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PRESTRESSED CONCRETE BRIDGE MEMBERS

PROGRESS REPORT NO. 24

PROBABLE FATIGUE LIFE OF PRESTRESSED
CONCRETE FLEXURAL MEMBERS

R. F. Warner
C. L. Hulsbos

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ABSTRACT

An investigation was conducted into the fatigue life of prestressed concrete flexural members. Two types of beam fatigue failure were considered, one resulting from fatigue of the strand reinforcement, the other corresponding to fatigue failure of the concrete in the compressive stress block. Attention was given primarily to the steel failure, which is the one more likely to occur in beams of normal design.

A method was developed for predicting the fatigue life of prestressed concrete flexural members failing by fatigue in the steel reinforcement under repeated constant cycle and cumulative damage loadings. A means of obtaining a lower bound estimate of beam fatigue life as limited by concrete fatigue failure is also described.

The work carried out in the investigation consisted of a program of fatigue tests on prestressed concrete beams, an experimental study of the fatigue properties of high strength steel strand reinforcement, and a theoretical
analysis of the stresses and deformations in a prestressed concrete flexural member under fatigue loading. The results of the theoretical analysis can be used together with the results of the experimental study of strand fatigue properties to predict the fatigue life of prestressed concrete flexural members. A comparison of predicted and observed fatigue lives for the test beams shows satisfactory correlation.
The term fatigue failure was first used by nineteenth century engineers to describe sudden, brittle failures which were observed in apparently sound metal machine parts. Upon investigation it was discovered that failure can be induced in materials by the repeated application of loads which are considerably smaller than the static strength. The number of load repetitions required to produce...
fatigue failure in a structural element or machine part of course depends upon the properties of the particular mate­rial and upon the magnitude of the loadings, but the number is usually many thousands. Although each load fluctuation by itself can cause no significant damage, the effect is progressive, with damage accumulating in the material as the number of loadings increases, until eventually the material is so weakened that the loads cannot be resisted.

Fatigue loading can exist in a variety of forms. The simplest is a continuous and regular variation of load between constant minimum and maximum levels. More com­plicated forms which are commonly met with in practice consist of repeated loads of varied magnitude occurring in random sequence at irregular intervals of time.

1.1. FATIGUE FAILURE IN PRESTRESSED CONCRETE BEAMS

Fatigue failure can occur in concrete, as well as in the metals. Prestressed concrete, being a combination of concrete and high strength steel, is also subject to the phenomenon. The possible modes of failure in fatigue of
prestressed concrete flexural members are comparable in some ways to the corresponding modes of failure under static loading, and a similar method of classification is useful. Thus, failures involving fatigue in the component materials may be referred to conveniently as "under-reinforced", "over-reinforced", or "balanced", depending upon whether the primary failure takes place in the steel, in the concrete, or in the two materials more or less simultaneously. It should be noted that a beam which is under-reinforced with respect to fatigue failure is not necessarily under-reinforced from the point of view of static ultimate strength.

In certain circumstances it may be possible also for a progressive bond failure to occur along the length of the beam as a result of fatigue loading. Bond-fatigue failure, like bond-failure under static loading, will occur only in regions where relatively steep moment gradients exist; it is therefore unlikely in flexural members in which the moment-to-shear ratio is large, and is more conveniently treated in association with a study of shear failure. In dealing with the basic modes of fatigue failure in flexure, attention will be restricted
in this study to fatigue in the tension steel and in the concrete compression region.

1.2 PREVIOUS INVESTIGATIONS

It is interesting to note that when prestressed concrete was introduced as a new method of construction, the possibility of its having poor fatigue properties was a matter of concern to many engineers. A number of beam fatigue tests were therefore conducted in Europe \(^1,2\), in England \(^3\), and later in the United States \(^4,5\). Although these initial investigations were little more than acceptance tests, they indicated that fatigue failure would not be a practical problem provided the repeated loadings were not large enough to cause cracking of the concrete. Since this state of affairs was realized in the designs at that time, interest in fatigue failure declined and subsequent work has not been sufficiently extensive to provide answers to the many questions concerning the effects of fatigue loading.

Unfortunately, however, this very lack of information on the fatigue resistance of prestressed concrete
members has made necessary the retention in design specifications of the stipulation of no cracking under load. This in turn places severe restrictions on possible improvements and refinements to design procedures. At present, for example, the economic use of partial pre-stressing techniques is often impossible because of code limitations on allowable bottom fiber concrete stresses.

Before current code requirements can be changed, far more information must be obtained on the behavior of prestressed concrete under fatigue loading. Knowledge of fatigue failure — which until recently was almost completely contained in the negative statement that fatigue failure will not occur if the loads are not sufficiently large to cause cracking — must obviously be broadened to allow quantitative estimates to be made of fatigue life and safety against fatigue failure under general loading conditions.

A bibliography and review of research on concrete fatigue has recently been published by Nordby (6), and it will not therefore be necessary to give a detailed account here of previous investigations. Nordby's review indicates that earlier studies of fatigue failure in
Prestressed concrete beams have consisted, in the main, of small experimental programs of beam fatigue tests. Because of the isolated nature of this work, the conclusions have often been limited to the performance of a particular type of beam under a particular condition of loading. Often, too, the published reports have been too brief even to yield any clear picture of the detailed sequence of events leading to failure.

Probably the most significant conclusion which can be drawn from previous experimental work is with respect to the type of failure which might be expected in beams of normal design. A study of the test reports indicates that under-reinforced fatigue failures are by far the most commonly occurring. Indeed, in the available literature, only one case of an over-reinforced failure is reported; Le Camus (7) was able to force a concrete fatigue failure by using a special reinforced concrete test beam of champignon design with a reduced concrete compression region. Next to the under-reinforced flexural fatigue failure, shear-fatigue failure is the most common fatigue failure reported in the literature.

The most important analytic study of the fatigue
strength of prestressed concrete beams has been made by Ekberg, et al. (8,9,10,11) Ekberg suggests that an estimate of fatigue resistance of a flexural member subjected to repeated loadings of constant magnitude can be made by using experimentally obtained fatigue-failure envelopes for the component materials of concrete and steel. Typical fatigue failure envelopes for high strength concrete and steel prestressing strand, taken from reference (8), are shown in Fig. 1. These figures indicate, for any given minimum stress level, the magnitude of the maximum stress level of a load cycle which would cause fatigue failure after one million applications.

The failure envelopes may be used, together with a theoretically obtained relation between applied moment and the resulting stresses in the beam, to determine those ranges of loading which would cause failure after one million repetitions. In order to obtain the steel stress-moment relations, Ekberg refers to an approximate analysis described by Colonnetti (12). Alternatively he suggests an approximation obtained by joining with straight lines the three easily computed points corresponding to zero moment, cracking moment, and ultimate
A relation between steel stress and applied moment, taken from reference (9), is shown in Fig. 2. An example will demonstrate the use of these figures. Assuming that the minimum moment in the beam, due to dead load only acting, is 20 percent, the corresponding minimum steel stress is found from Fig. 2 to be 48 percent. For a minimum stress level of 48 percent, Fig. 1 indicates a maximum stress level of 64 percent which, from Fig. 2, corresponds to a moment of 68 percent. Thus, a load cycle on the beam producing a moment cycle varying between 20 and 68 percent of the static ultimate moment could be expected to produce failure by steel fatigue after one million applications. A similar analysis, using the concrete fatigue envelope and a relation between applied moment and top fiber concrete stress, would yield the load cycle which would cause concrete fatigue failure after one million repetitions. When the load cycle corresponding to steel fatigue is smaller than that corresponding to concrete fatigue an under-reinforced failure will occur, and vice versa.

The method described above, being simple and
approximate, ignores several effects which will now be discussed in turn.

In the treatment of the concrete failure, conditions are considered only in the extreme concrete compression fiber, and the problem is treated as a case of fatigue failure of a concrete element under repeated axial loading. Concrete fatigue failure in fact takes place in the compression region of the beam in the presence of a stress gradient. This simplification however seems to be justified when it is realized that (a) primary concrete failure is extremely rare, and (b) in those rare cases where it does occur, consideration of the extreme conditions in the outer-most fiber will yield a lower bound to, and hence a conservative estimate of, fatigue strength.

Another effect which is not considered, but which might be of some importance, is the change which will occur in the stress-moment relations throughout the load history as a result of concrete creep and progressive bond breakdown around the tension cracks. The magnitude of the change, and the extent to which it will effect fatigue life, are questions which have not been investigated. Also
the methods suggested for obtaining the stress-moment relations are very approximate and can lead to considerable error in fatigue life predictions, especially in the case of low load and long fatigue life where a small error in stress will cause a very large error in the number of cycles to failure.

Perhaps the most important effect ignored in the approach is the essentially statistical nature of fatigue data. Constant cycle fatigue properties of a material cannot be represented adequately by three dimensional relations, such as S-N curves and fatigue envelopes, which involve only maximum and minimum stress levels and the number of cycles to failure. Recent studies of the fatigue properties of materials(13,14) clearly indicate that the variability inherent in the basic fatigue properties of materials requires that fatigue phenomena be treated in probabilistic terms.

In summarizing previous research on concrete fatigue, Nordby(6) concludes that --- "Most of the research up to this time (1958) has been exploratory and investigators now know what to look for." More specifically, previous research has indicated the primary importance of the
under-reinforced flexural fatigue failure. Ekberg's work, besides providing a very simple though very approximate method of estimating fatigue resistance under constant cycle loading, emphasizes clearly the importance of experimental studies of the fatigue properties of the materials as a basis for estimating the fatigue properties of the member.

Previous research into fatigue failure of prestressed concrete beams has been entirely restricted to the rather idealized situation of constant cycle repeated loadings. In practice it is of course far more usual for successive loadings to differ in magnitude, and for the load history to consist of a number of loads of different size, each with a different frequency of occurrence. Studies of the fatigue properties of prestressed concrete members under varied loading patterns have not yet been made, and important practical questions concerning the effect on fatigue life of a relatively small number of high overloads regularly mixed with the design loading have not been answered.

1.3 OBJECT AND SCOPE OF INVESTIGATION

In this investigation a method is developed for
predicting the probable fatigue life of under-reinforced prestressed concrete flexural members under repeated loadings of either constant or varied magnitude. The method is based on a theoretical analysis of the stresses and deformations in a concrete flexural member under repeated loadings and on an experimental study of the fatigue properties of a type of high strength steel used extensively in the United States in the manufacture of pretensioned prestressed concrete structures. To provide detailed information on beam behavior under fatigue loading and to check the accuracy of the method developed for predicting fatigue life, static and fatigue tests were conducted on eight prestressed concrete beams of rectangular section.

The fatigue properties of 7/16 inch diameter, seven wire, high strength prestressing strand were studied in an experimental investigation involving constant cycle tests, cumulative damage tests, and static tests on approximately 150 specimens. Equations were derived for the probable fatigue life of strand elements under repeated loadings of either constant or varied magnitude. The values of the test variables were chosen so that the
range of applicability of the resulting equations covers most practical situations.

On the basis of an assumed general pattern of beam behavior, an analysis was made of the deformations and stresses in a prestressed concrete flexural member under load. The resulting equations, together with the experimentally determined strand fatigue properties, provide a means of estimating the probable fatigue life of flexural members, as limited by steel fatigue failure. A method of obtaining a lower bound estimate of fatigue life as limited by concrete failure, by considering conditions in the extreme concrete fiber, is also indicated. A comparison of predicted and observed fatigue lives for the test beams shows satisfactory correlation.
CHAPTER 2  BEAM FATIGUE TESTS

2.1 INTRODUCTION

The prime purposes of the beam tests were to provide detailed information on the behavior of flexural members under repeated loadings and to obtain test data to check the accuracy of the methods developed in this investigation for the prediction of beam fatigue life.

Two beams were tested to failure statically, three were tested in fatigue under constant cycle loading, and three were tested in fatigue with varied repeated loadings. In addition to the beam tests, a number of tests were conducted on concrete cylinders to determine the stress-strain properties of the concrete and the effect on the stress-strain relation of a prior history of fatigue loading.

Beam fatigue test data from previous investigations provide very little quantitative information on beam
behavior under repeated loadings and so in this investigation particular attention was given to measurements of the effect of fatigue loading on beam deflections and concrete deformations. Fatigue loading was in all cases continued beyond the first wire failures to determine the post-failure behavior of the beams.

2.2 TEST SPECIMENS

The eight test beams were all twelve feet long, with a rectangular cross section approximately six inches wide and twelve inches deep. The longitudinal reinforcement consisted of three $\frac{7}{16}$ inch diameter high strength steel prestressing strands placed at a depth of eight inches below the top surface of the beam. In the first four beams manufactured, the nominal effective prestressing force in the strand was 60 percent of the static strength; in the other four beams the nominal value was 40 percent. The specimens were thus divided into two groups of four: F1 through F4, and F5 through F8. Apart from static tests to failure on one specimen from each group, the beams in the first group were used for the constant cycle fatigue tests, those in the second for the cumulative
damage tests.

Full details of the specimens are contained in Fig. 3 and Table 1. Actual dimensions of the beams given in Table 1 vary slightly from the nominal values. The effective prestressing forces, shown as $F_{se}$ in Table 1, also differ somewhat from the nominal values because of variability of creep and shrinkage effects in the concrete. Strains in the steel due to initial and effective prestressing forces, $\varepsilon_{si}$ and $\varepsilon_{se}$, are shown in Table 1 together with elastic and creep strains in the concrete at the steel level, $\varepsilon_{ce}$ and $\Delta \varepsilon_c$. Since it was impossible to test all the beams at the same age, the effect of variations in age at time of test was minimized by commencing the tests approximately 150 days after manufacture of the specimens. Actual ages at time of commencement of test are shown in Table 6.

Stirrup reinforcement was included in the shear spans of the beams, but not in the pure moment test regions. Six two-leg stirrups of 3/8-inch intermediate grade reinforcing bar were placed in the end regions of each beam at six inch spacings, as shown in Fig. 3.
Materials

The concrete used in the beams was made from 3/8-inch maximum size crushed limestone, fine Lehigh river sand, and type III, high early strength Portland cement. Sieve analyses of the aggregates are shown as grading curves in Fig. 4. The fineness modulus of the sand was 2.65. The specific gravities of sand and coarse aggregate were 2.65 and 2.69, respectively.

The same nominal concrete mix was used throughout, although slight changes were made in the water content of different mixes to adjust for variations in the moisture content of the sand. An attempt was made to keep the slump at approximately two inches. The concrete was mixed in six cubic foot batches in a horizontal drum, positive action mixer for three minutes and then transported in buggies to the prestressing bed. Five batches of concrete were used in the manufacture of each group of four beams.

In Table 2, details are given of the mix quantities and the regions in the test beams where the different batches were used. It will be noted that concrete from only one batch was placed in the test region of each beam.
in order to obtain greater uniformity. Results of static
tests conducted at the time of the beam tests on three
cylinders from each batch are also contained in Table 2.

The strand reinforcement used in the experimental
work of this investigation was obtained in two lots from
the manufacturer, John A. Roebling's Sons Corporation.
They were purchased as typical samples of high strength
steel prestressing strand and had gone through the normal
manufacturing processes of extrusion, cold drawing,
spinning, and stress relieving. Lot I was used in the
manufacture of the test beams and also for a small number
of static and fatigue tests to determine its properties;
the strand in Lot II was used in the experimental study
of strand fatigue properties described in Chapter 3.
Details of the chemical composition of both lots are
given in Table 3. The results of four constant cycle
fatigue tests conducted on specimens from Lot I are con-
tained in Table 5; these results are also plotted in Fig.
48 where they may be compared with the results of similar
fatigue tests conducted on Lot II specimens. Although
the mean static strength of the Lot I specimens is con-
siderably lower than for Lot II — 27.3 kips compared
with 28.56 kips — the fatigue strengths, stated as
percentages of static strength, agree quite well. As can be seen in Fig. 48 the results of the Lot I specimen tests are evenly distributed around the mean S-N line. The details of the test procedures used in the static and fatigue tests of Lot I strand are similar to those described in Chapter 3.

**Manufacture of Test Beams**

The beams were manufactured four at a time in a prestressing bed erected on the dynamic test bed in Fritz Engineering Laboratory. Three strands running the length of the bed were positioned and tensioned to the required initial prestressing force. Formwork and stirrups were assembled for four beams end to end along the strands, and the concrete was placed. After the concrete had set, the side forms were removed and the beam surfaces were prepared for deformation measurements. The concrete was then covered and kept moist. At an age of approximately five days the strand forces were gradually released, and the strand between the beams was burned off close to the concrete. The beams were stored for thirty days at a temperature of approximately
70 degrees F. under wet burlap and moisture-proof plastic sheeting. After this initial period of curing they were stored in the laboratory at room temperature and humidity.

**Measurement of Prestressing Force and Losses**

During the prestressing operations the strand forces were measured with dynamometers placed at each end of the strands. The dynamometers were made from eight inch lengths of pipe, one and one-half outside diameter and one-half inch inside diameter. SR-4 strain gages were attached to the outside surface, and the dynamometers were calibrated so that the strand forces could be determined from the strain readings on the side of the pipe.

A ten inch gage length Whittemore deformeter was used to measure elastic strains and creep and shrinkage strains on the sides of the beams. When the concrete in the beams had hardened, grids of small aluminum targets were cemented to both sides of the beams in the test region in the pattern shown in Fig. 6. The beams
lay in an east-west direction during casting and testing and for convenience the gage lengths for deformation measurements were labelled East (E), Right (R), and Right-Right (RR) proceeding in the direction east of the centerline, and West (W), Left (L), and Left-Left (LL) proceeding in the direction west of the centerline. Grid points were placed at the steel level on the north and south sides of the beam in the six sections RR through LL. Additional grid points were placed in the East and West Sections at six different levels, as shown in Fig. 6, so that in these regions the vertical distribution of strain could also be measured. Elastic and inelastic strains in the concrete obtained from deformeter readings made before and after release and during the curing process were used to determine the elastic and inelastic concrete prestress losses. The grid of gage points described above was also used for deformation measurements during testing of the beams.

2.3 TEST PROCEDURE

All static and fatigue tests were conducted in the
loading frame shown in Fig. 7. The beams were supported on a ten foot test span with a hinged support at one end and a rocker at the other to avoid any restraining effects. Two equal loads were applied symmetrically to the beams through steel distributor plates four inches wide and three-quarters of an inch thick. The distributor plates were grouted to the top surface of the concrete at sections three feet from each support. Twenty-two kip capacity Amsler jacks with spherical seatings at each end were used to apply both the static and dynamic loadings.

Static ultimate strength tests were conducted on beams F3 to F6; beams F1, F2 and F4 were tested in fatigue with constant cycles of loading; beams F5, F7 and F8 were tested in fatigue with a varied pattern of loading.

Static Ultimate Strength Tests

For the static tests to failure, a pendulum dynamometer was connected to the loading jacks to deliver pressure and also to measure load.
Loads were added to the beams in two kip increments. Concrete deformations were measured on the grid shown in Fig. 6 at each load increment; deflections were measured at the load points and at the centerline with Ames dial gages in contact with the lower surface of the test beam. As the loading became high, considerable creep occurred, and in order to allow the readings to settle down to relatively steady values a period of time was allowed to elapse between the application of a load increment and the measurement of deformations and deflections. The loading was increased until failure took place.

**Constant Cycle Fatigue Tests**

An Amsler pulsator was used to apply pressure to the jacks during the beam fatigue tests. The pulsator consists essentially of a pump which exerts a constant force in a pressure cylinder, and a piston within this cylinder which produces a sinusoidal variation in the cylinder pressure. The varying cylinder pressure is transmitted by pipes to the hydraulic jacks. Since friction losses throughout the system are extremely low, the oil pressure at the jack is used as an accurate
measure of the applied load. Maximum and minimum pressures in the jack piston are thus picked up and indicated by dial gages mounted on the pulsator. By reducing the amplitude of the sinusoidal pressure variation to zero the pulsator can be used to apply static loads. Operating frequencies of 250 and 500 cycles per minute are possible with the equipment. Fatigue loading was applied in the constant cycle beam tests at 250 cycles per minute.

Prior to the commencement of each fatigue test, two static tests were conducted on the beam to loads slightly higher than the maximum value to be used in the repeated load cycle. Fatigue loading was also interrupted at regular intervals in order to make additional static tests. Deformation and deflection measurements made during the static load tests indicated accurately the changes in the response of the beam which occurred in the previous sequence of fatigue loading. Crack development was recorded in each static test during an inspection of the beam while under maximum load. The fatigue loading was continued, with interruptions only for static tests, until failure.

In the first fatigue test, conducted on beam Fl,
dynamic deflections were initially measured with a small spring steel cantilever placed beneath the beam with its free end connected to the lower concrete surface by a thin, vertical aluminum rod. Deflections were recorded on brush equipment connected to strain gages on the upper and lower surfaces of the cantilever. Considerable "drift" occurred in the readings during the initial sequence of repeated loadings and prevented accurate measurements of dynamic deflections.

This apparatus was replaced by Ames dial gages. During fatigue loading the gages were taped down out of contact with the moving test beam. Extreme deflections under dynamic load were obtained by untaping the gage, holding the plunger, and allowing it to extend slowly upwards until it made contact with the lower surface of the test beam in its position of maximum deflection. Dial gages used in this way gave satisfactory performance and were used to measure deflections at the centerline and load points during both static and dynamic loading.

