Shell arch roof model subjected to two cases of a concentrated load, January 1951

B. Thurlimann
B. G. Johnston

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PROGRESS REPORT 213F

SHELL ARCH ROOF MODEL SUBJECTED TO
TWO CASES OF A CONCENTRATED LOAD

by

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Bruce G. Johnston

to

Roberts and Schaefer Engineering Company

Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Penna.

January 31, 1951
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Progress Report 213 F

Shell Arch Roof Model Subjected to Two Conditions of Concentrated Loading

by

Bruno Thürlimann and Bruce G. Johnston

Abstract

Experimental results of a shell arch roof model (Fig. 1), subjected to two conditions of concentrated loading (Fig. 4), are reported. Unlike the previously published Progress Reports 213C to 213E, no theoretical analysis is made. Both loads were applied at the center of the span, (1) to the middle rib and, (2) to one outer rib.

Introduction

Under usual conditions a shell roof is not acted upon by concentrated loads, but an exception is the case of a suspended crane track. The problem is of primary importance in other applications of shell structures, e.g. introduction of single loads into airplane fuselages, the problem of the action of the trunnion pins on hot metal ladles, etc. An analytical solution of these problems along the lines used in the Progress Reports 213C to 213E is possible.
In this report two tests on a model of a shell arch roof are described and some of the experimental results are presented in diagrams. No attempt is made to present a theoretical interpretation of the results nor to derive empirical formulas, other analysis problems having been given more priority by the sponsors.

Part I: Description of the Model and the Loading Apparatus *

Fig. 1 shows the dimensions of the model, which was a structural steel weldment. It represents one unit of an airplane hangar to an approximate scale of 1:30 and consists of a cylindrical shell stiffened by three ribs. The span of the shell is 10 feet and the distance between the ribs 12 in.

Figs. 2 and 3 are two pictures illustrating the model subjected to a concentrated load acting on the middle rib. A "Loading Ring" was placed between the upper horizontal member of a rectangular frame and the rib to measure the load produced by a mechanical jack acting between the lower horizontal member of the frame and the sub-structure supporting the model. The

* See Progress Report 213G for a complete description of all experimental work.
sensitivity of this apparatus was determined as ±50 lbs. which compares favorably to the magnitude of the applied loads.

The details of the application of the concentrated load may be seen in Fig. 3 and 4. Actually two forces nearly uniformly distributed over a length of 2 in. were acting on the rib. Fig. 4 gives, in addition, the total concentrated load applied to the middle rib (test T-8) and to the outer rib (test T-9) respectively.

Part II: Arrangement of the Recording Devices

Deflections were measured by Ames Dials, mounted on a special dial gage frame, which was simply supported at the abutments of the shell (see Fig. 2 and 3). The plungers of the gages were extended to the height of the ribs by rods (Fig. 3). The latter were fixed to the ribs by C-clamps and transmitted vertical deflections to the dials. Small horizontal movements of the ribs relative to the gage frame were possible due to the flexibility of the extension rods.

Further information about the measuring equipment, as to strain gages, level bars, and their locations,
may be found in Progress Report 213, VI, and Fig. 8 and 9.

Part III: Test Procedure

During fabrication, cold rolling of the shell to a circular shape and welding of the ribs to the shell produced internal stresses, the magnitude and distribution of which are beyond any exact control. In order to avoid plastic flow during the test the model was subjected several times to loads 10% higher than the actual test loads with the result that all strains became perfectly elastic during testing.

The test itself consisted in taking all readings of the instruments at an "initial load", the model being subjected to the weight of the loading apparatus only. Then a specified load was applied (Pg and Pg of Fig. 4 respectively) and a complete set of readings was made. The difference of the "load" and the "initial" readings gives the effect of the load on the corresponding gages. Accidental errors such as a mistake in reading, etc., were eliminated by repeating the procedure once. Good agreement between the two sets was obtained.
Part IV: Test Results

In Progress Report 213G, Appendix to 213F "Test Results for a Model of an Arch Roof under Two Cases of a Concentrated Load", all test results are tabulated.

