Connections for welded rigid portal frames, Ph.D. Dissertation, Lehigh University, (1950)

A. A. Topractsoglou

Follow this and additional works at: https://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports

Recommended Citation
https://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports/1401

This Technical Report is brought to you for free and open access by the Civil and Environmental Engineering at Lehigh Preserve. It has been accepted for inclusion in Fritz Laboratory Reports by an authorized administrator of Lehigh Preserve. For more information, please contact preserve@lehigh.edu.
CONNECTIONS FOR WELDED
RIGID PORTAL FRAMES

Volume 1

By
Anastasios Anthony Topraktsoglou

A DISSERTATION
Presented to the Graduate Faculty
of Lehigh University
in Candidacy for the Degree of
Doctor of Philosophy

Lehigh University
1980
Approved and recommended for acceptance as a dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

_________________________  ___________________________
Professor in Charge

Accepted, __________________

Special committee directing the doctoral work of Mr. ___________________________

_________________________  Chairman

_________________________

_________________________

_________________________
ACKNOWLEDGEMENT

The author is greatly indebted to Dr. Bruce G. Johnston for his supervision of the program and for his advice and help during the preparation of the dissertation.

The author takes this opportunity to express his sincere appreciation to Mr. Lynn S. Beadle for his help and suggestions which greatly added to the success of the present work.

The cooperation and readiness to help of Professor Cyril D. Jensen, Chairman of the Special Committee which directed the graduate studies and the work of the author is greatly appreciated.

The author extends his sincere appreciation to the members of his Special Committee, Professors William J. Eney, Hale Sutherland, Alvin G. Loeber and Clarence A. Shook, for their cooperation.

To his colleague, Mr. Jan M. Rusek, the author wishes to express his most sincere thanks for his unfailing help without which the present report would have been impossible.

Many thanks to Mr. Kenneth Harpel and the machine shop staff of Fritz Engineering Laboratory for their valuable cooperation.

The author wishes also to express his thanks to Mr. Eric Unfried for helping carry out the tests, to Mrs. Harvey
ACKNOWLEDGEMENT
(continuation)

Bickert for typing the manuscript and to Misses Frances Shamensk, Anna King, and to Misses Alex Smialova, Lewis Schneider, Robert Ketten, David Ching for their help either during the tests or in the preparation of this dissertation or both.

The test program was sponsored by the Welding Research Council, American Institute of Steel Construction and the United States Navy. The program was under the supervision and guidance of the Lehigh Project Subcommittee whose members are: Kesars. Amirikian, Boardman, Crowley, Epstein, Frankland, Grinter, Higgins, Husey, Johnston, Jones, Kreidler, Lawson, Maccutcheon, Newmark, Poole, Weiskopf, Spraragen, Vasta, Groover.

The SEIS rolled section used in the test was donated by Lehigh Structural Steel Company of Allentown, Pennsylvania, through Mr. Carl Kreidler to whom the author expresses his thanks.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. SYNOPSIS</strong></td>
<td></td>
</tr>
<tr>
<td><strong>II. INTRODUCTION</strong></td>
<td></td>
</tr>
<tr>
<td>Rigid Frames</td>
<td>1</td>
</tr>
<tr>
<td>Historical</td>
<td>3</td>
</tr>
<tr>
<td>Method of Analysis and Design</td>
<td>4</td>
</tr>
<tr>
<td>Knees and Connections</td>
<td>7</td>
</tr>
<tr>
<td>Welding</td>
<td>9</td>
</tr>
<tr>
<td>Need for Research and Previous Work</td>
<td>12</td>
</tr>
<tr>
<td>Knees for Portal Frames</td>
<td></td>
</tr>
<tr>
<td>Objectives of Research on Connection and the General Objective of the Project</td>
<td>12</td>
</tr>
<tr>
<td><strong>III. REQUIREMENTS FOR CONNECTIONS</strong></td>
<td>14</td>
</tr>
<tr>
<td><strong>IV. THEORETICAL ANALYSIS ON CONNECTIONS</strong></td>
<td>17</td>
</tr>
<tr>
<td>Elastic Stress Analysis of Knees</td>
<td>26</td>
</tr>
<tr>
<td>Rotation in Knees (elastic)</td>
<td>26</td>
</tr>
<tr>
<td>Initial Yielding in Square Knees of Types 5, 7, and 8</td>
<td>30</td>
</tr>
<tr>
<td><strong>V. TEST PROGRAM</strong></td>
<td>55</td>
</tr>
<tr>
<td>Test Specimens (Connections)</td>
<td>35</td>
</tr>
<tr>
<td>Coupon and Control Beam Tests</td>
<td>38</td>
</tr>
<tr>
<td>Moment Diagrams and Load Lines</td>
<td>39</td>
</tr>
<tr>
<td>VII. COST OF FABRICATION OF SPECIMENS</td>
<td>42</td>
</tr>
<tr>
<td>VII. TEST APPARATUS</td>
<td>45</td>
</tr>
<tr>
<td>Loading Methods</td>
<td>43</td>
</tr>
<tr>
<td>Lateral Supports</td>
<td>44</td>
</tr>
<tr>
<td>Instruments and Measurements</td>
<td>46</td>
</tr>
<tr>
<td>VIII. TEST PROCEDURE</td>
<td>49</td>
</tr>
<tr>
<td>IX. TEST RESULTS</td>
<td>49</td>
</tr>
<tr>
<td>N-Q Curves</td>
<td>49</td>
</tr>
<tr>
<td>Yielding</td>
<td>49</td>
</tr>
<tr>
<td>Strength and Rigidity</td>
<td>50</td>
</tr>
<tr>
<td>Stresses</td>
<td>52</td>
</tr>
<tr>
<td>Instability and Modes of Failure</td>
<td>55</td>
</tr>
<tr>
<td>X. DISCUSSION OF TEST RESULTS</td>
<td>59</td>
</tr>
<tr>
<td>Connection P</td>
<td>59</td>
</tr>
<tr>
<td>Connections A to N</td>
<td>60</td>
</tr>
<tr>
<td>XI. DESIGN RECOMMENDATIONS</td>
<td>65</td>
</tr>
<tr>
<td>Square Knees</td>
<td>65</td>
</tr>
<tr>
<td>Bracketed or Haunched Knees</td>
<td>68</td>
</tr>
<tr>
<td>Curved Knees</td>
<td>68</td>
</tr>
<tr>
<td>Types of Welds</td>
<td>70</td>
</tr>
<tr>
<td>Lateral Bracing</td>
<td>70</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS (continuation)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>XII. RECOMMENDATIONS FOR FURTHER RESEARCH</td>
<td>71</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>74</td>
</tr>
<tr>
<td>APPENDIX A = Control Beam Test</td>
<td>82</td>
</tr>
<tr>
<td>APPENDIX B = Coupon Tests</td>
<td>85</td>
</tr>
<tr>
<td>APPENDIX C = Welding Sequences</td>
<td>91</td>
</tr>
<tr>
<td>APPENDIX D = Influence of Connection Behavior on Frame Deflection</td>
<td>92</td>
</tr>
<tr>
<td>APPENDIX E = Connection Design Details and Further Test Results</td>
<td>97</td>
</tr>
<tr>
<td>VITA</td>
<td>100</td>
</tr>
<tr>
<td>FIGURES</td>
<td>Vol.2</td>
</tr>
</tbody>
</table>
CONNECTIONS FOR WELDED RIGID PORTAL FRAMES

