1953

Bond of prestressed strands, September 1953

D. H. Brown

Follow this and additional works at: http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports

Recommended Citation
http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports/1564

This Technical Report is brought to you for free and open access by the Civil and Environmental Engineering at Lehigh Preserve. It has been accepted for inclusion in Fritz Laboratory Reports by an authorized administrator of Lehigh Preserve. For more information, please contact preserve@lehigh.edu.
BOND OF PRESTRESSED STRANDS

by

Daniel H. Brown

in cooperation with

Professors W. J. Eney and K. E. Knudsen

A Report Submitted in Partial Fulfillment of the Requirements

of

C. E. 404 - Structural Research

Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania

1 September 1953
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>2</td>
</tr>
<tr>
<td>Test Procedure</td>
<td></td>
</tr>
<tr>
<td>(a) First pair of test beams</td>
<td>2</td>
</tr>
<tr>
<td>(b) Second pair of test beams</td>
<td>3</td>
</tr>
<tr>
<td>Results</td>
<td>5</td>
</tr>
<tr>
<td>Discussion</td>
<td>6</td>
</tr>
<tr>
<td>Conclusions</td>
<td>8</td>
</tr>
<tr>
<td>Future Research</td>
<td>9</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>10</td>
</tr>
<tr>
<td>References</td>
<td>11</td>
</tr>
<tr>
<td>Appendix</td>
<td></td>
</tr>
<tr>
<td>I Material properties</td>
<td>12</td>
</tr>
<tr>
<td>II Test beam designs</td>
<td>14</td>
</tr>
<tr>
<td>III Formulae for initial bond length and maximum bond stress</td>
<td>19</td>
</tr>
<tr>
<td>Index to Tables and Figures</td>
<td>22</td>
</tr>
</tbody>
</table>
ABSTRACT

Considerable research has been carried on in Europe investigating the many variables which affect the bond between pretensioned wires and concrete. A relatively new American innovation in the field of pretensioned—prestressed concrete is the use of 7-wire strand as a tensioning element. Little research on the bond of stranded wires has been carried on in either this country or in Europe. At Lehigh University the bond strength of 5/16" dia. 7-wire strand under various tension forces has been investigated. In addition, several pilot beams using stranded wires and one full-scale bridge member using 5/16" dia. 7-wire strand have been tested at the Fritz Engineering Laboratory of Lehigh University.

The program described herein was undertaken to determine the influence of the concrete strength on the bond strength of the strand. Measures of this variable are the initial bond length, maximum bond stress, and the behavior of the strand when subjected to live load.

It appears from these tests that 3/8" dia. 7-wire strand, when prestressed to 178,000 psi, may be safely used in beams as short as 12 feet without danger of excessive strand slip, provided certain precautions as to strand cover and concrete placement are observed.

* Numbers refer to list of references.
INTRODUCTION

The tests described in this report were conducted for the purpose of determining the effect of varying concrete strength on the bonding characteristics of 3/8" dia. 7-wire strand. Because of the unfavorable results obtained with pull-out tests(1) on 5/16" dia. 7-wire strand it was decided to employ beams for the present research. The main difficulty with the pull-out tests were that the strand tended to twist as it was pulled through the concrete; it was questionable whether this same type of failure can occur in an actual beam.

TEST PROCEDURE

(a) First pair of test beams (1E and 1W)

The original test program specified that several pairs of beams, each 8" wide, 10" deep, and 10 feet long with a single 3/8" dia. strand located 1" up from the lower surface, were to be constructed; each pair was to have a different concrete strength.

The first pair of beams were cast April 23, 1953 in the forms shown in Fig. 3. To more easily measure the strains in the concrete as it picked up stress from the strand, these beams were cast upside down as in Fig. 6. Considerable bleeding of the topmost concrete occurred and because of this, the concrete surrounding the strands probably had a somewhat lower strength than that indicated by the test cylinders. The test cylinders showed an average concrete strength of 4900 psi at 8 days and 6350 at 27 days (see Table 2).

After 7 days of moist curing the prestress of 178,000 psi or 14,700# per strand was released gradually, by releasing mechanical jacks. Ames dialS mounted on the strands and bearing against the end of the beam (Figs. 6 and 7) showed considerable slippage of the strands during the releasing operation.
The strands in both beams were released simultaneously in one operation. The slip that was measured by the Ames dials amounted to about 82% of the original elongation of the strands between the ends of the beams. In addition, cracks appeared at both ends of one beam which radiated from the strand toward the nearest surface. Strain readings on the concrete surface at the level of the strands, taken with a 10" Whittemore gage (Fig. 6) substantiated the high loss of prestress.

It was no surprise, therefore, that each beam, when subjected to a center load at 27 days after casting, cracked at a load of only 2700# on an 8-foot span. (See Fig. 8) In each case, the first strand slip (in addition to that measured during the releasing operation) was noted simultaneously with the first crack. If prestress is neglected, the computed tensile stress at the cracking load of 2700# was 485 psi, only about 8% of the average cylinder strength. After cracking, the load immediately dropped off and the maximum load carried after that time corresponded to an average bond stress of only about 30 psi.