In order to preserve the centering of the jacks on the distributor plates and the beam on its supports,
a minimum load of 4.5 kips per jack was always maintained on the beam. An attempt was made to obtain the same load cycle in all three constant cycle tests so that three replications would be obtained of the one fatigue test. A comparison of dynamic deflection readings with the load deflection curves obtained during the static tests provided an accurate measure of the actual loadings, including inertial effects, on the beams. Details of the applied loadings are contained in Table 6.

**Cumulative Damage Tests**

The cumulative damage tests were conducted in the same manner as the constant cycle tests, except that the fatigue loading varied between a constant minimum level and three different maximum levels. In order to mix the three load cycles evenly and at the same time have a repeated loading pattern which could be followed by the fatigue equipment, the load cycles were arranged in blocks which were repeatedly applied to the beam until completion of the test. Each block contained a total of 30,000 load cycles. As shown in Fig. 8, the load block contained, in order, 18,000 repetitions of the smallest load cycle, 9,000 repetitions of the
intermediate load, and 3000 repetitions of the maximum load cycle. Since the size of one load block was always very small with respect to the fatigue life of the beam, an even distribution of the three different loadings throughout the loading history was obtained.

The cumulative damage tests were conducted using the Amsler pulsator previously described. Since there was no programming arrangement on this equipment, all load changes were made manually. It was found that an experienced operator could change the loading from one level to the next within 200 cycles.

Static tests to loads slightly higher than the maximum load cycle were conducted prior to and interspersed through the fatigue loading in the manner described for the constant cycle tests. A minimum loading of 3.8 kips per jack was maintained on the beams to preserve centering. The fatigue loading was applied to beams F7 and F8 at the rate of 250 cycles per minute, to beam F5 at 500 cycles per minute. Apart from the regular static tests, rest periods of from four to six hours were introduced in each 24 hour period.

A comparison of dynamic deflection readings with
static load-deflection curves again provided an accurate measure of the loadings actually applied to the beams. An attempt was also made in these tests to obtain the same loadings on all three beams, but a theoretical estimate of the inertial loading effect for beam F5, tested at 500 cycles per minute, proved to be slightly inaccurate. The maximum load cycle applied to F5 was for this reason approximately 0.2 kips smaller than that used in F7 and F8. Values of the actual loadings are given in Table 6.

2.4 BEAM TEST RESULTS

Static Tests to Failure

Beams F3 and F6, which were tested statically, failed in a manner typical of under-reinforced beams by yielding of the steel and then crushing of the concrete in the outer compression fibers. Cracking and ultimate moments are given in Table 1.

The centerline deflections of the two beams are plotted against load in Fig. 9. In the higher load
range, F6, which had the lower prestressing force, deflected considerably more than F3, although its ultimate load is actually slightly higher.

The completed cracking patterns for the beams are shown in Figs. 10 and 11. Cracks were restricted almost entirely to the pure moment test region. One inclined crack however formed in the east shear span of each beam, but never at any time did shear failure appear likely. A strong tendency was observed in these tests, and also in the initial static tests conducted on the fatigue specimens, for the flexural cracks to follow a more or less vertical path to the level of the steel reinforcement, then to branch into two opposing inclined cracks. The tendency is clearly seen in the patterns recorded in Figs. 10 and 11.

The observed cracking patterns on the north and south sides were quite similar in the static tests, and indeed also in the fatigue tests, and concrete deformations at corresponding gage lengths on either side of the beam were nearly equal. Average top fiber concrete strains, extrapolated from the deformation readings in the East and West sections, are shown in
Fig. 12. Maximum and minimum values of concrete deformations at the steel level were recorded in sections R and RR in beam F3, and in the West and LL sections in beam F6, and are plotted against load in Fig. 13. Since at least one crack had formed within every gage length in each beam, measured concrete deformations were distributed approximately linearly with respect to depth.

**Beam Behavior under Fatigue Loading**

Beam behavior under fatigue loading followed a common pattern in all six fatigue tests and will be described in terms of deflections, deformations, and cracking patterns.

Deflections increased quite considerably under fatigue loading, particularly in the early load cycles. Deflections observed in the periodic static tests are shown, together with dynamic deflections, in Figs. 14 through 19 for the six specimens. In the figures, triangles are used to represent test points for the first static load cycle so that they will not be confused with test points for the second load cycle.
Dynamic deflections are shown by dashed lines. Due to unsatisfactory performance of the brush recording equipment, accurate dynamic deflection measurements were not obtained for beam F1 and hence are not shown in Fig. 14. The three dashed lines shown in Figs. 17, 18, and 19 correspond to the three different maximum load levels used in the cumulative damage tests. In the constant cycle tests, where the fatigue loading was more severe, deflections continued to increase, though at a decreasing rate, until the failure of the first wire. In the cumulative damage tests, however, there was a stronger tendency for the deflections to settle down to steady values. Indeed in the test of beam F8 deflections actually began to decrease slightly after 600,000 cycles of loading.

Concrete deformations measured on the sides of the beams were greatly influenced by the presence or absence of flexural cracks. When there were no cracks in a particular gage section, tensile deformations tended to be very small, and were very little influenced by fatigue loading. In beam F7, for example, cracks did not form in the West region and the deformations at the steel level in this gage length, at load 10.5 kips, are one-
tenth those in other regions, as is shown in Fig. 20. In beam F2, also, absence of cracks in region R resulted in relatively low deformations, as shown in Fig. 21. In Fig. 22 the distribution of concrete deformations in beam F7 in the East and West regions are plotted. Whereas the distribution is approximately linear in the section containing a flexural crack, there is a sharp discontinuity in the distribution below the neutral axis in the West region because of the absence of cracks in this section and the presence of cracks in adjacent sections. Concrete deformations at the steel level in the failure region at various stages of fatigue loading are shown for all beams in Figs. 23 through 28. Values for zero load have been obtained by extrapolation and are shown as dashed lines. Average top fiber concrete compressive strains, measured in the failure region, are shown for each beam in Figs. 29 through 34. Values for zero load have also been obtained by extrapolation and are shown by dashed lines.

The patterns of cracking which formed during the initial static tests were similar to those observed in the static ultimate tests. Some extension of the existing cracks took place during the fatigue loading,
particularly in the early load cycles, but no new cracks formed in any of the beams as a result of repeated loadings. Most of the crack development due to fatigue loading had taken place when approximately thirty thousand cycles of loading had been applied, after which development was almost nil until after some wires had snapped. The cracking patterns for all beams are shown in Figs. 10 and 11.

It is thus seen that after a short initial period in which deflections and deformations increased considerably and crack extension took place, the beams settled down — particularly beams F5, 7, and 8, on which the fatigue loading was less severe — to give a fairly consistent and constant response to load. No indication was given in any of the beams of whether or not wire fatigue failure was imminent.

The fracture of a wire in the beam could always be detected by a distinctive sound, together with a small but sudden increase in maximum deflection and a slight fall-off in load. The region containing the wire failure was determined from the deformation readings taken in the next static test. After sudden
increases in deflection and deformations which accompanied the initial wire failure, the beams tended to settle down with a consistent response to loading, but with slightly decreased rigidity. A considerable number of cycles often separated the first and second wire failures, but the interval separating successive failures tended to decrease as the number of failed wires increased. Thus the post-failure behavior of the beams consisted of an increasing rate of change of deflections, increased permanent set, and steadily decreasing rigidity. The beam fatigue tests were continued until beam rigidity was considerably reduced by the failure of four or five wires. When the wires began to fail in the beams the cracking patterns began to extend. A tendency was noted for those cracks which had already become inclined under the initial static loads to take almost horizontal paths and link together to form a continuous pattern running through a considerable portion of the test section at a level a little below the neutral axis. This tendency was particularly pronounced in beam Fl and is recorded in the completed cracking pattern for that beam in Fig. 10. Fatigue loading was continued on one beam, F7, until so many wires had
snapped that the static strength of the specimen had been reduced to the value of the maximum applied dynamic load. This test was terminated when concrete crushing was observed in the top fibers of the beam. Another beam, F8, was tested statically to failure after 5 wires had snapped due to fatigue loading, its static ultimate strength is recorded in Table 1.

**Beam Fatigue Test Results**

The results of the beam fatigue tests are recorded in Table 6. Values of the applied loading have been obtained by comparing dynamic deflections with the load deflection curves obtained at regular intervals during the static tests, and are average values taken over the entire history of loading up to failure. The terms $N_1$, $N_2$, etc. in Table 6 refer to the number of load cycles at which the first, second wire failure, etc. took place.

### 2.5 CONCRETE CYLINDER TESTS

Static ultimate strength tests were conducted at
the time of the beam tests on three cylinders from each concrete batch to obtain the values of concrete strength given in Table 2.

In addition to the static strength tests, an additional 30 cylinders were tested to determine stress-strain relations for the concrete and to observe the effect upon the stress-strain relation of a prior history of fatigue loading. Strain measurements were made with two six inch A9 electric resistance strain gages placed 180 degrees apart on the side of the cylinder. In tests involving large numbers of load applications, a Whittemore deformeter was used with aluminum targets cemented to the side of the cylinders to check the strain gage readings against drift.

The first load cycle was applied statically to allow strain readings to be made with a static strain indicator. The predetermined number of load cycles was then applied at a rate of 500 cycles per minute, and finally the specimen was tested statically to failure with strain readings being taken at regular intervals up to the ultimate load. Two different load cycles were used for pre-loading. Each cycle had a minimum
load level of 20 kips, while maximum load levels were 100 and 130 kips. Concrete fatigue tests currently being conducted in Fritz Engineering Laboratory indicate for this strength concrete that the smaller load cycle, 20 to 100 kips, may be regarded for all practical purposes as an understress, i.e. to have an infinite fatigue life. Fatigue tests to failure on three cylinders indicated an average fatigue life of 300,000 cycles for the 20 to 130 kip load cycle. Tests were conducted with pre-loadings of 0, 20, 30,000, and 100,000 cycles for each of the load cycles. An additional test was conducted with one million pre-loadings of the smaller cycle. Each test was replicated at least three times.

The strain readings from the final loading cycle to failure for each test are plotted in non-dimensional form in Figs. 35 through 42. The results of the initial static test were used to determine the value of the tangent modulus of elasticity at the commencement of the test, $E_{co}$. The amount of inelastic strain in the cylinder due to the repeated loadings, $\Delta \varepsilon_c'$, was measured prior to the final static test. Ultimate values of stress and strain measured during the static test to failure, $f'_c$ and $\varepsilon_u$, together with the initial modulus
of elasticity, were obtained during the final test. Mean values of these quantities are shown in the figures.

3.1. TEST VARIABLES

The strand fatigue tests are divided into two groups. The constant cycle tests comprising the first group were designed to provide an empirical relation between minimum and maximum stress level and probable fatigue life. In the cumulative damage tests comprising the second group, the specimen was subjected to a fatigue loading which fluctuated between a constant minimum level and either two or three different maximum levels. These tests provided data on the fatigue life of strand elements subjected to varied patterns of repeated loading.

Static tests were also conducted on a number of specimens to determine the stress-strain properties and static strength of the strand.

In the description of the strand fatigue tests and analysis of results which follow, maximum and minimum
stress levels and load levels are stated for convenience as percentages of the static ultimate strength.

**Constant Cycle Tests**

The constant cycle fatigue tests were conducted with minimum stress levels of 40 and 60 percent of the static ultimate strength. Various maximum stress levels were chosen to give fatigue lives varying between 50,000 and 5 million cycles for each minimum stress level. Apart from several tests which yielded fatigue lives outside of this main region of interest, at least six replications of each test were made. Details are given in Table 9 of the different values used for maximum and minimum stress levels and of the number of test replications. One test, with minimum and maximum stress levels of 60 and 80 percent, was replicated 20 times in order to obtain information not only on mean fatigue life but also on the manner in which the different values of fatigue life were distributed around the mean.

**Cumulative Damage Tests**

The strand cumulative damage tests were conducted in
a manner similar to the beam cumulative damage tests, by repeatedly applying to the specimen a block of load cycles until it failed in fatigue. A constant minimum stress level of either 40 or 60 percent was maintained in each block of load cycles, while the maximum stress level varied between two or three different values, as shown in Fig. 43. The smallest load cycle in any block was also the most frequently occurring and will be referred to as the design or predominant loading. The larger, less frequently occurring load cycles may be regarded as overloading. In some tests the design load was smaller than the fatigue limit indicated by the constant cycle test data, in others it was higher. The overloading, however, were always larger than the fatigue limit.

In one series of tests the main variable was the number of cycles contained in the load block. Otherwise, the size of the load blocks was chosen to be approximately one-tenth of the expected fatigue life.

In general, the cumulative damage tests were replicated either two or three times, but in one case ten replications were made to observe the distribution of the values about the mean.
3.2 SPECIMENS

Strand test specimens were taken from a fifteen hundred foot length of seven-sixteenth inch diameter strand, which was designated Lot II. The chemical composition of the steel is shown in Table 3. The strand was cut into seventy-four lengths approximately twenty feet in length; two specimens were taken from each length and were numbered consecutively in order of use. Thus, each specimen has a length number, prefixed by the letter L, and a test number, prefixed by the letter S; for example, L36-S45, etc. To minimize the effect of possible variations in material properties along the fifteen hundred foot sample, the test lengths were used in random sequence.

The specimens were held with a device which was designed to minimize stress concentrations and hence prevent premature fatigue failure in the gripping region. After a number of different methods had been tried, a gripping arrangement was finally adopted in which the force in the test piece was transmitted partly through a cement-grout bond anchorage and partly through a strand vise anchorage at the end of the specimen. Details of the grip are shown in Fig. 44.
The specimens were prepared in pairs. A twenty foot length of strand was tensioned to 70 percent of the static strength in a small prestressing frame, and the elements of the gripping devices were assembled around it. When the force in the strand was released, the strand vises at the end of each specimen retained a force in the test piece of approximately 45 percent of the static strength. A stiff sand-cement-water grout, of proportions 1.3:1.0:0.3, was then packed by hand around the strand and the transverse tension bolts. The specimens were left a minimum of twenty-four hours, and, just prior to testing, the transverse bolts were tightened. The spacing piece was removed only when load was applied to the specimen at the beginning of the test.

3.3 TEST PROCEDURE

A general view of the strand fatigue testing arrangement is shown in Fig. 45. The specimen was tested in a vertical position, with the lower end pinned to a solid base and the upper end pinned to a horizontal beam. The
beam was pinned to a supporting frame at one end and rested at the other on a 22 kip capacity Amsler jack. Dynamic load applied to the jack by an Amsler pulsator induced a dynamic reactive force in the test specimen. The loadings were applied at a rate of 500 cycles per minute.

In several tests dynamic strain measurements were made with SR-4 gages attached to the upper and lower surfaces of the beam and to individual wires in the specimen. A comparison of dynamic strains with strains measured under static loading indicated that inertial effects were negligible. The test set-up was calibrated so that the jack loads, indicated on dial gages attached to the pulsator, could be used as a measure of the specimen loads.

In the first fatigue tests, which were conducted with 60 percent minimum stress levels, the specimens were positioned halfway between the beam supports. In order to improve the accuracy with which the loads in the specimen were measured, the testing set-up was modified to allow specimens to be positioned at the quarter point closer to the jack. To maintain uniformity in the test
results, however, the remaining 60 percent minimum stress level tests were conducted at the half-point position, while all of the 40 percent minimum stress level tests were conducted at the quarter-point.

The static specimens were tested in a 300 kip capacity Baldwin Universal testing machine. The gripping arrangement developed for the fatigue tests was used also for all static strength tests. Load-strain curves were obtained from elongation measurements made over a 50 inch gage length with Ames dial gages. To compare the average strains measured in the strand with actual steel strains, several tests were conducted with strain gages attached to individual wires in the test piece.

3.4 TEST RESULTS

Static Tests

The results of the static ultimate strength tests on Lot II strand are contained in Table 7. All specimens failed in the open length of strand between the end grips. A mean load-strain relation, obtained from elongation
measurements on a 50 inch gage length, is shown in Fig. 46, where it is compared with a load-strain curve obtained using SR-4 gages attached to individual wires. The lower value of the modulus of elasticity of the strand, 28.0 x 10^6 p.s.i. as against 30.0 x 10^6 p.s.i. for the individual wire, is probably due to a tendency for the twisted wires to straighten slightly under load.

**Constant Cycle Fatigue Tests**

The constant cycle fatigue test results are contained in Table 8, where values of the minimum and maximum stress levels are given, in percentages of static strength, together with the number of load cycles at which the first wire in the strand fractured. The results are summarized, for purposes of analysis, in Table 9.

One of the six outside wires was always the first to fail in fatigue. Successive failures occurred in other outside wires until the remaining wires were so overstressed that they failed statically. Those wires which had failed in fatigue could be clearly distinguished by a typical fracture surface containing a crescent shaped
fatigue crack.

The number of load cycles separating the first and second wire failures was variable. Sometimes the first and second wires snapped almost simultaneously, with complete strand failure following quickly. On other occasions, usually in tests with smaller load cycles, the interval was large. Always, however, considerable elongation occurred in the specimen when the first wire failed.

In the majority of the specimens the failure section was in the open region between the gripping pieces. Whenever the failure was within the grips, a careful inspection was made to determine whether the strand had rubbed against the steel front end block of the grip. In one test, L1-S2, this had actually occurred because of incorrect grip alignment during manufacture and caused a considerable decrease in fatigue life. This test is marked with an asterisk in Table 8 and is not included in the analysis of the results.

The fatigue life of specimen L4-S6 was much lower than for other similar tests. An inspection of the failure section showed that fatigue had taken place in one of the wires in a region where a weldment had been made
during manufacture of the strand. This test result, indicated by a double asterisk in Table 8, is also discarded in the analysis of the results.

**Cumulative Damage Tests**

Fatigue failure under cumulative damage loading was similar to constant cycle fatigue failure. However, actual wire fracture only took place during the application of overloading. Even when several wires had already failed, further failures did not occur while loadings were being applied which were smaller than the fatigue limit.

The results of the cumulative damage tests are contained in Tables 10 through 15. A small number of the cumulative damage test specimens failed prematurely as a result of rubbing of the strand against the end block of the grip; the test results are given in the tables, but are marked by asterisks and are not used in the analysis of the results.

3.5 **ANALYSIS OF CONSTANT CYCLE FATIGUE TESTS**

Scatter is inherent in the results of all experimental
work. It is presented in the quantities being measured because of the essential variability of material properties; it is further introduced by imperfect methods of measurement and testing. Often the order of the scatter is small in comparison with the magnitude of the quantity being measured, in which case the quantity is adequately represented by the mean value. Thus, the static ultimate strength of the Lot II strand can be taken as 28.56 kips, the mean value of the test results in Table 7.

On some occasions, however, the deviations of results of similar tests from the mean value can be of the same order as the mean value itself. Such a situation has occurred in the constant cycle fatigue test data. For example in the group F data in Table 9, the fatigue life observed in twenty replications of the same test varied between 235,800 cycles for specimen L64-S37, and 40,900 cycles for specimen L16-S46. Although a portion of the scatter in fatigue test results can always be attributed to experimental technique, it is now generally recognized that considerable variability is inherent in the phenomenon of fatigue failure.

With scatter of such magnitude in the results of
similar tests, simple S-N curves and fatigue envelopes are clearly inadequate representations of fatigue properties. It is therefore necessary to associate variability with fatigue failure by treating the values of fatigue life observed in test replications as a sample taken from an infinite population of values which is distributed in some manner about a central or mean value and is represented by some distribution function. Thus, for any load cycle which might be applied to a specimen, we consider the probability of failure, \( P \), to vary between zero and unity, and with each value of \( P \) we associate a number \( N \), such that the probability is \( P \) that failure will occur at a number of cycles equal to or less than \( N \).

Several investigations have been conducted in order to obtain information on the shape of frequency distributions associated with the phenomenon of fatigue failure. Müller-Stock \(^{(15)}\) made 200 replications of a constant cycle fatigue test on steel specimens and obtained a distribution having a pronounced skew with a long right hand tail. Freudenthal \(^{(13)}\) obtained similar results and has shown, by a theoretical argument using several reasonable but approximate physical assumptions, that the
distribution should be approximately logarithmic-normal.

Weibull\(^{(16)}\) has suggested that although the log-normal distribution may fit test data well in the central region around the mean value, it may not represent extreme values very satisfactorily. In most cases, however, test data is not extensive enough to provide information on the distribution at a distance from the mean value, and the log-normal distribution has been used in a number of recent investigations\(^{(17)}\).

The log-normal distribution has the probability density function

\[
  f(X) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}, \quad (3.1)
\]

and cumulative distribution function

\[
  P = F(X) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx, \quad (3.2)
\]

where \(X = \log N\),

and \(\mu\) and \(\sigma\) are the mean and standard deviation of
the population of log N values. The functions \( f(X) \) and \( F(X) \) are completely determined when values for \( \mu \) and \( \sigma \) have been obtained.