I - SYNOPSIS

This paper presents experimental and analytical studies made on welded-rigid connections for portal frames. The experimental results were obtained from "Compression" tests of fifteen specimens of various types of connections.

The tests were carried to plastic range and ultimate failure. Deflections, rotations and strains were measured and are reported herein. Conclusions are drawn as to strength, rigidity, and economy of the different connections tested, and recommendations are made for the adequacy of the connections, for the design of knees, and for further research.

II - INTRODUCTION

The investigations carried out and reported herein concern knees which are a component part of rigid portal frames. Consequently, it is appropriate to consider briefly rigid frames.

Rigid Frames

"A rigid frame is a bent or a series of bends which depend for stability on continuity at the joints."1

1 R. Cross and N. D. Morgan, Continuous Frames of Reinforced Concrete, John Wiley and Sons, New York, 1933, p. 197.
A bent may be defined as a structure composed of columns supporting a transverse girder.

Some types of rigid frames are shown in Fig. 1(a) to (g). The columns and beams in the frame shown are rigidly connected. These are but the simpler types. Types like one-story and multi-story rigid frames, with or without side aisles, and other complicated forms are not shown.

Rigid frames offer certain advantages because of continuity (1).

Consider Fig. 2(a) and (c). In (a) the beam BD is simply supported on the columns which have to carry a direct force only, i.e. the vertical reaction. The maximum bending moment and deflection for this structure are those of a simply-supported beam; i.e. \( M = \frac{PL}{6} \) kip.in. and \( \Delta = \frac{PL^5}{48EI} \) in. Now, if the beam BD and the columns AB and DE are made continuous by providing a rigid joint, the columns will be obliged to work together with the beam in supporting the load \( P \). As a result, the moment at the center of the beam will be reduced by an amount depending on the relative stiffness between beam and columns. The continuity in Fig. 2(c) due to rigid joints results in smaller maximum bending moments in the beam as shown in Fig. 2(b) and (d). Consequently, the structure in (c) may be lighter and more

---

(1) Numbers in parenthesis refer to references at the end.
economical than the one in (a), depending on factors such as fabrication, erection, engineering costs and others.

Speaking on the same subject Griffith states:

Perhaps the most important factor contributing to the popularity of steel rigid frames has been their attractive appearance, and their adaptability to riveted or welded fabrication. These frames may be efficiently constructed for flat, gabled or curved roofs, and when so built of steel the economy and speed of erection is not surpassed with any other type of material or form of construction.

Historical

The rigid frame for roof and bridge construction was introduced into this country for the first time in the early twenties and was used in grade separations for the parkways of West Chester County, New York (2). For many years prior to this time, however, rigid frame construction was used in this country in the form of wind bracing for tie buildings.