(b) Second pair of test beams (2A and 2B)

Because of the excessive loss of prestress at release that was observed in the first pair of test beams it was decided for the second pair to use the same concrete strength and strand stress as in beams 1E and 1W but to increase the strand cover to 3". This was accomplished by increasing the depth of the beams from 10" to 12"; the length of each beam was also increased from 10 feet to 12 feet. Beams 2A and 2B were constructed right side up as shown in Fig. 4 with the cross-section indicated in Appendix III. The same instrumentation was used on the second pair of test beams as on the first with the exception that 2" Berry strain gage readings were not observed on the surface adjacent to the strand. The reason for this was that the 2"
Berry gage available was not sensitive enough to measure the small strains encountered. A ten-inch Whittemore gage was used to measure the concrete strains on one of the vertical faces at the level of the strands.

The strands were grasped at their ends by 3/8" Strandvise grips* which reacted against calibrated pipe dynamometers at one end of each strand and against mechanical jacks at the other. (Figs. 3, 4, and 5) At the mid-point of each strand on two wires diametrically opposed were located two type A 12-2 SR-4 strain gages which followed individual wires in a spiral fashion and were cemented to the wires with Duco cement. The gages were waterproofed with Neobon† and then surrounded with a plywood box 1-1/2" wide, 1-7/8" high, and 8" long (outside dimensions). A dummy gage was cemented to a short piece of strand which was then placed in a similar box and positioned near the active gages but not at the same cross-section. As the strands were tensioned, simultaneous readings were taken on the pipe dynamometers and on the strands, thus making it possible to calibrate the strand gages. The strand gages did not measure the true strain in the strand, it was necessary therefore to apply a correction to the gage strain to obtain the true strain. A value for the modulus of elasticity of 28,800,000 psi, which was taken from the manufacturer's curve, was used in the corrections. A set of calibration curves for the second pair of test beams are shown in Fig. 12 in the Appendix.

The concrete for beams 2A and 2B, while using considerably less cement per cubic yard, had very nearly the same strength at age 22 days as did beams 1E and 1W at age 27 days; see Tables 1 and 2. These beams were cast May 14, 1953, and, as in the first pair of test beams, the concrete was vibrated. Two days after pouring of the concrete the outside form was removed from each beam. After a total of 7 days of moist curing (from the time of pouring) the moist curing was stopped and small steel plugs were cemented to the outside beam surfaces for the Whittemore strain gage readings. After an addi-

* Manufactured by Reliable Electric Co., Chicago, Ill.
ional 7 days of air-dry curing (normal air of the laboratory) the strands were released gradually. Slip, concrete strains, and steel strains were measured.

Seven days after the strands had been released the beams were tested to destruction in a 300,000 lb. capacity universal testing machine. The beams were tested on a 10-foot span with the load being applied through a loading beam 1'-3" on either side of mid-span as shown in Fig. 9. Applied load, steel strain, and slip were noted; concrete strains were not observed. Deflections were measured to the nearest 0.01" by means of a wire and scale with mirror.

RESULTS

A derivation is presented in Appendix III which gives the initial bond length and the maximum bond stress after release of prestress in terms of the "slip" or "bond-developing movement" of the strand at release. Table 3 gives results of slip measurements made on beams 2A and 2B. The elastic shortening of the strand in the distance between the concrete face and the center of the gage bracket (Fig. 7) has been deducted from the slip readings. The concrete strains at the level of the strands at release have been plotted for each beam and are shown in Fig. 13. Note that the measured strains are about three times the predicted elastic strain. Shrinkage stresses were set up in the concrete due to the restraint imposed by the side and bottom forms and the strand during curing. When the strands were released and the beams removed from the forms, these shrinkage stresses were relaxed. In addition, some plastic flow due to the prestress in the concrete occurred during the 50 minutes from release until the strain measurements were made. These influences probably account for the great difference between the measured concrete strains at release and the predicted elastic strains.
During the destruction test beam 2A suffered a slip of its strand at the stressing end at a strand stress of about 260,000 psi. The maximum stress sustained by this strand was about 272,000 psi before the top concrete crushed. The strand in beam 2B did not slip at any stage of the destruction test and carried a stress of 273,000 psi before the concrete failure occurred. This information is summarized in Table 4. The average bond stresses shown in column I are the "net" average stresses required to resist the steel forces at midspan. A typical deflection curve is shown in Fig. 14. The plots of steel stress versus applied load for the first three cycles run on each beam are shown in Figs. 15 and 16. Fig. 17 is a plot of gages 26 and 28 during the loading to failure of beams 2A and 2B. Since the steel stresses exceed the proportional limit of the strand the strains have not been expressed in terms of stress except at the two points where the strand gages went out of range. Remembering that the loads in Fig. 17 do not include the weight of the loading beam, there is good agreement between the stresses in this figure and those shown in Table 4.

DISCUSSION

Prior to the placing of the concrete for the first pair of test beams (1E and 1W) the forms were oiled with machine oil. Some of this oil got onto the strands in the operation; the strands were therefore cleaned thoroughly three times with carbon tetrachloride and last with acetone and then a dry cloth. The wires appeared to be perfectly clean and dry. After the strands were released and the long ends cut off, it was noticed that the center wire was covered with a very thin layer of oil. However, no movement of the center wire with respect to the outside wires was observed at any stage of the work. It is thus doubtful that the excessive loss of prestress could be laid to the condition of the steel surface.
Several factors probably contributed to the extensive slipping of the strands in the first two beams: one would be the water gain, mentioned previously, which lowered the strength of the concrete around the strands; combined with this weakness, the one-inch cover was apparently insufficient to resist the stresses imposed by the 3/8" dia. strand. The beams were kept moist for seven days, the strands were released on the seventh day, thus, shrinkage before release was kept at a minimum.

