Column design in continuous structures

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Welded Continuous Frames and Their Components

COLUMN DESIGN IN CONTINUOUS STRUCTURES

by

M. Ojalvo and V. Levi

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ABSTRACT

A new method for the design of columns in multi-story continuous frames is developed in this report. The method applies to planar frames adequately braced against sway. Rotational restraint furnished by members framing into the column in question is considered.

The method is illustrated by the design of an interior column. Charts necessary for the application of the method are included in the Appendixes.
I. INTRODUCTION

This report is concerned with the design of columns in multi-story structures having rigid beam-to-column connections. The beams may be designed plastically. The method is applicable when the following conditions are satisfied:

1. The columns and beams are of a ductile material such as steel.

2. Horizontal loads on the structure are resisted by a system of braced bays or by shear walls which keep the maximum drift per story height (horizontal displacement) within the limits prescribed as tolerable in modern practice. A drift of 0.002 ft. per ft. of story height has been suggested as acceptable.\(^1\)

3. The floor framing arrangement results in column bending about only one principal axis.

4. The columns are prismatic.

In conventional design practice beams are selected on the basis of an elastic stress analysis and an allowable
(design) stress. Columns are designed as follows:

1. Sizes (sections) are assumed.

2. An approximate elastic moment distribution is performed to determine the moments at the ends of the column.

3. Columns are checked by an empirical interaction equation (such as the AISC equation) or against an allowable stress.

However, a frame usually does not fail elastically. Furthermore, interaction equations are based upon the assumption of a pinned-end beam-column, while the actual column is integral with the remainder of the structure. This can result in sections larger than those required. An allowable stress criterion for column design is even less rational, since column failure is due to instability and not due to over-stress. Because of the foregoing reasons a more rational method based on the ultimate strength of a subassemblage of the structure is proposed in this report. This subassemblage includes the column to be designed, and all the members framing into it. The loading and end conditions most unfavorable to the design are assumed.
II. TYPES OF COLUMN FAILURE

Rational design of columns is impossible without an understanding of the possible types of failure. This section briefly describes the basic types of column failure.

One type of failure for thin walled columns is local buckling of a highly compressed portion. This kind of buckling changes the shape of the cross section. It can occur in the flange or web of a rolled wide-flange member or in the walls of a pipe column. When local buckling occurs prior to the overall collapse of the column there is a reduction in the rigidity of the column and a corresponding decrease in its capacity to resist additional loads.

Another type of failure is buckling by bifurcation. The term buckling is used to refer to a type of failure characterized by an abrupt change of the equilibrium configuration. Examples of this type of failure are the buckling of an axially loaded column by bending about one of the principal axes, lateral-torsional buckling of a beam-column with primary bending about the major principal axis, and torsional buckling of a column subjected to a compressive force applied through the shear center.
Another type of column failure is due to excessive bending (i.e. instability without bifurcation). For this case the column deforms into a bent configuration from the very beginning of loading and the initial deformed shape corresponds to the one existing at ultimate load.

The design method of this report considers only column failure by excessive bending about one of the principal axes of the column. Failure by this kind of inelastic instability may be investigated by considering several magnitudes of a single kind of deformation. Each magnitude or degree of deformation corresponds to the equilibrium configuration of the column for certain applied moments and axial load. By considering the axial load to remain constant in an unrestrained beam-column, for example, the maximum end moments consistent with equilibrium can be found. References 2 and 3 are representative of two different approaches to the solution of this type of problem.

A designer must consider all possible types of column failure in order to achieve a safe design. Consequently, in the proposed method it is necessary to restrict the column sections to those that are not likely to fail by either local
or lateral-torsional buckling. The use of circular or box sections will eliminate the possibility of failure by lateral-torsional buckling. If open sections are used, they could be oriented so that they are bent about their minor principal axis or, if they are bent about their major axis, adequate lateral bracing must be provided. To eliminate the possibility of local buckling, the unsupported length to thickness ratios of the walls of the column cross section must be suitably limited. These limiting ratios are adequately defined in the Commentary on Plastic Design. (4)
III. DESIGN LOADING

A column should be designed for the loading condition most unfavorable to it. This is achieved by considering several possible loading conditions.

Figure 1 shows the distribution of live loading that is likely to be least favorable to the interior column AB. The spans AC and BF have only dead load acting on them. This loading results in single curvature bending of the column AB.