The first frames built were of reinforced concrete, but later steel frames were erected (3), (4), (5), (6). At present, rigid frames of steel are extensively used in subways, grade separations, industrial buildings, gymnasiuims, auditoriums, etc. In recent years, rigid frames have achieved great popularity. Their even more extensive

---

use in the future is no longer a matter of doubt. During the last twenty years, engineers in this country became more acquainted with this type of structure. With the experience gained, the analysis of the simpler types of rigid frames is regarded as routine standard in engineering design practices.

Method of Analysis and Design

In order to obtain a good perspective of current trends in the design of rigid frames, it would be helpful to review very briefly the methods of analysis and design of rigid frames.

Elastic: The analysis of complicated types of rigid frames may become a task requiring considerable ingenuity. However, any method of analysis for indeterminate structures such as Moment Distribution, Column Analogy, Slope Deflection or Method of Work may be used to analyze rigid frames (1), (2), (7), (8).

When the loading, span, and height are fixed, the rigid frame is first proportioned arbitrarily. Then, it is analyzed with one of the methods mentioned above and the induced stresses checked. In no instance are the stresses in any component member of the frame allowed to exceed the elastic limit of the material or a given allowable stress. Members with higher stresses than the allowable are redesigned. If after the revisions, the frame is considerably different than the original a new analysis may be made (8), (9).
Plastic: In present-day engineering practice the elastic analysis is the method in use. It results in safe structures and is time-proven. However, structures designed by this method may not be the most economical. John F. Baker speaking on elastic design goes so far as to say:

"The orthodox method, which is irrational in many respects, cannot lead to the most economical use of steel."3. Therefore, during the last twenty years, a new design method has been proposed by some prominent engineers (10), (11), (12), (13). Based on the ultimate load-carrying capacity, this method utilizes the reserve strength stored in the different component members if they are allowed to be stressed to the yield point (14), (15), (16). *

As illustration of the above, consider the fixed-ended beam shown in Fig. 3(a) loaded at third points. As the load is increased, the extreme fibers of the beam at the support will reach the yield point G_y of the material first. The bending moment diagram then is as shown dotted in Fig. 3(b). When this condition is reached, the load on the third points of the beam will be:

---


*) The material is structural (mild) steel.
\[
P_y = \frac{M_y L}{2} = \frac{3E_y}{2} \frac{3L}{6}
\]

As the load is increased, plastic zones will be developed at the ends of the beam, and this condition will continue until the ends become completely plastic. As the load is further increased, the full plastic moment \( M_p \) at the ends will remain constant, and a plastic zone will be formed at the center. The final stage is reached when the center becomes fully plastic and the beam is no longer able to resist additional load. When this stage is reached the structure deflects continuously under the load. It is then said that the structure, having been reduced to a kinematic mechanism, has collapsed.

The extent of the plastic zones and the bending moment diagram at this stage are shown in Fig. 5(a) and (b). Since both at the ends and center the moment is \( M_p \), the plastic hinge moment, the collapse load is:

\[
P_p = \frac{6M_p}{L} \quad (2)
\]

or from expressions (1) and (2)

\[
\frac{P_p}{P_y} = \frac{4}{3} \times \frac{M_p}{M_y} = \frac{4}{3} \times \text{shape factor}
\]

The shape factor for I-beams or WF sections is between 1.1 and 1.2. Taking an average minimum value of 1.125, we have:

\[
P_p = \frac{4}{3} \times 1.125 \quad P_y = 1.3 \quad P_y
\]
that is 50 per cent higher than \( P_o \), the maximum load for elastic design.

The analysis and design of rigid frames with the method of "plastic" or "limit" design is rather simple. The loads on the structure, which will cause "plastic hinges" to be developed and reduce the frame to a mechanism, are the limit loads. For instance, in Fig. 4(a) and (b), the frame has been reduced to a mechanism by the development of the plastic hinges at B and D or at C and D. At such a loading stage, the structure is said to have reached its carrying capacity. A "load factor" is applied to the limit loads to obtain the working loads the structure will carry. Frames designed by this method are reported to be more economical than those designed by the elastic method (11).

Knees and Connections

It was pointed out above that in a frame the component members (beams and columns) work together as integral parts of the structure to support the loads. To obtain this working together, the members are joined rigidly to each other with rigid connections (or joints) and knees.

In general a connection or a joint is any fastening arrangement by which different members in a structure are connected. Some connection types are shown in Fig. 6(a)
and (b), where (a) are joints for tier buildings, (b) a corner and an interior connection for a rigid frame.

In rigid frames the connections used are of the general types shown in Fig. 6(b), the knee and the interior connection with different shapes and designs. Knees will be defined as rigid connections for portal frames or as corner connections in multi-span rigid frames.

Welding

The connections for rigid frames may be riveted, bolted or welded. The last of these three methods is comparatively new, but has already gained the recognition of the engineering profession.

The success of structural welding can be attributed to (a) satisfactory performance, (b) proved economies and (c) research (4).

The advantages of welding are described in the Welding Handbook (17) as follows:

Due to the resulting homogeneity of the completed work, welding is particularly well suited to rigid frame construction. It also offers the advantages of (1) a compactness of joint with an attendant saving in connection material and (2) a simplicity of joint which, if the steel work is left exposed to view, enhances the general appearance of the building interior. Because of these advantages, it is in the rigid-frame type of structures that welding has been most effectively used in building construction.4

---

A. Amirilian, Head Designer, Bureau of Yards and Decks, speaking on the same subject states:

There has been ample evidence, too, regarding the savings due to the use of welded framings. These economies have been proved by competitive bids, as well as by actual construction costs.  