When the strands were released the ends rotated as they slipped into the concrete, following the grooves of the individual wires. During the destruction tests, the strand ends also rotated as they slipped.

Beams 2A and 2B apparently had sufficient cover and quality of concrete to resist the prestressing forces. The two beams were similar in all respects save two: during the tensioning of the strands, which was done simultaneously for both beams, the two Strandvises holding the strand in beam 2A failed, both failures occurring at the same moment. The stress in the strand was approximately 170,000 psi at the time of failure. As there was apparently no harm done to either the gages or the strand, new Strandvises were mounted and the tensioning operation completed. The second difference was in the slump of the concrete surrounding the strands of the two beams: Table 1 shows a one-inch lower slump in beam 2A than in beam 2B. The strands used in beams 2A and 2B were wrapped with newspapers and kept covered until the beams were ready to be poured. For that reason there was no need to clean these strands prior to placing the concrete.

Table 3 shows considerably better bonding of the strand in beam 2B than in beam 2A. The maximum bond stresses found in beam 2B agree well with the maximum bond stress of 1060 psi for 0.2" dia. smooth wire determined in a test made in England(3). Results of tests conducted in Switzerland to determine average bond values for different sizes of wires are shown in Table 5.
In view of these results, those found for beam 2B do not seem unreasonable.

The method used in these tests for computing the initial bond length and bond stresses is a simple one since only the slip of the strand at release is required plus an assumption as to the shape of the steel stress versus position curve (Fig. 1). The strand slip is easily measured and could be done occasionally as a quality control procedure during manufacture in much the same way that concrete cylinders are used.

In the English investigations(3) it was found that an exponential function described the steel stress vs. position curve. This curve and a third degree parabola are plotted in Fig. 18 for an assumed initial bond length of 28 inches. Not that the exponential function approaches the $y = 1.00$ line asymptotically; the English experimenters found that at the point where the actual bond stress was zero, the theoretical bond stress was 13.5 psi. It appears from Fig. 18 that the third degree parabola may be a fair assumption as to the manner in which the steel stress varies along the length of the strand. It should be noted that in the first half of the initial bond length 88% of the steel stress has been recovered, 98% of the stress is recovered in the first 3/4 of the initial bond length.

CONCLUSIONS

From the results obtained with the last pair of test beams, it appears that for the concrete cover, strength, and beam length used, and the magnitude of steel prestress imposed, that 3/8" dia. 7-wire strand has satisfactory bonding qualities. From a comparison of the behavior of Beams 2A and 2B, which were constructed as identical as possible, it is obvious that the bonding characteristics are extremely sensitive to factors seemingly beyond the control of the fabricator. Even though the strand in beam 2A suffered end slip, the strand forces in both beams exceeded the minimum guaranteed ultimate strength for this diameter strand.
The method employed to compute the initial bond length and bond stresses, while as yet requiring experimental verification, has the advantage over other methods in its simplicity and in not affecting the natural behavior of the strand. The strand slip at release also appears from these tests to predict the strands behavior under live load, since in each case, the beam end showing the greatest slip of strand at release of prestress also was the first end at which the strand slipped during the destruction test.

FUTURE RESEARCH

The original objective of this research, i.e., the effect on the bond strength of varying concrete strengths, still remains to be accomplished. Additional beams, constructed in exactly the same manner as the last pair of test beams, containing concrete having strengths of say 5000 psi and 7000 psi, would establish the effect of this variable. Some other variables which should be considered in a truly comprehensive study of the bond of prestressed strands are: strand diameter, strand cover, steel quality, (stress-relieved or regular grade), strand spacing, strand stress, strand surface condition, concrete curing conditions, elapsed time after pouring before release, etc.

Experimental verification of the assumption that the strand stress after release varies as a third degree parabola (Fig. 1 and 18) should be made. This could be done by using a small column, say 4" by 4" in cross-section, prestressed by a 3/8" dia. 7-wire strand. Strains measured over a short gage length on the concrete would provide the desired information.
ACKNOWLEDGEMENTS

This research was made possible as the result of a grant by the Lehigh University Institute of Research, Dr. H. A. Neville, Director. Additional help was received from the John A. Roebling's Sons Co. which donated the strand used in the tests and also from the Reliable Electric Company which furnished the Strandvises; this assistance is appreciated. The author also wishes to thank Mr. Kent Preston of Roebling's for his interest in this work.

The supervision and guidance given by Professors W. J. Eney and K. E. Knudsen and the great help given by Cesar Buenaventura, Hector Daiutolo, Alexis Smislova, Alfred Roesli, Kenneth Harpel and others, which made this work possible, is gratefully acknowledged.

