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STEPPED COLUMNS:

A SIMPLIFIED DESIGN METHOD

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## STEPPED COLUMNS: A SIMPLIFIED DESIGN METHOD

### Summary

A simple method is presented for the design of stepped columns which presents, with respect to the classical effective length method, some advantages (mainly swiftness and precision), when designing members in compression and bending.

The method is based on a simplified model with two degrees of freedom. It is possible to obtain the ultimate interaction domains for stepped members, taking into account the effects of both geometrical and mechanical imperfections and of the loading path.

Some of these domains are presented, and compared with available numerical results.

## Introduction

The problem of how to determine the ultimate load carrying capacity of stepped steel columns has been exhaustively -- even if not extensively -- treated in the literature.

Only limited research has, however, been carried out on the behavior of these structural elements when taking into account both the non-linearity of the constitutive law of the material and the geometrical non-linearity.

Most of the preceeding studies /1,6/ dealt with the problem of the determination of the elastic critical load of axially compressed members, with various conditions of end restraints and loading.

The only attempt, to the author's knowledge, to determine the load carrying capacity in the elastoplastic range for a stepped column is a work of Barnes and Mangelsdorf /7/; the paper, however, doesn't consider compression and bending, which is the most frequently occurring stress state for these members.

It may be concluded that the only aspect to be investigated so far is that related to the elastic behavior of stepped columns, and that when determining the ultimate load carrying capacity of such elements, reference is usually made to the effective length parameter.

Design practice /8,11/ reflects the theoretical state of the research. The tendency is basically to design stepped members carrying out separate checks for the two shafts, by using the effective length method and the axial thrust-bending moment interaction formulas which are valid for members with uniform cross-section.

With reference to the AISE Recommendation, /8/ such formulas can be written as:

$$P/P_{c,n} + C_m M/[M_p (1 - P/P_E)] \leq 1 \quad (1)$$

where  $P$  is the total axial thrust in the shaft (upper or lower),  $M$  is the maximum first order bending moment,  $C_m$  is a reduction coefficient  $\leq 1$  which is a function of the bending moment's distribution,  $M_p$  is the fully plastic bending moment of the profile,  $P_{c,n}$  and  $P_E$  are the ultimate and critical elastic loads, calculated on the base of the effective slenderness ratio of the shaft under consideration.

Several general and specific critical considerations may be developed about this kind of approach; in particular it should be noted that:

- the effective length is derived from the critical multiplier of the axial loads acting on the column, and is thus linked to a prefixed value of the ratio of these loads; the effective length is therefore different for different load combinations. (the methods based on calculating the effective length of the structural members, lose than (at least in part), their advantage of being easy and quick to apply, when the ultimate limit state design method is adopted, which implies to take into account a number of different load combinations).

- design methods based on the concept of effective length do not fit well for considering the interaction between the column segments in operative terms. Thus they require a series of separate checks.

On the basis of these considerations, the author (believing that the correct way to deal with the problem of determining the load carrying capacity of these members is the approach developed by preceding international research into the behavior and stability of members with uniform cross-section and axial load) has performed a numerical study /12,13/ following step-by-step the response of a stepped member (affected by both geometrical and mechanical imperfections) during a number of different loading paths, up to the attainment of the collapse situation.

Ultimate interaction domains, for the elements considered in the study were numerically obtained in terms of the two vertical loads  $P_1$  and  $P_2$  (respectively applied on the upper and lower shafts) and compared with those deducible on the base of design methods based on the effective length and formula (1).

It was pointed out that:

- The shapes of the interaction domains obtained numerically are very similar to those obtained making reference to the effective length concept.
- For simple compression members, there is a close agreement between the numerical results, and those obtained by the effective length method, which enables a fundamentally correct evaluation of the ultimate load carrying capacity for stepped columns.
- For members in compression and bending:
  - a. With a method based on the concept of effective length (which is implicitly linked with the concept of instability of equilibrium as a bifurcation problem), it is possible to understand correctly which situation is associated with the collapse of the structural element, but it is not possible to appreciate the effect of the geometrical imperfections on the behavior of the member and on the shape of its ultimate interaction domain. (The author pointed out /13/ that this effect is relevant and different in the two shafts).
  - b. The method based on the effective length tends to be always on the safe side when the collapse situation is reached in the lower shaft (the situation of greatest practical interest), whilst it tends sometimes to be on the unsafe side when the collapse occurs in the upper shaft.
- The safety factor assumed using a method based on the effective length concept is not homogeneous, and is a function of the vertical loads' ratio.

The knowledge of the ultimate interaction domains has the advantage of allowing the safety margin associated with the various load combinations (which can occur during the life of the structure) to be appreciated in global terms in design checks. If reference is made to these ultimate domains, methods based on axial thrust-bending moment interaction formulas such as (1) are decidedly

complex, from the computational point of view, since for every load combination they require:

- the calculation of the effective length
- the solution of the interaction formula with regard to the axial load

Furthermore, in order to obtain a better precision in the solution, also, the reduction coefficient  $C_m$  should be defined for the different values of the ratio between the applied loads.

A simple approach was then proposed /12,13/ based on the use of an interaction formula directly written in terms of the applied vertical loads, of the type:

$$\left( \frac{P_1}{P_{1C,M}} \right)^\alpha + \left( \frac{P_2}{P_{2C,M}} \right)^\alpha = 1 \quad (2)$$

In (2),  $P_{1C,M}$  and  $P_{2C,M}$  are the maximum values of the loads  $P_1$  and  $P_2$  sustainable by the column in the presence of a single vertical load;  $P_{1C,M}$  and  $P_{2C,M}$ , implicitly take into account the possible transverse actions.

The use of such formula requires on the one hand the definition of the value of the exponent  $\alpha$ , and on the other hand the availability of a sufficiently simple method for determining the loads  $P_{1C,M}$  and  $P_{2C,M}$ ; it has thus the implicit advantage of using formula (1) (i.e. determining the coefficient  $C_m$ ) only for calculating  $P_{1C,M}$  and  $P_{2C,M}$ , that is when one of the two vertical loads is absent.

In the case of members with uniform cross-section, it was shown in a preceding papers /12/ that it is possible, with a certainly acceptable degree of approximation, to adopt  $\alpha = 1.0$  for elements subjected to centric vertical loads, and  $\alpha = 0.9$  for elements subjected to eccentric vertical loads.

In the same paper, it was however pointed out the coefficient  $C_m$ 's obvious influence on the domain's intersection with the load axis (that is on the values of  $P_{1C,M}$  and  $P_{2C,M}$ ).