The compactness of welded connections results in rigidity with the least amount of material and cost. Consequently, it is advantageous to use welded connections in rigid frames. At a meeting of the Structural Steel Committee of Welding Research Council that approved initiating the Lehigh investigations of continuous welded frames, A. Amirilian stated, "To properly exploit the full advantages of welding fully continuous construction must be utilized."

Need for Research and Previous Work

Structural Engineering is an applied science which is based mainly on certain fundamental principles of mechanics. However, in developing the various theories used in Structural Analysis and Design, certain assumptions are made. The validity of these assumptions and the soundness of the theories based on them must be checked by experiments. A theory has to be verified by experiment to be of practical use.

---

Therefore, it is of utmost importance that the assumed behavior of welded connections be checked by tests. The demand for information on the actual behavior of welded connections became more imperative with the advance of the welding processes which made structural welding more economical.

Realizing this need, various research projects investigating column to beam connections were carried out in this country and in England.

In general, welded beam-to-column connections are of three types:

a. flexible  
b. semi-rigid  
c. rigid

A considerable number of tests have been carried out at Lehigh University on flexible and semi-rigid types of beam-to-column connections used mainly in tier building construction.

The tests were carried out mainly on actual connections as used in practice (19), (20), (21), (22). The same subject was also studied by photo-elastic methods (23). Experiments on a full-scale building frame with semi-rigid connections were carried out by Johnston and Mount (24).

A literature survey showed that data are still inadequate for rigid connections which have not been investi-
gated to a great extent. In England, Hendry and others reported on rigid connections for frames (25), (26), (27), (23). Some further research was carried out in this country and reported in (29), (30), (31), (32). Tests on full-scale and model rigid frames were carried out both in England and in this country (33), (34), (35).

However, the information obtained from the investigations on connections listed above is not sufficient. None of these investigations deal with the behavior of the connections when the connection has entered the plastic range. Consequently, the results of these scattered investigations are not enough to draw final conclusions even for elastic behavior, let alone plastic behavior.

Connections are a very important component of a rigid frame since they are the ties between beams and columns, transferring the stresses from one to the other. They can be looked on as "traffic circles" for stresses. Therefore, their behavior influences the general behavior of a rigid frame to a great extent.

In view of the interest shown in plastic design, it is of importance that the behavior of welded rigid connections be investigated beyond the elastic limit and to ultimate failure. It is with this purpose in mind that the present series of tests was carried out in Fritz Engineering
Laboratory. The data obtained could, however, be useful for studies, both in the elastic and the plastic range.

Knees for Portal Frames

The present preliminary investigations were restricted to certain types of knees for welded portal frames. Fig. 5 shows various portal frames and the types of knees that could be used. The knees shown were either types already used in existing structures, or types suggested by authorities in this field. The beams in the frames shown are horizontal, but they could also be sloping as in gabled roof frames for industrial buildings and gymnasiums.

Objectives of Research on Connections and the General Objective of the Project

A review of the literature on welded connections showed that there is very little information on the plastic behavior of rigid connections. Therefore, the objectives of the present investigations were set-up with plastic design in mind. The objectives of the connection test can be summarized as follows:

1. To study the basic behavior of rigid connections, both in elastic and plastic range with emphasis in the second.
2. To study the rigidity of connections.
3. To study the strength of connections.
4. To obtain information on the cost of fabrication of connections.
5. To study the influence of design details within several connection types.

6. To observe the mode of failure of rigid connections.

7. To compare the various types investigated.

8. To obtain stresses and check the stress analysis theories.

9. To draw conclusions.

The connection investigations reported herein are part of a five-year program on "Welded Continuous Frames and Their Components". The ultimate objective of the project is to study the carrying capacities of steel frames loaded in the plastic region. Dr. Bruce Johnston, under whose direction work has been carried on this far, has summarized the objectives of the project as follows:

1. To determine the behavior of steel beams, columns, and continuous welded connections with emphasis on plastic behavior, and the development of theories to predict such behavior.

2. To determine how to proportion various types of welded continuous frames to develop the most balanced resistance in the plastic range and thus reach the greatest possible collapse load.

3. To determine procedures of analysis that will enable one to calculate the collapse loads of welded continuous frames and verify the analysis by suitable tests.

4. To determine procedures of analysis that will enable one to calculate the elastic and permanent deformations in welded continuous frames in the range intermediate between elastic limit and collapse loads.
6. To explore limitations in the application of plastic range design over and above deformation limitation, namely, fatigue, local buckling, lateral buckling, etc.

6. To develop practical design procedures for the utilization of reserve plastic strength in the design of continuous welded frames.

XII — REQUIREMENT FOR CONNECTIONS

Requirements for connections vary according to the type of structure in which they are used. The following is a short review of the requirements in various types.

**Pin-connected Joints:** It is assumed that no moment can be transferred by a pin-connected joint, since the members joined are free to rotate. The pin should be able to transfer the direct forces in the members through shear.