D. H. Brown
1 Sept. 1953
REFERENCES

1. Mayo, Loewer and Eney
   Progress Report No. 2 - Prestressed Concrete Bridge Members
   TEST OF A PRETENSIONED CONCRETE BEAM CONTAINING 5/16" DIA.
   BONDED STRANDS (Bond Test)
   Lehigh University - Institute of Research

   Progress Reports 1, 2 (Beam Test), 3, and 5 - Prestressed
   Concrete Bridge Members
   Lehigh University - Institute of Research

3. G. Marshall, B.Sc., Ph.D., University of Leeds
   END ANCHORAGE AND BOND STRESSES IN PRESTRESSED CONCRETE
   "Magazine of Concrete Research", December 1949, p. 123

4. Dr. Ritter: Polytechnic Institute of Zurich
   (Publication unknown)
APPENDIX

I Material Properties

(a) Steel Strand Reinforcement:

The reinforcement in all four test beams consisted of 3/8" nominal diameter 7-wire strand, stress-relieved grade, as manufactured by John A. Roebling's Sons Co. The manufacturer's stress-strain curve for this strand is shown in Fig. 10.

To obtain bond stresses for this strand it is necessary to determine the "effective" perimeter in contact with the concrete. In a previous study(1) it was decided that the effective perimeter is the average of the outside and inside perimeters. This is shown in the sketch, Fig. 11, where the average perimeter was found to be 1.348".

Each strand in the four test beams was prestressed with a force of 14,700# giving a prestress of 178,000 psi.

(b) Concrete:

The concrete used in the first pair of test beams (1E and 1W) contained the following mix proportions per cubic yard:

990 lb.  Cement
1390 lb. Concrete Sand
1360 lb. 1/2" Crushed Stone
48.5 gal. Water

Average Slump = 4"

The concrete used in the second pair of test beams (2A and 2B) contained the following mix proportions per cubic yard:

793 lb.  Cement
1450 lb. Concrete Sand
1390 lb. 1/2" Crushed Stone
45.5 gal. Water

Average Slump = 3"
The actual mixes varied throughout beams 2A and 2B as is shown by Table 1. The upper mixes were purposely of lower slump concrete so that there would be a minimum of water gain.

Table 2 shows the results of the tests made on the concrete cylinders representing the concrete in the test beams.
II Test Beam Designs

(a) First pair of test beams. (1E and 1W)

Prestresses:

Prestress in steel = 178,000 psi  
\[ F_1 = 178,000 \times 0.0826 = 14,700 \]  

\[ I_c = \frac{8 \times 10^3}{12} = 667 \text{ in}^4 \]  

\[ A_c = 8 \times 10 = 80 \text{ in}^2 \]  

\[ f_{b,t} = \frac{F_1}{A_c} \cdot \frac{F_1 \cdot e}{Z_{b,t}} \]  

\[ f_{b,t} = \frac{14,700}{80} \cdot \frac{14,700 \cdot 4}{133} = 183 \cdot 440 = 623 \text{ psi compression} \]  

\[ f_{t,t} = 183 - 440 = -257 \text{ psi tension} \]  

Dead Load Stresses:

\[ W_G = 8 \cdot 10 \times 150 = 83.3 \text{ #/ft.} \]  

\[ M_G = \frac{wL^2}{8} = \frac{83.3 \times 8 \times 8 \times 12}{8} = 8000 \text{ #} \]  

\[ f_{G,b} = \frac{M_G}{Z_{t,b}} = \frac{8000}{133} = 60 \text{ psi} \]
Stresses at centerline section:

\[ \begin{align*}
\text{Prestress} & \quad -257 \\
\text{Dead Load} & \quad +60 \\
\text{Stresses at centerline} & \quad -197 \\
\text{Stresses due to } M_L = 75,000" & \quad +563 \\
\text{Combined Stresses (at crack-opening load)} & \quad -563 \\
& \quad 0
\end{align*} \]

Percentage of reinforcement = \( \frac{0.0826 \cdot 100}{9 \cdot 8} = 0.115 \% \)

Shear Stresses:

Shear at zero bottom fiber stress = 1560#

1st moment of area above N. A. = \( Q = 8 \cdot 5 \cdot 2.5 = 100 \text{ in}^3 \)

\[ v = \frac{V Q}{I_c b} = \frac{1560 \cdot 100}{667 \cdot 8} = 29.2 \text{ psi} \]

Prestress at N. A. = 183 psi compression = \( f \)

\[ f_d = \text{diagonal tension} = \frac{f}{2} - \sqrt{\frac{f^2}{2} - (\frac{f}{2})^2} \]

\[ f_d = 91.5 - \sqrt{29.2^2 - 91.5^2} = 91.5 - 96.0 = -4.5 \text{ psi} \]

Ultimate Moment:

Percentage of reinforcement is low, therefore (provided good bond exists) steel failure will occur before concrete failure.

Guaranteed minimum ultimate strength of 3/8" @ strand = 20,900#

Based on a rectangular concrete stress distribution at ultimate load and \( f_d = 6350 \text{ psi} \) (see Table 2), the depth of the concrete compression zone required to balance the steel force is \( \frac{20,900}{8 \cdot 6350} = 0.41" \).