In order to investigate the suitability of the log-normal distribution to the constant cycle fatigue test results of this investigation a \( \chi^2 \) goodness-of-fit test was conducted on the 20 replications of the group F data.\(^*\) The details of the \( \chi^2 \) test are contained in Table 16. A \( \chi^2 \) value of 1.2 was obtained which was well within the .05 significance level. A second \( \chi^2 \) test was conducted using all of the test data contained in Table 9. The data for different load cycles were grouped together by making a change of variable from log N to Z,

\[
Z = \frac{\log N - \log N}{D} \tag{3.3}
\]

and \( \log N \) and D are the mean and standard deviation of the set of data being grouped. This change of variable reduces each set of data to one with a mean of zero and standard deviation of unity. A plot of the grouped constant cycle fatigue test data is compared with the log-normal distribution in Fig. 47. The details of the \( \chi^2 \) test for the grouped data are contained in Table 17. The \*For a description of the \( \chi^2 \) test, see p. 85, Ref. 18.
\( \chi^2 \) value of 10.70 is again well within the 0.05 significance level value.

The assumption of a log-normal distribution is apparently in reasonable agreement with the test data and will be made throughout this investigation.

In Fig. 48 fatigue life \( N \) has been plotted on logarithmic scale against maximum stress level for the constant cycle fatigue data. Although the tests were not designed primarily to indicate values of the fatigue limit, \( S_L \), approximate values of 71 and 55 percent have been obtained for the 60 and 40 percent minimum stress levels, respectively, by extrapolation.

In Fig. 49 the two sets of data have been plotted together using variables \( R = (S_{\text{max}} - S_L) \) and \( \log N \). A mean line has been fitted to this data by using a relation of the form

\[
\log N = \frac{a_1}{R} + a_2 + a_3 R.
\]

The method of least squares was used to obtain the following three simultaneous equations for the evaluation of the open parameters \( a_1 \), \( a_2 \) and \( a_3 \);
Equations 3.4 and 3.5 provide values for the mean

\[ \alpha_1 \sum_{i=1}^{n} \frac{1}{R_i^2} + \alpha_2 \sum_{i=1}^{n} \frac{1}{R_i} + n\alpha_3 = \sum_{i=1}^{n} \frac{\log N_i}{R_i} \]

\[ \alpha_1 \sum_{i=1}^{n} \frac{1}{R_i} + n\alpha_2 + \alpha_3 \sum_{i=1}^{n} R_i = \sum_{i=1}^{n} \log N_i \]

\[ n\alpha_1 + \alpha_2 \sum_{i=1}^{n} R_i + \alpha_3 \sum_{i=1}^{n} R_i^2 = \sum_{i=1}^{n} \left( \log N_i \right) R_i. \]

Solution of these equations yields the relation

\[
\log N = \frac{1.4332}{R} + 5.5212 \div 0.0486 R, \quad (3.4)
\]

where \( R = S_{\text{max}} - S_L \).

Assuming a linear variation of \( S_L \) between the 40 and 60 percent minimum stress level values of 55 and 71 percent, the following equation is obtained for the fatigue limit;

\[
S_L = 0.8 S_{\text{min}} + 23. \quad (3.5)
\]

Equations 3.4 and 3.5 provide values for the mean
fatigue life corresponding to any stress amplitude in the region under consideration. Since values of both the mean and standard deviation are required to specify completely the log-normal frequency distribution, it is now necessary to obtain appropriate values for the standard deviation corresponding to each stress amplitude.

The best unbiased estimate of the standard deviation of the population is given by

$$D = \left[ \frac{1}{n-1} \sum_{i=1}^{n} (\log N - \bar{\log N})^2 \right]^{\frac{1}{2}}$$

where $n$ is the number of replications and $\bar{\log N}$ is the mean value for the sample. Values of $D$ for the seven sets of test data are plotted against $R$ in Fig. 50. Considerable variation occurs among the points. A change in the position of the test specimens in the loading rig from center to quarter-point has reduced the scatter of the 40 percent minimum stress level test results quite considerably. However, a fairly consistent trend is followed. Both the quarter-point and center-point set-up data yield reasonably linear variations of $D$ with $R$. The
larger values corresponding to the center-point load position are probably due in part to larger experimental errors associated with that set-up.

The use of anything but the simplest relation is unwarranted by the test data available, and, for the purposes of this investigation, a straight line variation is assumed and fitted to the seven points. A least squares fit to these points yields for the standard deviation,

\[ D = 0.2196 - 0.0103 R. \]  \hspace{1cm} (3.6)

The S-N-P relation is thus given by the equations

\[ P = F(X) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx, \hspace{1cm} (3.2) \]

where

\[ X = \log N; \]

\[ \mu = \log N = \frac{1.4332}{R} + 5.5212 = 0.0486R, \hspace{1cm} (3.4) \]

where

\[ R = S_{\max} - (0.8 S_{\min} + 23); \]

and

\[ \sigma = D = 0.2196 - 0.0103 R. \]  \hspace{1cm} (3.6)
Values of $P$ corresponding to values of $X$ in Eq. 3.2, and vice versa, can of course be obtained most easily from standard tables. (18)

It should be noted that the above equations have been derived for the following ranges of variables:

\[
40 \leq S_{\text{min}} \leq 60
\]

\[
0 < R < 15
\]

3.6 ANALYSIS OF CUMULATIVE DAMAGE TESTS

Several procedures for predicting fatigue life under varied repeated loadings have been suggested in previous investigations, and it will be convenient to review the more important of these before considering the results of the experimental work conducted in this investigation.

Review of Cumulative Damage Theories

A general, quantitative theory of fatigue failure must obviously be based on assumptions which describe, at least approximately, the fundamental physical and
metallurgical changes which take place in a material subjected to fatigue loading. It will shortly be seen that investigators are not yet in agreement even on the essential nature of the fatigue failure mechanism, let alone on general principles which yield quantitative data on fatigue life. Extensive metallurgical studies will be necessary before satisfactory progress can be made towards this goal.

Quantitative information on the fatigue properties of materials must therefore come at present from engineering studies which are phenomenological and experimental in nature, and hence restricted in application.

One of the earliest, simplest, and most widely known procedures for predicting mean fatigue life under repeated loadings was suggested by Palmgren (19) and later by Miner (20). In this approach the cycle ratio, $r_i$, is defined for a stress amplitude $S_i$ as

$$r_i = \frac{n_i}{N_i},$$

where $n_i$ is the number of cycles of $S_i$ loading which have been applied to the specimen, and $N_i$ is the mean fatigue
life corresponding to $S_1$. It is assumed that fatigue damage accumulates in the specimen in direct proportion to the sum of the cycle ratios. Damage is complete and failure takes place when the summation is equal to unity, i.e., for $q$ different stress amplitudes, when

$$\sum_{i=1}^{q} r_i = \sum_{i=1}^{q} \frac{n_i}{N_i} = 1$$

(3.7)

Two series of tests conducted by Miner on aluminum alloy specimens yielded mean summation values of 1.05 and 0.98, with extreme values of 1.49 and 0.61.

Tests conducted by other investigators have in some cases yielded results differing considerably from unity. Dolan, Richart, and Work (21) conducted rotating beam tests on steel and aluminum specimens with blocks of load cycles as shown in Fig. 51. To summarize briefly the results of their tests it is convenient to consider two cases: (a) tests in which the smallest stress level in the block is less than the fatigue limit, i.e. is an understress; (b) tests in which the smallest stress level is larger than the fatigue limit and is an over-stress.
In the first case, large numbers of understresses had no apparent effect on the fatigue life of specimens of SAE 2340 steel and aluminum. On the other hand, damage was accelerated in specimens of SAE 1045 and 4340 steels, when the understresses were mixed with overstresses. In the second case, with all the stresses in the load block above the fatigue limit, the fatigue life was frequently found to be close to the S-N value of the minimum stress level when maximum and minimum stress levels were close together. An interesting result of their experimental work was that load blocks A and G, shown in Fig. 51, gave quite similar fatigue lives. Values of the sum of cycle ratios for the tests varied between 0.18 and 23.0, with only a small number giving the value of unity. However, since the tests were not designed to provide quantitative data on scatter of test results, no conclusions can be drawn on how much of the variation is due to inherent and experimental variability and how much due to inapplicability of the linear summation procedure.

Richart and Newmark (22) suggested a more general relation between damage and cycle ratio, of the form
\[ D = r^n \]

where the exponent \( n \) depends upon the stress level. This approach unfortunately is restricted by the necessity of determining the \( D-r \) relations experimentally. In a review of research on fatigue, Grover et al.\(^{(17)}\) found that the test data of Richart and Newmark gave \( \sum \frac{n}{N} \) values not too much out of line from those of Miner's tests. They further observed that the experimental evidence which is in disagreement with the linear concept consists of tests with only two or three different stress amplitudes, and that better correlation may be obtained if the stress amplitudes were applied in random sequence and with at least five different stress levels.

Within the last few years considerable importance has been attached to studies of fatigue life of structural parts under spectrum and random type loadings. Interest in such studies at present comes mainly from the aircraft industry which faces problems involving fatigue of airframe parts subjected to high frequency, randomly varying loads.
In a discussion of acoustic fatigue failure under spectrum loading, Freudenthal\(^{(23)}\) refers to the existence of two basic fatigue mechanisms. In the so-called high level mechanism, corresponding to the short life part of the S-N relation, the mechanism of deformation is akin to that observed in uni-directional static deformation. In the other basic mechanism, which applies to the flatter, low level portion of the S-N relation, less strain hardening and no significant deformation takes place, but a multitude of fine slip bands form, congregated in striations. The location of the transition range from one mechanism to the other is, according to Freudenthal, affected by a large number of variables and cannot be pin-pointed accurately.

The principal effect of the existence of two mechanisms is an interaction between fatigue damage at different stress amplitudes. Freudenthal suggests that interaction between stress levels within the high level region will be slight, but that intermittent high stress cycles will accelerate the propagation of cracks at the low stress levels. Thus, a so-called non-propagating crack at the lower amplitudes is likely to start to
propagate as a result of a few high level stress cycles. Freudenthal claims that intermittent cycles of overloads must be expected to shorten the fatigue life under low stress amplitudes far beyond the immediate damage associated with them.

In order to make quantitative estimates of fatigue life, Freudenthal suggests that the interaction effect be taken into account by constructing a fictitious S-N diagram for the particular load spectrum to be considered. Interaction factors, $W_i$, corresponding to each stress amplitude $S_i$, take the form of simple ratios between mean constant cycle fatigue life $\overline{N_i}$, and the fatigue life $N_i'$ obtained when the stress amplitudes of the load spectrum above $S_i$ are interspersed with $S_i$ in the overall ratio defined by the spectrum. To obtain a reasonable approximation to the S-N' diagram, the real S-$\overline{N}$ relation at the 50 percent probability level is assumed to be of the form

$$\frac{\overline{N_i}}{N_i} = \left(\frac{S_i}{S_1}\right)^{-\gamma}$$

which plots as a straight line on double logarithmic
representation, and passes through a reference point 
\((N_i, S_i)\). By choosing this point to be the boundary 
between high level and low level fatigue — thus be-
tween slight and strong interaction — and by changing 
the exponent \(\gamma\) to \(\rho\), the S-N' relation

\[
\frac{N_i'}{N_i} = \left( \frac{S_i}{S_1} \right)^{-\rho}
\]

is obtained for the region \(N_i' \geq N_i\). In the region 
\(N_i' < N_i\), where interaction is slight, the two S-N
relations for \(N_i\) and \(N_i'\) are assumed to coincide.

Experimental figures quoted by Freudenthal for
aluminum alloy and 4340 steel, with \(8 < \gamma < 16\), and an
exponentially shaped load spectra, shows \(\rho\) values
varying between 4 and 8, the lower values being
characteristic of more severe spectra and shorter
fatigue lives.

In a discussion of Freudenthal's work, Coffin\(^{24}\)
agrees that interaction will occur between different
stress levels, but disagrees on its cause. Referring
to tests conducted on specimens of AISI type 347 stain-
less steel he points out that the hypothesis of high
and low level fatigue mechanisms is not supported by observations of the stress-strain characteristics observed during test. Coffin attributes interaction to mechanically induced structural changes in the metals which take place during the high loading.

In a recent study of cumulative damage, Liu and Corten (25) assume a simplified physical mechanism of fatigue crack development which begins with a so-called nucleation period in which permanent fatigue damage is initiated by the formation of a number of damage nuclei. The nuclei then extend and join to form fatigue cracks which propagate at a rate that increases as the number of cycles of applied load increases. Damage is assumed to occur at stress levels which are lower than the minimum stress required to initiate damage, the rate of propagation depending upon the stress level. Liu and Corten express the fatigue damage caused by N cycles of stress amplitude as

\[ D = m c N^a \]

where \( m \) is the number of damage nuclei, \( c \) is a coefficient of crack propagation, and \( a \) is a constant to be evaluated. Now the damage at failure, \( D_f \), is the same no matter what
the stress history, and so, considering two constant
cycle load histories $S_1$ and $S_2$,

$$D_f = m_1 a_1 N_1^{a_1} = m_2 c_2 N_2^{a_2}$$

In treating a load history containing two different
levels, it is assumed that the damage nuclei are initiated
only at the upper stress level, $S_1$, but the damage con­
tinues at both levels. Letting $C = c_2/c_1$ and assuming
$a_1 = a_2 = a$, Liu and Corten develop the following expres­
sion for the fatigue life, $N_g$,

$$N_g = N_1 \frac{1}{\alpha + C \frac{1}{\alpha}} \left( 1 - \alpha \right)$$

where $\alpha$ is the proportion of $S_1$ cycles. When $q$ different
stress levels are employed, the expression is generalized
to

$$N_g = N_1 \frac{1}{\sum_{i=1}^{q} \alpha_i \frac{1}{\alpha_i}} \left( 1 - \alpha \right)$$

(3.8)

Extensive cumulative damage tests were conducted on
tensile wire specimens of 2024-T4 and 7075-T6 aluminum
alloy and hard drawn steel. The terms $C^{1/a}$ was evaluated empirically as

$$C^{1/a} = \left[ \frac{S_2 - S_o}{S_1 - S_o} \right]^d, \quad (3.9)$$

with values of the constants $S_o$ and $d$ given as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>$d$</th>
<th>$S_o$ p.s.i.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T4 Alloy</td>
<td>5.778</td>
<td>0</td>
</tr>
<tr>
<td>7075-T6 Alloy</td>
<td>3.300</td>
<td>15,000</td>
</tr>
<tr>
<td>Hard Drawn Steel</td>
<td>5.98</td>
<td>0</td>
</tr>
</tbody>
</table>

Although Eqs. 3.8 and 3.9 give a somewhat better correlation in the test data of Liu and Corten than does Eq. 3.7, it is interesting to note that extreme values quoted for $\sum \frac{n}{N}$ for the different tests are 1.7 and 0.8, which are comparable with Miner's values, even though this experimental work is considerably more extensive. The mean value of $\sum \frac{n}{N}$ for the test data given is 1.12 with a standard deviation of 0.213. It is interesting also to note that when two different stress levels in a load block were close together, observed fatigue lives tended to be less than the values predicted by the linear summation theory, whereas the opposite trend was observed.
in the results of Dolan, Richart, and Work.

In this brief review of work on cumulative damage, the phenomenological nature of the various approaches is clearly to be seen. Even in the more recent work, such as that of Liu and Corten, in which quantitative studies are based on assumed failure mechanisms, so many simplifying assumptions have to be made that the work remains essentially phenomenological and empirical. The theories and procedures reviewed were all concerned with estimating the mean fatigue life of specimens under varied load cycles. Apart from brief discussion on the scatter of test results and possible forms which the frequency distributions might take, no work has been done to provide quantitative information on the variability of fatigue life under varied load cycles.

Mean Fatigue Life Under Varied Load Cycles

In Tables 10 through 15 a comparison is made between fatigue lives observed in the tests and values predicted by Eq. 3.7 and by Eqs. 3.8 and 3.9. The values of \( \sum \frac{n}{N} \)

observed in the tests were quite close to unity, indicating
reasonable agreement between experimental and predicted values. The mean of the $\sum \frac{n}{N}$ values for all tests is 0.97, with extreme values of 0.48 and 1.65 and a standard deviation of 0.224. This value for the standard deviation is quite comparable to the values given in Table 9 for the standard deviation of the quantity $\frac{N_e}{N_t}$ obtained in the constant cycle tests. Since

$$\sum_{\text{experimental}} \frac{n}{N} \approx \frac{N_e}{N_t},$$

where $N_e$ is the observed fatigue life and $N_t$ is the value predicted by the linear summation theory, the variabilities of the cumulative damage tests and constant cycle tests, as measured by the standard deviations, are of a similar order. It therefore appears reasonable to attribute the observed scatter in $\sum \frac{n}{N}$ to inherent variability in the test data rather than to inapplicability of the theory.

It will be noted that there is no evidence at all, in the test data, of the damaging effect of understresses, even when mixed with overloading. On the contrary, there is a slight but fairly distinct tendency for understresses,
i.e. stresses lower than the fatigue limit, to improve fatigue resistance. This may be seen in the results of tests 3AA, 3BA, and 3DA, which are contained in Table 10, and the results in Table 11, where the summation values are always a little above unity. That this improvement is actually due to the presence of the understresses and is not simply the beneficial effect of intermittent application of the overloads is indicated by tests 4AA and 5AA. These tests, in which the understresses are of zero amplitude — i.e. correspond to rest periods — gave summation values slightly less than unity. The evidence is of course insufficient to establish a definite trend of improved fatigue life with the presence of understresses, however it does seem reasonable to assume in the following that understresses will not contribute to fatigue damage.

Although no interaction effect can be observed between high and low stress levels, tests 5CA and 6BA, in which the stress blocks contain three different overstresses, yield summation values considerably less than unity and might indicate an interaction effect. For these two tests, Eqs. 3.8 and 3.9 yield better results than Eq. 3.7. However, the two other tests with three
overstresses, 5BA and 6AA, both have summation values greater than unity. No definite trend is therefore indicated.

In view of the very reasonable agreement between test results and values predicted by the linear theory, Eq. 3.7 will be used in this investigation for the prediction of mean fatigue life of strand reinforcement under varying cycles of repeated loading.

Probable Fatigue Life Under Varied Cycles of Repeated Loading

It was seen earlier that fatigue life under constant cycle loading is distributed log-normally, at least to first approximation, about the mean value. It seems reasonable to expect a log-normal distribution to apply approximately also to fatigue life under varied load cycles. If the log-normal assumption were made, probable fatigue life would be established by the value of the mean fatigue life, given by Eq. 3.7, together with a value which would have to be estimated for the standard deviation. Instead, however, of assuming a log-normal distribution and proceeding to study possible methods of
estimating the standard deviation, a direct approach is made, in the following, by generalizing the linear accumulation theory so that it may be applied at all probability levels.

Considering a load history which consists of two stress levels, $S_1$ and $S_2$, occurring in the proportions $a$ and $(1-a)$, the mean fatigue life of a strand is given by the equation

$$\sum \frac{n}{N} = 1$$

or

$$\frac{a N(0.5)}{N_1(0.5)} + \frac{(1-a) N(0.5)}{N_2(0.5)} = 1$$  \hspace{1cm} (3.10)$$

where $N_1(0.5)$, $N_2(0.5)$, and $N(0.5)$ are the mean fatigue lives corresponding to $S_1$, $S_2$, and the combined loading respectively.

In general, considering possible conditions where the linear accumulation theory may not yield satisfactory estimates of mean fatigue life, a cumulative damage theory for mean fatigue life would be used which provides a relation of the form
where \( 0 \leq a \leq 1 \). Equation 3.11 describes a relation between \( N(0.5) \) and \( a \), as shown in Fig. 52. However the fatigue lives corresponding to \( S_1 \) and \( S_2 \) actually consist of distribution functions with ranges of \( N_1 \) and \( N_2 \) values corresponding to different probability levels, as shown also in Fig. 52. In order to obtain curves corresponding to probability levels other than 0.5, it appears reasonable to assume that the form of the \( N-a \) relation will not alter with the probability level, and that Eq. 3.11 may be generalized to

\[
\Phi [N(P), a] = 0 \tag{3.12}
\]

to apply to all probability levels. It will be noted that although the fatigue lives at \( S_1 \) and \( S_2 \) may be log-normally distributed, the distribution obtained from 3.12 for values of \( a \) other than zero and unity will not, in general, be log-normal.

Equation 3.10 may be rearranged as

\[
N(0.5) \left\{ a \left[ N_2(0.5) - N_1(0.5) \right] + N_1(0.5) \right\} - N_1(0.5) \cdot N_2(0.5) = 0 \tag{3.13}
\]
and generalized according to 3.12 to

\[ N(P) \left\{ a \left[ N_2(P) - N_1(P) \right] + N_1(P) \right\} - N_1(P) \cdot N_2(P) = 0 \quad (3.14) \]

Equation 3.14 allows the fatigue life to be determined for any probability level and any combination of \( S_1 \) and \( S_2 \). In Fig. 53 a diagram has been constructed similar to Fig. 52, using Eq. 3.14 and \( N \) values corresponding to a 60 percent minimum stress level and 80 and 85 percent values for \( S_1 \) and \( S_2 \). These load cycles were used in cumulative damage test 3FA, the results of which are contained in Table 15. The predicted cumulative frequency distribution is compared with the distribution of ten replications in Fig. 53.

This number of test replications is of course too small to provide justification for the generalization from 3.11 to 3.12, but in view of the complete lack of other test data, the reasonableness and simplicity of the procedure, and the very fair correlation between these few tests and the predicted distribution, it will be adopted in this investigation.