**Flexible Connections:** Beam-to-beam or beam-to-column connections used in standard steel building construction are connections of this type. These connections must be capable of carrying the required end reaction and must be flexible enough to allow with safety the full end rotation which might be expected in a freely supported simple beam. The beam end moments which would be developed through the use of this type of connections are a small percentage of the full fixed-end moment and they are usually not considered. (36). See Fig. 7(a).
Semi-rigid Connections: They are connections which transmit bending moment with some degree of rotation between the axes of the members connected, thereby providing a degree of restraint somewhere between complete flexibility and full rigidity. The bending moment diagram for a beam with such connections is given in Fig. 7(b). Many connections assumed as "flexible" are inherently "semi-rigid".

Rigid Connections: Rigidity in connection is a relative term. In steel building construction rigid connections are those in which the relative rotation between the end of the beam and the column is reduced to a minimum. See Fig. 7(c). Such connections should be capable of developing the fixed-end moment of the beam.

For knees of portal frames the connection requirements are different. Failure in rigid frames usually means excessive deformations after the structure is reduced to a mechanism.\(^6\) See Fig. 4(a) and (b). However, before this stage is reached the deflections have to be relatively small. Rotations in a knee larger than certain minimum values will result in additional deflections which are undesirable. Therefore, the first requirement in a knee is rigidity. Since there is no "hundred-percent" rigid

---

\(^6\) When a frame keeps on deflecting under constant load it is said that it has been reduced to a mechanism. See page 6.
Knee, the required rigidity will be defined relative to the rolled sections (members) joined by the knee. "A knee should be at least as rigid as the weakest of the members it joins." The rigidity of knees can be evaluated by Moment-Rotation curves, as shown later.

A frame reaches its maximum carrying capacity when it develops a certain number of "plastic hinges". Therefore, a knee should be strong and efficient enough to develop the plastic hinges at the end of the rolled section, Fig. 3, section AA, or in the knee itself. The place where the plastic hinge is to be developed depends on the design.

High rigidities and strengths can be obtained by use of more material. However, the connection must be so designed as to develop the required rigidities and strengths with the least cost in material and fabrication.

To illustrate further, consider Fig. 10 where the Moment-Rotation Curves of two connections B and C are compared to the rotations of the rolled section (beam) over "an equivalent length" given by curve A. The equivalent lengths (AB + BC) of some typical connections are shown in Fig. 9. Knee C develops higher moments than the rolled section and shows less rotation than the rolled section; therefore it is satisfactory. Knee B is unsatisfactory because it develops neither the plastic moment or
"Hinge value", nor the rigidity of the rolled section. If knee C behaved as shown in D (dotted), it would be regarded as unsatisfactory also since it does not carry the maximum load through large rotations up to $\phi_1$ which may be required in developing the hinge at point C, Fig. 4(b). Fatigue resistance and aesthetics are additional requirements which in certain cases may become very important.

The requirements for Portal Frame Rigid Knees, therefore, are as follows: rigidity, strength, economy, rotation characteristics after the maximum moment is reached, resistance to fatigue and aesthetics.

IV - THEORETICAL ANALYSIS OF CONNECTIONS

The following are some theoretical considerations regarding stresses and deformations in knees.

Elastic Stress Analysis of Knees

Some theories for an elastic stress analysis of rigid knees have been proposed. The following is a very short account of these theories for square and curved knees.

Square Knees: F. Bleich (37) has proposed approximate methods for stress analysis and design of square knees. He states that for square knees:

....where the ratio of the length of the restraining arm to its depth is equal or larger than one, the Navier theory yields sufficiently accurate results and one may determine the fiber stresses and shear stresses according to the conventional theory.
Curved Knees: The two methods proposed for the analysis of this type of knees are as follows:

a. Meich's Method (37). In any rational analysis for such knees, two factors must be taken into consideration:
   1. The curvature of the centerline of the knee
   2. The rapid change in cross-section at the knees.

For the derivation of the analysis, circular (or cylindrical) sections are used as in the wedge theory of the Theory of Elasticity. The curved centerline of the beam is defined as the line connecting the centers of gravity of the circular sections, see Fig. 11(a). 0b and 0a are the two tangents, and abc is the circular arc. The method of derivation is based on the assumption that "the elongation of a fibre perpendicular to the cylindrical section, due to the fibre stress $G$ caused by a bending moment, is proportional to the length of the arc $v". Proceeding with the above assumption and considering the behavior of a small element of the curved bar, a basic relation for fiber stress is obtained:

$$G = \frac{1}{\cos \alpha} \left[ \frac{H}{A} - \frac{H}{\rho A} \frac{E_v}{\rho} \frac{\rho}{\rho + v} \right]$$  \hspace{1cm} (3)$$

where $A =$ the total area of the cylindrical cross-section.

$2 \alpha =$ the angle between the tangents at the end of the cylindrical section.

$N, H =$ the normal force and the moment at the section.
\[ \rho = \text{the radius of curvature of the centerline.} \]
\[ v = \text{the distance from the centerline.} \]
\[ Z = \text{a magnitude analogous to the moment of inertia defined by the integral:} \]
\[ Z = \rho \int \frac{v^2}{\rho + v} \, dA \]

If \( \rho > 2d \) (\( d = \text{depth of the cross-section} \)), \( Z \) may be replaced by the moment of inertia \( I \).