The total ultimate moment is \( (9 - 0.41/2) \cdot 20,900 = 184,000" \)

Net moment produced by live load = 184,000 - \( M_g \) = 176,000#

Load required to produce this moment = 7330#
(b) Second pair of test beams. (2A and 2B)

Prestresses:

Prestress in steel = 178,000 psi

\[
F_1 = \frac{178,000 \cdot 0.0826}{12} = 14,700\text{#}
\]

\[
I_c = \frac{8 \cdot 12^3}{12} = 1152\text{ in}^4
\]

\[
A_c = 8 \cdot 12 = 96\text{ in}^2
\]

\[
f_{b,t} = \frac{F_1}{A_c} \leq \frac{F_1 \cdot e}{Z_{b,t}}
\]

\[
f_{b,F1} = \frac{14,700}{96} \leq \frac{14,700 \cdot 3}{192} = 153.1 \leq 230 = 383\text{ psi compression}
\]

\[
f_{t,F1} = 153 - 230 = -77\text{ psi tension}
\]

Dead Load Stresses:

\[
W_G = \frac{8 \cdot 12}{144} \cdot 150 = 100\#/\text{ft}
\]

\[
M_G = \frac{w \cdot L^2}{8} = \frac{100 \cdot 10 \cdot 10 \cdot 12}{8} = 15,000\#\text{in}
\]

\[
f_{c,b} = \frac{15,000}{192} = 78\text{ psi}
\]
Stresses at centerline section:

\[
\begin{align*}
\text{Prestress} & \quad \text{Dead Load} = \text{Stresses at centerline} \\
\text{Stresses due to } M_L = 58,500" \text{ (Applied load = 2600#)} \\
\text{Stresses due to } M_L = 153,600" \text{ (Applied load = 6820#)} \\
\text{Stresses at cracking load (Total applied load = 9420#)}
\end{align*}
\]

Assumed modulus of rupture = 0.13 \( f_c \) = 0.13 • 6200 = 800 psi
Percentage of steel = \( \frac{0.0826}{9 \cdot 8} \) = 0.115%

Shear Stresses:

Shear at zero bottom fiber stress = 1300#
1st moment of area above N. A. = \( Q = 8 \cdot 6 \cdot 3 = 144 \text{ in}^3 \)
\[
\nu = \frac{V Q}{I_c b} = \frac{1300 \cdot 144}{1152 \cdot 8} = 20.4 \text{ psi}
\]
prestress at N. A. = 153 psi compression = \( f \)
\[
f_d = \text{diagonal tension} = \frac{f}{2} - \sqrt{v^2 + (\frac{f}{2})^2}
\]
\[
f_d = 76.5 - \sqrt{20.4^2 + 76.5^2} = 76.5 - 79.2 = -2.7 \text{ psi}
\]
Elastic strain in concrete at level of strand due to release of prestress:
\[
f_c = \frac{9}{12} \cdot 304 \cdot 1 = 229 \cdot 1 = 229 \text{ psi (at level of strand)}
\]
\[
E_c \text{ at release} = 4.65 \cdot 10^6 \text{ psi (see Table 2)}
\]
\[
\varepsilon_c \text{ at release} = \frac{229}{4.65 \cdot 10^6} = 49.3 \cdot 10^{-6}
\]
Ultimate Moment:

Percentage of reinforcement is low, therefore (provided good bond exists) steel failure will occur before concrete failure.

Guaranteed minimum ultimate strength of 3/8" Ø strand = 20,900#

Based on a rectangular concrete stress distribution at ultimate load and $f'_c = 6200$ psi (see Table 2), the depth of the concrete compression zone required to balance the steel force is $\frac{20,900}{8 \cdot 6200} = 0.42''$

The total ultimate moment is $= (9 - 0.42/2) \cdot 20,900 = 184,000#''$

Net moment produced by live load $= 184,000 - M_G = 169,000#''$

Load required to produce this moment $= 7500#$. 
III Formulae for Initial Bond Length and Maximum Bond Stress

The following derivation shows the relationship between the movement of the strand end into the concrete at release of prestress and the maximum bond stress together with the initial bond length.

Notation: The notation used is that recommended by Joint ACI-ASCE Committee 323 as published in ACI Journal, October 1952, with the following additions:

- \( a \) = initial bond length: distance from the end of the member to the point on the strand where the bond stress is essentially zero. (After release of prestress and before the member is loaded.)
- \( b \) = bond-developing movement: movement of the strand end into the concrete member as the pretensioned strand is released.
- \( p \) = perimeter of steel in "effective" contact with the concrete.

FIG. 1 VARIATION OF STEEL STRESS \((f_s)\) VS. POSITION FROM BEAM END

FIG. 2 VARIATION OF BOND STRESS \((u)\) VS. POSITION FROM BEAM END
The relationship that the \( u \) vs. \( x \) curve is proportional to the derivative of the \( f_s \) vs. \( x \) curve is evident from the following equilibrium sketch of a segment of a bonded wire under differential stress:

\[
\begin{align*}
\frac{\Delta F}{dx} &= 0 \\
\frac{u_x}{dx} - f_s A_s - (f_s \frac{df_s}{dx}) A_s &= 0 \\
u_x &= \frac{df_s}{dx} \frac{A_s}{p}
\end{align*}
\]

Stated in words, the bond stress at any point is equal to the slope of the steel stress curve at that point times \( \frac{A_s}{p} \).

Looking at Fig. 1 it can be seen that the relative movement between \( m \) and \( n \) is equal to the unit shortening that has taken place in the strand at this point multiplied by \( dx \). By assuming that at point "c" the strand does not move with respect to the concrete (since by definition point \( c \) is the point of zero bond stress, it follows that there can be no movement at that point) it is possible to equate the summation of all of these small incremental shortenings over the \( dx \) distances to the movement of the strand end into the concrete member.