When \( q \) different stress levels are combined with relative frequencies of occurrence \( a_i \), Eq. 3.7 may be
generalized in the above manner to yield
\[
q \sum_{i=1}^{q} \frac{a_i N(P)}{N_i(P)} = 1
\]
or
\[
N(P) = \frac{1}{\sum_{i=1}^{q} \frac{a_i}{N_i(P)}} \tag{3.15}
\]
for any probability level P. Equation 3.15 will be used in this investigation, together with the constant cycle S-N-P relation, represented by Eqs. 3.2, 3.4, and 3.6 to predict the probable fatigue life of strand specimens.
CHAPTER 4 BEAM BEHAVIOR UNDER REPEATED FLEXURAL LOADINGS

4.1 INTRODUCTION

To use the information obtained in the previous chapter for the prediction of the fatigue life of a pre-stressed member containing strand reinforcement, it is necessary to know, or to be able to predict, the response of the beam to load. In particular, it must be possible to determine the relation between steel stress and applied moment in any given load cycle, so that the loading history of the beam can be transformed into the corresponding stress history for the steel.

In this analytical study, a detailed analysis is first made of the response of prestressed concrete members of rectangular section with the steel reinforcement placed in one horizontal layer. The more complicated cases of beams with the reinforcement distributed between several levels and beams with I sections are then treated briefly in turn.
Loading is considered in two stages; zero moment to $M_{on}$ and $M_{on}$ to static ultimate moment, where $M_{on}$ is the moment at which cracks begin to open. In the first loading stage, increments of strain in the steel and concrete are relatively small and linear stress-strain relations are assumed for both materials. Previously formed flexural cracks are closed in this initial loading stage by the internal prestressing force, and so the cracked regions are assumed to behave elastically provided the stresses remain compressive, i.e., provided $M_{on}$ is not exceeded.

Conditions in the second stage of loading are considerably more complicated. The analysis of beam behavior is based on a consideration of the following:

(a) Stress-strain relations for concrete and steel,
(b) An assumed pattern of deformation in the beam in the region of flexural cracking,
(c) Equilibrium of internal forces.

The results of the experimental work of Chapter 2 of this report are used in several instances in the treatment of the second loading stage. The concrete
In the first loading stage, linear relations will be assumed between stress and strain for both concrete and steel, and strains at different levels in the beam will be assumed to vary linearly with depth. Considering the steel-concrete transformed section of a rectangular beam, we determine the position of the centroidal axis as

\[ \frac{1}{4} \left( 1 - \psi \right) \]

where \( \psi \) is a bond parameter which measures the degree of bond breakdown between strand and concrete in the beam near a flexural crack. The stress-strain data obtained from the cylinder tests are used as a basis for the choice of an equation for the concrete stress-strain relation. An idealized pattern of beam deformation is assumed which describes, approximately, the concrete deformations observed in the beam tests, and, finally, a bond parameter \( \psi \), which measures the degree of bond breakdown between strand and concrete in the beam near a flexural crack, is evaluated empirically from the deformation measurements made on the beams during the fatigue tests.

4.2 FIRST LOADING STAGE, \( M \leq M_{\text{on}} \)

In the first loading stage, linear relations will be assumed between stress and strain for both concrete and steel, and strains at different levels in the beam will be assumed to vary linearly with depth. Considering the steel-concrete transformed section of a rectangular beam, we determine the position of the centroidal axis as
\[ \bar{x} = \frac{A_c \cdot e}{A_c + (m-1)A_s} \]

where \( e \) is the distance from the center of gravity of the concrete area, \( A_c \), to the center of gravity of the steel area, \( A_s \), and \( \bar{x} \) is the distance from the center of gravity of the steel area to the centroidal axis. The moment of inertia of the steel-concrete transformed section with respect to the centroidal axis is

\[
I = A_c \left[ \frac{h^2}{12} + (e-\bar{x})^2 + (m-1) \cdot \frac{A_s}{bh} \cdot \bar{x}^2 \right]
\]

and taking tensile stresses positive, the top and bottom concrete fiber stresses and the steel stress induced by moment \( M \) are, respectively,

\[
f^t_{cL} = -\frac{M}{I} \left[ \frac{h}{2} + e - \bar{x} \right]
\]

\[
f^b_{cL} = +\frac{M}{I} \left[ \frac{h}{2} - e + \bar{x} \right]
\]

\[
f_{sL} = +m \cdot \frac{M}{I} \bar{x}
\]

If the prestressing force in the steel prior to the
The application of the n-th load cycle is $F_n$, the corresponding stresses in the unloaded beam are

$$f_{ct} = -F_n \left[ \frac{1}{A_c} - \frac{e}{I_c} \right],$$

$$f_{cb} = -F_n \left[ \frac{1}{A_c} + \frac{e}{I_c} \right],$$

$$f_{cs} = \frac{F_n}{A_s},$$

where $I_c$ is the moment of inertia of the rectangular concrete section about its center of gravity. The total stresses at moment $M$ in the n-th cycle are therefore

$$f_{cn} = -F_n \left[ \frac{1}{A_c} - \frac{e}{I_c} \right] - \frac{M}{I} \left[ \frac{h}{2} + e - \overline{x} \right] \quad (4.1)$$

$$f_{cn} = -F_n \left[ \frac{1}{A_c} + \frac{e}{I_c} \right] + \frac{M}{I} \left[ \frac{h}{2} - e + \overline{x} \right] \quad (4.2)$$

$$f_{sn} = \frac{F_m}{A_s} + \frac{M}{I} \bar{x} \quad (4.3)$$

For the first cycle of loading, i.e. when $n = 1$, the value of $F_{se}$ will either be known by measurement or estimated in the design calculations. Cracking will
take place in this initial load cycle when $f_{c1}^b$ becomes equal to the modulus of rupture of the concrete, i.e.

when

$$F_{se} \left[ \frac{1}{A_c} + \frac{e}{I_c} \frac{h}{2} \right] + \frac{M}{I} \left[ \frac{h}{2} - e + \overline{X} \right] = f'_t$$

Thus, the value of $M_{01}$ in the first load cycle is

$$M_{01} = I \cdot \frac{f'_t - f_{cF}^b}{\frac{1}{2} h - e + \overline{X}} \quad (4.4)$$

In subsequent load cycles, i.e. when $n > 1$, cracks will begin to open when the value of $f_{cn}^b$ is zero; thus,

$$M_{on} = I \cdot \frac{-f_{cF}^b}{\frac{1}{2} h - e + \overline{X}} \quad (4.5)$$

for

$$n > 1.$$
beam is for most of the time greater than or less than $M_{on}$; imperfect closing of the cracks may tend to increase the value slightly; creep in the steel strand will decrease $F_n$. Appropriate values of $F_n$ must be chosen in each particular instance on the basis of an analysis for creep and shrinkage losses and an estimation of other possible effects.

4.3 SECOND LOADING STAGE, $M > M_{on}$

In the second loading stage, several details must be considered in the analysis of beam behavior. These are discussed in the following sections.

Concrete Stress-Strain Relation

A cubic parabola of the form

$$F = \alpha_0 + \alpha_1 E + \alpha_2 E^2 + \alpha_3 E^3,$$  \hspace{1cm} (4.6)

where stress and strain are expressed by the non-dimensional terms

$$F = \frac{f_c}{f'_c} \quad \text{and} \quad E = \frac{\varepsilon_c}{\varepsilon_u}$$

and the $\alpha$ terms are open parameters, is used here to
represent the loading portion of the concrete stress-strain relation. With the following conditions fulfilled,

(a) \( F = 1 \) when \( E = 1 \)

(b) \( \frac{dF}{dE} = 0 \) when \( E = 1 \)

(c) \( F = 0 \) when \( E = 0 \)

(d) \( \frac{dF}{dE} = \alpha \) when \( E = 0 \)

Eq. 4.6 becomes

\[
F = \alpha E + (3-2\alpha) E^2 + (\alpha-2) E^3
\]

(4.7)

The parameter \( \alpha \) is the initial slope of the curve and hence represents, in non-dimensional form, the tangent modulus of elasticity at zero load at the beginning of the \( n \)-th load cycle; i.e.,

\[
\alpha = E \frac{\varepsilon_u}{f_c}
\]

(4.8)

For Eq. 4.7 to represent a monotonically increasing curve for values of \( E \) between zero and unity, a limitation must be placed on the possible values of \( \alpha \). If the initial slope is too steep, the curve reaches a maximum value at a smaller value of \( E \) and then becomes a minimum at \( E = 1 \).
It must therefore be stipulated that

$$\frac{d^2F}{dE^2} \leq 0 \text{ at } E = 1.0,$$

which leads to the result

$$\alpha \leq 3.$$

Equation 4.7 exhibits a concave-up section in the initial load range when

$$\frac{d^2F}{dE^2} < 0 \text{ at } E = 0,$$

i.e., when

$$\alpha < 1.5.$$

In Fig. 54, Eq. 4.7 has been plotted for values of $\alpha$ varying from 0.5 to 3.0.

An experimental study of the stress-strain relation for concrete and the effect of pre-loadings on the relation is described in section 2.5 of this report.

The test points are plotted on dimensionless coordinates in Figs. 35 through 42, and compared with curves obtained using Eq. 4.7 with suitable values of $\alpha$. The stress-strain data for the specimens without prior loading,
shown in Fig. 35, follow quite well the cubic parabola. The test points in Figs. 36, 37, 39, and 40, obtained from specimens which had been subjected previously to light fatigue loadings, also fit quite well to the plotted curves. When the fatigue loading had been intense — for example in Fig. 38 test points are shown for specimens which had been subjected to approximately one-third of the number of load cycles required to cause fatigue failure — there is a distinct tendency for the stress-strain relation to assume a concave-up region in the initial and lower load ranges. Even in such cases a cubic parabola provides a reasonable approximation for the stress-strain relation; a more complicated equation is certainly not justified when account is taken of the considerable variation which is observed between replications of the same test, even when differences in values of maximum stresses and strains have been removed in the figures by non-dimensionalizing.

It was seen earlier that the maximum value which can be given to $\alpha$ is 3. The maximum value of $\alpha$ used in Figs. 35 through 42 for the correlation of test data for high strength concrete is 2.0; it appears that values of $\alpha$ less than 3 will be adequate for most types of concrete.
It should be noted that when the experimental curve has an initial concave-up section, the best fit equation will not necessarily be obtained by substituting for $\alpha$ the observed initial slope, but by choosing $\alpha$ to provide the best fit at all load levels.

Equation 4.7 will be used in the following work to represent the stress-strain relation for concrete subjected to axial loading. In applying the stress-strain relation obtained from axially loaded test pieces to the concrete in the compressive stress block of a flexural member, several important effects must be considered.

In previous studies\(^{(26,27)}\) of the problem of predicting static ultimate flexural strength from the concrete stress-strain relation, it has been found that the general stress-strain characteristics in axial compression are applicable to conditions involving a compressive stress gradient provided account is taken of the unloading portion of the stress-strain relation at high loads. The unloading phenomenon is not normally observed in axial tests on concrete cylinders since sudden failure is induced at maximum load by the release of energy stored in the testing machine.\(^{(28)}\) Hognestad, Hanson, and McHenry
Another effect which must be considered when using cylinder test data in the prediction of beam behavior is the variation in concrete strength between beam and cylinder. In static ultimate strength theory the strength of the concrete in the beam is usually written as $k_3f'_c$, where $f'_c$ is the cylinder strength. In most ultimate

$$F = 1 - \frac{\beta}{\gamma^2} (E-1)^2$$

in the range

$$1 \leq E \leq 1 + \gamma.$$
strength studies, the product of the factor $k_3$ and another factor $k_1$, is evaluated, such that $k_1k_3f_c^\ell$ is the average stress in the concrete compressive stress block at failure. Reliable information on the relation between concrete strengths in the beam and in the cylinder has not yet been published. Hognestad, Hanson, and McHenry (26) evaluated $k_3$ empirically from a series of eccentric column tests as

$$k_3 = \frac{3900 + 0.35f_c^\ell}{3000 + 0.82f_c^\ell} - \left(\frac{f_c^\ell}{26000}\right)^2$$  \hspace{1cm} (4.11)

Unfortunately the data used to derive Eq. 4.11 were all obtained using the one size of test specimen and the equation does not take into account one of the prime variables, namely, the relative size of the two pieces of concrete. In ultimate strength theory for reinforced concrete columns a value for $k_3$ of 0.85 is commonly used to account for size effect, poorer concrete compaction in the column, etc. The value of 0.85 for $k_3$ will be adopted in this investigation, because of the lack of more reliable information. It is thus assumed that the stress-strain curve shown in Fig.
55 can be applied to the compression concrete in the flexural member provided the quantity \( \frac{f_c}{f'_c} \) is replaced by \( \frac{f_c}{k_3 f'_c} \). For convenience, the term \( F \), which henceforth will be used in connection with the concrete in the beam, will be taken to be

\[
F = \frac{f_c}{k_3 f'_c} \quad (4.12)
\]

The area under the curve represented by Eq. 4.7 between zero and \( E_1 \), for \( E_1 \ll 1.0 \), is

\[
A = \int_0^{E_1} FdE
\]

i.e.,

\[
A = \frac{\alpha}{2} E_1^2 + \frac{3-2\alpha}{3} E_1^3 + \frac{\alpha-2}{4} E_1^4 \quad (4.13)
\]
With the interval between $E_1$ and the center of gravity of the area defined as $k_2E_1$,

$$
k_2E_1 = E_1 - \frac{\int_0^E \int_0 F \, dE \, dE}{E_1}$$

and so

$$k_2 = 1 - \frac{\frac{\alpha}{3} + \frac{3-2\alpha}{4} E_1 + \frac{\alpha-2}{5} E_1^2}{\frac{\alpha}{2} + \frac{3-2\alpha}{3} E_1 + \frac{\alpha-2}{4} E_1^2} \quad (4.14)$$

**Deformations in the Beam**

To represent quantitatively the deformations in the beam for loadings larger than $M_{on}$, an idealized deformation condition is assumed in which evenly spaced vertical tension cracks break the beam into a series of blocks. Tensile deformations in the lower portion of the beam are concentrated at the crack, while compressive deformations
are concentrated in a region above the crack. The deformation of the beam is thus pictured as a series of slight kinks at the cracked sections, each kink consisting of slight rotations of the adjacent blocks about the neutral axis, as shown in Fig. 56. The deformation at one cracked section is given quantitatively by the discontinuity at the steel level, $\Delta l_b$, and the deformation in the concrete top fiber above the crack, $\Delta l_t$.

The discontinuity $\Delta l_b$ essentially represents the width of the crack at the steel level. Since infinite strains cannot exist in the steel reinforcement, full or partial breakdown of bond must be assumed to occur between concrete and steel over some finite distance $l_b$ on either side of the crack. The integral of the strain increment in the steel, $\Delta \varepsilon_s(x)$, over the interval $-l_b < x < +l_b$ is equal to the value of the tensile deformation $\Delta l_b$, i.e.

$$\Delta l_b = \int_{-l_b}^{+l_b} \Delta \varepsilon_s(x) \, dx$$

It is reasonable to assume further that the function
\( \Delta \xi_s(x) \) is symmetric, with a maximum value of \( \Delta \xi_{s1} \) at the crack, and of the form

\[
\Delta \xi_s(x) = \Delta \xi_{s1} F(\xi)
\]

where

\[
\xi = \frac{x}{l_b}
\]

Then,

\[
\Delta l_b = 2 \int_0^{l_b} \Delta \xi_s(x) \, dx,
\]

and

\[
\Delta l_b = 2 l_b \Delta \xi_{s1} \int_0^1 f(\xi) \, d\xi. \tag{4.15}
\]

At the cracked section the total steel strain is

\[
\xi_{s1} = \xi_{SF} + \xi_{CF} + \Delta \xi_{s1}, \tag{4.16}
\]

where \( \xi_{SF} \) and \( \xi_{CF} \) are, respectively, the tensile strain in the steel and the compressive strain in the concrete at the steel level due to \( F_n \).
The deformation of the beam is most easily pictured by imagining the deformation $\Delta l_t$ as a discontinuity in the concrete top fibers obtained by overlapping the portions of adjacent blocks above the neutral axis, as shown in Fig. 56. The value of $\Delta l_t$ is equal to the sum of the compressive strains in the concrete top fibers.

It is assumed that the concrete strains $\varepsilon_c(x)$ are distributed symmetrically around the cracked section in a manner which is represented by the equation

$$\varepsilon_c(x) = \varepsilon_{c1} f(\eta)$$

where $\varepsilon_{c1}$ is the maximum value of $\varepsilon_c(x)$ which occurs at the cracked section,

$$\eta = \frac{x}{l_c},$$

and $2l_c$ is the spacing between cracks. Equating $\Delta l_c$ to the integral of the strains, one obtains, as before,

$$\Delta l_t = 2 \int_0^{l_c} \varepsilon_c(x) \, dx,$$

and
\[ \Delta l_t = 2 \ell_c \epsilon_{c1} \int_0^1 f(\eta) \, d\eta \quad (4.17) \]

It is now assumed that the deformations in the beam over the length \( 2 \ell_c \), that is, the deformations which have previously been assumed to be concentrated at one cracked section, are linearly distributed in the vertical direction; thus,

\[ \frac{\Delta l_t}{\Delta l_b} = \frac{kd}{(1-k)d} \quad (4.18) \]

where \( kd \) is the depth to the neutral axis.

Substituting for the deformations, one obtains

\[ \frac{\epsilon_{c1}}{\Delta \epsilon_{s1}} \frac{l_c}{l_b} \left( \frac{1}{\int_0^1 f(\eta) \, d\eta} \right) = \frac{k}{1-k} \quad (4.19) \]

For convenience, a dimensionless parameter \( \psi \) is defined as
It will be noted that the distribution of internal stresses at a cracked section during a particular load cycle will depend upon the magnitude of previous loadings. When the loading under consideration is considerably larger than previous ones, a field of tension stresses will exist in the concrete immediately below the neutral...
axis of stress. In some circumstances, especially at lower loads, the concrete tensile stresses are an important consideration in the equilibrium of the section. When, however, a previous loading has been greater than the one under consideration, cracks will have extended above the present level of the neutral axis and there will be no concrete tensile stresses at the section. In the present analysis it is assumed that over loadings are evenly distributed through the life of the member and it is reasonable to ignore concrete tensile stresses in the equilibrium considerations.

Assuming a linear distribution of strain above the crack, one obtains for the total compressive force,

$$C = b \int_0^{kd} f_c \, dy$$  \hspace{1cm} (4.23)

With

$$F = \frac{f_c}{k_3 f'_c}$$

and

$$E = \frac{\varepsilon_c}{\varepsilon_u}$$
Eq. 4.23 may be written as

\[
C = \frac{bd k_3 f'_c k}{E_1} \left[ \int_0^{E_1} Fd\xi \right], \tag{4.24}
\]

where \( E_1 \) is the extreme fiber value of \( E \).

Equations 4.13 and 4.24 together yield

\[
C = \frac{bd k_3 f'_c k}{E_1} \left[ \frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3 \right]. \tag{4.25}
\]

Horizontal forces may now be equated to yield

\[
\frac{f_{sl} A_s}{bd k_3 f'_c} = k \left[ \frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3 \right]; \tag{4.26}
\]

and equating internal and external moments,

\[
M_1 = f_{sl} A_s d (1 - k_2 k), \tag{4.27}
\]

where \( k_2 \) is given in terms of \( E_1 \) in Eq. 4.14.
Evaluation of Bond Parameter

The non-dimensional term $\psi$ is defined in Eq. 4.20 in terms of the lengths $l_b$ and $l_c$ and the integrals of $f(\xi)$ and $f(\eta)$. Considering the integrals as they appear in Eq. 4.19, we observe that they are, in effect, averaging factors which specify the shapes of the distributions of the strain increment in the steel, $\Delta \varepsilon_s(x)$, and the concrete top fiber strain $\varepsilon_c(x)$. Thus, the terms $\Delta \varepsilon_{s1} \int_0^1 f(\xi) \, d\xi$ and $\varepsilon_{c1} \int_0^1 f(\eta) \, d\eta$ are, respectively, the average strain increment in the steel over the length $l_b$ and the average concrete top fiber strain over the length $l_c$. The values of the integrals lie between zero and unity.

The crack spacing, $2l_c$, the length of the bond break-down, $l_b$, and the distribution of the steel strain increment $\Delta \varepsilon_s(x)$ all depend directly upon the bonding properties of the strand and concrete. The distribution of $\varepsilon_c(x)$ will depend to a large extent upon the crack spacing and hence indirectly upon the bond properties. The term $\psi$ may therefore be regarded as a parameter which is intimately associated with the quality of the strand-concrete.
bond.

Provided an over-loading of reasonable magnitude has occurred on the member, new cracks are not likely to form as a result of fatigue loading; \( l_c \) may well be considered constant over the major portion of the beam fatigue life. The maximum value of the ratio \( \frac{l_c}{l_b} \) is unity, which will occur only when bond has been broken along the entire beam. It will be noted that small values of this ratio, i.e., values considerably less than unity, represent a situation favorable to the formation of an intermediate crack, and it may be concluded that the \( \frac{l_c}{l_b} \) ratio, after the initial loading sequence, will have a value greater than, but in the vicinity of, unity and will decrease in value towards unity as the repeated loadings further break down bond and increase \( l_b \).