To obtain extreme fiber stresses formula (3) could be simplified as follows:

Since \( A = \frac{I}{\rho^2} \), the formula for maximum flange stress, when \( v = c \), is:

\[ G = \sec \alpha \left[ \frac{I}{A} - \frac{I}{I} \left( \frac{c}{\rho} + \frac{\rho}{\rho + c} \right) \right] \]

For the type of sections commonly used in rigid frame construction, \( I^a = \frac{2}{3} c^a \) is a good approximation. See Fig. 11(b). Using this in the last equation

\[ G = \sec \alpha \left[ \frac{I}{A} - \frac{I}{I} \left( \frac{2}{3} \frac{c}{\rho} + \frac{\rho}{\rho + c} \right) \right] \]

Let \( K = \frac{\rho}{\rho + c} + \frac{2}{3} \frac{c}{\rho} = \frac{1}{1 + \frac{c}{\rho}} + \frac{2}{3} \frac{c}{\rho} \) (Plot \( K \) against \( \frac{I}{\rho} \))

Then,

\[ G = \left[ \frac{I}{A} - \frac{I}{I} K \right] \sec \alpha \quad (4) \]

\( ^{a) \text{Proposed by H. D. Ritschel.} } \)
Since the direct stress is small, Eq. (4) can be written as

\[ G = \frac{N}{A} - \frac{M}{I} \cdot K \sec \alpha \]  
(5)

K can be directly obtained from tables or graphs. Equation (5) is a suitable formula to be used for analysis and design. The above relation can be applied to the analysis of connection as shown in Fig. 11(c). The N and M shown acting on the cylindrical section AB are negative.

b. William Osgood has proposed a method of analysis based on the Wedge Theory as applied to the problem of tapered beams (39). The following, in a very condensed form, are the results of the theory:

Given the tapered beam shown in Fig. 12(a) loaded with \( P_1, P_2 \) and \( N \), the author derives the following expressions for normal forces caused by the external loads at any circular section A-E:

\[ G_{r_1} = \frac{P_1 \sin \theta}{t_\theta (\alpha - \sin \alpha \cos \alpha) + 2A \sin \alpha} \]  
(6)

\[ G_{r_2} = \frac{P_2 \cos \theta}{t_\theta (\alpha + \sin \alpha \cos \alpha) + 2A \cos \alpha} \]  
(7)

\[ G_{r_3} = \frac{2M \sin 2\theta}{t_\theta (\sin 2\alpha - 2 \alpha \cos 2\alpha) + 4 \alpha \alpha \sin 2\alpha} \]  
(8)

where \( G_{r_1}, G_{r_2} \) and \( G_{r_3} \) = normal (fiber) stresses due to \( P_1, P_2 \) and \( N \)

\( \theta \) = angle between centerline and any fiber

\( \alpha \) = angle between centerline and extreme fiber
When the two flanges have different cross-sections, then
the extreme fiber angles become $\alpha_1$ and $\alpha_2$, and the above
expressions can be modified to (6a):

$$ F_1 = \frac{P_1 \sin \theta}{(A_0 \sin^2 \alpha_1 + A_2 \sin^2 \alpha_2) + \frac{P_2}{2} (2 \alpha_1 + 2 \alpha_2 - \sin 2 \alpha_1 - \sin 2 \alpha_2)} $$  

$$ F_2 = \frac{-P_2 \cos \theta}{(A_0 \cos^2 \alpha_1 + A_2 \cos^2 \alpha_2) + \frac{P_2}{2} (2 \alpha_1 + 2 \alpha_2 + \sin 2 \alpha_1 + \sin 2 \alpha_2)} $$  

$$ F_3 = \frac{N \sin \theta}{\rho (A_0 \sin 2 \alpha_1 + A_2 \sin 2 \alpha_2) + \frac{P_2}{2} \left[ \frac{1}{2} (\sin 2 \alpha_1 + \sin 2 \alpha_2) - (\alpha_1 \cos 2 \alpha_1 + \alpha_2 \cos 2 \alpha_2) \right]} $$  

where $A_0$ = Area of outside flange
$A_2$ = Area of inside flange

The theory and equations derived for straight flanges
are applied as an approximation to beams with curved flanges.
An example of the use of the theory is its application to
rigid knees with curved inner flanges. Consider the point
A in the knee shown in Fig. 12(b). If the tangent is drawn
at this point, its intersection with the horizontal straight
flanges gives the center of the circular arc AB whose radius
is $\rho$. The forces $F_1$, $F_2$, and $N$ are calculated from $\rho$ and $\nu$

as follows:
\[ P_1 = V \cos \alpha - H \sin \alpha \]
\[ P_2 = V \cos \alpha + V \sin \alpha \]
\[ M = v \left[ a = \frac{h \cos^2 \alpha - R(1 - \cos 2\alpha)}{\sin 2\alpha} \right] \tag{9} \]
\[ \rho = \frac{h + R(1 - \cos 2\alpha)}{\sin 2\alpha} \]

Different circular sections can be considered and the extreme fiber stresses can be calculated by use of equations (6), (7) and (8). In case the flanges are of different areas, equations (6a), (7a) and (8a) must be used instead. For each case the forces \( P_1 \), \( P_2 \), and \( M \) must be calculated.