It has been shown that single curvature bending is the most critical for beam columns. However, the loading condition conducive to the worst bending of a column does not result in the largest possible compressive load because the compressive load on AB is somewhat smaller than it would be if the live load were applied over span AC as well. The possibility therefore exists that the most critical condition of loading occurs when the live load acts on all spans with the exception of BF. In order to limit the work required to design a single column, the compressive load acting on AB is computed on the basis of live loads on all spans. The bending moments are computed on the basis of live loads on
all spans except AC and BF. All beam and column loads are multiplied by a factor of 1.85 which is in accordance with the recommendation of the Plastic Design Manual of the A.I.S.C. (6).

Although it seems unlikely that a value of 1.85 times the dead load will be available on the unloaded span to help stabilize the column, it also seems unrealistic to assume that 1.85 times the dead load on the fully loaded span will be available to help bend the column. As a compromise a uniform factor of 1.85 is used for all loading.
IV. THEORETICAL BASIS OF THE PROCEDURE

The basis of a procedure for the design of interior restrained columns will be discussed next.

The computations for the actual collapse load in a continuous frame must consider the entire structural action of the building, and not only the action of the members immediately adjacent to the column. This involves extensive computations, and but slightly affects the resulting design. To simplify the analysis for the adequacy of an interior column such as AB in Fig. 1, the following assumptions are made:

1. Only members framing directly into AB will be considered.

2. Ends A and B of column AB are assumed to be symmetrically loaded and symmetrically restrained (i.e. $\theta_A = \theta_B$ and $M_{AB} = M_{BA}$). Since symmetry can not exist except for equal spans on either end of the column and for antisymmetric loading, the column is designed for the end conditions which subject it to the greatest end moments.
3. The restraining moment of a member such as AC is computed on the assumption that joints C and A rotate equal amounts but in opposite directions (i.e. as $\theta_A$ rotates clockwise, $\theta_C$ rotates an equal amount counterclockwise). In addition it is assumed that joint D, at the far end of the fully loaded span AD, rotates the same amount as joint A but in the opposite sense.

4. The net bending moment supplied by the beams at the column ends A and B are partially resisted by the columns above and below these points. It is assumed that the ratio $M_{AB}/M_{AG}$ is equal to $L_{AG}/L_{AB}$, where $L_{AG}$ and $L_{AB}$ are the lengths of the members AG and AB.

Consider column AB in Fig. 1 as a restrained member subjected to axial load and bending moments. As joints A and B rotate, the partially loaded spans CA and BF reduce the net moment which the column must resist at A and B. The design method considers this reduction of moments. Another feature of the method, however, is that it accounts for equal
The end rotations of intersecting columns and beams in the inelastic as well as in the elastic range of structural action.

Whether the consideration of only the members adjacent to the column is conservative depends on the conditions assumed for their ends. These end conditions are specified by the four simplifying assumptions previously stated.

The adequacy of the first simplifying assumption depends on the other three. The second simplifying assumption tends to make the column design conservative. The third simplifying assumption also tends to make the column design conservative for if joints C and A rotate in opposite directions, the resisting moment $M_{AC}$ is reduced. On the other hand, the opposite rotations of joints D and A tend to increase the moment $M_{AD}$. The assumed rotations at C and D therefore serve to increase the column moment $M_{AB}$. The fourth assumption regarding the distribution of net beam moments to the column above is more difficult to evaluate. Whether or not it contributes to a conservative design depends on various factors ($L/r$, $P/P_y$, end conditions). The error for the assumed distribution, however, will not be appreciable. A conservative designer may possibly wish to ignore the
rotational restraints of the columns above and below by assuming that they do not contribute to the resistance of the unbalanced beam moments.

The four preceding assumptions reduce the analysis of an interior column of a rectangular frame to the analysis of column AB in the subassemblage shown in Fig. 2. It is important to recognize that in general for each interior column four such subassemblages are possible. The one which yields the most unfavorable unbalanced joint moment will be used. Appropriate assumptions should similarly be made for the design of an exterior column.

The structural action of the subassemblage in Fig. 2 may best be visualized with the curves of Fig. 3. The column curve gives the relation between end rotation and applied end moments for a given compressive load. The beam curve gives the net moment to be resisted by column AB of Fig. 2 for any rotation of joint A. This net moment consists of the algebraic sum of the beam end moments at joint A multiplied by a factor $\alpha$. The factor $\alpha$ is the proportion of the net beam moment which is resisted by column AB and is equal to $\frac{L_{AG}}{L_{AB} + L_{AG}}$ for the loading conditions assumed in
Fig. 2. The rest of the net moment is resisted by the column above. At zero rotation the net moment will be $\alpha(M_F\mid_{AD} - M_F\mid_{AC})$, where $M_F$ is the moment for fixed ends. Equilibrium will be obtained only when the net moment from the beams and the column end moment are equal. This occurs when the joint has rotated to values of $\theta$ for which the curves in Fig. 3 intersect. If an equilibrium configuration is impossible, there will be no intersection of the beam and column curves. When the curves intersect, one intersection will represent a stable equilibrium configuration and the other an unstable one.