Relative movement \( m \) to \( n \) = \( \frac{f_{so} - f_x}{E_s} \) \( dx \) \hspace{1cm} (1)

As suggested by Dr. Ritter(4), a third degree parabola will be used to define the \( f_s \) vs. \( x \) curve.

\[
\begin{align*}
f_{so} - f_x &= \left[ \frac{a - x}{a} \right]^3 \cdot f_{so} \hspace{1cm} (2) \\
f_x &= f_{so} \left[ 1 - (1 - \frac{x}{a})^3 \right] \hspace{1cm} (3)
\end{align*}
\]
\[
b = \int_0^a \frac{f_{so} - f_x}{E_s} \, dx
\]

Thus, the bond-developing movement equals the cross-hatched area shown in Fig. 1 divided by \(E_s\).

An expression will now be obtained for the bond stress in terms of the bond-developing movement.

\[
u_x = \frac{df_s}{dx} \frac{A_s}{p} = \frac{d}{dx} \left( \int_0^a \frac{f_{so}}{E_s} \left( 1 - \left( 1 - \frac{x}{a} \right)^3 \right) \right) \frac{A_s}{p}
\]

\[
u_x = \frac{A_s}{p} \int -3 \left( 1 - \frac{x}{a} \right)^2 \left( - \frac{1}{a} \right) f_{so}
\]

\[
u_x = 3 \frac{f_{so}}{a} \frac{A_s}{p} \int 1 - \frac{x}{a}^2
\]

Substituting \(a = \frac{4 E_s b}{f_{so}}\) in the above expression gives a formula for the bond stress at any point in terms of the bond-developing movement.

\[
u_x = \frac{3 f_{so}^2 A_s}{4 E_s b p} \int 1 - \frac{x}{a} \frac{f_{so}}{4 E_s b}^2
\]

\[
u_{max} = \nu_x \text{ at } x = 0
\]

\[
u_{max} = \frac{3 f_{so}^2 A_s}{4 E_s b p}
\]
TABLES AND FIGURES

TABLE 1 Concrete Mixes Used in Beams 2A and 2B  28
2 Summary of Concrete Cylinder Tests  29
3 Initial Bond Length and Bond Stresses  31
4 Steel Stresses During Destruction Test Based on Crack Height  33
5 Final Average Bond Stresses (EMPA)  38

FIGURE 1 Variation of $f_s$ vs. position from beam end.  19
2 Variation of bond stress vs. position from beam end.  19
3 Forms ready for casting of beams 1E and 1W.  23
4 Forms ready for casting of beams 2A and 2B.  23
5 Detail of pipe dynamometer.  23
6 Concrete strain readings being taken on beams 1E and 1W.  24
7 Ames gages mounted to measure slip.  24
8 Beam 1E after failure by a center load of 2700#.  25
9 Beam 2B supporting a total load of 95% of the ultimate load.  25
10 Manufacturer's stress-strain curve for strand reinforcement.  26
11 Determination of effective bond perimeter.  27
12 Calibration curves for gages in beams 2A and 2B.  30
13 Predicted and observed concrete strains at release - Beams 2A and 2B.  32
14 Load-deflection curves for beam 2B.  34
15 Increase of steel stress with applied load, cycles 1-3, Beam 2A.  35
16 Increase of steel stress with applied load, cycles 1-3, Beam 2B.  36
17 Increase of steel strain with applied load, cycle 4, Beams 2A and 2B.  37
18 Two possible variations of strand stress.  39
FIG. 3  FORMS READY FOR CASTING OF BEAMS 1E and 1W.  
Note mechanical jacks and Strandvise grips 
which are attached to the strands.

FIG. 4  FORMS READY FOR CASTING OF BEAMS 2A and 2B.  
Note pipe dynamometers in foreground 
against which the Strandvise grips are 
bearing.

FIG. 5  DETAIL OF PIPE DYNAMOMETER
FIG. 6 CONCRETE STRAIN READINGS BEING TAKEN ON BEAMS 1 E AND 1 W WITH 10" WHITEMORE AND 2" BERRY STRAIN GAGES PRIOR TO RELEASE OF PRESTRESS

FIG. 7 AMES GAGES MOUNTED TO MEASURE SLIP DURING RELEASE OF PRESTRESS
FIG. 8 BEAM 1E AFTER FAILURE BY A CENTER LOAD OF 2700#.

FIG. 9 BEAM 2B SUPPORTING A TOTAL LOAD OF 95% OF THE ULTIMATE LOAD. Note that only one crack has developed.
Tensile Test
on
3/8 in. Diameter 7 Wire Strand
for
Prestressed Concrete
Ultimate Tensile Strength - 276,000 psi
For Each Strand

FIG. 10 MANUFACTURER'S STRESS-STRAIN CURVE FOR STRAND REINFORCEMENT
Center wire: 0.126" Ø
Outside wires: 0.122" Ø

Outside perimeter = \( \pi \cdot 0.370 \)
= 1.163"

Inside perimeter = \( 6 \pi \cdot 0.122 \cdot \frac{240}{360} \)
= 1.533"

Average perimeter = \( \frac{1.163 + 1.533}{2} \) = 1.348"

FIG. 11 DETERMINATION OF EFFECTIVE BOND PERIMETER OF 3/8" DIAMETER 7-WIRE STRAND
**TABLE 1**  CONCRETE MIXES USED IN BEAMS 2A & 2B

