1 + n \frac{f_c}{\sigma_y}}\right]
\]

\[0 < \mu \quad 0 < \xi \quad \frac{t_f}{d_s} \leq \eta\]

A13.2 Moment Resistance

\[
M_{13} = \left[\frac{\sigma_y A_s d_s}{1 + \mu \frac{d_c}{d_s} - \eta}\right] \left\{ \frac{A_c}{A_s} \frac{f_c}{\sigma_y} \left( \frac{1}{2} + \frac{\xi}{2} + \frac{\xi}{3} \frac{d_c}{d_s} - \frac{\xi^2}{6} \frac{d_c}{d_s} \right) + \frac{2 \mu \xi}{3} \frac{d_c}{d_s} + \frac{\mu^2}{3} \left[\frac{d_c}{d_s}\right]^2 - \frac{\mu^2}{6} \left[\frac{d_c}{d_s}\right]^2 - \frac{n \xi}{2} \frac{d_c}{d_s} - \frac{\mu}{3} \frac{d_c}{d_s} - \frac{2 \mu^2}{3} \frac{d_c}{d_s} - \frac{n}{2} \right.
\]

\[+ \frac{n \xi}{6} \frac{d_c}{d_s} + \frac{\mu \xi}{6} \frac{d_c}{d_s} - \frac{\mu^2}{6} \frac{d_c}{d_s} - \frac{n \xi}{6} \frac{d_c}{d_s} - \frac{\mu^3}{6} \left[\frac{d_c}{d_s}\right]^2 + \frac{\mu^2}{6} \frac{d_c}{d_s} + \frac{A_f}{A_s} \left( \frac{1}{6} \frac{d_s}{t_f} - \frac{1}{2} \right) \]

\[+ \frac{n}{12} \frac{d_s}{t_f} - \frac{\mu}{2} \frac{d_c}{d_s} + \frac{n}{4} \frac{d_s}{t_f} \right) + \frac{w d_s}{A_s}
\]

\[\left( \frac{t_f}{d_s} - \frac{1}{2} - \frac{\mu}{2} \frac{d_c}{d_s} + \frac{\mu}{2} \frac{d_c}{d_s} \frac{t_f}{d_s} + \frac{n}{2} \frac{t_f}{d_s} \right) \left\}
\]
A13.3 Curvature $\phi$

$$\phi = \frac{\varepsilon_y}{d_s + \mu d_c - \eta d_s}$$

A13.4 Compressive Force in the Concrete Slab

$$C = f'_c A_c \left[ \frac{1}{2} + \frac{\xi}{2} - \frac{\mu}{2} \right]$$
Outline of Example Solution

The moment curvature relations were determined for the section shown in Fig. 4. An outline of the solution for this section follows:

Step 1 - Location of Neutral Axis in the elastic range and determination of the moment of inertia of the section.

\[
\bar{y} = \frac{\sum A y}{\sum A} = \frac{\frac{A_c (d_a + d_c)}{n} + \frac{A_s (d_g)}{2}}{\frac{A_c}{n} + A_s} = 11.60 \text{ in.}
\]

\[
I = \int \frac{y^2 dA}{A} = 587.7 \text{ in}^4
\]

Step 2 - Determination of the yield moment \(M_y\)

\[
\sigma = \frac{M_c}{Y}
\]

\[
M_y = \sigma \frac{I}{c} = \frac{39(587.7)}{11.60} = 1975 \text{ k-in.}
\]

Step 3 - Determination of the curvature at \(M_y\) \(\phi_y\)

\[
\phi = \frac{M}{EI}
\]

\[
\phi_y = \frac{M_y}{EI} = \frac{1975}{(30,000 \times 10^3)587.7} = 112.0 \times 10^{-6} \text{ radians}
\]
Alternate approach considering strains

\[
\varepsilon_y = \frac{\sigma_y}{E} = \frac{39 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = 1.3 \times 10^{-3} \text{ in/in.}
\]

\[
\phi_y = \frac{\varepsilon_y}{\varepsilon_y} = \frac{1.3 \times 10^{-3}}{11.60} = 112.0 \times 10^{-6} \text{ radians}
\]

Step 4. Determination of the Plastic Moment \( (M_p) \)

\[
M_p = \sigma_y A_s \left( d_c + \frac{d_s}{2} - \frac{\sigma_y A_s}{2b f'_c} \right)
\]

\[
M_p = 2860 \text{ k-in.}
\]

Step 5. Determination of the Curvature at the Plastic Moment \( (\phi_p) \)

\[
\phi_p = \frac{\varepsilon_y}{\mu d c}
\]

\[
= \frac{1.3 \times 10^{-3}}{(0.397)(4)} = 817.8 \times 10^{-6} \text{ radians}
\]
Step 6. Determination of the possible stress distributions in the elastic plastic region.