In Fig. 3 point $m$ represents a stable equilibrium configuration because a small increase in the rotation (deformation) would generate a resisting moment in the column that is larger than the net unbalanced beam moment. This causes the joint to rotate back to the equilibrium position designated by point $m$. At $n$ a small increase in the rotation results in a net beam moment larger than the resisting column moment at a joint. The increase of the rotation thereby causes a moment unbalance tending to increase the rotation further. In general, when there are intersections, the point of intersection corresponding to the smaller end rotation will represent
the stable configuration. It follows that an adequate column is one for which the beam and column curves intersect. If the column and beam curves are tangent, the column is on the verge of collapse when subjected to 1.85 times the design loads and the column may be considered to have an adequate factor of safety.

The heart of any design method is a design criterion which tests the adequacy of a trial cross section. The criterion for this method is the existence of a value of $\Theta$ for which the column end moment $M_{AB}$ is equal to or larger than the net beam moment $\alpha\left\{M_{AD} - M_{AC}\right\}$. This is equivalent to showing that there is an intersection of the beam and column $M-\Theta$ curves.

In general the beams and columns may be expected to behave in a non-linear manner as indicated by the curves in Fig. 3. The beams would have considerable inelastic deformations at distributed loads equal to the load factor times the normal design loads, and the columns would be expected to behave in a non-linear manner due to both the inelastic deformations and the axial loads which amplify them.
V. DESIGN PROCEDURE FOR STEEL STRUCTURES

The beams of a structure made of A-7 steel can be designed according to the concepts of simple plastic theory. At the critical design condition for a column such as AB in Fig. 1, plastic hinges will have occurred in the beams AD and BE at A, D, E, and B. The moments \( M_{AD} \) and \( M_{BE} \) increase up to the plastic moment value \( M_p \) and remain constant and equal through further rotations of joints A, B, C, D, E, and F. The unbalanced moment which the columns must resist at joint A is \( \{ M_p \}_{AD} - M_{AC} \). If \( \alpha_u \) designates the magnitude \( \frac{M_{AB}}{M_{AB} + M_{AG}} \) the resultant moment \( M_{AB} \) is:

\[
M_{AB} = \alpha_u \left( M_p \right)_{AD} - M_{AC} \]  \hspace{1cm} (1)

Similarly \( \alpha_L \) designates the ratio \( \frac{M_{BA}}{M_{BA} + M_{BH}} \), and:

\[
M_{BA} = \alpha_L \left( M_p \right)_{BE} - M_{BF} \]  \hspace{1cm} (2)

\( M_{AC} \) and \( M_{BF} \) are functions of the fixed end moments due to the dead load on spans AC and BF respectively and the end rotations \( \theta \) (assumed to be equal in magnitude at A,B,C,
and F). If the dead and live distributed loads are equal on all beams framing into column AB, two different sub-assemblages need to be considered. These are shown in Fig. 4a and 4b. $M_{AB}$ is the larger column moment as obtained from Fig. 4a, and $M_{BA}$ is the larger column moment as obtained from Fig. 4b. It is possible to determine the larger moment between $M_{AB}$ and $M_{BA}$. The adequacy of the column can then be checked for the assemblage corresponding to the larger of these two. In the following it will further be assumed that $M_{AB}$ is larger than $M_{BA}$.

The left hand side of Eq. 1 gives the end moment of a beam-column which is symmetrically bent and subjected to an axial load equal to the one on the actual column. The end moments $M_{AB}$ are a function of the end rotations $\Theta$, of the column. Curves of column end moment $M_{AB}$ vs. end rotation $\Theta$ may be constructed in accordance with the methods outlined in Ref. 3. The column curve of Fig. 3 is one such curve for a particular column cross section and axial load.