<table>
<thead>
<tr>
<th>BATCH</th>
<th>CEMENT #</th>
<th>WATER #</th>
<th>SAND* #</th>
<th>GRAVEL #</th>
<th>SLUMP in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>52.9</td>
<td>23.3</td>
<td>98.7</td>
<td>92.7</td>
<td>3-1/2</td>
</tr>
<tr>
<td>2</td>
<td>49.9</td>
<td>22.0</td>
<td>98.7</td>
<td>92.7</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>49.9</td>
<td>22.0</td>
<td>98.7</td>
<td>92.7</td>
<td>1-1/2</td>
</tr>
<tr>
<td>4</td>
<td>46.9</td>
<td>20.7</td>
<td>98.7</td>
<td>92.7</td>
<td>1/2</td>
</tr>
<tr>
<td>5</td>
<td>44.9</td>
<td>19.8</td>
<td>98.7</td>
<td>92.7</td>
<td>2-3/4</td>
</tr>
<tr>
<td>6</td>
<td>52.9</td>
<td>23.3</td>
<td>98.7</td>
<td>92.7</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>52.9</td>
<td>23.3</td>
<td>98.7</td>
<td>92.7</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>52.9</td>
<td>23.3</td>
<td>98.7</td>
<td>92.7</td>
<td>1-1/2</td>
</tr>
<tr>
<td>9</td>
<td>46.9</td>
<td>20.7</td>
<td>98.7</td>
<td>92.7</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>46.9</td>
<td>20.7</td>
<td>98.7</td>
<td>92.7</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>23.4</td>
<td>10.4</td>
<td>49.3</td>
<td>46.3</td>
<td>-</td>
</tr>
</tbody>
</table>

*Contains 2# of free water, except for batch 11 which had 1#.

---

**STRENGTH—**

<table>
<thead>
<tr>
<th>Beam 2A</th>
<th>Beam 2B</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Strand—

<table>
<thead>
<tr>
<th>Beam 2A</th>
<th>Beam 2B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

**POSITIONS OF BATCHES IN BEAMS**
TABLE 2
SUMMARY OF CONCRETE CYLINDER TESTS

<table>
<thead>
<tr>
<th>BEAM</th>
<th>EVENT</th>
<th>AGE (days)</th>
<th>CYLINDER</th>
<th>$f'_c$ (psi)</th>
<th>$E_c$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E &amp; 1W</td>
<td>At Release</td>
<td>8</td>
<td>A1</td>
<td>4450</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>8</td>
<td>A3</td>
<td>5000</td>
<td>4250</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>8</td>
<td>A4</td>
<td>5260</td>
<td>4510</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>8</td>
<td>Avg.</td>
<td>4900</td>
<td>4380</td>
</tr>
<tr>
<td></td>
<td>Destruction Test</td>
<td>27</td>
<td>A2</td>
<td>7070</td>
<td>4800</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>27</td>
<td>A6</td>
<td>6470</td>
<td>4760</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>27</td>
<td>A7</td>
<td>5510</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>27</td>
<td>Avg.</td>
<td>6350</td>
<td>4780</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>A8</td>
<td>6990</td>
<td>-</td>
</tr>
<tr>
<td>2A &amp; 2B</td>
<td>At Release</td>
<td>15</td>
<td>B1</td>
<td>5710</td>
<td>4600</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>15</td>
<td>B6</td>
<td>5320</td>
<td>4700</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>15</td>
<td>Avg.</td>
<td>5500</td>
<td>4650</td>
</tr>
<tr>
<td></td>
<td>Destruction Test</td>
<td>22</td>
<td>B2</td>
<td>6360</td>
<td>4900</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>22</td>
<td>B7</td>
<td>6100</td>
<td>4860</td>
</tr>
<tr>
<td></td>
<td>&quot;</td>
<td>22</td>
<td>Avg.</td>
<td>6200</td>
<td>4880</td>
</tr>
</tbody>
</table>

Note: Cylinder B2 was taken from batch 2, 2nd pair of test beams, cylinder A4 was taken from batch 4, 1st pair of test beams, etc.
FIG. 12  CALIBRATION CURVES FOR SR-4 GAGES IN BEAMS 2A AND 2B
TABLE 3. INITIAL BOND LENGTH AND BOND STRESSES
BEAMS 2A AND 2B

<table>
<thead>
<tr>
<th>BEAM</th>
<th>EVENT</th>
<th>MEASURED SLIP</th>
<th>INITIAL BOND LENGTH</th>
<th>AVERAGE BOND STRESS</th>
<th>MAXIMUM BOND STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(b) (inches)</td>
<td>(a) (inches)</td>
<td>(psi)</td>
<td>(psi)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stressing end</td>
<td>Anchor end</td>
<td>Stressing end</td>
<td>Anchor end</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>2A</td>
<td>On form immediately after release</td>
<td>0.087</td>
<td>0.054</td>
<td>57.0</td>
<td>35.3</td>
</tr>
<tr>
<td>2A</td>
<td>20 minutes after release (simply supported)</td>
<td>0.094</td>
<td>0.056</td>
<td>61.5</td>
<td>36.6</td>
</tr>
<tr>
<td>2A</td>
<td>3 hours after release</td>
<td>0.099</td>
<td>0.053</td>
<td>64.7</td>
<td>37.9</td>
</tr>
<tr>
<td>2B</td>
<td>On form immediately after release</td>
<td>0.044</td>
<td>0.048</td>
<td>28.8</td>
<td>31.4</td>
</tr>
<tr>
<td>2B</td>
<td>20 minutes after release (simply supported)</td>
<td>0.046</td>
<td>0.051</td>
<td>30.1</td>
<td>33.4</td>
</tr>
<tr>
<td>2B</td>
<td>3 hours after release</td>
<td>0.048</td>
<td>0.051</td>
<td>31.4</td>
<td>33.4</td>
</tr>
</tbody>
</table>

NOTE:
For definitions of (a) and (b), see Section III of Appendix.