It has been tried, but it hasn't yet been possible, to extend the same approach to columns with variable cross-section, because coefficient  $\alpha$  has a very wide range of variation, and it is influenced by too many parameters. Some research is still going on, trying to determine the values of  $\alpha$  to be used in (2), in the case of stepped elements.

Because of the uncertainties introduced while calculating  $P_{1C,M}$  and  $P_{2C,M}$  (i.e., calculating the reduction coefficient  $C_m$  of (1)) and because of the problems arisen while trying to determine the values of  $\alpha$ , the simple method presented in this paper has been set up which enables the ultimate interaction domains for stepped elements to be determined in a very simple and easy way.

## The Model

### The Equilibrium Equations

From preceding studies /12,13/, it has been noticed that the collapse of a stepped member is mainly associated with two different and non-correlated situations: the collapse of the upper shaft or the collapse of the lower shaft; in both cases, however, the collapse situation is reached in the most stressed section of the shaft.

The collapse situation of these elements, seems then to be caused more by local buckling in a well defined area of one of the shafts (the most stressed cross section) rather than by global instability of the whole member.

It is then possible to predict "where" in the shaft, but it is not possible to know a-priori in which one of the two shafts the collapse will occur, this last fact depending on the loading conditions.

Starting from these considerations, in this paper the behavior of stepped columns is simulated with a simple model with two degrees of freedom; the deformability of the element is concentrated in the two most stressed cross-sections and the interaction between the two shafts is disregarded.

If the column is considered as simply cantilevered at its lower edge (a simplifying and conservative scheme when dealing with mill-building columns, because the rotational restraint effect of the roof structure is ignored) the most stressed section in each shaft is its lower section, and the ultimate load carrying capacity of the stepped element can be determined making reference to the simple model shown in Fig. 1. ( It is assumed the presence of adequate bracings preventing the out-of-plane buckling of the column ).

The model consists of two rigid bars and two cells in which the deformability has been concentrated. The two shafts, having respectively a length  $L_1$ , a cross-section with area  $A_1$  and moment of inertia (with respect to the center of gravity)  $I_1$  (upper shaft) and a length,  $L_2$ , a cross-section with area  $A_2$  a moment of inertia (with respect to the center of gravity)  $I_2$  (lower shaft), are connected together taking into account an eccentricity  $e_{12}$  between them.

Two vertical loads  $P_1$  and  $P_2$  are applied respectively with an eccentricity  $e_1$  and  $e_2$ , at the top of each shaft, together with two horizontal forces  $F_1$  and  $F_2$ . In addition, a horizontal force  $H$ , proportional by a constant coefficient  $\xi$  to the vertical load  $P_2$  may be present at the top of the lower shaft :  $H = \xi P_2$

The two degrees of freedom of the model may be identified with the relative rotation  $v_1$  between the upper and the lower shaft and with the absolute rotation  $v_2$  of the lower shaft with respect to the vertical axis. Initial geometrical imperfections  $f_{01} = v_{01} L_1$  and  $f_{02} = v_{02} L_2$  have been assumed respectively at the top of the upper and of the lower shaft,  $v_{01}$  and  $v_{02}$  being the initial values of  $v_1$  and  $v_2$ , respectively.

The equilibrium conditions for the model in a displaced configuration, characterized by two rotations  $v_1$  and  $v_2$  can be derived by equating in each cell the internal bending moments to the external ones due to the applied loads.

Two equations can be written:

$$P_1 e_1 + P_1 L_1 (v_1 + v_2) + F_1 L_1 = K_1 (v_1 - v_{01}) \quad (3)$$

$$P_2 e_2 + P_2 L_2 v_2 + F_2 L_2 + H L_2 + F_1 (L_1 + L_2) + P_1 (e_1 + e_{12}) + P_1 [(L_1 + L_2)v_2 + L_1 v_1] \\ = K_2 (v_2 - v_{02}) \quad (4)$$

When the external loads, the initial out of straightness and the bending stiffnesses  $K_1$  and  $K_2$  of the two shafts are known, (3) and (4) form a system of linear equations in which the unknowns are the two rotations  $v_1$  and  $v_2$ , that is the parameters which define the equilibrium configurations of the model. The collapse situation may be reached either in the upper or in the lower shaft. In the first case, rotation  $v_1$  is equal to the ultimate limit rotation  $v_{1 \text{ lim}}$ , and  $v_2$  may be whatever (but less than  $v_{2 \text{ lim}}$ ), while in the second case rotation  $v_1$  may be whatever (but less than  $v_{1 \text{ lim}}$ ), and  $v_2$  is equal to  $v_{2 \text{ lim}}$ .

#### Equivalence Between Model and Real Column

The parameters which govern the behavior of the model must be defined so that there is a complete equivalence between the model and the simulated real element.

To reach this aim, equating for each step the Euler elastic critical load and the ultimate limit bending moment, it is imposed that the discrete model and the continuous real member have the same global elastic deformability, and that they locally reach their ultimate strength under the same bending stresses.

So, for each step, two equations may be written from which the two unknown parameters (the bending stiffness  $K$ , and the ultimate limit rotation  $v_{\text{lim}}$ ) can be determined.

In each shaft of the model, the Euler critical load can be defined respectively as:

$$P_{\text{cr1}} = K_1 / L_1 \quad \text{and} \quad P_{\text{cr2}} = K_2 / L_2$$

while the ultimate limit bending moment, in the elastic range, can be defined respectively as:

$$M_{\text{PL1}} = K_1 (v_{1 \text{ lim}} - v_{01}) \quad \text{and} \quad M_{\text{PL2}} = K_2 (v_{2 \text{ lim}} - v_{02})$$

For the real column, the Euler critical loads of the two shafts are respectively:

$$P_{\text{cr1}} = \Pi^2 E I_1 / 4 L_1^2 \quad \text{and} \quad P_{\text{cr2}} = \Pi^2 E I_2 / 4 L_2^2$$

where  $E$  is the Young modulus.

The ultimate limit bending moment, is not a constant, in a cross-section of a member which is subjected to variable axial loads, but it is different, for different values of the axial load.