The right hand side of Eq. 1 is also a function of $\Theta$ because of $M_{AC}$.

$$M_{AC} = M_{F} + f(\Theta)$$

......(3)
In Eq. 3 \[ M_F \int_{AC} \] is the fixed end moment \( \left( w_{DL} \frac{L^2}{I^2} \right)_{AC} \). The term \( f(\Theta) \) depends upon the length to depth ratio of AC, the cross section, the stress strain diagram, the dead load, and the residual stresses, if any, in member AC. For rolled steel wide-flange beams of A-7 steel the moment curvature relationship is sufficiently well defined* so that curves of beam moment \( M_{AC} \) vs. \( \Theta \) may be constructed for different length to depth ratios and dead load fixed end moments. In general a different curve must be constructed for each cross section. In the beam curves of Appendix A the non-dimensional ratio \( \frac{M_F \int_{AC}}{M_P} \) is designated as \( k \).

To check the adequacy of a column it is necessary to have \( M_C - \Theta \) curves for the columns at various slenderness ratios \( (L/r) \) and average compressive stresses. It is also necessary to have \( M_B - \Theta \) curves for the beams for various fixed end moments and length-depth \( (L/d) \) ratios.

These curves may be nondimensionalized in terms of the yield stress of steel, \( \sigma_y \) and the moment that would just cause yielding in a member \( M_y = S \sigma_y \). These curves are shown in Appendixes A and B. The curves take into

*For most WF sections the distribution of area over the cross section is similar.
account the detrimental effects of residual stresses which are assumed to be equal to 0.3 \( \sigma_y \) in the flange tips.\(^{(4)}\)

The design procedure is as follows:

1. Design beams by simple plastic theory.

2. Assume a column section.

3. Assume several values of \( \theta \).

4. Determine:
   
   a. \( M_C \) ... from column moment-rotation curves (Appendix B)
   
   b. \( M_p \) ... the plastic moment in the fully loaded beam
   
   c. \( M_B \) ... from beam-moment curves for the beam subjected to dead load only (Appendix A)

5. If for any value of \( \theta \)

\[
M_C \left[ \begin{array}{c} \text{AB} \end{array} \right] \geq \alpha \left[ \begin{array}{c} M_p \left[ \text{AD} \right] - M_B \left[ \text{AC} \right] \end{array} \right] \quad \cdots \cdots (4)
\]

the column is adequate.

6. If the column is inadequate it will soon become apparent that Eq. 4 will not be satisfied for any \( \theta \), and a larger column is selected.

7. If the column is adequate a lighter section may be chosen, and its adequacy is tested. The procedure is repeated until an inadequate section is found.

8. The lightest adequate section is used.
VI. ILLUSTRATIVE EXAMPLE

An interior column of a multi-story multi-bay frame is designed. The column to be designed is designated AB in Fig. 5. It is restrained by beams AD and BF with spans of 22', and by beams AC and BE with spans of 16'. Column AB has a length of 12'-6", as have the columns above and below it. All beams are subjected to a dead load of 1 k/ft. and to a live load of 1.4 k/ft. The total vertical load possible at A is equal to 182k. The story of column AB is the fourth from the top of the frame.

VI.1 Design of Beams

\[ \sigma_y = 33 \text{ ksi} \quad \text{Load factor} = 1.85 \quad M_p = \frac{wL^2}{16} \]

\[ M_p = \frac{1.85 \times (2.40)(22)^2}{16} = 134.2 \text{ ft-kips} \]

Use 14WF34, \( M_p \uparrow_{\text{AD}} = 149.9 \text{ ft-kips} \)

\[ M_p \downarrow_{\text{AC}} = \frac{1.85 \times (2.40)(16)^2}{16} = 71 \text{ ft-kips} \]

Use 12B22, \( M_p \downarrow_{\text{AC}} = 80.7 \text{ ft-kips} \)

\[ \frac{L}{d} = 16 \]
VI.2 Design of Column AB

Axial load = 1.85 (182) = 338 kips

Try a 12WF65 column with the beams framing perpendicular to the web (Minor Axis Bending).

\[ M_p = S \sigma_y \cdot \text{(Shape factor)} \]

\[ M_p = 29.1 \left( \frac{33}{12} \right) 1.5 = 120 \text{ ft-kips} \]

(Shape factor \( \approx 1.5 \) for minor axis bending).

\[ \frac{P}{F_y} = \frac{338}{33(19.11)} = 0.535 \]

\[ r = 3.02, \quad \frac{L}{r} = \frac{12.5(12)}{3.02} = 49.7 \]

\( \alpha = 1/2 \) (Because of equal heights)

Assume \( \theta = 0.01 \)