To illustrate the use of the wedge theory formulas (6), (7), and (8), the strains in Type 4 connections tested are calculated below. (See Fig. 12(a) and Fig. 17)

From the bottom of the bracket a line is extended until it hits the upper flange of the beam at \( C \). The forces \( P_1 \), \( P_2 \) and the moment \( M \) at this point are calculated. With center point \( C \), three circular arcs are drawn through points \( A \), \( B \), and \( C \), and the radii are found to be 20.3 in., 16.3 in. and 17.2 in., respectively. The angle \( \alpha \) is 22.5° for all three points. The angle \( \theta \) is +22.5° for the tension flange (points \( A \) and \( B \)) and -22.5° for the compression flange (point \( C \)). The stresses at these points are calculated as follows:
Point C: $\alpha = 22.5^\circ = 0.3927$ radians

and \sin \alpha = 0.3927; \cos \alpha = 0.9239

Substituting in expression (9),

$$P_1 = 0.3927 \; P; \quad P_2 = 0.9239 \; P; \quad H = 13.62 \; P$$

From eqs. (6), (7), and (8),

$$G_{p_1} = \frac{0.3927 \; P \times (-0.3927)}{1/4 \; \times 17.2(0.3927 - 0.3927 \times 0.9239) + 2 \times 1/4 \; \times 4(0.3927)}$$

$$G_{p_1} = 0.312 \; P$$

$$G_{p_2} = \frac{0.9239 \; P \times 0.9239}{4.3(0.707) + 2 \times (0.9239)^2}$$

$$G_{p_2} = 0.174 \; P$$

and,

$$G_{p_3} = \frac{2 \times 13.62 \; P \times (-0.707)}{17.2(4.3(0.707 - 2 \times 0.393 \times 0.707) + 4 \times 1 \times 0.393 \times 0.707)}$$

$$G_{p_3} = 0.635 \; P$$

Therefore,

$$G = G_{p_1} + G_{p_2} + G_{p_3} = (-0.312 + 0.174 + 0.635) \; P = -1.121 \; P$$

and \( \varepsilon = \frac{1.121 \; P}{80.4} \times 10^{-6} = 0.00013 \times 10^{-6} \) P

The same procedure followed for the other two gages gives:

$$G = (+0.285 - 0.185 + 0.503) \; P = +0.444 \; P$$

and \( \varepsilon = 0.0010 \times 10^{-6} \) for point A,

$$G = (+0.321 - 0.180 + 0.691) \; P = 0.652 \; P$$

and \( \varepsilon = 0.0020 \times 10^{-6} \) for point B.
Rotations in Knees (elastic)

It is of great importance to know the rotations that take place in a knee area. Such rotations, as was stated earlier, must be smaller than a certain minimum. The following are some methods by which rotations in certain types of knees are found in the elastic range.

Square Knees without Diagonal Stiffeners (Types 5, 7, 8): Consider the connection shown in Fig. 13(a). The rotations in the knee ABCD will be found by making the following assumptions:

a. The bending moment \( M \) at the section DA is taken entirely by the flanges. In the knee shown, \( M = V(\frac{d}{2} - \frac{t}{2}) \)
and the flange force \( P = V(\frac{d}{2} - \frac{t}{2}) \).

b. Shear, \( V = 0.707P \), is taken by the web and the normal force is disregarded.

c. The flange force varies with a straight-line variation between D and C, with minimum at D and zero at C.

d. Stress concentrations are not considered.

e. Navier's assumptions for bending assumed to apply.

Fig. 13(a) shows the forces on the knees. The shearing stress in horizontal sections is:

\[ T = \frac{V\left(\frac{d}{2} - \frac{t}{2}\right)}{A_w} \]

Therefore, \( \phi_1 = \frac{T}{G} = \frac{V\left(\frac{d}{2} - \frac{t}{2}\right)}{A_w G} \) (See Fig. 13(b)) (10)
The bending moment in the knee at any section $x$ inches from line $CD$ is

$$M = (V \frac{b}{d}) \frac{x}{2} + V (\frac{b}{d} - \frac{1}{2}) x = \frac{V b}{d} x$$

The rotation between $AB$ and $CD$ is given as:

$$\frac{1}{2} \int_{0}^{d} \frac{V b}{d} x \, dx = \frac{V b}{2d} \int_{0}^{d} x \, dx = \frac{1}{2} \frac{V b}{d} \frac{d^2}{2} = \frac{V bd}{4d}$$

Total rotation $\phi = \frac{V}{A_p G} \left( \frac{b}{d} - \frac{1}{2} \right) + \frac{V bd}{4d}$  \hspace{1cm} (11)

**Example.** Formula (11) will be used to find the rotations in connection $F$, Type 7. (See Fig. 17)

Given: $d = 15.48$ in., $I = 102.6$ in$^4$

$f = 64$ in.$,$

$r = 30 \times 10^6$ lb./sq.in.,

$E = 30 \times 10^6$ lb./sq.in., $A_p = 0.27 \times 3 = 2.16$ in$^2$

\[
\phi = 0.707 \left[ \frac{34}{2.16 \times 11.3 \times 10^6} + \frac{86 \times 15.48}{4 \times 30 \times 10^6 \times 102.6} \right]
\]

\[
\phi = 0.707 \times 10^{-6} F(0.226 + 0.092) = 0.317 \times 10^{-6} \text{ radians}
\]

**Square knees with diagonal stiffeners (Types 2, 8B):**

Rotations due to shear in the square knee $ABCD$ reinforced with diagonal stiffeners (Fig. 14) will be found by making the following assumptions:

a. The thrust of the two compressive forces $\frac{V_l}{b_2}$ and $\frac{V_2}{b_2}$, where $L$ = distance between section $PA$ and the external load, is taken up by the stiffener at the point $A$. 
b. The stress on the stiffener varies from maximum at A to zero at C.

c. Stress concentrations and rotations due to bending are disregarded.