For the cross-section, a linear interaction domain can be assumed (on the safe side), of the kind:

$$\frac{M}{M_u} + \frac{N}{N_u} = 1$$

where  $M_u$  is the maximum bending moment sustainable by the cross-section in absence of axial load at the plastic adaptation limit state (i.e.,  $M_u = \Psi f_y S$ , where the coefficient  $\Psi$  ( $\geq 1$ ) amplifying the section modulus  $S$ , is called the plastic adaptation coefficient, and is:  $1 \leq \Psi \leq \alpha$ , where  $\alpha$  is the shape factor of the cross section/ $14/$ ,  $f_y$  is the yield stress of the material) and  $N_u$  is the maximum axial load sustainable by the cross section, in absence of bending moment (i.e.,  $N_u = f_y A$ ).

When the value of the axial load in the shaft is known, then it is possible to define:

$$M_{PL1} = M_{u1} \left( 1 - \frac{P_1}{P_{1u}} \right) \quad \text{and} \quad M_{PL2} = M_{u2} \left( 1 - \frac{P_1 + P_2}{P_{2u}} \right)$$

where  $M_{u1} = \Psi_1 f_y S_1$ ,  $M_{u2} = \Psi_2 f_y S_2$ ,  $P_{1u} = f_y A_1$ ,  $P_{2u} = f_y A_2$ .

By equating the corresponding expressions, the four unknown parameters are determined:

$$5a) \quad v_1 \text{ lim} = \left( 1 - \frac{P_1}{P_{1u}} \right) \frac{M_{u1}}{K_1} + v_{01} \quad 5b) \quad K_1 = \Pi^2 EI_1 / 4L_1$$

$$6a) \quad v_2 \text{ lim} = \left( 1 - \frac{P_1 + P_2}{P_{2u}} \right) \frac{M_{u2}}{K_2} + v_{02} \quad 6b) \quad K_2 = \Pi^2 EI_2 / 4L_2$$

It must be noted that posing the equivalence of the Euler elastic critical loads separately in the various steps doesn't imply that the same equivalence exists between the whole model and the real structure. The operating way was however forced, because the critical load of the model depends on the ratio of the bending stiffness  $K_1$  and  $K_2$  of the two steps, which are a-priori unknown.

The approximation here introduced, can however be disregarded in the present work, because of the assumption of disregarding the overall buckling of the member.

### The Ultimate Interaction Domains

It is possible to reduce equations (3) and (4) to two expressions respectively of the kind  $v_1 = v_1(v_2)$  and  $v_2 = v_2(v_1)$ , by solving equations (3) with respect to  $v_1$  and equation (4) with respect to  $v_2$ .

Substituting in equation (4), equation (3) solved with respect to  $v_1$ , the following expression for  $v_2$  is obtained:

$$v_2 = \frac{1}{K_2 - P_1(L_1+L_2) - P_2L_2 - \frac{P_1^2 L_1^2}{K_1 - P_1 L_1}} \left[ K_2 v_{02} + P_1(e_1 + e_{12}) + F_1(L_1+L_2) + \right. \\ \left. F_2L_2 + \xi P_2L_2 + P_2e_2 + P_1L_1 \frac{P_1e_1 + F_1L_1 + K_1 v_{01}}{K_1 - P_1L_1} \right] \quad (7)$$

When the geometrical characteristics of the column are known, this expression gives a value of  $v_2$  as a function of the external loads. The collapse situation is reached in the lower shaft when the loading condition is such that  $v_2$  is greater than (or equal to)  $v_{2 \text{ lim}}$  [eq. (6a)]. Equating  $v_2$  to  $v_{2 \text{ lim}}$  and varying the values of the vertical loads, equation (7) describes a curve in the plane  $P_1 \div P_2$ . This curve defines in the same plane  $P_1 \div P_2$  an admissible region: all the points contained in the area bounded by the coordinate axis and by the curve represent admissible loading conditions for the lower shaft. The points on the curve represent loads' combinations which cause the limit situation to be reached in the lower shaft.

The points external to this admissible area represent loads' combinations which cannot be sustained by the column and cause the collapse of the lower shaft.

Analogously substituting in equation (3) equation (4) solved with respect to  $v_2$ , an expression is reached:

$$v_1 = \frac{1}{K_1 - P_1L_1 - \frac{P_1^2 L_1^2}{K_2 - P_2L_2 - P_1(L_1+L_2)}} \left[ P_1e_1 + F_1L_1 + K_1 v_{01} + \right. \\ \left. + P_1L_1 \frac{K_2 v_{02} + P_1(e_1 + e_{12}) + F_1(L_1 + L_2) + F_2L_2 + \xi P_2L_2 + P_2e_2}{K_2 - P_2L_2 - P_1(L_1 + L_2)} \right] \quad (8)$$

which, when the geometrical characteristics of the column are known, defines the value of  $v_1$  as a function of the external loads.

The collapse situation in the upper shaft is reached when the loading conditions is such that  $v_1$ , is greater than (or equal to)  $v_{1 \text{ lim}}$  [eq (5a)].

Equating  $v_1$  to  $v_{1 \text{ lim}}$  and varying the values of the vertical loads, also equation (8) describes a curve in the plane  $P_1 \div P_2$ : all the points contained in the region bounded by the coordinate axis and the curve represent admissible loading conditions for the upper shaft. On the contrary, the column cannot sustain loads' combinations represented by points external to the admissible region, without collapse of the upper shaft.

If the two shafts have different cross sectional properties than the two curves represented by equations (7) and (8) intersect each other. The ultimate interaction domain for the column is the intersection of the two admissible regions for the two shafts, and the boundary of the domain is the envelope of the two curves.

If the column has a constant cross section, the two curves don't intersect, and the region bounded by equation (7), which is completely contained into that bounded by equation (8), turns out to be the ultimate interaction domain of the element.

### Design Considerations

Reliable results cannot, of course, be expected from the model as it is. In fact, even if it is possible to evaluate in a substantially correct way the global behavior of the column, the real stiffness of the stepped member cannot be correctly evaluated using the model, because of the rough simplifying assumptions on which the model itself is based.

Furthermore, even if it is possible to take into account the effect of the initial out-of-straightness on the shape of the ultimate domains, the model cannot take into account the effect of the residual stresses, an effect that preceding papers /12,13,15/ have shown to be relevant, on stepped members as well as on prismatic members. It is possible to partially reduce the approximation introduced with the initial assumptions by normalizing the domains obtained over the maximum values  $P_{1uc}^*$  and  $P_{2uc}^*$  of the centric vertical loads sustainable by the model (respectively at the top of the whole column, and at the top of the lower shaft), in absence of the other loads (both vertical and horizontal).

Thus it is possible to reduce the ultimate interaction domains in a non-dimensional form, in the plane  $P_1/P_{1uc}^* \div P_2/P_{2uc}^*$ . These domains however, because in a non-dimensional form, cannot yet be used by the designer for practical applications.