On the basis of these experimental values the fiber stresses $\sigma_u$ and $\sigma_L$ in the ribs were computed and plotted in Fig. 5. The $T_2$-forces, i.e., the direct forces in the shell in circumferential direction are shown for a cross section in the middle of the span* (Fig. 6). Finally the deflections of the ribs as recorded by the Ames dials are given in Fig. 7.

Part V: Some Conclusions Resulting from the Tests

Figs. 5 to 7 exhibit clearly the high local effects of concentrated loads on structures composed of relatively thin parts, in the present case of a thin shell and slender ribs. In the test T-8, concentrated load on the middle rib, the stresses become evenly spread out in the region between the abutments and the quarter point. Any local effect of the concentrated load has disappeared for this region and three loads $\frac{P_8}{3}$ acting on each of the ribs at the center

* $T_2$-forces are defined in Progress Report 213B, Fig. 3b.
would cause an equivalent stress distribution in this part. Test T-9, concentrated load on an exterior rib, leads to similar results. This effect should certainly be considered in a theoretical analysis in order to get some simplifications.

Considering the similitude between an actual structure and the present model, having a scale factor \( n \), some relations may be derived.

Table: Relation between the Model and a Structure

\[
\begin{align*}
n &= \frac{l_{\text{St}}}{l_{\text{Mod}}} \quad \text{(scale factor)} \\
e &= \frac{E_{\text{St}}}{E_{\text{Mod}}} \\
m &= \frac{P_{\text{St}}}{P_{\text{Mod}}} 
\end{align*}
\]

<table>
<thead>
<tr>
<th>A. Given Relations:</th>
<th>Model</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Length</td>
<td>( l_{\text{Mod}} )</td>
<td>( l_{\text{St}} = nl_{\text{Mod}} )</td>
</tr>
<tr>
<td>2. Modulus of Elast.</td>
<td>( E_{\text{Mod}} )</td>
<td>( E_{\text{St}} = e E_{\text{Mod}} )</td>
</tr>
<tr>
<td>3. Load</td>
<td>( P_{\text{Mod}} )</td>
<td>( P_{\text{St}} = m P_{\text{Mod}} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Derived Relations:</th>
<th>Model</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Stress</td>
<td>( \sigma_{\text{Mod}} )</td>
<td>( \sigma_{\text{St}} = \frac{m}{n^2} \sigma_{\text{Mod}} )</td>
</tr>
<tr>
<td>2. Deflection</td>
<td>( \delta_{\text{Mod}} )</td>
<td>( \delta_{\text{St}} = \frac{m}{en} \delta_{\text{Mod}} )</td>
</tr>
</tbody>
</table>
An example will show the use of the table. On the basis of the test results of the model the stresses and the deflection of an actual structure subjected to a concentrated load, acting on the middle rib at the center of the span, may be calculated:

<table>
<thead>
<tr>
<th>Structure:</th>
<th>Reinforced concrete</th>
<th>$E_{St} = 3 \times 10^6$ lb/in$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrated load</td>
<td>$P_{St} = 120,000$ lbs.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model:</th>
<th>Steel</th>
<th>$E_{Mod} = 30 \times 10^6$ lb/in$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrated load</td>
<td>$P_{Mod} = 5040$ lbs.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale factor:</th>
<th>$n = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratios:</td>
<td>$\frac{m}{P_{Mod}} = \frac{120,000}{5040} = 23.8$</td>
</tr>
<tr>
<td></td>
<td>$e = \frac{1}{10} = 0.1$</td>
</tr>
</tbody>
</table>

Experimental values of the model:

(Test T-6, Concentrated Load on Middle Rib)