From charts in Appendix A with \( k = 0.49 \) and \( L/d = 16 \)
\[
\frac{M_{AC}}{M_y} = 0.947
\]

\[
M_{AC} = 0.947 \times (70.8) = 67 \text{ ft-kips}
\]

\[
\frac{1}{2} \cdot \left\{ M_P \right\}_{AD} - M_{AC} \right\} = \frac{1}{2} \cdot (149.9 - 67) = 41.5 \text{ ft-kips}
\]

From charts in Appendix B

\[
\frac{M_C}{M_y} \right\}_{AB} = 0.43
\]

\[
M_C \right\}_{AB} = \frac{0.43(120)}{1.5} = 34.4 \text{ ft-kips}
\]

\[
M_C \right\}_{AB} < \frac{1}{2} \cdot \left\{ M_P \right\}_{AD} - M_{AC} \right\}, \text{ i.e. } 34.4 \text{ ft-kips} < 41.5 \text{ ft-kips}
\]

Try \( \theta = 0.02 \)

\[
\frac{M_{AC}}{M_y} = \frac{1.14}{1.135}
\]

\[
M_{AC} = \frac{1.14 \times 120}{1.135} = 114 \text{ ft-kips}
\]

\[
\frac{M_C}{M_y} \right\}_{AB} = 0.5, \quad M_C \right\}_{AB} = \frac{0.5(120)}{1.5} = 40 \text{ ft-kips}
\]

\[
\frac{1}{2} \cdot \left\{ M_P \right\}_{AD} - M_{AC} \right\} = \frac{1}{2} \cdot (149.9 - 80.4) = 34.8 \text{ ft-kips}
\]

\[
M_C \right\}_{AB} > \frac{1}{2} \cdot \left\{ M_P \right\}_{AD} - M_{AC} \right\}, \text{ i.e. } 40 \text{ ft-kips} > 34.8 \text{ ft-kips}
\]

Therefore 12WF65 is adequate.
A lighter rolled steel column would have a flange width smaller than 12". It is unlikely that a smaller flange width can be adequate.

The design of column AB, by the AISC rule with end moments determined by an approximate continuous elastic analysis, results in a 12WF72. The continuous elastic design herein used is described in Ref. 7. The method of this report results in a saving in steel weight of 9.7% for column AB.
VII. SUMMARY

A design method for columns of multi-story frames with rigid connections is developed. This is achieved by considering the elasto-plastic behavior of a subassemblage which includes the column and the beams adjacent to it. The column is designed for a loading and assumed joint rotations most unfavorable to it. The detrimental effect of residual stresses in the members is also considered.

The procedure consists of choosing several column cross sections and checking their adequacy. A column is safe if an end rotation $\theta$ can be found such that the resisting moment of the column is greater than or equal to the net moment applied by the beams.

Only columns which are not likely to fail by lateral-torsional buckling can be used. The method is illustrated by the design of an interior WF column bent about its weak axis.
This study is part of the general investigations "Welded Continuous Frames and Their Components" currently being carried out at the Fritz Engineering Laboratory of the Civil Engineering Department of Lehigh University under the general direction of Lynn S. Beedle. The investigation is sponsored jointly by the Welding Research Council and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, American Iron and Steel Institute, Office of Naval Research, Bureau of Ships, and Bureau of Yards and Docks.

The authors wish to express special thanks to Prof. Theodore V. Galambos for constructive suggestions and to Mr. Y. Fukumoto for the preparation of the charts in the Appendixes.
IX. NOMENCLATURE

D.L. = Dead load

d = Depth of beam

k = (F.E.M.)/\(M_p\) for the unloaded beam

L.L. = Live load

L = Length of a member

\(M, M_B, M_C\) = Bending moment at the end of a member

\(M_{F, AC}\) = Fixed end moment for beam AC

\(M_p\) = Plastic moment

P = Axial load in the column

\(P_y\) = Area of column multiplied by \(\sigma_y\)

r = Radius of gyration

S = Section modulus

w = Distributed load

\(\alpha\) = A factor equal to the ratio of unbalanced beam moment to column end moment at a joint

\(\Theta\) = Rotation at the end of a column or beam

\(\sigma_{rc}\) = Maximum residual compressive stress at the flange tips of a rolled steel WF member