In the idealized case of perfect bonding - in so far as the analysis of deformations instead of strains is applicable - length \( l_b \) becomes equal to \( l_c \) and both become infinitesimal, uniform strain distributions exist in both the concrete and steel, and so \( \psi = 1.0 \).
When there is no bond between steel and concrete, for example in an unbonded post-tensioned beam, crack spacing becomes large, but \( l_b \) is again equal to \( l_c \). Whereas a state of almost uniform strain must exist in the steel and

\[
\int_0^1 f(\xi) \, d\xi \rightarrow 1.0,
\]

the concrete strains in the top fiber will be largely concentrated in the region above the crack and

\[
\int_0^1 f(\eta) \, d\eta \rightarrow 0.
\]

Practical limitations, however, such as friction between cable and concrete ensure reasonably high values of this integral and hence of \( \psi \).

For conditions intermediate between the two extremes just considered, values of \( \psi \) will not necessarily lie between zero and unity. There is no reason why values of \( \psi \) in excess of unity should not exist.
In view of the lack of basic information on the bonding properties of strand and concrete and the scattersome nature of the phenomenon, theoretical evaluation of $l_c$, $l_b$, and the integral terms is not at present feasible. Measurements of concrete deformations during beam tests can however be used to evaluate $\psi$ empirically.

In the beam fatigue tests described in Chapter 2 of this report, static tests were conducted at regular intervals during the fatigue loading, and the concrete deformation measurements on the sides of the beams provide experimental values of $k$ for different values of moment $M_1$. It will be noted that the concrete elastic strains corresponding to the effective prestressing force $F_n$ must be added to the strains measured during the loading operation in order to obtain the correct values of $k$. Thus, considering $M_1$ and $k$ as known quantities, Eqs. 4.14, 4.22, 4.26, and 4.27, together with the steel stress-strain relation, may be used to evaluate $f_{sl}$, $\varepsilon_{sl}$, $k_2$, $E_1$, and $\psi$. $\psi$ values obtained in this way from the beam fatigue tests are shown in Table 18. Computations were made for values of $k_3$ equal to 0.85 and 1.00, at loads of 12.5 kips for beams $F_1$, $F_2$, and $F_4$, and 10.5 kips for beams $F_5$, $F_7$, and $F_8$. $\psi$ values were also computed for loads a little higher than the cracking loads, but the values were not reliable because values of the term $(\varepsilon_{sl} - \varepsilon_{SF} - \varepsilon_{CF})$ on which $\psi$
directly depends, were very small and could not accurately be determined. Values of \( \psi \) shown in Table 18 are grouped around a value of unity. It will be noted that, after a preliminary sequence of loading of approximately 10 percent of the fatigue life, there is little change in \( \psi \) values with the number of applied load cycles. Although all beams were manufactured from similar materials, average values of \( \psi \) for the different beams vary between values of 0.6 to 1.6. In view of the scatter in the values and their grouping around the value of unity the most appropriate value for beams of this type appears to be

\[
\psi = 1.0.
\]

Values of \( \psi \) considerably different from unity of course may well be required for beams of different design, different concrete strength, etc. The effect of variations in \( \psi \) values on the steel stresses will be considered later in section 5.4.

Stresses in a Rectangular Section

The equations derived for the cracked rectangular
section are now summarized.

\[
\frac{f_{sl} A_s}{b d k_3 f_c} = k \frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3
\]  \hspace{1cm} (4.26)

\[
M_1 = f_{sl} A_s d (1 - k_2 k).
\]  \hspace{1cm} (4.27)

\[
\varepsilon_{sl} = \varepsilon_{SF} + \varepsilon_{CF} + \frac{1 - k}{k} \varepsilon_{c1} \gamma
\]  \hspace{1cm} (4.22)

\[
k_2 = 1 - \frac{\frac{\alpha}{2} + \frac{3-2\alpha}{3} E_1 + \frac{\alpha-2}{4} E_1^2}{\frac{\alpha}{3} + \frac{3-2\alpha}{4} E_1 + \frac{\alpha-2}{5} E_1^2}
\]  \hspace{1cm} (4.14)

These equations, together with the steel stress-strain relation, may be used to evaluate, for a moment $M_1$ in the $n$-th load cycle, the unknowns $f_{sl}$, $\varepsilon_{sl}$, $k$, $k_2$, and $E_1$. It will be noted however that the value of $F_n$ for the $n$-th load cycle must be known or estimated since it is used to determine $\varepsilon_{SF}$ and $\varepsilon_{CF}$. In general, the calculations for the stress moment relation will be simplified if values of $f_{sl}$ and $\varepsilon_{sl}$ are first chosen and substituted in Eqs. 4.26 and 4.22 to obtain $k$, $E_1$, and hence, through Eq. 4.14, $k_2$. The corresponding values
of $M_1$ may then be obtained by substitution of values in Eq. 4.27. The process may then be repeated to obtain different points on the $M_1 - f_{s1}$ curve.

In Appendix 1 a set of calculations for the stress-moment relation for beam F7 is given, for a $k_3$ value of 0.85 and a $\psi$ value of 1.0.

When a large number of stress-moment calculations are to be made, it may be convenient to plot Eqs. 4.26 and 4.22 in the form of an intercept chart on coordinates of $E_1$ and $k$, for appropriate values of $a$. Equation 4.22 may be divided throughout by $\epsilon_u$ and rearranged to the form

$$\frac{1}{\psi \epsilon_u} \left[ \epsilon_{s1} - \epsilon_{sF} - \epsilon_{cF} \right] = E_1 \frac{1 - k}{k} \quad (4.22a)$$

Equations 4.26 and 4.22a now contain the terms $E_1$ and $k$ in the right hand sides, while their left hand sides are functions of either $f_{s1}$ or $\epsilon_{s1}$. Each equation may be used to plot a family of curves on the $E_1$-$k$ coordinates. The resulting intercept chart provides values directly for $k$ and $E_1$ without a trial and error procedure.
Steel at Several Different Levels in the Beam

The equations in the previous section were derived for a rectangular section in which all of the steel reinforcement lies at a depth $d$ below the top surface. When the steel lies at $z$ different levels, there will be $z$ different values of stress and strain for the steel. Letting the depths of the different steel layers be $d_1, d_2, \ldots, d_z$, the corresponding steel stresses be $f_{s1}, f_{s2}, \ldots, f_{sz}$, and the depth to the neutral axis be $a$, one obtains for the two equilibrium equations,

\[ M = f_{s1} A_{s1} (d_1 - k_2 a) + f_{s2} A_{s2} (d_2 - k_2 a) + \cdots + f_{sz} A_{sz} (d_z - k_2 a) \]  

\[(4.28)\]

and

\[ k \left[ \frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_2 + \frac{\alpha-2}{4} E_3 \right] = \frac{1}{bd} k_3 \begin{bmatrix} f_{s1} A_{s1} \\ +f_{s2} A_{s2} \\ \cdots \\ \cdots \\ +f_{sz} A_{sz} \end{bmatrix} \]

\[(4.29)\]
If it is again assumed that deformations are
linearly distributed in the vertical direction, and the
tensile deformations at the z steel levels are \( \Delta l_{b1} \),
\( \Delta l_{b2} \), \( \ldots \) \( \Delta l_{bz} \), then

\[
\Delta l_t = \frac{1}{\frac{d_1}{a} - 1} \Delta l_{b1} \\
= \frac{1}{\frac{d_2}{a} - 1} \Delta l_{b2} \\
\ldots \ldots \ldots \ldots \\
\ldots \ldots \ldots \ldots \\
= \frac{1}{\frac{d_z}{a} - 1} \Delta l_{bz}
\]

Substitution of Eqs. 4.15 and 4.17 in the above
set yields

\[
\varepsilon_{c1} = \frac{1}{\frac{d_1}{a} - 1} \Delta \varepsilon_{s1} \gamma_1^{-1} \\
= \frac{1}{\frac{d_2}{a} - 1} \Delta \varepsilon_{s2} \gamma_2^{-1} \\
\ldots \ldots \ldots \ldots \\
\ldots \ldots \ldots \ldots \\
= \frac{1}{\frac{d_z}{a} - 1} \Delta \varepsilon_{sz} \gamma_z^{-1}
\]
If it is further assumed that the bond parameter will not vary with the steel level, the following compatibility equations are obtained

\[
\varepsilon_{s1} = \varepsilon_{sF1} + \varepsilon_{cF1} + \left( \frac{d_1}{a} - 1 \right) \varepsilon_{c1} \\
\varepsilon_{s2} = \varepsilon_{sF2} + \varepsilon_{cF2} + \left( \frac{d_2}{a} - 1 \right) \varepsilon_{c1} \\
\vdots \\
\varepsilon_{sz} = \varepsilon_{sFz} + \varepsilon_{cFz} + \left( \frac{dz}{z} - 1 \right) \varepsilon_{c1}
\]

These equations, together with Eqs. 4.28, 4.29, 4.14, and the stress-strain relation for the steel, must be used to obtain by a trial and error procedure the different values of steel stress.

In most practical situations, however, the solution of these simultaneous equations is impracticable and the simplifying assumption must be made that all strands are grouped at the center of gravity of the steel.

4.4 BEAMS WITH I SHAPED SECTIONS

In the case of beams of I section, loading must be
considered in three stages; zero moment to $M_{O_1}$, $M_{O_1}$ to $M_t$, and $M_t$ to static ultimate moment, where $M_{O_1}$ is the moment at which cracks begin to open and $M_t$ is the moment at which the depth to the neutral axis is equal to the depth of the top flange.

Linear stress-strain relations may be assumed for the first loading stage and a simple elastic analysis of the section, similar to the analysis in section 4.2, provides values for steel stress and cracking moment. In the third loading stage the neutral axis will lie in the top flange and the steel stresses may be determined from an analysis of the beam assuming a rectangular section with the width equal to the width of the upper flange.

In the second loading stage, $M_{O_1} < M_1 < M_t$, the beam is cracked and the neutral axis lies in the web. Letting the value of the dimensionless strain term $E$ in the top concrete fiber be $E_1$, and assuming a linear distribution of compressive strains, one obtains for the value of $E$ at the bottom level of the top flange

$$E = \frac{kd - t}{kd} E_1$$

where $t$ is the flange thickness. The compressive force
is therefore

\[
C = b' \int_{0}^{kd} f_c \, dy + (b-b') \int_{(kd-t)}^{kd} f_c \, dy
\]

i.e.

\[
C = k_2 \frac{f_c' d}{E_1} \cdot k \left[ b' \left( \int_{0}^{E_1} FdE + (b-b') \int_{(1-\frac{t}{kd}) E_1}^{E_1} FdE \right) \right]
\]

(4.30)

The center of gravity of the compressive force is a distance \(k_2 kd\) from the top fiber, and

\[
k_2 = \frac{\int_{0}^{E_1} \frac{EFdE + (1 - \frac{b'}{b})}{b} \int_{(1-\frac{t}{kd}) E_1}^{E_1} \frac{EFdE}{E_1} \int_{0}^{E_1} FdE + (1 - \frac{b'}{b}) \int_{(1-\frac{t}{kd}) E_1}^{E_1} FdE}{(4.31)}
\]
Equations for horizontal equilibrium and moment equilibrium are

\[
f_{s1} A_s = k \frac{f_c'}{d} \int b \frac{b'}{E_1} F dE + \frac{(b-b')}{E_1} FdE + (1-\frac{t}{kd})E_1
\]

\[
M_1 = f_{s1} A_s d (1-k_2 k).
\]

An analysis of the deformations in the beam yields the compatibility condition represented by Eq. 4.22. It should however be noted that values of the bond parameter \(\gamma\) may well vary considerably from those for rectangular sections because of a possible change in the shape of the concrete strain distribution in the top fibers, as represented by

\[
\int_0^1 f(\eta) d\eta.
\]
The integrals appearing in Eqs. 4.31 and 4.32 may be evaluated without difficulty in terms of $E_1$ and $k$, and then Eqs. 4.22, 4.27, 4.31 and 4.32, together with the steel stress-strain relation, may be used to solve for steel stress by a trial and error procedure. The main difference between these calculations and those for a rectangular section is that $k_2$ is now a function of $k$ as well as $E_1$. It is convenient to begin the calculations by assuming a steel stress $f_{s_1}$, and make trial values of $k$ until the correct value of $\varepsilon_{s_1}$ is given by Eq. 4.22. The moment corresponding to $f_{s_1}$ is then obtained by substitution in Eq. 4.27.
5.1 UNDER-REINFORCED BEAMS

In the preceding two chapters an experimental study was made of the fatigue properties of high strength steel prestressing strand and a theoretical analysis was made of the steel and concrete stresses in members subjected to flexural loadings. The results obtained from these two studies provide a means for calculating probable beam fatigue life as limited by the fatigue strength of the strand reinforcement.

In order to predict the fatigue life of a given beam, it is necessary first to determine, from the known or assumed load history, the corresponding stress history for the reinforcing steel. To make the transformation from load history to stress history, use is made of stress-moment curves which may be computed for any
particular beam cross section using the equations developed in Chapter 4. If the response of the beam to load remains constant throughout the major portion of its fatigue life, only one stress-moment relation has to be obtained. If, however, the response of the beam varies as a result of the fatigue loading, the load history must be broken into a number of intervals, the size of the interval depending upon the rate of change of beam response, and a stress-moment relation must be computed for each interval.

It was seen in the beam fatigue tests described in Chapter 2 that, after an initial sequence of repeated loadings during which considerable changes took place in the deflections, deformations, and cracking patterns, the beams settled down to a fairly consistent response to the repeated loadings. Values of $\gamma$ computed in Chapter 4 from deformation measurements on the beams under test also remained fairly constant after the initial sequence of loadings. The results of these tests thus indicate that a single stress-moment relation would normally be sufficient — at least for beams similar to those tested — for determining the stress history, and
in the following discussion it will be assumed that the response of the beam to load remains constant.

When the beam is subjected only to repeated load cycles of constant magnitude, the stress history will consist of repeated stress cycles of constant magnitude. After the magnitude of the stress cycle has been determined from the stress-moment relation, Eqs. 3.4 and 3.6 may be used to determine the mean fatigue life, $\bar{N}$, and the standard deviation of fatigue life, $D$, for a single strand element subjected to this stress cycle. These two values may be used in Eq. 3.2 to determine the number of cycles, $N$, corresponding to any probability level $P$.

If there are $u$ similar strands present in the beam section at the same level, then the probability of beam failure at or before $N$ cycles is

$$Q = 1 - (1 - P)^u. \quad (5.1)$$

Should the strands be placed at $z$ different levels, with $u_1, u_2, \ldots, u_1, \ldots, u_z$ strands in the first, second, ... $i$-th, ... $z$-th levels, then the probability of
beam failure at or before \( N \) cycles is

\[
Q = 1 - (1-P_1)^{u_1} (1-P_2)^{u_2} \ldots (1-P_i)^{u_i} \ldots (1-P_z)^{u_z},
\]

(5.2)

where \( P_i \) is the probability of failure at or before \( N \) cycles for an individual strand subjected to the repeated stress cycles which occur in the steel at the \( i \)-th level.

In the case of a beam subjected to cumulative damage loading, the load history may be expressed as a curve relating load magnitude and relative frequency of occurrence (load-frequency distribution), a load-frequency histogram, or a block of load cycles as shown in Fig. 8. In each case the load history can be expressed, either exactly or approximately, as a block of load cycles, and the stress moment relation may then be used to make the transformation into a corresponding block of stress cycles. Equation 3.15 will then indicate, for a strand element subjected to this repeated load block, the probability of failure, \( P \), corresponding to any number of load cycles, \( N \). Equations 5.1 or 5.2 may be used to determine from \( P \) the probability of fatigue failure of the beam at \( N \) cycles.
5.2 COMPARISON WITH TEST RESULTS

Strand fatigue failure took place in all of the beam fatigue tests described in Chapter 2, and a comparison may now be made between observed and predicted mean fatigue lives.

Stress-moment relations were computed for the six beams and were used to determine the magnitude of the stresses in the reinforcement in the beams under the test loadings. In making the computations, a $\psi$ value of 1.0 and a $k_3$ value of 0.85 were adopted. Creep-relaxation losses in the steel were not measured during the beam tests; a value of 4 percent was however adopted, on the basis of figures quoted by Kommendant\(^{(30)}\) for prestressing wires. Values of applied moments and corresponding steel stresses are shown for the six beams in Table 19.

Since the beams contained three strands, values of $u = 3$ and $Q = 0.5$ are substituted in Eq. 5.1 to give a value of 0.206 for $P$. Thus, the mean fatigue life of a beam is equal to the fatigue life at the 0.206 probability level of a single strand subjected to the stress
history of the steel in the beam. The relation between the stress interval $R$ and $\log N$ has been determined for the 0.206 probability level from Eqs. 3.2, 3.4, and 3.6, and is plotted in Fig. 49.

Use of the 0.206 probability line in Fig. 49, together with values computed for $R$, provides values of predicted mean fatigue life for beams $F_1$, $F_2$, and $F_4$, which were subjected to constant cycle loading. In the case of beams $F_5$, $F_7$, and $F_8$, which were all subjected to cumulative damage loading, the 0.206 probability line provides values of $N(0.206)$ which may be substituted in Eq. 3.15 to give the predicted mean fatigue life. The complete calculations for fatigue life of beam $F_7$ are shown in Appendix I. Although a comparison of the values of computed and predicted fatigue lives in Table 19 shows a slight trend for the method to over-estimate fatigue life, agreement is generally quite good, especially considering the variability of the phenomenon being studied.

5.3 OVER-REINFORCED BEAMS

In determining the fatigue life of under-reinforced
beams, strand fatigue test data may be used directly in the beam calculations since the state of stress in the strand in the beam is essentially simple tension. Also, since the strands are present in the beam as discrete elements, the "size effect" involved in the prediction of the fatigue life of strands from the fatigue data for one strand is taken into account quite simply, using Eq. 5.1. A study of fatigue failure in the concrete compression zone of the beam, however, is complicated by both size effect and the presence of the stress gradient.

A statistical approach to the size effect and stress gradient problems has been made by Fowler (29), but his work, being concerned with materials such as steel which exhibit similar stress-strain properties in tension and compression, is not directly applicable to concrete. A considerable amount of work, both analytic and experimental, will be required before concrete fatigue life under stress gradients can be predicted using fatigue test data obtained from axially loaded specimens.

A simple lower bound estimate of the fatigue life of over-reinforced concrete beams can however be obtained
by determining the fatigue life of a piece of plain concrete similar to the concrete in the beam, with cross sectional area equal to the area of the concrete stress block, and subjected to a pattern of repeated stresses which are uniform over the cross section and equal in value to the stresses in the extreme fiber of the beam.

Stress-moment relations for the concrete top fiber may be determined using the equations derived in Chapter 4. The non-dimensionalized concrete top-fiber strain, $E_1$, is evaluated during the steel stress computations; the corresponding value of concrete stress is given by Eq. 4.7. The stress history for the concrete top fiber may then be obtained from the stress-moment relation and the known or assumed load history. Fatigue test data obtained from axially loaded test specimens may then be used to estimate a lower bound value for beam fatigue life.

5.4 DISCUSSION

Before a summary is made of the results of this
investigation, several important aspects of the study of under-reinforced fatigue failure will be discussed, in order to emphasize limitations involved in the present approach.

Limited Applicability of Strand Fatigue Data

It was noted previously that quantitative information on material fatigue properties must at present come from experimental studies, and therefore that such information is restricted in application. It is important to emphasize the limited applicability of the strand fatigue test data obtained in Chapter 3 of this report. All of the strand tests were conducted on unrusted 7/16 inch diameter strand, obtained in two lots from the manufacturer. Although little variation was observed between the fatigue properties of the two lots, considerable variation might be expected between the products of different manufacturers, and, quite possibly, in the product of one manufacturer over a period of time. Strand which has been stored for some time and allowed to rust will have poorer fatigue properties; some differences in the fatigue properties of strand of different
sizes must also be expected.

More fatigue tests are obviously required to investigate each of these effects. Such fatigue tests may well indicate the advisability of using an equation for mean fatigue life more conservative than Eq. 3.4; they most certainly will indicate values for standard deviation much greater than those represented by Eq. 3.6.

The experimental study described in Chapter 3 was concerned with the fatigue properties of the strand in the life region between 50,000 cycles and 5 million cycles. Approximate values for fatigue limit were adopted, on the basis of an extrapolation of the mean S-N curves, and were used in the derivation of Eq. 3.4. Some error in the values of the mean fatigue limit does not however influence significantly the "fit" of Eq. 3.4 in the finite life region under consideration. A different type of test (17) would of course be required to establish accurate values for the probable fatigue limit of the material.
Size Effect

Equations 5.1 and 5.2 take into account the influence of the number of strands in the beam, and, in effect, represent an allowance for size effect for the amount of steel in the cross section. There is another size effect to be considered in the longitudinal direction. If the steel stress were uniform along the length of the beam the entire size effect for the steel would be represented by the following equation,

\[ Q = 1 - (1-P)\alpha \beta \varepsilon \]

where \( \varepsilon \) is the ratio of the length of the beam to the length of the strand test specimen. However, an examination of the deformations measured in the test beams indicates that steel stress varies greatly along the length of the beam, even in regions of constant moment, and in fact will acquire a maximum value only at the widest crack. This can be seen in Figs. 20, 21, and 22.