Preceding studies /12,13/ has shown that by using the effective length concept, it is possible to evaluate with a good precision (at least from an engineering point of view) the values of  $P_{1uc}$  and  $P_{2uc}$  for the real column, entering with the values of the equivalent length (calculated separately for the upper shaft and for the lower shaft /4,6/) on the stability curves for the upper and lower shaft respectively.

Once the two values  $P_{1uc}$  and  $P_{2uc}$  for the real column have been calculated, in an extremely fast and easy way, it is possible to render the non-dimensional domains previously obtained in a dimensional form, ready for being used by the designer.

By following this way, it is also possible, although indirectly, to conglobate into the model the effect of residual stresses on the ultimate value of the load carrying capacity of the member.

A short interactive computer program has been set up, which enables the numerical solution of equations (7) and (8) to be obtained, for the different loads' combinations considered. Once the statical and geometrical properties of a stepped member are entered as input datas, the code automatically furnishes as output result the ultimate interaction domains in the non-dimensional form, in the plane  $P_1/P_{1uc}^* \div P_2/P_{2uc}^*$ .

### Comparison With the Numerical Results

Some comparisons have been done (even if in a limited number of available cases), between the domains obtained in former works /12,13/ with a numerical simulation method /16/.

In the following figures (2-6), the domains are shown in a non-dimensional form, in the plane  $P_1/P_{1uc} \div P_2/P_{2uc}$ . The numerical domains have been non-dimensionalized on the values  $P_{1uc}$  and  $P_{2uc}$  obtained by the numerical simulation /12,13/, while the domains obtained by using the simplified model have been normalized over the values  $P_{1uc}^*$ ,  $P_{2uc}^*$  obtained using the model itself.

It is possible to see that there is a good agreement between the results of the numerical simulation, and those obtained using the simplified model. Only in the case when horizontal forces of the wind are present, the difference is significant, although contained under the 8%, and however always on the safe side (Fig. 4). A comparison has also been done with some of the domains obtainable by using formula (2) as proposed in /12/ (Fig. 2 and 3).

### Conclusion

In this paper a simple method is presented for determining the ultimate interaction domains for stepped columns. The method requires the use of the effective length concept only for calculating the ultimate values of the centric axial load applied at the top of the lower shaft ( $P_{2uc}$ ), and of the whole column ( $P_{1uc}$ ).

These values are then used for rendering in a dimensional form the ultimate interaction domains determined in a non-dimensional form using a simple model with two degrees of freedom.

Using this model, it is possible to take into account the effect of both mechanical and geometrical imperfections and of the loading path, on the shape of the ultimate interaction domains for stepped structural members. It is possible to obtain the ultimate domains avoiding all the difficulties connected with the use of methods based on the effective length concept and axial thrust-bending moment interaction formulas [like (1)], which require long calculations when dealing with members in compression and bending.

The method presented in this paper represents also an overcoming of that proposed in /12/ and /13/, based on equation (2), and whose results are heavily influenced by the values adopted for the coefficient  $C_m$  of equation (1), when calculating the values of  $P_{1C,M}$  and  $P_{2C,M}$  to be used in equation (2).

The method was checked only in a limited number of cases, whose numerical results were available to the author at the time of the publication.

These cases refer to some of the examples presented in /12/ and /13/, that is prismatic members, with stepwise axial loads, whose cross-section is a HE200A profile, and stepped columns, with ratio between the moments of inertia  $I_2/I_1 = 10$ , and ratio between the lengths of the two shafts  $L_2/L_1 = .6, 1.0$ . Therefore, before any use or application of the method in standard design practice, more extensive research and check works (both numerical and experimental) are required.

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- Fig. 7 : FORTRAN Source for equation ( 8 )
- Fig. 8 : FORTRAN Source for equation ( 7 )