1. Fiber stresses of the middle rib at the center of the span:

   Upper fiber: $(\sigma_u)_{Mod} = -33,450$ lb/in$^2$

   Lower fiber: $(\sigma_L)_{Mod} = 9,612$ lb/in$^2$

2. Deflection of the middle rib at the center of the span

   $\delta_{Mod} = 0.1303$ in
Derived values for the structure:

1. Fiber stress: \( \sigma_{st} = \frac{m}{n^2} \sigma_{mod} = \frac{23.8}{30^2} \sigma_{mod} \)

   Upper fiber: \( (\sigma_u)_{st} = 684.6 \text{ lb/in}^2 \)

   Lower fiber: \( (\sigma_L)_{st} = 254.2 \text{ lb/in}^2 \)

2. Deflection: \( \delta_{st} = \frac{23.8}{0.1 \cdot 30} \delta_{mod} \)

   \( \delta_{st} = 1.034 \text{ in.} \)

Hence, the middle rib of a hangar roof (Rapid City type, \( n = 1:30 \)) is able to carry a load of 60 tons having a deflection of about 1 in., the stresses being 684.6 lb/in\(^2\) in compression and 254.2 lb/in\(^2\) in tension (actually to be taken by the reinforcing steel).

Conclusions

The tests showed that a shell arch roof is able to carry considerable concentrated loads when applied to the ribs. As indicated in the introduction this fact could be of importance if the construction of crane tracks suspended from the ribs is considered.

Concerning testing of actual structures the usual tests made during decentering of the form work are not reliable, for the load carried by the form work just
prior to its removal is not necessarily identical to the dead load of the structure (shrinkage of the concrete and motions of the form work occur during hardening of the concrete). Furthermore, dead load produces relatively low stresses in the structure (see Progress Report 213D) which are difficult to measure with reasonable accuracy. Loading conditions similar to the one described in this report should give much better results. One or a few single loads could be applied to an arch rib in a very simple way. It would be desirable to get some experimental results on a full-scale structure built in reinforced concrete. The analysis and particularly the theoretical derivation of the effective width could be checked experimentally under actual conditions.
SHELL ROOF MODEL
SHOWING MEASURED
DIMENSIONS.
ALL DIMENSIONS IN
INCHES

FRONT VIEW 1" = 10"
TOP VIEW 1" = 10"
SECTION A-A 1" = 4"
SECTION B-B 1" = 4"
SECTION C-C 1" = 4"
T-8:
Concentrated Load
on the Middle Rib:

\[ P_8 = 5040 \text{ lbs.} \]

T-9:
Concentrated Load
on an Exterior Rib:

\[ P_9 = 2540 \text{ lbs.} \]

Detail of the Application of the Load to the Rib

Clearance for
Gages and Wires

Fig. 4
TEST T-9

Remote Rib

Middle Rib

Test T-9

Loaded Outer Rib

Probable exp. stress in upper fiber
Probable exp. stress in lower fiber
× Measured stress in upper fiber
○ Measured stress in lower fiber

Ω = 0.5866

Loaded Middle Rib

Test T-8

EXPERIMENTAL FIBER STRESSES OF THE RIBS

Outer Ribs
TEST T-9
(Loaded Outer Rib)

Probable Experimental $T_2$-Forces
× Measured $T_2$ at this Point
○ Symmetrical Point to × about $Q$

TEST T-8
(Loaded Middle Rib)

EXPERIMENTAL $T_2$-FORCES
IN THE CENTER OF THE SPAN

Fig. 6
**Probable Experimental Deflection Line**
- Measured at this Point
- Symmetrical Point to about \( P \)

Remote Rib

Middle Rib

Loaded Outer Rib

\( P = 2540 \text{ lbs.} \)

Outer Rib

Loaded Middle Rib

\( P = 5040 \text{ lbs.} \)

**TEST T-9**

**TEST T-8**

**Fig. 7**

EXPERIMENTAL DEFLECTIONS OF THE RIBS