Average bond stress = \( \frac{\text{Effective prestressing force}}{(a) \cdot \text{effective perimeter}} \)

Maximum bond stress = \( 3 \cdot \text{Avg. bond stress} \)
FIG. 13 PREDICTED AND OBSERVED CONCRETE STRAINS AT RELEASE AS OBTAINED FROM 10" WHITTEMORE STRAIN GAGE MEASUREMENTS
## TABLE 4. STEEL STRESSES DURING DESTRUCTION TESTS BASED ON CRACK HEIGHT

<table>
<thead>
<tr>
<th>BEAM</th>
<th>EVENT</th>
<th>TOTAL APPLIED LOAD - Lbs.</th>
<th>CRACK HEIGHT ABOVE STEEL (in.)</th>
<th>FORCE IN STEEL AT CENTERLINE (Lbs.)</th>
<th>Steel Stress (psi)</th>
<th>AVG. BOND STRESS (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
</tr>
<tr>
<td>2A</td>
<td>First slip (stressing end)</td>
<td>7455</td>
<td>8.0</td>
<td>1750</td>
<td>19,800</td>
<td>21,550</td>
</tr>
<tr>
<td>2A</td>
<td>Anchor end started slipping</td>
<td>7725</td>
<td>8.3</td>
<td>1740</td>
<td>20,100</td>
<td>21,840</td>
</tr>
<tr>
<td>2A</td>
<td>Concrete began crushing</td>
<td>8045</td>
<td>8.5</td>
<td>1720</td>
<td>20,700</td>
<td>22,420</td>
</tr>
<tr>
<td>2B</td>
<td>Concrete began crushing</td>
<td>8075</td>
<td>8.5</td>
<td>1720</td>
<td>20,800</td>
<td>22,520</td>
</tr>
</tbody>
</table>

(1) Includes 125#/ for weight of loading beam.

(2) From measurements on test beams.

(3) \( \frac{M_G}{L} \times \frac{9 - \text{crack ht.}}{2} \times \text{crack ht.} \)

(4) \( 45^\circ \times \frac{\text{Load}}{L} \times \frac{9 - \text{crack ht.}}{2} \times \text{crack ht.} \)

(5) Steel stress = \( \frac{\text{Steel Force}}{0.0826} \)

(6) Avg. Bond Stress = \( \frac{\text{Steel Force}}{1.548 \times 72} \) = 0.0103 \times \text{Steel Force}
FIG. 14 LOAD-DEFLECTION CURVES FOR BEAM 2B

Note: Applied Load = P

CENTERLINE DEFLECTION (inches x 10^{-2})

Ultimate Load = 7950#
Fig. 15 INCREASE OF STEEL STRESS DUE TO APPLIED LOAD (EXCLUDING LOADING BEAM) FOR BEAM 2A
FIG. 16 INCREASE OF STEEL STRESS DUE TO APPLIED LOAD (EXCLUDING LOADING BEAM) FOR BEAM 2B
FIG. 17 INCREASE OF STEEL STRAIN DUE TO APPLIED LOAD (EXCLUDING LOADING BEAM) FOR BEAMS 2A AND 2B

Same loading system as shown in figure 15

Maximum load = 7920#

Maximum load = 950#

Gage 26, Beam 2A

Gage 28, Beam 2B

STRAIN (Per cent)

APPLIED LOAD (lbs.) Excluding Loading Beam

(Cycle 4) Beam 2A

Beam 2B (Cycle 4)
**TABLE 5. FINAL AVERAGE BOND STRESSES**

<table>
<thead>
<tr>
<th>TYPE OF WIRES</th>
<th>DIAMETER (in.)</th>
<th>FINAL AVERAGE BOND STRESSES (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round</td>
<td>0.06</td>
<td>465</td>
</tr>
<tr>
<td>&quot;</td>
<td>0.08</td>
<td>250</td>
</tr>
<tr>
<td>&quot;</td>
<td>0.12</td>
<td>170</td>
</tr>
<tr>
<td>Notched</td>
<td>0.12</td>
<td>720</td>
</tr>
<tr>
<td>Round</td>
<td>0.20</td>
<td>140</td>
</tr>
<tr>
<td>Strand</td>
<td>4X0.10</td>
<td>425</td>
</tr>
</tbody>
</table>

*From EMPA, Bericht Nr. 162, Zurich, 1950:

\[ f_{si} = 170,000 \text{ psi} \]
\[ f'_{c} = 6,400 \text{ psi} \]
FIG. 18  TWO POSSIBLE VARIATIONS OF STRAND STRESS
VS. POSITION ALONG BEAM

\[ y = k \sqrt{1 - e^{-0.145x}} \]
(English Investigation)

\[ y = k \sqrt{1 - \left(1 - \frac{x}{a}\right)^3} \]
(Third degree parabola)