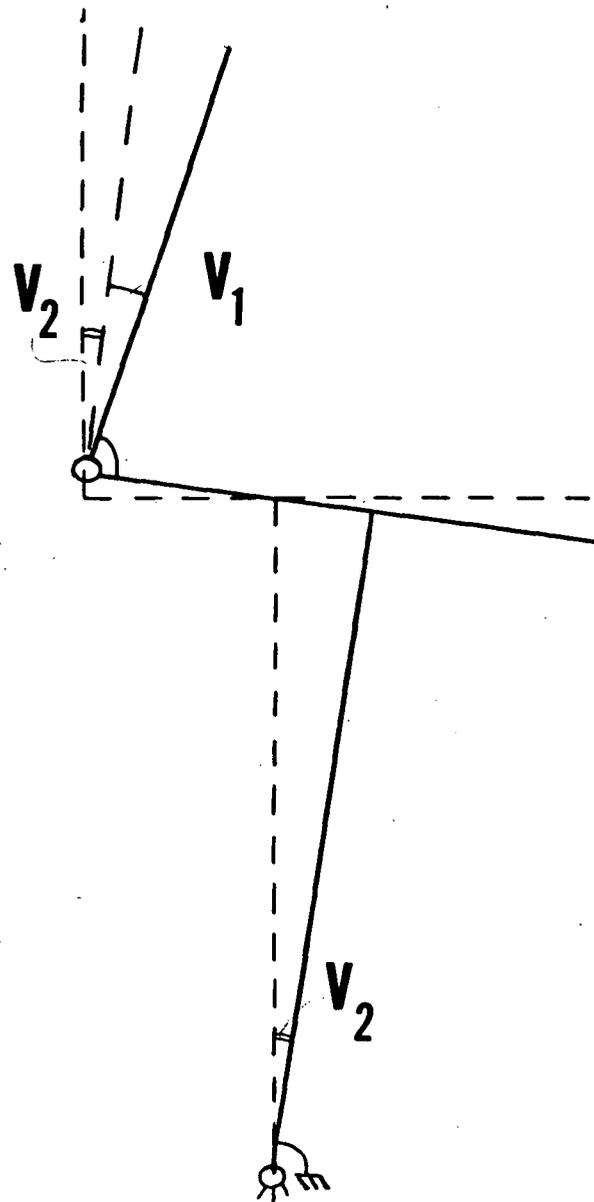
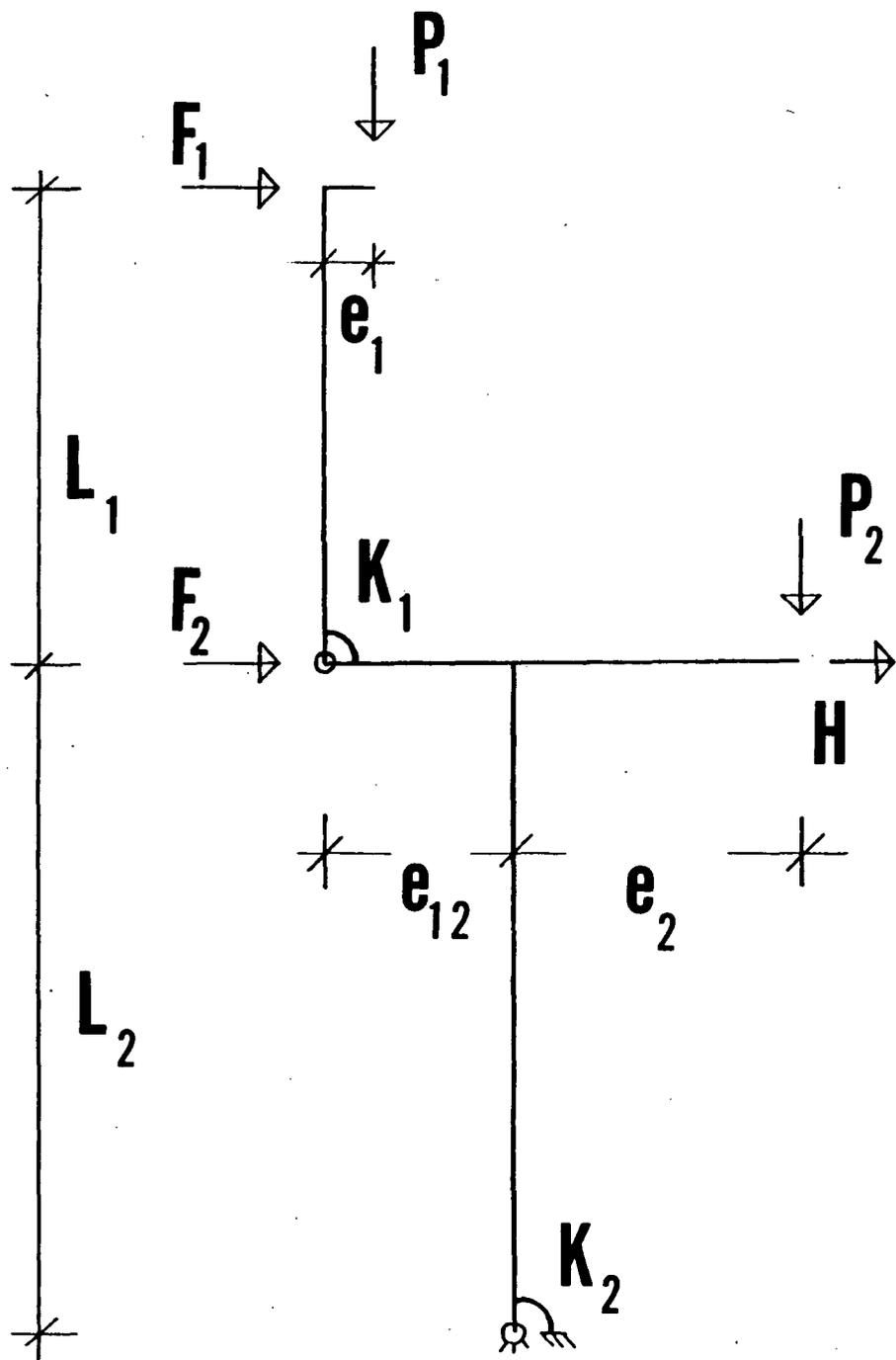