The neutral axis in the elastic range and at the yield moment is in the top flange of the steel beam. The stresses in the extreme fibers of the steel section and the concrete slab at the yield moment are 39 ksi and 1.46 ksi respectively.

The neutral axis is in the slab when the section reaches the plastic moment (step 4). Since the neutral axis must move from its position at $M_y$ to that at $M_p$, the possible stress distributions in the elastic plastic region between $M_y$ and $M_p$ are stress distribution 5, 6, 7, 8, 9, 10, 11, 12 and 13.

Step 7. Solution of the possible stress distributions in the elastic plastic region.

A. Stress distribution 5. The equation for $\alpha$ is solved and yields

\[
\alpha = 0.4711 + 0.0016\eta^2
\]

For this stress distribution $\eta$ has the limits

$0 < \eta \leq 1.0$
Step 7 (Cont'd)

By assuming various values for $\eta$ between zero and one we see that $a$ varies between 0.471 and 0.473. Using the values obtained for $a$ and $\eta$, the moment, $E$ Force, and curvature are determined.

These values are listed in Table II on page 54. The stress in the extreme concrete fiber with $a = 0.473$ and $\eta = 1.0$ is 1.49 ksi.

B. Since the neutral axis is still in the top flange ($a < 0.5$) and the concrete is elastic, it is easily seen that the next stress distribution is stress distribution 6.

By assuming various values of $\eta$ starting with $\eta = 0.033$ we see that $\eta = 0.2813$ when $a = 0.5$. At this point the neutral axis moves into the concrete slab. The stress in the extreme concrete fiber with $a = 0.5$ and $\eta = 0.2813$ is 1.85 ksi.
C. The next stress distribution is therefore stress distribution 10.

By assuming various values of $\eta$ starting from 0.2813, values of $\mu$, which give the position of the neutral axis are determined. The limit of application of this equation is the point at which the stress in the extreme concrete fiber reaches $f'_c$ or 3.6 ksi. The values for $\mu$ and $\eta$ to cause the extreme fiber of the concrete to reach $f'_c$ are 0.279 and 0.83 respectively.

The neutral axis is now in the concrete slab, the extreme concrete fibers have reached $f'_c$ and the steel beam has yielded to 0.83 of its depth.

D. Stress distribution 12 is therefore the next stress distribution.
By assuming values of \( \eta \) between 0.83 and 0.967 the location of the neutral axis \((\mu)\) may be determined. The moment and curvature for each point is then computed using these values.

When yielding of the steel beam has reached a depth of 0.967 \(d_s\) or the underside of the flange the neutral axis is still in the slab and the extreme fibers of the concrete have reached \(f'_c\).

E. Stress distribution 13 is therefore the next stress distribution and the last one prior to reaching \(M_r\).

By assuming values of \( \eta \) between 0.967 and 1.0 the location of the neutral axis, the moment, and the curvature may be determined.

A summary of the pertinent equations and quantities determined using the various stress distributions are given in Tables I and II pages 50 and 54.
APPENDIX C

The Plastic Moment of A Composite Section

Plastic Moment - Case I (Neutral Axis in Concrete Slab)*

Strains

\[ \frac{x}{\varepsilon_u} = \frac{d_c}{\varepsilon_u + \varepsilon_y} \]

\[ x \leq \frac{d_c}{1 + \varepsilon_y/\varepsilon_u} \]

Equilibrium:

\[ f_c'b \times = \sigma_y A_x \]

\[ x = \frac{\sigma_y A_x}{f_c'b} \]

Limiting Ratio

\[ \frac{A_s}{A_c} \leq \frac{f_c'}{\sigma_y} \left( \frac{1}{1 + \varepsilon_y/\varepsilon_u} \right) \]

\[ \frac{d_c}{d_s} \geq \frac{(\varepsilon_u + \varepsilon_y)}{(\varepsilon_{st} - \varepsilon_y)} \]

Plastic Moment

\[ M_p = T \left( \frac{d_s}{2} + d_o - \frac{x}{2} \right) \]

\[ M_p = \sigma_y A_s \left( d_c + \frac{d_s}{2} - \frac{\sigma_y A_s}{2b f_c'} \right) \]

* Taken from notes of Bruno Thurlimann CEHill "Selected Topics in Concrete Structures".
### TABLE I

Summary of Equations Obtained in Example Solution

1. Stress Distribution Five

   A. Location of the neutral axis
   
   \[ a = 0.4711 + 0.0016\eta^2 \]
   
   \[ 0 < \eta \leq 1.0 \]