An accurate analysis of the longitudinal size effect would involve the determination of the steel stress at each section along the beam, the calculation of the probability of failure in each increment of beam...
length, and finally, the combined probability of failure for the entire length. Such a procedure, even if it were possible to evaluate accurately the variation in steel stress along the beam, is clearly not feasible. In this investigation it has been assumed that failure will always occur in the region of maximum steel stress, which will exist at the widest crack. Empirical values of $\psi$ shown in Table 18, were accordingly obtained only from deformation measurements in the gage length in which failure eventually occurred. In all six beam tests the wire failures took place in the gage length which gave the largest tensile deformation readings.

Considering Eq. 5.1, it is seen that the likelihood of fatigue failure increases greatly with the number of strands in the cross section. It should, however, be remembered that beam fatigue failure has here been associated with first wire failure. When there is a very large number of discrete steel elements present in the cross section, the consequences of failure of one or even several of them is far less serious, and it may be necessary in such a situation arbitrarily to define beam fatigue failure as the failure of some proportion of the total number of elements of steel.
Variability in Response of Beam to Load

An examination of the results obtained in Chapter 3 indicates extreme sensitivity of strand fatigue life to small changes in the maximum and minimum stress levels. At the 60 percent minimum stress level, for example, a change in maximum stress level of only 14 percent, from 71 to 85, is sufficient to change the mean fatigue life from infinity to approximately 70,000. This sensitivity becomes more pronounced, of course, in the range of large N and small S values, where the mean curve is approaching its asymptotic value. Computations for beam fatigue life show a like sensitivity of beam fatigue life to small variations in the loading, particularly in the maximum load level, and also to small errors in the computed steel stresses.

In the stress computations a number of factors are involved which cannot be evaluated precisely in most practical situations, and it is important to observe the effect of variations in these quantities on beam fatigue life. The quantities $k_\gamma$, $\psi$, and prestress losses are particularly important in this respect. Although losses due to concrete creep and shrinkage can be measured accurately in laboratory test beams, accurate
prediction of these quantities, especially under field conditions, is almost impossible because of inherent variability in concrete properties. In addition to the concrete losses, a certain amount of loss occurs due to creep and relaxation in the steel. It is difficult to measure steel losses in the laboratory, and in the case of the beam tests conducted in this investigation a 4 percent loss was assumed on the basis of figures given by Kommendant (30). While losses in the prestressing force do not materially affect the maximum steel stress level in the loaded beam, they directly affect the minimum steel stress level corresponding to the beam in the unloaded state.

To observe the variation in values of predicted fatigue life, stress calculations were made for beam F7 using $k_3$ values of 0.85 and 1.0, $\nu$ values of 0.7, 1.0, and 1.3, and steel losses of 2 and 4 percent. Stress–moment relations were plotted for each calculation, values were thus obtained for steel stresses in the beam due to the applied loadings, and values of mean fatigue life were then determined from Eq. 3.15. Results of five different sets of calculations are contained in
Table 20, which shows the effect on fatigue life of variations of the parameters from the previously assumed values of $k = 0.85$, $\gamma = 1.0$, and 4 percent steel losses. Variations in the factors of the order considered are seen to vary the mean fatigue life by 20 to 30 percent. It should however be noted that beam F7 was subjected to one particularly heavy overloading which caused a large proportion of the fatigue damage in the beam. The value of the stress interval $R$ for this overload is large, in the range of 7 to 9; in cases where the $R$ value is small, the corresponding variation in beam fatigue life, due to variations in $k$, $\gamma$, and steel loss, will be larger, and may well exceed 100 percent.

Since it will not be possible in a practical situation to predict any of these factors with exactitude, variability in predicted beam fatigue life is likely to be considerably greater even than that indicated by the variability in the strand fatigue data. In such a situation, it would seem advisable to treat not only the fatigue properties of the materials as random variables but also the response of the beam to
Thus, quantities such as $f_c$, $k_3$, $a$, $F_n$, and $\psi$ would be considered not as single valued parameters but as statistics with associated frequency distributions. Such a procedure, however, is clearly not feasible until very extensive experimental work is conducted to determine the frequency distributions for each random variable.

The reasonable agreement obtained between predicted mean and observed fatigue life for the test beam does however indicate the appropriateness of the methods developed in this investigation. By adopting suitably conservative values for parameters which are not known exactly, the equations may be used to check the safety against fatigue failure of partially pre-stressed members which are cracked under load.

**Effect on Beam Fatigue Life of Repeated Overloadings**

In the cumulative damage tests on beams F5, F7, and F8, the predominant load level produced approximately zero stress in the concrete at the bottom fiber; the first overload, $P_{01}$, was large enough to cause the
tension cracks to open, and produced a stress in the steel approximately equal to the fatigue limit; the second overload, \( P_{o2} \), opened the crack further and caused an overstress of considerable magnitude in the strand. In beam F5 the steel stress level corresponding to load \( P_{o1} \) was just below the fatigue limit and hence, according to the findings of Chapter 2, did not cause fatigue damage. Failure was brought about in this beam by the repeated application of load \( P_{o2} \). Load \( P_{o1} \) produced stress levels in beams F7 and F8 above the fatigue limit and contributed significantly to fatigue damage.

The reasonable correlation of theory with experiment for these three beams, together with the conclusion of Chapter 2 that stress levels smaller than the fatigue limit do not contribute to strand fatigue, indicates that loadings which cause opening and closing of the tension cracks will only begin to affect beam fatigue life when they produce overstresses in the steel reinforcement. The information obtained in this investigation may be used to estimate beam fatigue life due to repeated overloading.
5.5 SUMMARY AND CONCLUSIONS

An investigation was conducted into the fatigue life of prestressed concrete beams subjected to either constant cycle or cumulative damage loadings. Attention was given primarily to beams which are under-reinforced with respect to fatigue failure, i.e., to beams in which fatigue of the tension steel would precede fatigue in the concrete compression zone.

An experimental study was made of the fatigue properties of 7/16 inch diameter high strength prestressing strand. An empirical relation between maximum and minimum stress level and probable fatigue life was developed from the constant cycle test data. The results of cumulative damage tests showed good correlation with mean fatigue life predicted by Miner's theory. A generalized form of Miner's theory was developed to apply at all probability levels.

A theoretical analysis was made of the behavior of prestressed concrete beams under repeated loadings. Equations were derived for the stresses in the steel and in the extreme concrete compressive fibers in members of rectangular and I-shaped sections subjected to repeated loadings.
A method of determining the probable fatigue life of under-reinforced members was indicated, which uses the data obtained from the strand fatigue tests, together with the equations derived in the analysis of beam behavior.

Static and fatigue tests were conducted on eight prestressed concrete beams of rectangular section. Although considerable changes in deformations and deflections took place in the early load cycles, the beams settled down quickly to a consistent response to load which was maintained over the major portion of the load history. Steel fatigue failures occurred in all beams which were fatigue tested. Satisfactory agreement was obtained between computed mean fatigue life and observed fatigue life.

Finally, a method was indicated for obtaining a lower bound estimate for the fatigue life of over-reinforced members by using the equations derived in the theoretical analysis to determine the stress history of the concrete in the extreme compression fiber, and applying data on concrete fatigue life obtained from fatigue tests on axially loaded specimens.

The following conclusions are indicated by the experimental and theoretical work comprising this investigation:
(1) The response of a prestressed concrete beam may be expected to vary considerably as a result of the application of fatigue loading. This variation is probably due to creep effects, changes in the concrete stress-strain relation, and progressive bond breakdown between the tension steel and surrounding concrete. However, after an initial sequence of repeated loadings, representing perhaps ten percent of the fatigue life, the beam settles down to a fairly regular and consistent response to load.

When the fatigue loading is particularly severe, a continuous change in beam response may occur up to failure. Such severe fatigue loading would rarely be encountered under field conditions, and consequently, in most cases the fatigue properties of a member may be studied by assuming a constant response of beam to load.

(2) The fatigue failure of under-reinforced beams occurs by successive fracture of the elements of steel reinforcement in the beam. A considerable number of load cycles may separate the first and second steel failures, but the interval separating successive failures will tend to decrease as the number of failed elements increases. Failure of each steel element is accompanied by a corresponding decrease in beam rigidity.
When the total area of steel reinforcement is contained in a small number of elements, it is advisable to associate beam fatigue failure with failure of the first steel element. When there are a large number of steel elements present in the section, beam fatigue failure may better be defined arbitrarily as the failure of some proportions of the elements. The proportion would be chosen from a consideration of allowable decreases in beam rigidity and factor of safety against static load which could be allowed to occur as a result of the steel failures.

(3) The fatigue life of an under-reinforced member subjected to a known load history may be estimated using the fatigue properties of the reinforcing steel, together with an analysis of the response of the beam to load.

An example solution for the probable fatigue life of an under-reinforced prestressed concrete beam, using the procedure proposed in this report, is included in the appendix. All of the formulas are listed and explained as they are used in the solution.

(4) Quantitative information on material fatigue properties must at present come from experimental studies, and such
information is therefore restricted in application. Because of the variability inherent in material fatigue properties, simple S-N curves and fatigue envelopes are inadequate representations of constant cycle fatigue properties. Statistical interpretation of strand fatigue test data is necessary in any satisfactory treatment.

(5) The results of the investigation of strand fatigue properties indicated that stress cycles in the loading history which are smaller than the fatigue limit will not contribute to fatigue failure in the strand. (It should be noted that the opposite conclusion has been obtained by other investigations working with other materials.) Thus, beam loadings which cause flexural cracks to open will not shorten beam fatigue life provided the stresses induced in the strand reinforcement are smaller than the fatigue limit. The use of partial prestressing techniques should not therefore lead to problems of premature fatigue failure, provided a conservative estimate of the stresses in the reinforcement, together with steel data, indicates adequate fatigue life for the beam.
6. NOMENCLATURE

\[ A_c \] area of concrete section

\[ A_s \] cross sectional area of longitudinal tension steel

\[ b \] width of rectangular beam, width of top flange of I beam

\[ b' \] width of web of I beam

\[ d \] effective depth of beam

\[ D \] standard deviation of \( \log N \)

\[ e \] distance from center of gravity of \( A_s \) to center of gravity of \( A_c \)

\[ E \] non-dimensionalized concrete strain; \( E = \frac{\varepsilon_c}{\varepsilon_u} \)

\[ E_c \] modulus of elasticity of concrete

\[ E_{cn} \] modulus of elasticity of concrete in the n-th load cycle

\[ E_s \] modulus of elasticity of steel

\[ E_1 \] value of \( E \) at extreme concrete compression fiber

\[ f_{sl} \] total steel stress in steel for \( M > M_{on} \)

\[ F \] non-dimensionalized concrete stress; \( F = \frac{f_c}{f'_c} \)

for concrete in cylinders; \( F = \frac{f_c}{k_3 f'_c} \) for concrete in beams
\( F_n \)  
prestressing force in beam during the n-th load cycle

\( F_{se} \)  
prestressing force in test beam just prior to first load cycle

\( F_{si} \)  
initial prestressing force in steel prior to transfer

\( h \)  
full depth of concrete section

\( I \)  
moment of inertia of steel-concrete transformed section about centroidal axis

\( I_c \)  
moment of inertia of concrete area about its centroidal axis

\( k \)  
dimensionless factor defining depth to neutral axis at a cracked section

\( k_2 \)  
dimensionless factor defining center of gravity of compressive force in concrete compressive stress block

\( k_3 \)  
dimensionless factor relating concrete strength in beam and cylinder

\( l_b \)  
distance from crack over which full or partial bond breakdown occurs

\( 2l_c \)  
crack spacing

\( \Delta l_c \)  
total deformation at the top fiber of the beam over length \( 2l_c \)
\( \Delta l_b \)  total deformation at the steel level of the beam over length \( 2l_b \).

\( m = \frac{E_S}{E_c} \)

\( M \)  applied moment

\( M_{on} \)  moment in \( n \)-th load cycle at which cracks begin to open

\( M_{ol} \)  cracking moment in first load cycle

\( M_{ult} \)  static ultimate moment of beam

\( N_u \)  number of cycles

\( \bar{N} \)  mean fatigue life

\( \log N \)  mean of \( \log N \)

\( p \)  proportion of steel in cross section; \( p = \frac{A_S}{bd} \)

\( P \)  probability of strand failure at or before \( N \) cycles

\( Q \)  probability of beam failure at or before \( N \) cycles

\( R \)  stress interval; \( R = S_{\text{max}} - S_L \)

\( S_L \)  fatigue limit corresponding to \( S_{\text{min}} \)

\( S_{\text{max}} \)  maximum stress level in a repeated load cycle

\( S_{\text{min}} \)  minimum stress level in a repeated load cycle

\( u \)  number of strands in the beam at depth \( d \)

\( X \)  \( \log N \)
\( X \) \( \log N \)

\( z \) number of levels of steel in beam

\( z = \log N - \log N \)

\( D \)

\( \alpha \) dimensionless quantity defining the shape of the concrete stress-strain relation; \( \alpha = E_{cn} \frac{\varepsilon_u}{f'_c} \)

\( \beta \) dimensionless parameter defining the shape of the concrete stress-strain relation

\( \gamma \) dimensionless parameter defining the shape of the concrete stress-strain relation

\( \varepsilon \) strain

\( \varepsilon_c \) concrete strain

\( \Delta \varepsilon_c \) inelastic concrete strain at the steel level due to creep and shrinkage losses

\( \varepsilon_{ce} \) elastic concrete strain at the steel level due to prestressing force \( F_{se} \)

\( \varepsilon_{cF} \) elastic strain in concrete at the steel level due to prestressing force \( F_n \)

\( \varepsilon_{cl} \) concrete strain in top fiber of the beam

\( \varepsilon_s \) steel strain
\[ \epsilon_{se} \] steel strain corresponding to prestressing force \( F_{se} \)

\[ \epsilon_{si} \] steel strain corresponding to initial prestressing force \( F_{si} \)

\[ \epsilon_{sF} \] steel strain due to prestressing force \( F_n \)

\[ \epsilon_{sl} \] total steel strain at the cracked section at moment \( M_1 > M_{on} \)

\[ \Delta \epsilon_s \] strain increment; \( \Delta \epsilon_s = \epsilon_{sl} - \epsilon_{sF} - \epsilon_{cF} \)

\[ \Delta \epsilon_{sl} \] strain increment at the cracked section

\[ \Delta \epsilon_{su} \] strain increment at the cracked section corresponding to \( M_{ult} \)

\[ \epsilon_u \] concrete strain in cylinder at \( f'_c \)

\[ \epsilon_{u} \] concrete top fiber strain at \( M_{ult} \)

\[ \gamma = \frac{x}{l_c} \]

\[ \zeta = \frac{x}{l_b} \]

\[ \gamma \] bond parameter; \( \gamma = \frac{l_c}{l_b} \left( \frac{\int_0^1 f(\eta) d\eta}{\int_0^1 f(\zeta) d\zeta} \right) \)
7. APPENDIX  EXAMPLE SOLUTION OF FATIGUE LIFE OF AN UNDER-REINFORCED BEAM

The procedure proposed in this report is illustrated by a numerical calculation of the probable fatigue life of test beam F7.

7.1 BEAM AND MATERIAL PROPERTIES

\[
\begin{align*}
  b &= 6.31'' \\
  d &= 8.00'' \\
  h &= 12.06'' \\
  \bar{K} &= 1.92'' \\
  e &= 1.97'' \\
  A_s &= 0.3267 \text{ in}^2 \\
  P_s &= 0.00648 \\
  I_c &= 920 \text{ in}^4 \\
  I &= 929.4 \text{ in}^4 \\
  m &= 6.4 \text{ (first load cycle)} \\
  f_c' &= 6.22 \text{ ksi} \\
  \epsilon_u &= 0.0023^{**} \\
  F_1 &= F_n = 36.30^k \\
  \epsilon_{SF} &= 0.00397^* \\
  \epsilon_{CF} &= 0.00014^{**} \\
\end{align*}
\]

*Including measured concrete creep and shrinkage losses and 4 percent steel creep loss.

**Values of \( \epsilon_u \) and \( \epsilon_{CF} \) from test measurements. In design calculations, values of \( \epsilon_u \) and \( \epsilon_{CF} \) may be estimated.
7.2 FIRST LOADING STAGE, \( M = M_{on} \)

a) Cracking Moment in First Load Cycle, \( M_{01} \)

Take \( f'_t = 0.10 \) \( f_c' = 0.622 \) ksi

\[
 f_{cF}^b = -Fse \left[ \frac{1}{A_c} + \frac{e}{I_c} \frac{h}{2} \right]
\]

\[
 f_{cF}^b = -36.30 \left[ \frac{1}{76.09} + \frac{1.97 (6.03)}{920} \right]
\]

\[
 = -0.950 \text{ ksi}
\]

\[
 M_{01} = I \frac{f'_t - f_{cF}^b}{\frac{h}{2} - e + \delta}
\]

\[
 = 929.4 \left( \frac{1.572}{5.98} \right)
\]

\[
 = 244 \text{ in-k}
\]

\[
 f_s = \frac{Fse}{A_s} + m \frac{M_{01}}{I} \frac{\delta}{\delta}
\]

\[
 = 110 + 6.4 \left( \frac{244}{929.4} \right) 1.92
\]

\[
 = 113.2 \text{ ksi}
\]
b) Cracking Moment, \( n > 1 \)

\[
M_{on} = I \frac{-f_c^b}{h^2 - e + \bar{x}}
\]

\[
= \frac{929.4 \times (0.950)}{5.98}
\]

\[
= 147.6 \text{ in} \cdot \text{k}
\]

\[
f_s = 110 + \frac{6.4 \times (147.6) 1.92}{929.4}
\]

\[
= 112.2 \text{ ksf}
\]

7.3 SECOND LOADING STAGE, \( M > M_{on} \)

The stress-moment calculations in the second loading stage are shown in tabular form at the end of this section. Details of computations are as follows.

**Column**

1. Choose steel stresses, \( f_{s1} \), at suitable intervals.

2. Obtain corresponding steel strains, \( \varepsilon_{s1} \), from stress-strain curve.

3. Compute \( \varepsilon_{s1} - \varepsilon_{SF} - \varepsilon_{CF} \); for BM F7, \( (\varepsilon_{SF} + \varepsilon_{CF}) = 0.00411 \).

4. Compute \( \frac{(\varepsilon_{s1} - \varepsilon_{SF} - \varepsilon_{CF}) \psi}{bd k_3 f'_c} \)

5. Compute \( \frac{A_s \cdot f_{s1}}{bd k_3 f'_c} \)
6. Make trial values of \( k \) until equations 4.26 and 4.22a are satisfied simultaneously.

(Compute value of \( \alpha \) using Equation 4.8, \( \alpha = \frac{E_{cn}E_u}{f_c^t} \).

Note that the average value of Young's modulus, \( E_{cn} \), will usually be less than the initial value of the first load cycle, \( E_{co} \). In this example, \( E_{cn} = 3.61 \times 10^6 \) psi is the average of the results of the concrete stress-strain tests with pre-loadings, see Figures 37, 38, 40, 41, and 42. Thus,

\[
\alpha = \frac{3.61 \times 10^6 \times 0.0023}{6220} = 1.38, \ \text{Take } \alpha = 1.40. \]

\[
\frac{f_{sl} A_s}{bd k_3 f_c^t} = k \left[ \frac{\alpha}{2} E_1 + \frac{3-2\alpha}{3} E_1^2 + \frac{\alpha-2}{4} E_1^3 \right] \tag{4.26}
\]

\[
\left( \frac{\varepsilon_{sl} - \varepsilon_{sF} - \varepsilon_{cF}}{E_u \psi} \right) = E_1 \cdot \frac{1-k}{k} \tag{4.22a}
\]

To evaluate quickly the right side of Eq. 4.26 it is convenient to plot \( k \) against \( E_1 \) for various values of the quantity \( \frac{f_{sl} A_s}{bd k_3 f_c^t} \).