Fig. 1

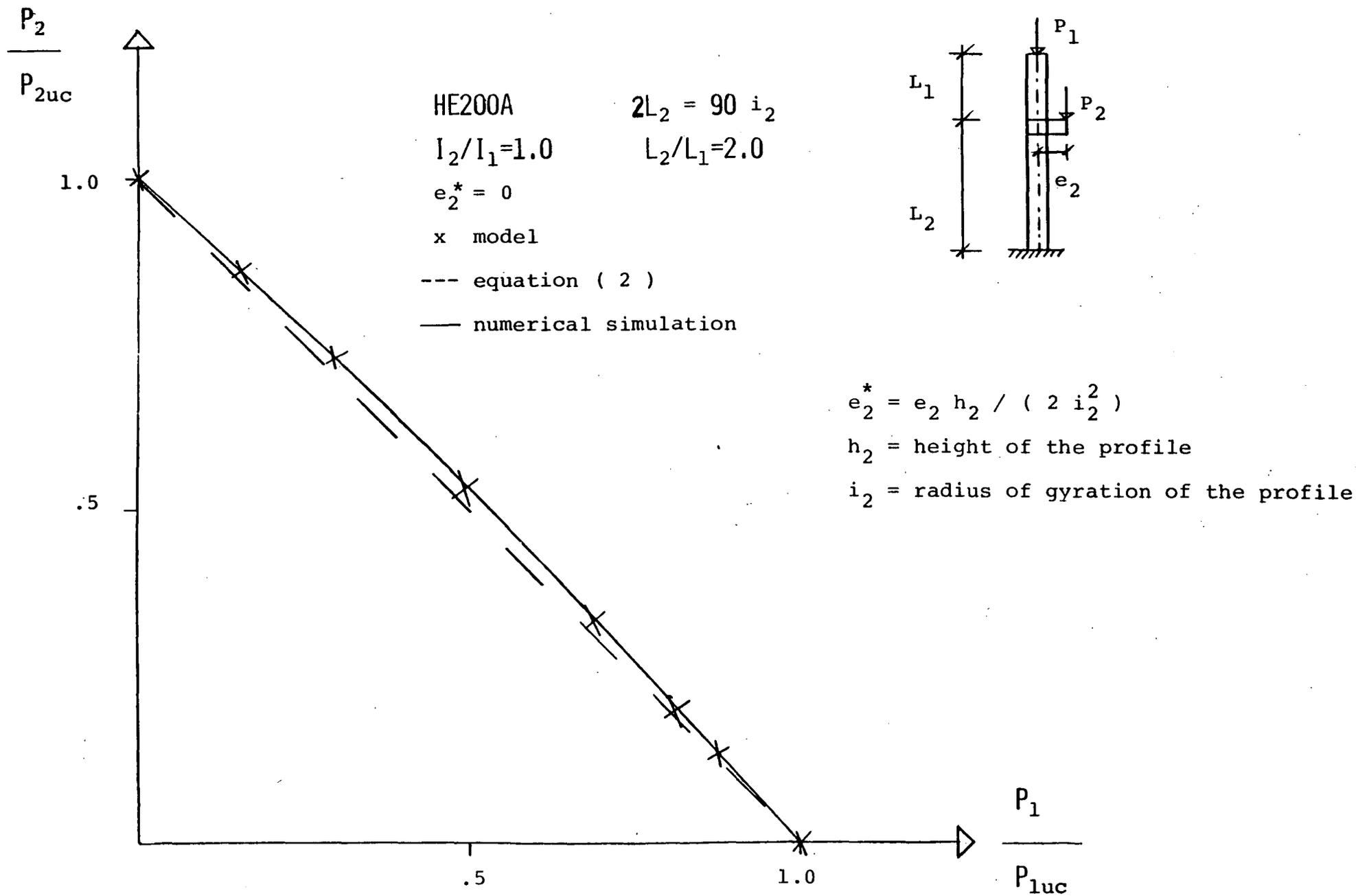


Fig. 2

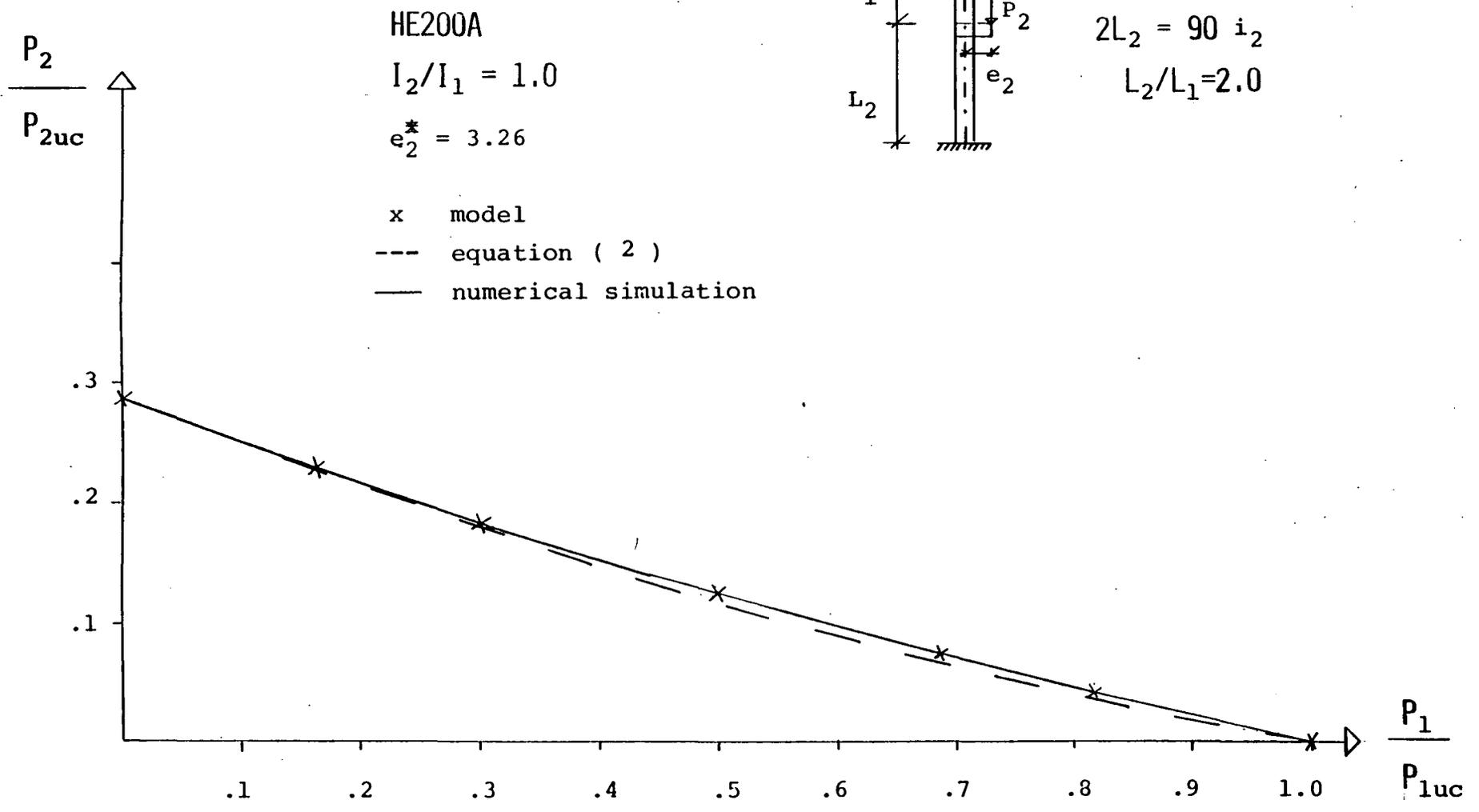


Fig. 3

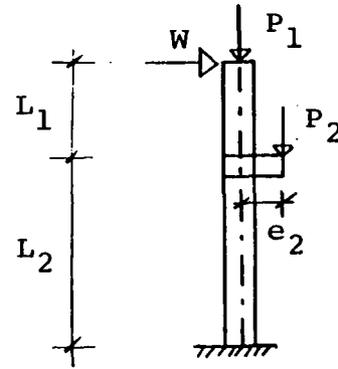
HE200A

$$I_2/I_1=1.0$$

$$e_2^* = 3.26$$

x model

— numerical simulation



$$2L_2 = 90 i_2$$

$$L_2/L_1=2.0$$

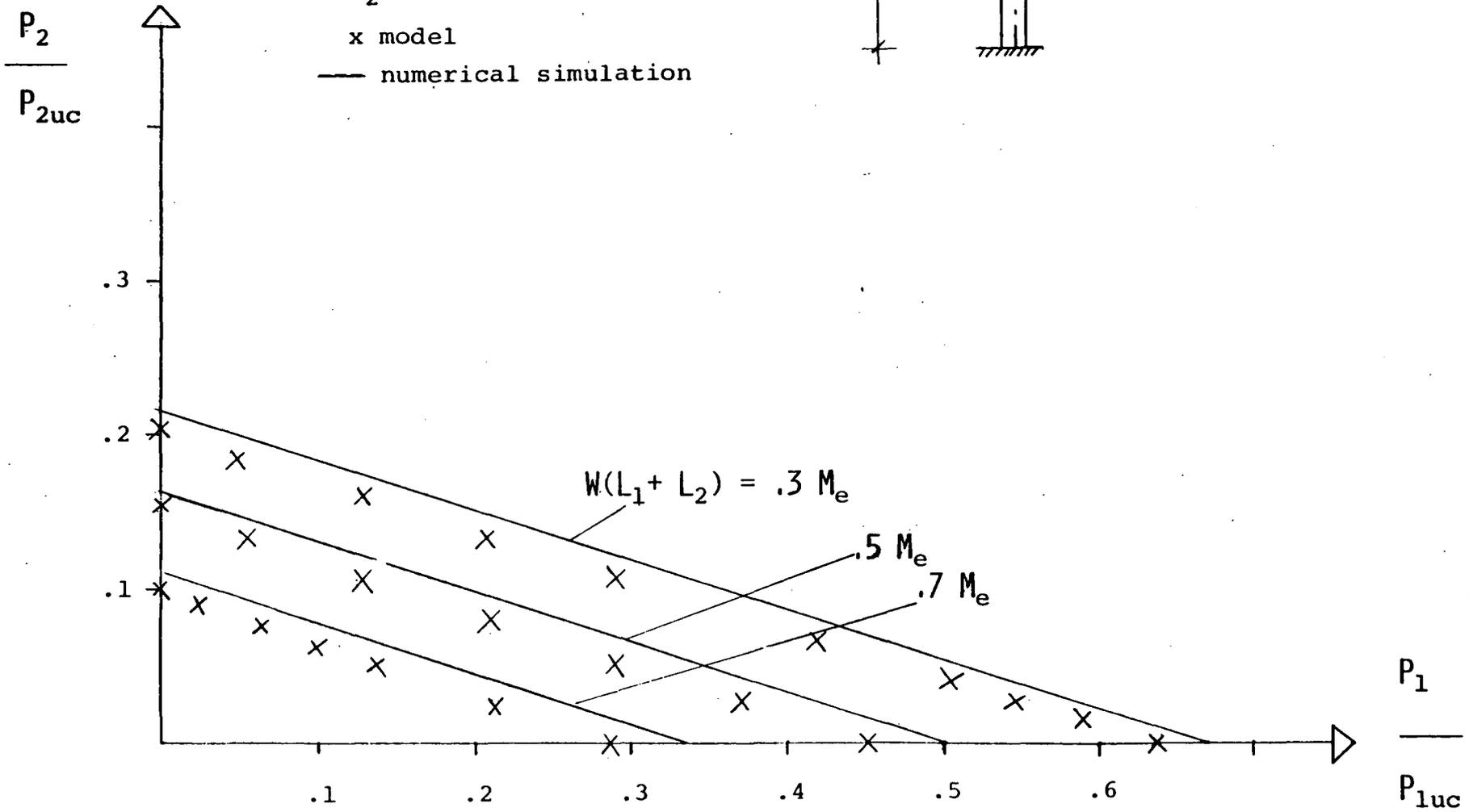


Fig. 4

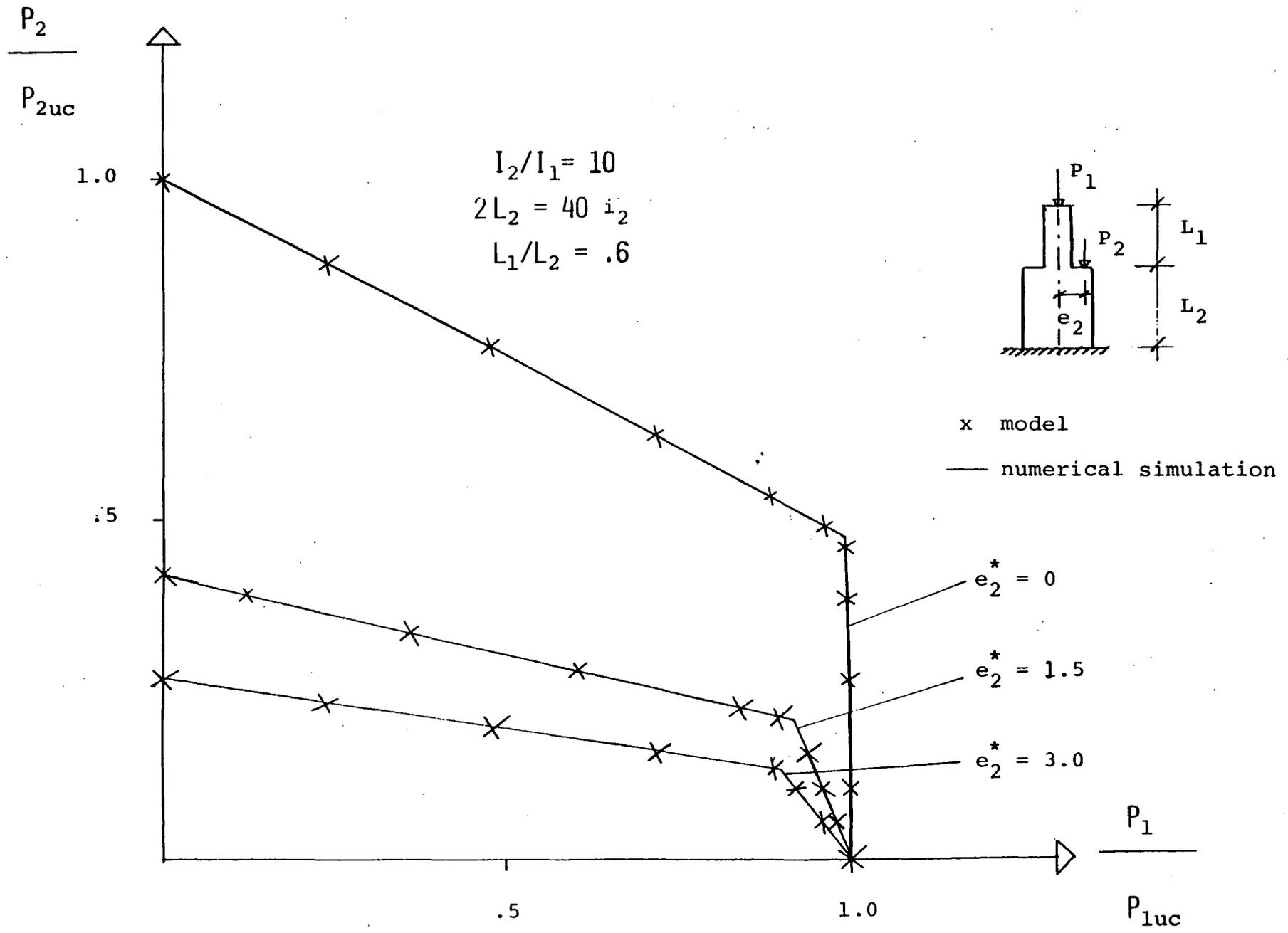


Fig. 5

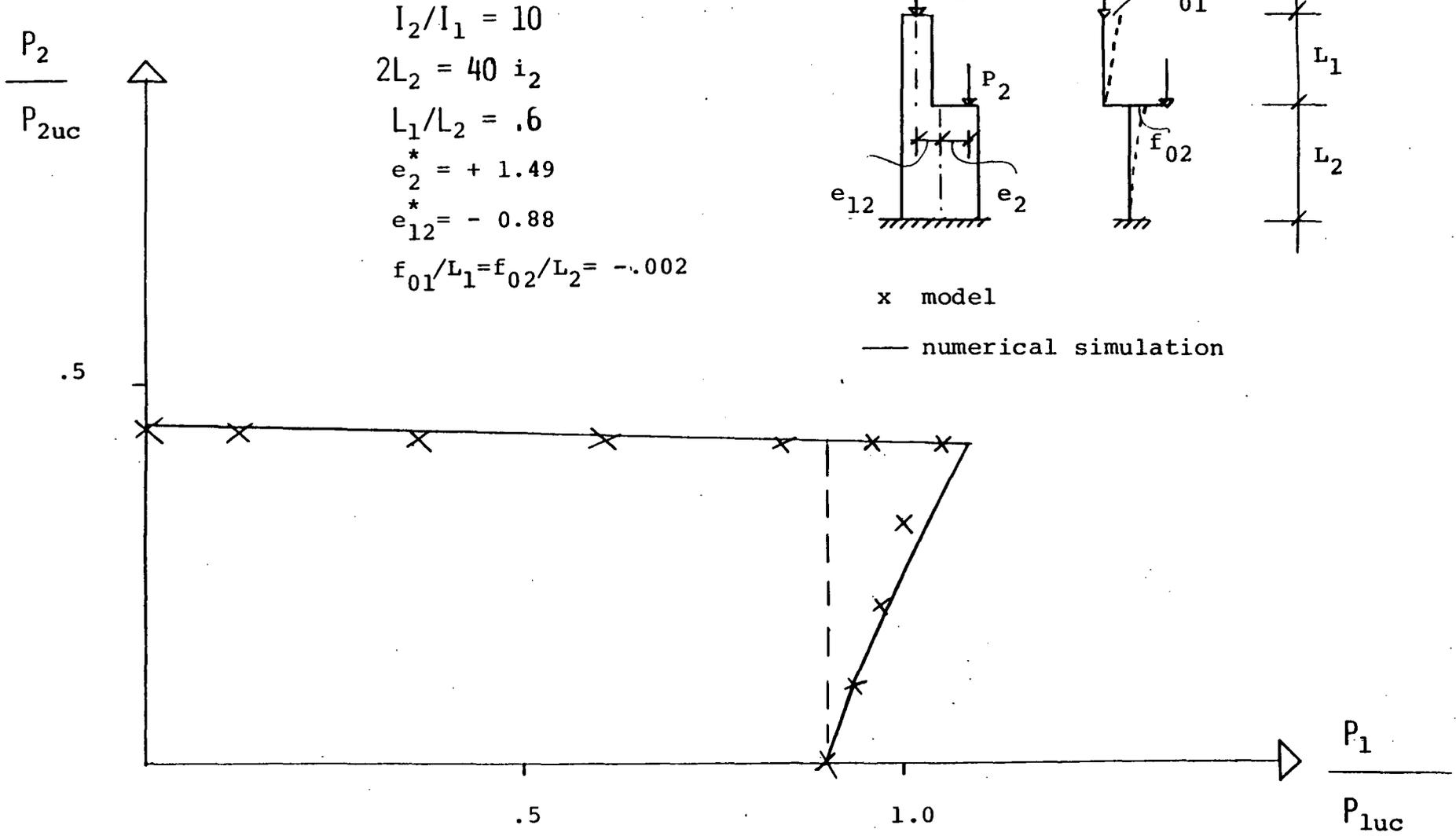


Fig. 6

```

3100      SUBROUTINE SUB1(P1,P2,FUNZ)
3110      COMMON/UNO/AA1,AA2,AL1,AL2,AI1,AI2,UM1,UM2,V01,V02,E1,E2