   B. Moment Resistance
   
   \[ M_5 = \left[ \frac{3714.42}{1 + 2\alpha - \eta} \right] \left\{ 3.82375 - 5.9184\mu_4 \right\} \]

   C. Curvature
   
   \[ \phi = \frac{2.6 \times 10^{-3}}{11.95 + 23.9\alpha - 0.8\eta} \]

   D. Compressive Force
   
   \[ C = 748.8k\sqrt{\frac{4}{3} - 2\alpha} \]
   
   \[ \frac{1}{1 + 2\alpha - 0.066\eta} \]

2. Stress Distribution Six

   A. Location of the neutral axis
   
   \[ a = 0.4694 + 0.0939\eta + 0.05277\eta^2 \]
   
   \[ 0.033 \leq \eta \]

   B. Moment Resistance
   
   \[ M_6 = \left[ \frac{3714.42}{1 + 2\alpha - 2\eta} \right] \left\{ 4.1371 - 6.59\alpha + 0.1187\eta^3 + 0.0107\eta \right\} \]
2. Stress Distribution Six (Cont'd)

C. Curvature $\phi$

$$\phi = \frac{2.6 \times 10^{-3}}{11.95 + 23.9\mu - 23.9\eta}$$

D. Compressive Force

$$C = 748.8k \left[ \frac{\mu - 2\alpha}{3} \right] \left[ \frac{1+2\alpha-2\eta}{\mu-\eta} \right]$$

3. Stress Distribution 10

A. Location of the neutral axis

$$\xi^2 - 2.83\mu + 0.448\eta^2 + 0.7973\eta - 0.2586 = 0$$

B. Moment Resistance

$$M_{10} = \left[ \frac{371.4.42}{1 + \mu - \eta} \right] \left\{ 0.4166 - 1.091\mu + 0.3984\mu^2 + 0.0444\mu^3 \right\} + 0.0053\eta + 0.0599\eta^3$$

C. Curvature

$$\phi = \frac{1.3 \times 10^{-3}}{11.95 + 4\mu - 11.95\eta}$$

D. Compressive Force

$$C = 124.8 \left[ \frac{(1-\mu)^2}{1+\mu-\eta} \right] \left[ \frac{1+\mu-\eta}{3} \right]$$

4. Stress Distribution 12

A. Location of the neutral axis

$$\xi^2 + \mu - 3.57 - 3.947\eta + 2.6759\eta^2 - 3.9266\eta + 1.2558 = 0$$

$$\xi = 2.769\eta - 1.923\mu - 1.769$$
Table I (Cont'd)

4. Stress Distribution 12 (Cont'd)

B. Moment Resistance

\[
M_{12} = \left[ \frac{3714.42}{1 + \frac{\mu}{3} - \eta} \right] \begin{cases} 
1.278 - 1.344\eta + 1.349\xi - 1.349\eta\xi \\
-0.944\mu - 0.531\mu^2 + 1.266\mu\eta + 0.572\mu\xi \\
+0.122\mu^2\eta - 0.122\mu\eta\xi - 0.122\xi^2 + 0.122\eta\xi^2 \\
+0.041\mu^2\xi - 0.041\mu^2 - 0.041\mu^3 + 0.06\eta^3 
\end{cases}
\]

C. Curvature

\[
\phi = \frac{1.3 \times 10^{-3}}{11.95 + 4\mu - 11.95\eta}
\]

D. Compressive Force

\[
C = 691.2 \left[ \frac{1}{2} + \frac{\xi}{2} - \frac{\mu}{2} \right]
\]

5. Stress Distribution 13

A. Location of the neutral axis

\[
\mu^2 + \mu \left[ 3.5707 - 3.94\eta \right] - 1.756\eta^2 + 4.642\eta - 2.8856 = 0 \\
\xi = 2.769\eta - 1.923\mu - 1.769
\]

B. Moment Resistance

\[
M_{13} = \left[ \frac{3714.42}{1 + \frac{\mu}{3} - \eta} \right] \begin{cases} 
1.349\xi + 0.57 \mu\xi - 1.349\eta\xi + 0.122\eta^2 \\
-0.122\xi^2 + 0.041\mu^2 + 4.161 - 0.944\mu \\
-0.5327\mu^2 - 5.833\eta + 1.230\mu\eta + 0.041\mu^2\xi \\
-0.1229\mu\eta^2 - 0.041\mu^3 + 0.1229\mu^2\eta + 1.66\eta^3 
\end{cases}
\]