7. Determine \( k_2 \) from Eq. 4.14; again it is convenient to plot \( k_2 \) against \( E_1 \).

8. Value of \( E_1 \).

9. Value of \( E_{c1} \).

10. Obtain \( k \) \( k_2 \), i.e., Column 6 x column 7.

11. Hence \((1 - k \cdot k_2)\).
12. Compute \( M_1 \) from
\[
M_1 = f_{s1} A_s d (1 - k_2 k)
\] (4.27)

7.4 MEAN FATIGUE LIFE OF BEAM

Values of \( f_{s1} \) and \( M_1 \) may now be used to plot a stress-moment curve and hence obtain the following stresses corresponding to the applied load:

<table>
<thead>
<tr>
<th>Load Kips</th>
<th>Moment in.k</th>
<th>Stresses %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\text{min}} ) = 3.00</td>
<td>136.8</td>
<td>44.4</td>
</tr>
<tr>
<td>( P_{\text{pred}} ) = 7.05</td>
<td>253.7</td>
<td>51.1</td>
</tr>
<tr>
<td>( P_{\text{o1}} ) = 9.09</td>
<td>327.3</td>
<td>60.3</td>
</tr>
<tr>
<td>( P_{\text{o2}} ) = 10.37</td>
<td>373.8</td>
<td>67.2</td>
</tr>
</tbody>
</table>

\[
S_L = 0.8 (S_{\text{min}}) + 23
= 0.8 (44.4) + 23
= 58.6
\]

\( S_{\text{pred}} - S_L \) = negative, therefore an understress

\[
S_{\text{o1}} - S_L = 60.3 - 58.6 = 1.7
\]

\[
S_{\text{o2}} - S_L = 67.2 - 58.6 = 8.6
\]

From the 0.206 probability line in Fig. 49, the following
values of $\log N$ and hence $\overline{N}$ are obtained.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\log N$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.6</td>
<td>5.144</td>
<td>139,400</td>
</tr>
<tr>
<td>1.7</td>
<td>6.225</td>
<td>1,678,000</td>
</tr>
</tbody>
</table>

Substituting values in Eq. 3.15:

$$N(P) = \frac{1}{\sum a_i \frac{N_i}{N_i(P)}} \quad (3.15)$$

$$N(0.206) = \frac{1}{\frac{0.1}{139,400} + \frac{0.3}{1,678,000}}$$

$$= \frac{10^6}{0.718 + 0.179}$$

$$N(0.206) = 1.12 \times 10^6$$

The mean fatigue life of the beam is equal to the number of cycles for which the probability of fatigue failure in one strand is 0.206. Thus the predicted mean fatigue life of the beam is $1.12 \times 10^6$ cycles.
Cracked Section: \( \psi = 1.0 \) \( k_3 = 0.85 \) \( \alpha = 1.4 \)

<table>
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<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_{s1}</td>
<td>( \epsilon_{s1} )</td>
<td>( \epsilon_{s1} - (A)^* )</td>
<td>( \epsilon_{s1} - (A) )</td>
<td>( A_s ) ( f_{s1} ) ** ( \gamma )</td>
<td>( k )</td>
<td>( k_2 )</td>
<td>( E_1 )</td>
<td>( \epsilon_c )</td>
<td>( k_k )</td>
<td>( l - k_k )</td>
<td>( M^{***} )</td>
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<tr>
<td>120</td>
<td>0.00434</td>
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<td>0.00023</td>
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<td>0.0224</td>
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<td>0.856</td>
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</table>

*(A) = \( \epsilon_{sF} + \epsilon_{cF} = 0.00411 \)

** \( A_s \) \( \frac{f_{s1}}{bd \ k_3 \ \gamma} \) = .001227 \( \text{in}^2 \) \( \kips \)

*** \( M = f_s A_s d (1 - k k_2) \)

\( A_s d = 0.3267 \) \((8) = 2.62 \)
9. TABLES AND FIGURES
<table>
<thead>
<tr>
<th>Beam</th>
<th>b (in.)</th>
<th>d (in.)</th>
<th>h (in.)</th>
<th>$f_c^*$ (ksi)</th>
<th>P</th>
<th>$\varepsilon_{si}$</th>
<th>$\varepsilon_{ce}$</th>
<th>$\Delta\varepsilon_c$</th>
<th>$\varepsilon_{se}$ (kips)</th>
<th>$M_{ol}$ (in-kips)</th>
<th>$\varepsilon_u$</th>
<th>$\varepsilon_{su}$</th>
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<td>419</td>
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<td>241.2</td>
<td></td>
<td>(503.0)</td>
</tr>
</tbody>
</table>

*Average f' in test section

Notes: All strains in in/in $\times 10^{-5}$
All moments in in-kips
TABLE 2 - DETAILS OF CONCRETE MIXES

<table>
<thead>
<tr>
<th>Beams</th>
<th>Batch</th>
<th>Cement lb</th>
<th>Water lb</th>
<th>Sand lb</th>
<th>Gravel lb</th>
<th>Slump in</th>
<th>Static Strength of Cylinders at Time of Beam Tests</th>
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</thead>
<tbody>
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<td>V</td>
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<td>I</td>
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<td>2-1/8</td>
<td>6310</td>
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<td>V</td>
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<td>386</td>
<td>386</td>
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<td>6670</td>
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TABLE 3 - RANGE OF CHEMICAL COMPOSITION OF STRAND STEEL

<table>
<thead>
<tr>
<th>Content, Percent</th>
<th>Lot I</th>
<th>Lot II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>0.68 - 0.85</td>
<td>0.68 - 0.80</td>
</tr>
<tr>
<td>Manganese</td>
<td>0.40 - 0.75</td>
<td>0.40 - 0.75</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.04 Max.</td>
<td>0.04 Max.</td>
</tr>
<tr>
<td>Sulfur</td>
<td>0.05 Max.</td>
<td>0.05 Max.</td>
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TABLE 4 - STATIC TESTS, LOT I STRAND

<table>
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<th>Specimen No.</th>
<th>Pult, lbs.</th>
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<tbody>
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<td>BS-1</td>
<td>27,300</td>
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<tr>
<td>BS-2</td>
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<tr>
<td>BS-3</td>
<td>27,400</td>
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<tr>
<td>BS-4</td>
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Mean Pult = 27,300 lb.
Standard Deviation = 96 lb.

TABLE 5 - FATIGUE TESTS, LOT I STRAND
(Stress in Percent of Static Ultimate Stress)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Smin</th>
<th>Smax</th>
<th>N</th>
<th>log N</th>
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</thead>
<tbody>
<tr>
<td>BS-9</td>
<td>42</td>
<td>63</td>
<td>220,000</td>
<td>5.3424</td>
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<td>BS-10</td>
<td>40</td>
<td>70</td>
<td>122,000</td>
<td>5.0864</td>
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<td>BS-11</td>
<td>40</td>
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<td>926,000</td>
<td>5.9666</td>
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<td>60</td>
<td>169,000</td>
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<td>56.8</td>
<td>1,119,000</td>
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</tr>
<tr>
<td>Beam</td>
<td>Age (days)</td>
<td>Rate of Loading</td>
<td>Jack Loads, kips*</td>
<td>Failure Section</td>
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<td>------</td>
<td>------------</td>
<td>----------------</td>
<td>------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>F1</td>
<td>167</td>
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<td>2</td>
<td>170</td>
<td>250</td>
<td>4.50 12.10</td>
<td>E</td>
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<tr>
<td>3</td>
<td>161</td>
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<td>(Static Test)</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>169</td>
<td>250</td>
<td>4.50 12.10</td>
<td>W</td>
</tr>
<tr>
<td>F5</td>
<td>180</td>
<td>500</td>
<td>3.80 7.08 9.14</td>
<td>W</td>
</tr>
<tr>
<td>6</td>
<td>156</td>
<td></td>
<td>(Static Test)</td>
<td>L</td>
</tr>
<tr>
<td>7</td>
<td>196</td>
<td>250</td>
<td>3.80 7.05 9.09</td>
<td>L</td>
</tr>
<tr>
<td>8</td>
<td>168</td>
<td>250</td>
<td>3.80 7.12 9.08</td>
<td>E</td>
</tr>
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</table>

*Including dynamic effect, estimated from deflection readings
### TABLE 7 - STATIC TESTS, LOT II STRAND

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<tr>
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<td>28,620</td>
</tr>
<tr>
<td>L 2 - S 4</td>
<td>28,675</td>
</tr>
<tr>
<td>L 3 - S 5</td>
<td>28,650</td>
</tr>
<tr>
<td>L41 - S15</td>
<td>28,450</td>
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<tr>
<td>L57 - S20</td>
<td>28,500</td>
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<tr>
<td>L13 - S51</td>
<td>28,600</td>
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<tr>
<td>L70 - S52</td>
<td>28,520</td>
</tr>
<tr>
<td>L70 - S53</td>
<td>28,450</td>
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</tbody>
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Mean $P_{ult} = 28,560$ lb.

Standard Deviation = 89 lb.
### TABLE 8 - CONSTANT CYCLE STRAND TEST RESULTS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{min}}$</th>
<th>$S_{\text{max}}$</th>
<th>N</th>
<th>log N</th>
</tr>
</thead>
<tbody>
<tr>
<td>L 1 - S 2</td>
<td>60</td>
<td>70</td>
<td>460,200*</td>
<td>----</td>
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<tr>
<td>L 2 - S 3</td>
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<td>80</td>
<td>234,400</td>
<td>5.36996</td>
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<tr>
<td>L 4 - S 6</td>
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<td>80</td>
<td>33,000**</td>
<td>----</td>
</tr>
<tr>
<td>L 5 - S 8</td>
<td>60</td>
<td>75</td>
<td>425,500</td>
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</tr>
<tr>
<td>L 5 - S 9</td>
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<td>70</td>
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<td>6.51930</td>
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<tr>
<td>L 9 - S10</td>
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<td>75</td>
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<tr>
<td>L17 - S11</td>
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<td>70</td>
<td>5,440,600*</td>
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<td>$S_{\text{max}}$</td>
<td>N</td>
<td>log N</td>
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<td>-----------</td>
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Table 8 - Continued

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<th>$S_{\text{max}}$</th>
<th>N</th>
<th>log N</th>
</tr>
</thead>
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<td>71,000</td>
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<td>715,000</td>
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$\rightarrow$ No failure

* Premature failure in grip. Not included in analysis.

** Failure at weldment. Not included in analysis.
<table>
<thead>
<tr>
<th>Group</th>
<th>Stress Levels, % Static Ult.</th>
<th>No. of Replications</th>
<th>Fatigue Life</th>
<th>Log Fatigue Life</th>
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<tbody>
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<td>$S_{\text{max}}$</td>
<td>$\bar{N}$</td>
<td>$D_N$</td>
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<td>6</td>
<td>89,200</td>
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<td>65</td>
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<td>150,400</td>
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<td>C</td>
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<td>60</td>
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<td>D</td>
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<td>81,900</td>
</tr>
<tr>
<td>F</td>
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<tr>
<td>G</td>
<td>60</td>
<td>75</td>
<td>7</td>
<td>705,630</td>
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</table>

$\bar{N}$ = Mean fatigue life  
$D_N$ = Standard deviation of $N$  
$\log \bar{N}$ = Mean of $\log N$  
$D$ = Standard deviation of $\log N$
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Stress Level, % Ult.</th>
<th>Block Shape</th>
<th>$N_e$</th>
<th>$\sum \frac{n}{N}$</th>
<th>Exp'tal N/Pred.N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_{\text{min}}$</td>
<td>$S_{\text{pred}}$</td>
<td>$S_{\text{OL}}$</td>
<td>$\alpha$</td>
<td>$\beta/\alpha$</td>
</tr>
<tr>
<td>3AA-1</td>
<td>L43-S48</td>
<td>60</td>
<td>65</td>
<td>85</td>
<td>30,000</td>
<td>0.25</td>
</tr>
<tr>
<td>3AA-2</td>
<td>L48-S64</td>
<td>60</td>
<td>65</td>
<td>85</td>
<td>30,000</td>
<td>0.25</td>
</tr>
<tr>
<td>3BA-1</td>
<td>L41-S55</td>
<td>60</td>
<td>70</td>
<td>85</td>
<td>30,000</td>
<td>0.25</td>
</tr>
<tr>
<td>3BA-2</td>
<td>L68-S58</td>
<td>60</td>
<td>70</td>
<td>85</td>
<td>30,000</td>
<td>0.25</td>
</tr>
<tr>
<td>3BA-3</td>
<td>L40-S121</td>
<td>60</td>
<td>70</td>
<td>85</td>
<td>30,000</td>
<td>0.25</td>
</tr>
<tr>
<td>3CA-1</td>
<td>L54-S56</td>
<td>60</td>
<td>75</td>
<td>85</td>
<td>22,500</td>
<td>0.25</td>
</tr>
<tr>
<td>3CA-2</td>
<td>L68-S59</td>
<td>60</td>
<td>75</td>
<td>85</td>
<td>22,500</td>
<td>0.25</td>
</tr>
<tr>
<td>3DA-1</td>
<td>L43-S49</td>
<td>60</td>
<td>65</td>
<td>85</td>
<td>22,500</td>
<td>0.40</td>
</tr>
<tr>
<td>3DA-2</td>
<td>L55-S70</td>
<td>60</td>
<td>65</td>
<td>85</td>
<td>22,500</td>
<td>0.40</td>
</tr>
<tr>
<td>3EA-1</td>
<td>L54-S57</td>
<td>60</td>
<td>75</td>
<td>85</td>
<td>15,000</td>
<td>0.40</td>
</tr>
<tr>
<td>3EA-2</td>
<td>L20-S67</td>
<td>60</td>
<td>75</td>
<td>85</td>
<td>15,000</td>
<td>0.40</td>
</tr>
<tr>
<td>3FA-1</td>
<td>L44-S54</td>
<td>60</td>
<td>80</td>
<td>85</td>
<td>15,000</td>
<td>0.25</td>
</tr>
<tr>
<td>3FA-2</td>
<td>L33-S68</td>
<td>60</td>
<td>80</td>
<td>85</td>
<td>15,000</td>
<td>0.25</td>
</tr>
</tbody>
</table>

$N_e$ = Observed fatigue life

$N_L$ = Fatigue life predicted by Eq. 3.7

$N_G$ = Fatigue life predicted by Eqs. 3.8 and 3.9

*Failure in grip - not included in analysis
### TABLE 11. - STRAND CUMULATIVE DAMAGE TESTS WITH TWO MAXIMUM STRESS LEVELS

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Stress Level, % Ult.</th>
<th>Block Shape</th>
<th>N&lt;sub&gt;e&lt;/sub&gt;</th>
<th>∑&lt;sub&gt;n/N&lt;/sub&gt;</th>
<th>Exp'tal N/Pred.N</th>
<th>N&lt;sub&gt;e&lt;/sub&gt;/N&lt;sub&gt;L&lt;/sub&gt;</th>
<th>N&lt;sub&gt;e&lt;/sub&gt;/N&lt;sub&gt;G&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>3AA-1</td>
<td>L43-S48</td>
<td>60 65 85</td>
<td>30,000 0.25</td>
<td>357,300</td>
<td>1.08</td>
<td>1.10 1.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AA-2</td>
<td>L48-S64</td>
<td>60 65 85</td>
<td>30,000 0.25</td>
<td>385,700</td>
<td>1.15</td>
<td>1.20 1.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AB-1</td>
<td>L58-S85</td>
<td>60 65 85</td>
<td>300,000 0.25</td>
<td>221,000*</td>
<td>--</td>
<td>-- --</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AB-2</td>
<td>L45-S88</td>
<td>60 65 85</td>
<td>300,000 0.25</td>
<td>96,500*</td>
<td>--</td>
<td>-- --</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AB-3</td>
<td>L12-S92</td>
<td>60 65 85</td>
<td>300,000 0.25</td>
<td>540,000</td>
<td>1.11</td>
<td>1.67 2.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AB-4</td>
<td>L66-S91</td>
<td>60 65 85</td>
<td>300,000 0.25</td>
<td>550,000</td>
<td>1.24</td>
<td>1.70 2.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AC-1</td>
<td>L63-S100</td>
<td>60 65 85</td>
<td>10,000 0.25</td>
<td>349,000</td>
<td>1.07</td>
<td>1.08 1.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3AC-2</td>
<td>L61-S103</td>
<td>60 65 85</td>
<td>10,000 0.25</td>
<td>390,000</td>
<td>1.20</td>
<td>1.20 1.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N<sub>e</sub> = Observed fatigue life

N<sub>L</sub> = Fatigue life predicted by Eq. 3.7

N<sub>G</sub> = Fatigue life predicted by Eqs. 3.8 and 3.9

*Failure in grip - not included in analysis
### TABLE 12 - STRAND CUMULATIVE DAMAGE TESTS WITH TWO MAXIMUM STRESS LEVELS

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Stress Level, % Ult.</th>
<th>Block Shape</th>
<th>Ne</th>
<th>( \sum \frac{n}{N} )</th>
<th>Exp'tal N/Pred.N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( S_{\text{min}} )</td>
<td>( S_{\text{pred}} )</td>
<td>( S_{\text{ol}} )</td>
<td>( \alpha )</td>
<td>( \beta/\alpha )</td>
</tr>
<tr>
<td>4AA-1</td>
<td>L11-S86</td>
<td>40</td>
<td>40</td>
<td>70</td>
<td>30,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4AA-2</td>
<td>L66-S90</td>
<td>40</td>
<td>40</td>
<td>70</td>
<td>30,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4BA-1</td>
<td>L14-S82</td>
<td>40</td>
<td>60</td>
<td>70</td>
<td>30,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4BA-2</td>
<td>L45-S89</td>
<td>40</td>
<td>60</td>
<td>70</td>
<td>30,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4BB-1</td>
<td>L62-S81</td>
<td>40</td>
<td>60</td>
<td>70</td>
<td>150,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4BB-2</td>
<td>L58-S84</td>
<td>40</td>
<td>60</td>
<td>70</td>
<td>150,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4BC-1</td>
<td>L14-S83</td>
<td>40</td>
<td>60</td>
<td>70</td>
<td>10,000</td>
<td>0.4</td>
</tr>
<tr>
<td>4BC-2</td>
<td>L12-S93</td>
<td>40</td>
<td>60</td>
<td>70</td>
<td>10,000</td>
<td>0.4</td>
</tr>
</tbody>
</table>

\( N_e \) = Observed fatigue life  
\( N_L \) = Fatigue life predicted by Eq. 3.7  
\( N_G \) = Fatigue life predicted by Eq. 3.8 and 3.9
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Stress Level, % Ult.</th>
<th>Block Shape</th>
<th>Ne</th>
<th>Σ n/N</th>
<th>Exp'tal N/Pred. N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S_min</td>
<td>S_pred</td>
<td>S_o1</td>
<td>S_o2</td>
<td>α</td>
</tr>
<tr>
<td>5AA-1</td>
<td>L46-S98</td>
<td>60</td>
<td>60</td>
<td>80</td>
<td>85</td>
<td>60,000</td>
</tr>
<tr>
<td>5AA-2</td>
<td>L39-S107</td>
<td>60</td>
<td>60</td>
<td>80</td>
<td>85</td>
<td>60,000</td>
</tr>
<tr>
<td>5BA-1</td>
<td>L61-S102</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
<tr>
<td>5BA-2</td>
<td>L39-S106</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
<tr>
<td>5BA-3</td>
<td>L53-S125</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
<tr>
<td>5CA-1</td>
<td>L63-S101</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
<tr>
<td>5CA-2</td>
<td>L8-S104</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
<tr>
<td>5CA-3</td>
<td>L40-S120</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
<tr>
<td>5CA-4</td>
<td>L26-S123</td>
<td>60</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>30,000</td>
</tr>
</tbody>
</table>

Ne = Observed fatigue life

NL = Fatigue life predicted by Eq. 3.7

NG = Fatigue life predicted by Eqs. 3.8 and 3.9

*Failure in grip - not included in analysis
TABLE 14 - STRAND CUMULATIVE DAMAGE TESTS WITH THREE MAXIMUM STRESS LEVELS

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Stress Level, % Ult.</th>
<th>Block Shape</th>
<th>Ne</th>
<th>(\sum \frac{n}{N} )</th>
<th>Exp'tal N/Pred.N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(S_{\text{min}})</td>
<td>(S_{\text{pred}})</td>
<td>(S_{\text{ol}})</td>
<td>(S_{\text{o2}})</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>6AA-1</td>
<td>L60-S95</td>
<td>40</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td>20,000</td>
</tr>
<tr>
<td>6AA-2</td>
<td>L19-S96</td>
<td>40</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td>20,000</td>
</tr>
<tr>
<td>6BA-1</td>
<td>L60-S94</td>
<td>40</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td>30,000</td>
</tr>
<tr>
<td>6BA-2</td>
<td>L19-S97</td>
<td>40</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td>30,000</td>
</tr>
<tr>
<td>6BA-3</td>
<td>L27-S119</td>
<td>40</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td>30,000</td>
</tr>
<tr>
<td>6CA-1</td>
<td>L46-S99</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>30,000</td>
</tr>
<tr>
<td>6CA-2</td>
<td>L67-S109</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>30,000</td>
</tr>
</tbody>
</table>

\(N_e\) = Observed fatigue life  
\(N_L\) = Fatigue life predicted by Eq. 3.7  
\(N_G\) = Fatigue life predicted by Eqs. 3.8 and 3.9
### TABLE 15 - STRAND CUMULATIVE DAMAGE TESTS WITH TWO MAXIMUM STRESS LEVELS

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Specimen No.</th>
<th>Stress Level, % Ult. (S&lt;sub&gt;min&lt;/sub&gt;, S&lt;sub&gt;pred&lt;/sub&gt;, Sol)</th>
<th>Block Shape (N&lt;sub&gt;e&lt;/sub&gt;, B/α, ∑n/N)</th>
<th>Experimental N/Predicted N (N&lt;sub&gt;e&lt;/sub&gt;/N&lt;sub&gt;L&lt;/sub&gt;, N&lt;sub&gt;e&lt;/sub&gt;/N&lt;sub&gt;G&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3FA-1</td>
<td>L44-S54</td>
<td>60 80 85</td>
<td>15,000 0.25 155,700 1.17</td>
<td>1.18 1.48</td>
</tr>
<tr>
<td>3FA-2</td>
<td>L33-S68</td>
<td>60 80 85</td>
<td>15,000 0.25 101,300 0.75</td>
<td>0.76 0.96</td>
</tr>
<tr>
<td>3FA-3</td>
<td>L52-S113</td>
<td>60 80 85</td>
<td>15,000 0.25 110,350 0.83</td>
<td>0.86 1.05</td>
</tr>
<tr>
<td>3FA-4</td>
<td>L52-S112</td>
<td>60 80 85</td>
<td>15,000 0.25 139,450 1.05</td>
<td>1.05 1.32</td>
</tr>
<tr>
<td>3FA-5</td>
<td>L34-S111</td>
<td>60 80 85</td>
<td>15,000 0.25 134,950 1.02</td>
<td>1.02 1.28</td>
</tr>
<tr>
<td>3FA-6</td>
<td>L34-S110</td>
<td>60 80 85</td>
<td>15,000 0.25 101,250 0.75</td>
<td>0.76 0.96</td>
</tr>
<tr>
<td>3FA-7</td>
<td>L35-S117</td>
<td>60 80 85</td>
<td>15,000 0.25 158,500 1.19</td>
<td>1.20 1.51</td>
</tr>
<tr>
<td>3FA-8</td>
<td>L42-S114</td>
<td>60 80 85</td>
<td>15,000 0.25 101,350 0.76</td>
<td>0.76 0.96</td>
</tr>
<tr>
<td>3FA-9</td>
<td>L35-S116</td>
<td>60 80 85</td>
<td>15,000 0.25 157,250 1.18</td>
<td>1.19 1.50</td>
</tr>
<tr>
<td>3FA-10</td>
<td>L42-S115</td>
<td>60 80 85</td>
<td>15,000 0.25 131,250 0.98</td>
<td>0.99 1.25</td>
</tr>
</tbody>
</table>

N<sub>e</sub> = Observed fatigue life

N<sub>L</sub> = Fatigue life predicted by Eq. 3.7

N<sub>G</sub> = Fatigue life predicted by Eqs. 3.8 and 3.9
TABLE 16 - $\chi^2$ GOODNESS OF FIT TEST

CONSTANT CYCLE STRAND FATIGUE TESTS - GROUP F

<table>
<thead>
<tr>
<th>Interval</th>
<th>0</th>
<th>E</th>
<th>0-E</th>
<th>$(0-E)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\infty &lt; Z &lt; -0.675$</td>
<td>3</td>
<td>5</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>$-0.675 \leq Z &lt; 0$</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$0 \leq Z &lt; +0.675$</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$+0.675 \leq Z &lt; \infty$</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sum$</td>
<td>20</td>
<td>20</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

$Z = \frac{\log N - \bar{\log N}}{D}$

$O$ = Observed number of test points within interval of $Z$ values

$E$ = Expected number of test points within interval of $Z$ values

$\chi^2 = \frac{\sum(O-E)^2}{E} = \frac{6}{5} = 1.20$

For three (3) degrees of freedom, $\chi^2_{0.05} = 7.82$.