3120      +,E12,F1,F2,ALFA,CK1,CK2,PU1,PU2,ALT
3130      A=CK1-P1*AL1
3140      B=CK2-P2*AL2-P1*ALT
3150      C=CK2*V02+P2*E2+F2*AL2+ALFA*P2*AL2+F1*ALT+P1*(E1+E12)
3160      D=CK1*V01+P1*E1+F1*AL1
3170      TETA1=(D+P1*AL1*C/B)/(A-(P1*AL1)**2/B)
3180      TETA1L=(1.-P1/PU1)*UM1/CK1
3190      FUNZ=ABS(TETA1-V01)-ABS(TETA1L)
3200 C    WRITE(6,123)A,B,C,D,TETA1,TETA1L,FUNZ,P1,P2
3210      123 FORMAT(10(2X,E10.4))
3220      RETURN
3230      END

```

Fig. 7

```

2950     SUBROUTINE SUB2(P1,P2,FUNZ)
2960     COMMON/UNO/AA1,AA2,AL1,AL2,AI1,AI2,UM1,UM2,V01,V02,E1,E2
2970     +,E12,F1,F2,ALFA,CK1,CK2,PU1,PU2,ALT
2980     A= CK1-P1*AL1
2990     B=CK2-P2*AL2-P1*ALT-(P1*AL1)**2/A
3000     C=CK2*V02+P2*E2+F2*AL2+ALFA*P2*AL2+F1*ALT+P1*(E1+E12)
3010     D=(CK1*V01+P1*E1+F1*AL1)/A
3020     TETA2=C+P1*AL1*D
3030     TETA2=TETA2/B
3040     TETA2L=(1.-(P1+P2)/PU2)*UM2/CK2
3050     FUNZ=ABS(TETA2-V02)-ABS(TETA2L)
3060 C    WRITE(6,123)A,B,C,D,TETA2,TETA2L,FUNZ,P1,P2
3070     123 FORMAT(10(2X,E10.4))
3080     RETURN
3090     END

```

Fig. 8