C. Curvature

\[
\phi = \frac{1.3 \times 10^{-3}}{11.95 + 4\mu - 11.95\eta}
\]
5. Stress Distribution 13 (Cont'd)

D. Compressive Force

\[ C = 691.2 \left( \frac{1}{2} + \frac{\varepsilon}{2} - \frac{\mu}{2} \right) \]

6. Plastic Moment

A. Moment Resistance

\[ M_p = \sigma_y A_s \left[ \frac{d_S}{2} + d_C - \frac{\sigma_y A_s}{2bfC} \right] \]

\[ = 2860 \text{ k in} \]

B. Curvature

\[ \phi = \frac{1.3 \times 10^{-3}}{0.3974 (4)} = 817.82 \times 10^{-6} \text{ radians} \]

C. Compressive Force

\[ C = T = \sigma_y A_s = (39)(7.97) = 310.83^k \]
### TABLE II

**SUMMARY OF VALUES DETERMINED FROM THE VARIOUS STRESS DISTRIBUTIONS**

<table>
<thead>
<tr>
<th>Stress Distribution</th>
<th>( \eta_d_s ) (in)</th>
<th>( \alpha )</th>
<th>( \mu )</th>
<th>( \zeta )</th>
<th>Moment ( M_y ) (k-in)</th>
<th>Curvature ( \phi ) (radians ( \times 10^{-6} ))</th>
<th>Compressive Force in Slab ( C ) (kips)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>817.82</td>
<td>310.83</td>
</tr>
</tbody>
</table>

\( M_p \)
Fig. 1 - STRESS-STRAIN RELATIONS FOR CONCRETE

Fig. 2 - STRESS-STRAIN RELATIONS FOR STEEL
Fig. 3. STRESS DISTRIBUTIONS IN THE ELASTIC-PLASTIC RANGE
Concrete Slab:
- $b_c = 48$ in.
- $d_c = 4$ in.
- $f'_c = 3600$ psi

Steel Beam:
- Section - 12WF27
- $A_s = 7.97$ in$^2$
- $w_d = 2.87$ in$^2$
- $d_s = 11.97$ in.
- $f_y = 39.0$ ksi
- $A_f = 5.29$ in$^2$

Composite section:
- $I = 587.7$ in$^4$
- $m = 45.1$ in$^3$
- $n = 10$

Ratios:
- $\frac{d_c}{d_s} = \frac{4}{12} = 0.33$
- $\frac{d_s}{d_c} = \frac{12}{4}$
- $\frac{d_c}{t_f} = \frac{4}{0.4} = 10$
- $\frac{d_s}{t_f} = \frac{12}{0.4} = 30$
- $\frac{t_f}{d_s} = \frac{0.4}{12}$
- $\frac{A_c}{A_s} = \frac{192}{7.97} = 24$
- $\frac{t_f}{d_c} = \frac{0.4}{4}$
- $\frac{f'_c}{f_y} = \frac{3.6}{39} = 0.092$
- $\frac{A_f}{A_s} = \frac{5.29}{7.97} = 0.6637$
- $\frac{w_d}{A_s} = \frac{2.87}{7.97} = 0.3601$

**Fig. 4** Dimensions of Composite Section Used in Example Solution
Fig. 5 - MOMENT CURVATURE RELATIONS FOR EXAMPLE SOLUTION
Fig. 6 COMPARISON OF COMPUTED QUANTITIES AND TEST RESULTS
Fig. 7 VARIATION OF THE COMpressive FORCE IN THE SLAB
Fig. 8 VARIATION OF SHEAR FLOW ALONG THE LENGTH OF THE SPECIMEN
Composite Beam Subjected to Any Loading

A. Using the Internal Couple Method to Resolve the Stress Diagram into Equivalent Forces:

\[ M_{\text{max}} = M_u \]

\[ C = T = \frac{M_u}{d} \]

B. Considering the Slab as a Free Body and using the Elastic Formula \( \frac{V_m}{I} \) to Compute the Shear Flow

\[ S_e = \frac{V_m m}{I} \]

C. Integrating the Shear Flow over the Length "a" and Summing the Horizontal Forces to Check Equilibrium

\[ \int_0^a S_e \, dx - C = \frac{M_u m}{I} - \frac{M_u}{d} \leq 0 \]

D. Error produced by using the Elastic Formula \( \frac{V_m}{I} \)

\[ \frac{\int_0^a S_e \, dx}{C} = \frac{md}{I} \leq 1 \]

Fig. 9 COMPUTATION OF SHEAR FLOW AT \( M_p \)
**Fig. 10** CALCULATED CONNECTOR FORCES AT $M_p$