*The $\chi^2$ test is described on page 85, Ref. 18.*
TABLE 17 - $\chi^2$ GOODNESS OF FIT TEST*

CONSTANT CYCLE STRAND FATIGUE TESTS - GROUPS A THROUGH G

<table>
<thead>
<tr>
<th>Interval</th>
<th>0</th>
<th>E</th>
<th>0-E</th>
<th>(0-E)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-$\infty$ &lt; Z &lt; -1.220</td>
<td>4</td>
<td>6.33</td>
<td>-2.33</td>
<td>5.46</td>
</tr>
<tr>
<td>-1.220 &lt; Z &lt; -0.766</td>
<td>7</td>
<td>6.33</td>
<td>-0.67</td>
<td>0.44</td>
</tr>
<tr>
<td>-0.766 &lt; Z &lt; -0.430</td>
<td>5</td>
<td>6.33</td>
<td>-1.33</td>
<td>1.78</td>
</tr>
<tr>
<td>-0.430 &lt; Z &lt; -0.140</td>
<td>4</td>
<td>6.33</td>
<td>-2.33</td>
<td>5.44</td>
</tr>
<tr>
<td>-0.140 &lt; Z &lt; +0.140</td>
<td>9</td>
<td>6.33</td>
<td>+2.67</td>
<td>7.12</td>
</tr>
<tr>
<td>+0.140 &lt; Z &lt; +0.430</td>
<td>4</td>
<td>6.33</td>
<td>-2.33</td>
<td>5.44</td>
</tr>
<tr>
<td>+0.430 &lt; Z &lt; +0.766</td>
<td>11</td>
<td>6.33</td>
<td>+4.67</td>
<td>21.70</td>
</tr>
<tr>
<td>+0.766 &lt; Z &lt; +1.220</td>
<td>10</td>
<td>6.33</td>
<td>+3.67</td>
<td>10.34</td>
</tr>
<tr>
<td>1.220 &lt; Z &lt; $\infty$</td>
<td>3</td>
<td>6.33</td>
<td>-3.33</td>
<td>10.11</td>
</tr>
<tr>
<td>$\sum$</td>
<td>57</td>
<td>57</td>
<td>+5.33</td>
<td>67.83</td>
</tr>
</tbody>
</table>

$Z = \frac{\log N - \log N_D}{D}$

0 = Observed number of test points within interval of Z values

E = Expected number of test points within interval of Z values

$\chi^2 = \frac{\sum(O-E)^2}{E} = \frac{67.83}{6.33} = 10.70$

For eight (8) degrees of freedom, $\chi^2_{0.05} = 15.51$

*The $\chi^2$ test is described on page 85, Ref. 18.
<table>
<thead>
<tr>
<th>Beam</th>
<th>Percentage of Fatigue Life</th>
<th>$\psi$ for $k_3 = 1.0$</th>
<th>$\psi$ for $k_3 = 0.85$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1*</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>4.4</td>
<td>1.72</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>26.7</td>
<td>1.58</td>
<td>1.32</td>
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<tr>
<td></td>
<td>100.0</td>
<td>1.46</td>
<td>1.21</td>
</tr>
<tr>
<td>F2*</td>
<td>0</td>
<td>1.71</td>
<td>1.42</td>
</tr>
<tr>
<td></td>
<td>15.3</td>
<td>1.26</td>
<td>1.06</td>
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<td></td>
<td>36.7</td>
<td>1.14</td>
<td>0.95</td>
</tr>
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<td></td>
<td>61.0</td>
<td>1.16</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>91.5</td>
<td>1.14</td>
<td>0.95</td>
</tr>
<tr>
<td>F4*</td>
<td>0</td>
<td>1.32</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>18.0</td>
<td>0.89</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>43.2</td>
<td>0.87</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>79.1</td>
<td>0.82</td>
<td>0.69</td>
</tr>
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</table>

*Computed from deformations measured at $M = 450$ kip inches.
Table 18 - Continued

<table>
<thead>
<tr>
<th>Beam</th>
<th>Percentage of Fatigue Life</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$k_3 = 1.0$</td>
</tr>
<tr>
<td>F5**</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>4.6</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>15.4</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>30.8</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>46.2</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>61.6</td>
<td>0.69</td>
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<tr>
<td></td>
<td>86.3</td>
<td>0.69</td>
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<tr>
<td>F7**</td>
<td>0</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>2.6</td>
<td>1.25</td>
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<td>7.7</td>
<td>1.14</td>
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<tr>
<td></td>
<td>77.2</td>
<td>1.14</td>
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</table>

**Computed from deformation readings at $M = 378$ kip inches.
Table 18 - Continued

<table>
<thead>
<tr>
<th>Beam</th>
<th>Percentage of Fatigue Life</th>
<th>( \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( k_3 = 1.0 )</td>
</tr>
<tr>
<td>F8**</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2.6</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
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<td>13.2</td>
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<td>21.1</td>
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<td>29.1</td>
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<td>39.7</td>
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<td></td>
<td>52.8</td>
<td>1.15</td>
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<td>66.0</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>90.0</td>
<td>1.15</td>
</tr>
</tbody>
</table>

**Computed from deformation readings at \( M = 378 \) kip inches.
### TABLE 19 - COMPARISON OF PREDICTED AND OBSERVED BEAM FATIGUE LIVES

<table>
<thead>
<tr>
<th>Beam</th>
<th>Moments, in-k.</th>
<th>Stresses, % Static Ult.</th>
<th>$\bar{N}_p$ x 10^6</th>
<th>$N_e$ x 10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{min}$</td>
<td>$M_{pred}$</td>
<td>$M_{01}$</td>
<td>$M_{02}$</td>
</tr>
<tr>
<td>F1</td>
<td>162</td>
<td>436</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>F2</td>
<td>162</td>
<td>436</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>F4</td>
<td>162</td>
<td>436</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>F5</td>
<td>136.8</td>
<td>254.6</td>
<td>329</td>
<td>366.4</td>
</tr>
<tr>
<td>F7</td>
<td>136.8</td>
<td>253.7</td>
<td>327.3</td>
<td>373.8</td>
</tr>
<tr>
<td>F8</td>
<td>136.8</td>
<td>256.3</td>
<td>327.0</td>
<td>375.2</td>
</tr>
</tbody>
</table>

**Notes:**

* Understress

$\bar{N}_p$ = Predicted mean fatigue life

$N_e$ = Observed fatigue life
TABLE 20 - EFFECT OF PARAMETERS $k_3$, $\psi$, % STEEL LOSSES ON MEAN FATIGUE LIFE*

<table>
<thead>
<tr>
<th>$k_3$</th>
<th>$\psi$</th>
<th>% Steel Losses</th>
<th>$S_{\text{min}}$</th>
<th>$S_{01}$</th>
<th>$S_{02}$</th>
<th>$S_L$</th>
<th>$N \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.7</td>
<td>4</td>
<td>44.4</td>
<td>58.8</td>
<td>65.0</td>
<td>58.6</td>
<td>1.96</td>
</tr>
<tr>
<td>0.85</td>
<td>1.0</td>
<td>4</td>
<td>44.4</td>
<td>60.3</td>
<td>67.2</td>
<td>58.6</td>
<td>1.12</td>
</tr>
<tr>
<td>0.85</td>
<td>1.3</td>
<td>4</td>
<td>44.4</td>
<td>61.1</td>
<td>68.7</td>
<td>58.6</td>
<td>0.78</td>
</tr>
<tr>
<td>0.85</td>
<td>1.0</td>
<td>2</td>
<td>45.4</td>
<td>60.3</td>
<td>67.2</td>
<td>59.3</td>
<td>1.47</td>
</tr>
<tr>
<td>1.00</td>
<td>1.0</td>
<td>4</td>
<td>44.4</td>
<td>59.9</td>
<td>66.8</td>
<td>58.6</td>
<td>1.32</td>
</tr>
</tbody>
</table>

*Computation made for Beam F7
FIG. 1 - FATIGUE FAILURE ENVELOPES FOR ONE MILLION LOAD CYCLES

**NOTE:** All stresses in percent of corresponding static ultimate stress.
FIG. 2  - STEEL STRESS - MOMENT RELATION
FIG. 4 - GRADING CURVES FOR FINE AND COARSE AGGREGATES

FIG. 3 - DETAILS OF BEAM TEST SPECIMENS
FIG. 5 - LOAD VERSUS STRAIN, 7/16 INCH DIA. STRAND - LOT I

Pult. = 27.3 kips
Deformations measured over 50 inch Gage Length

FIG. 6 - GRID LAYOUT FOR MEASUREMENT OF CONCRETE DEFORMATIONS (SOUTH FACE)
FIG. 7 - BEAM TEST SET-UP
FIG. 8 - LOAD BLOCK FOR BEAM CUMULATIVE DAMAGE TESTS

FIG. 9 - CENTER-LINE DEFLECTIONS, STATIC ULTIMATE TESTS - F3, F6
Notation

<table>
<thead>
<tr>
<th>Letter</th>
<th>No. Cycles</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>10,000</td>
</tr>
<tr>
<td>c</td>
<td>25,000</td>
</tr>
<tr>
<td>d</td>
<td>100,000</td>
</tr>
<tr>
<td>e</td>
<td>200,000</td>
</tr>
<tr>
<td>f</td>
<td>225,000</td>
</tr>
<tr>
<td>z</td>
<td>At completion of Test</td>
</tr>
</tbody>
</table>

Note: Crack extensions recorded at 12.5 kips

S - South Face
B - Bottom Face
N - North Face

FIG. 10 - CRACKING PATTERNS
FIG. II - CRACKING PATTERNS

Notation

<table>
<thead>
<tr>
<th>Letter</th>
<th>No. Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>30,000</td>
</tr>
<tr>
<td>c</td>
<td>90,000</td>
</tr>
<tr>
<td>d</td>
<td>1,200,000</td>
</tr>
<tr>
<td>e</td>
<td>At completion of Test</td>
</tr>
</tbody>
</table>

Note: Crack extensions recorded at 10.5 kips

S - South Face
B - Bottom Face
N - North Face
Concrete Strain (in./in.)

Concrete Deformation, in Inches per Ten Inch Gage Length

FIG. 12 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, STATIC ULTIMATE TESTS – F3, F6

FIG. 13 - CONCRETE DEFORMATIONS AT THE STEEL LEVEL, STATIC ULTIMATE TESTS – F3, F6
FIG. 14 - MID-SPAN DEFLECTIONS, STATIC LOADS — BEAM F1

FIG. 15 - MID-SPAN DEFLECTIONS, STATIC AND DYNAMIC LOADS — BEAM F2
FIG. 16 - MID-SPAN DEFLECTIONS, STATIC AND DYNAMIC LOADS - BEAM F4

FIG. 17 - MID-SPAN DEFLECTIONS, STATIC AND DYNAMIC LOADS - BEAM F5
FIG. 18 - MID-SPAN DEFLECTIONS, STATIC AND DYNAMIC LOADS - BEAM F7

FIG. 19 - MID-SPAN DEFLECTIONS, STATIC AND DYNAMIC LOADS - BEAM F8
FIG. 20 - CONCRETE DEFORMATIONS AT THE STEEL LEVEL - BEAM F7

FIG. 21 - CONCRETE DEFORMATIONS AT THE STEEL LEVEL - BEAM F2
Note: All deformations in inches $\times 10^4$, per 10 inch Gage Length

FIG. 22 - CONCRETE DEFORMATIONS IN EAST AND WEST SECTIONS, BEAM F7
FIG. 23 - CONCRETE DEFORMATIONS AT STEEL LEVEL, WEST SECTION - BEAM F1

FIG. 24 - CONCRETE DEFORMATIONS AT STEEL LEVEL, EAST SECTION - BEAM F2
FIG. 25 - CONCRETE DEFORMATIONS AT STEEL LEVEL, WEST SECTION - BEAM F4

FIG. 26 - CONCRETE DEFORMATIONS AT STEEL LEVEL, WEST SECTION - BEAM F5
FIG. 27 - CONCRETE DEFORMATIONS AT STEEL LEVEL, L SECTION - BEAM F7

FIG. 28 - CONCRETE DEFORMATIONS AT STEEL LEVEL, EAST SECTION - BEAM F8
FIG. 29 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, WEST SECTION - BEAM F1

FIG. 30 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, EAST SECTION - BEAM F2
FIG. 31 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, WEST SECTION - BEAM F4

FIG. 32 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, WEST SECTION - BEAM F5
FIG. 33 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, WEST SECTION - BEAM F7

FIG. 34 - TOP FIBER CONCRETE COMPRESSIVE STRAINS, EAST SECTION - BEAM F8
FIG. 35 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: NONE

\[
F = \frac{f_c}{f'_{cu}}
\]

\[
E = \frac{\varepsilon}{\varepsilon_u}
\]

- \( f'_{c} = 6920 \text{ psi} \)
- \( \varepsilon_u = 2.508 \times 10^{-3} \text{ in./in.} \)
- \( E_{co} = 4.40 \times 10^6 \text{ psi} \)

\( \alpha = 1.70 \)

FIG. 36 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 20 CYCLES, 20-130 KIPS

\[
F = \frac{f_c}{f'_{cu}}
\]

\[
E = \frac{\varepsilon}{\varepsilon_u}
\]

- \( f'_{c} = 6670 \text{ psi} \)
- \( \varepsilon_u = 2.137 \times 10^{-3} \text{ in./in.} \)
- \( \Delta\varepsilon_c = 0.255 \times 10^{-3} \text{ in./in.} \)
- \( E_{co} = 4.33 \times 10^6 \text{ psi} \)
- \( E_{cn} = 4.60 \times 10^6 \text{ psi} \)

\( \alpha = 1.60 \)
FIG. 37 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 30,000 CYCLES, 20-130 KIPS

FIG. 38 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 100,000 CYCLES, 20-130 KIPS
FIG. 39 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 20 CYCLES, 20-100 KIPS

\[ F = \frac{f_c}{f'c} \]
\[ E = \frac{\varepsilon}{\varepsilon_u} \]
\[ f'c = 6900 \text{ psi} \]
\[ \varepsilon_u = 2.425 \times 10^{-3} \text{ in./in.} \]
\[ \Delta \varepsilon_c = 0.13 \times 10^{-3} \text{ in./in.} \]
\[ E_{co} = 4.34 \times 10^6 \text{ psi} \]
\[ E_{cn} = 4.66 \times 10^6 \text{ psi} \]

\[ \alpha = 2.0 \]

FIG. 40 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 30,000 CYCLES, 20-100 KIPS

\[ F = \frac{f_c}{f'c} \]
\[ E = \frac{\varepsilon}{\varepsilon_u} \]
\[ f'c = 6250 \text{ psi} \]
\[ \varepsilon_u = 2.367 \times 10^{-3} \text{ in./in.} \]
\[ \Delta \varepsilon_c = 0.136 \times 10^{-3} \text{ in./in.} \]
\[ E_{co} = 3.89 \times 10^6 \text{ psi} \]
\[ E_{cn} = 3.68 \times 10^6 \text{ psi} \]

\[ \alpha = 1.7 \]
FIG. 41 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 100,000 CYCLES, 20-100 KIPS

$F = \frac{f_c}{f_{c0}}$
$E = \frac{\epsilon}{\epsilon_u}$

$f'_c = 6700 \text{ psi}$
$\epsilon_u = 2.48 \times 10^{-3} \text{in./in.}$
$\Delta\epsilon'_c = 0.137 \times 10^{-3} \text{in./in.}$
$E_{co} = 4.48 \times 10^6 \text{ psi}$
$E_{cn} = 3.50 \times 10^6 \text{ psi}$

$\alpha = 1.70$

FIG. 42 - CONCRETE STRESS-STRAIN RELATION
PRE-LOADING: 1,000,000 CYCLES, 20-100 KIPS

$F = \frac{f_c}{f_{c0}}$
$E = \frac{\epsilon}{\epsilon_u}$

$f'_c = 6250 \text{ psi}$
$\epsilon_u = 2.167 \times 10^{-3} \text{in./in.}$
$\Delta\epsilon'_c = 0.370 \times 10^{-3} \text{in./in.}$
$E_{co} = 4.16 \times 10^6 \text{ psi}$
$E_{cn} = 3.75 \times 10^6 \text{ psi}$

$\alpha = 1.60$
FIG. 43 - LOAD BLOCKS FOR CUMULATIVE DAMAGE TESTS ON STRANDS
FIG. 44 - STRAND GRIPPING DEVICE

SECTION THRU A-A

1. Strand
2. Steel Clamps
3. Strandvise
4. Grout
5. Spacer Block
6. Transverse Tension Bolts
FIG. 45 - STRAND FATIGUE TEST SET-UP
Solid Line obtained from deformations measured over 50 Inch Gage Length

Dashed Line obtained from strain gages attached to individual wires

\[ E = 28.0 \times 10^6 \text{ psi} \]

Load (% Pull) vs. Strain, Inches per Inch

FIG. 46 - LOAD VERSUS STRAIN, 7/16 INCH DIA. STRAND, LOT II
Test data grouped by change of variable:

\[ z = \frac{\log N - \log \bar{N}}{D} \]

\[ \log \bar{N} = \text{Mean log N of sample} \]
\[ D = \text{Standard deviation of sample} \]

FIG. 47 - FREQUENCY DISTRIBUTION OF GROUPED CONSTANT CYCLE TEST DATA
Note: All stresses in percent of static ultimate stress

- $S_{\text{min.}} = 60$
- $S_{\text{min.}} = 40$

Lot I Strand Tests ($S_{\text{min.}} = 40$)

**FIG. 48 - MAXIMUM STRESS LEVEL, VERSUS FATIGUE LIFE, CONSTANT CYCLE TESTS**
\[
\log N = \frac{1.4332}{R} + 5.5212 - 0.0486R
\]

where \( R = S_{\text{max}} - S_L \)

\( S_L = 0.8 S_{\text{min}} + 23 \)

Note: All stresses in percent of static ultimate stress

FIG. 49 - R VERSUS \( \log N \)
\[ D = 0.2196 - 0.0103R \]

where \( R = S_{\text{max}} - S_L \)

\[ S_L = 0.8 S_{\text{min}} + 23 \]

Note: All stresses in percent of static ultimate

FIG. 50 - \( R \) VERSUS STANDARD DEVIATION
FIG. 51 - LOAD BLOCKS USED IN CUMULATIVE DAMAGE TESTS BY DOLAN ET AL.
FIG. 52 - CUMULATIVE DAMAGE THEORY AT PROBABILITY LEVEL P
FIG. 53 - PREDICTED AND OBSERVED FREQUENCY DISTRIBUTIONS FOR CUMULATIVE DAMAGE TEST 3 FA
FIG. 54 - CUBIC PARABOLA FOR VARIOUS VALUES OF $\alpha$

$$F = aE + (3 - 2a)E^2 + (a - 2)E^3$$

FIG. 55 - COMPLETE STRESS - STRAIN RELATION FOR CONCRETE

$$F = aE + (3 - 2a)E^2 + (a - 2)E^3$$

$0 \leq E \leq 1.0$
\( \ell_b = \) length of full or partial bond breakdown
\( \ell_c = \) crack spacing

**FIG. 56 - IDEALIZED DEFORMATION CONDITION AT CRACKED SECTION**
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