\(\sigma_y\) = Yield stress
X. APPENDIX A
ELASTIC LIMIT

\[ \frac{M}{M_y} \]

\[ \frac{L}{d} = 24 \]
\[ \frac{L}{d} = 22 \]
\[ \frac{L}{d} = 20 \]
\[ \frac{L}{d} = 18 \]
\[ \frac{L}{d} = 16 \]

\[ kM_p = 0.200 M_p \]
\[ f = 1.14 \]
\[ k = \frac{W L^2}{12 M_p} \]

\[ \sigma_{rc} = 0.3 \sigma_y \]
\[ \sigma_y = 33 \text{ ksi} \]
\[ E = 30 \times 10^3 \text{ ksi} \]
Elastic Limit

\[ M = \frac{M_y}{L_d} \]

- \( L_d = 24 \)
- \( L_d = 22 \)
- \( L_d = 20 \)
- \( L_d = 18 \)
- \( L_d = 16 \)

Equations:

\[ \tau_c = 0.3 \sigma_y \]
\[ \sigma_y = 33 \text{ ksi} \]
\[ E = 30 \times 10^3 \text{ ksi} \]

Elastic Modulus:

\[ kM_p = 0.216 M_p \]
\[ k = \frac{wL^2}{12 M_p} \]

Factor:

\[ f = 1.14 \]
$\frac{M}{M_y}$

ELASTIC LIMIT

$\sigma_{rc} = 0.3 \sigma_y$
$\sigma_y = 33 \text{ ksi}$
$E = 30 \times 10^3 \text{ ksi}$

$W$

$kM_p = 0.3 M_p$
$k = \frac{WL^2}{12M_p}$

$\theta$

$\frac{L}{d} = 24$
$\frac{L}{d} = 22$
$\frac{L}{d} = 20$
$\frac{L}{d} = 18$
$\frac{L}{d} = 16$

$1.14$
The diagram illustrates the relationship between the moment ratio $M/M_y$ and angular deflection $\theta$ for different values of $L/d$. The curves show the elastic limit behavior for various ratios of the moment to the yield moment, $M/M_y$. The equations given are:

- $\sigma_{rc} = 0.3\sigma_y$
- $\sigma_y = 33$ ksi
- $E = 30 \times 10^3$ ksi
- $kM_p = 0.456 M_p$
- $f = 1.14$
- $k = \frac{wL^2}{12 M_p}$
\[
\frac{M}{M_y} = 0.5 \quad kM_p = 0.5 \quad M_p = 0.570 \quad M_y
\]

\[
f = 1.14
\]

\[
k = \frac{wL^2}{12M_p}
\]

\[
\sigma_t = 0.3 \sigma_y
\]

\[
\sigma_y = 33 \text{ ksi}
\]

\[
E = 30 \times 10^3 \text{ ksi}
\]
8 WF 31
Weak Axis Bending

\[ \sigma_{rc} = 0.3 \sigma_y \]
\[ \sigma_y = 33 \text{ ksi} \]
\[ E = 30 \times 10^3 \text{ ksi} \]
Weak Axis Bending

\[ \frac{M}{M_y} = 0.01 \text{ for } L/r = 40 \]

\[ \frac{M}{M_y} = 0.02 \text{ for } L/r = 50 \]

\[ \frac{M}{M_y} = 0.03 \text{ for } L/r = 60 \]

\[ \frac{M}{M_y} = 0.04 \text{ for } L/r = 70 \]

\[ P = 0.7 P_y \]

\[ \sigma_{rc} = 0.5 \sigma_y \]

\[ \sigma_y = 33 \text{ ksi} \]

\[ E = 30 \times 10^3 \text{ ksi} \]
B WF 31
Weak Axis Bending
$\sigma_{rc} = 0.3 \sigma_y$
$\sigma_y = 33$ ksi
$E = 30 \times 10^3$ ksi
XII. FIGURES
UNFAVORABLE LOADING FOR COLUMN AB

Fig. 1

\[ M_{CA} = M_{AC} \]

\[ M_{DA} = M_{AD} \]

\[ M_{EB} = M_{BE} = M_{AD} \]

\[ M_{FB} = M_{BF} = M_{AC} \]

DESIGN SUBASSEMBLAGE

Fig. 2
\[ \alpha (M_{AD} - M_{AC}) \]

BEAM CURVE

\[ M = M_{AB} = M_{BA} \]

COLUMN CURVE

MOMENT VS JOINT ROTATION RELATIONSHIPS

FIG. 3
POSSIBLE DESIGN SUBASSEMBLAGES

FIG. 4
D.L. = 1.00 k/ft
L.L. = 1.40 k/ft

DESIGN EXAMPLE

Fig. 5
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