From assumption b it follows that the stiffener stresses cause uniform shear $T_{g}$ in the web of the knee; see Fig. 14(b).

The total contraction in the stiffener is

$$\Delta z = \frac{\sigma_{\text{max}}}{2E} \times h$$

(12)

where $h = \sqrt{h_{1}^{2} + h_{2}^{2}}$ and $\sigma_{\text{max}} = 0.7071\sqrt{\left(\frac{1}{h_{1}} + \frac{1}{h_{2}}\right)}$

Area of Stiffener

Consider now the web without any stiffener. Due to shearing forces this web will take the shape ABC'D'. From the distorted web, (Fig. 14(c)), the following relations are obtained:

$$C^{t}C^{n} = h_{g}; \quad E^{n} = h_{2} \sin \gamma = h_{2} \gamma; \quad C^{n} = h_{1} - h_{3} \gamma$$

where $\gamma$ = the shearing strain.

The change in diagonal length is

$$C^{t}C^{n} = \Delta L = \sqrt{h_{1}^{2} + h_{2}^{2}} - \sqrt{h_{2}^{2} + (h_{1} - h_{2} \gamma)^{2}}$$

$$\Delta L = \frac{2h_{1}h_{2} \gamma - h_{2}^{3} \gamma^{3}}{\sqrt{h_{1}^{2} + h_{2}^{2} + \sqrt{h_{2}^{2} + (h_{1} - h_{2} \gamma)^{2}}}} = \frac{h_{2} \gamma (2h_{1} - h_{2} \gamma)}{2h}$$

Since this change in length must be equal to the contraction in the stiffener

$$\frac{\sigma_{\text{max}}}{2E} \times h = \frac{2h_{1}h_{2} \gamma}{2h}$$
Quitting the term with $y^2$ as very small and solving for $y$
the shear rotation is found as

$$y = \frac{G_{\text{max}} h^3}{2 h_1 h g}$$  \hspace{1cm} (13)

Example. Equation (13) will be used to find the rotations in connection A, Type 2. (See Fig. 17.)

Since $h_1 = h_2$, $h^3 = 2h_1^3$
Therefore,

$$y = \frac{G_{\text{max}} h^3}{2 h_1 h g} = \frac{G_{\text{max}}}{E} = \frac{\sqrt{G}}{E}$$

where $G_0$ = stress in the compression flange at the re-entrant angle $= 2.317 P$

Substituting,$$y = \frac{\sqrt{G} \times 2.317 P}{29.4 \times 10^6} = 0.111 \times 10^{-6} P \text{ radians.}$$

If rotations are measured between points which are about one inch outside the knee on the stiffener, the rotations in these additional lengths must be considered. These have been calculated and found to be equal to $0.036 \times 10^{-6} P$. The rotations due to bending in the knee (diagonal stiffener omitted) is $0.068 \times 10^{-6} P$.

Therefore, total computed rotation $\theta = (0.111 + 0.036 + 0.068) 
\times 10^{-6} P = 0.214 \times 10^{-6} P$

Bracketed and Curved Knees (Types 3A and 4). The following is a method for obtaining rotations in such knees by use of the ordinary curved-beam theory. Consider a
Type 5A curved knee, Fig. 15(a), acted on by $N_0$, $V_0$, and $M_0$. These forces will produce rotation over the whole knee area from $A'A$ to $B'B'$. Now, consider a regular curved beam with the radius of the inner flange equal to the radius of the knee $R$. See Fig. 16(b). This beam, shown dotted in (a) also, should have approximately the same rotations as the knee in (a).

The internal forces at any section $CC'$, making $\Theta$ degrees with plane $BB'$ should be:

\[ N = N_0 + N_1 + N_2 = N_0 - N_0(\rho = \rho \cos \Theta) + V_0 \sin \Theta \]

\[ U = U_1 + N_2 = N_0 \cos \Theta + V_0 \sin \Theta \]

\[ V = V_1 + V_2 = N_0 \sin \Theta - V_0 \cos \Theta \]

and since $N_0 = V_0 = 0.707P$ for the connections tested

\[ \begin{aligned} 
N &= 0.707P \left( \sin \Theta + \cos \Theta \right) \\
V &= 0.707P \left( \sin \Theta - \cos \Theta \right) \\
M &= N_0 + 0.707P \rho (1 + \cos \Theta - \sin \Theta) 
\end{aligned} \]  

\[ (14) \]

The rotation at any section due to moment is

\[ \Delta \phi_1 = \frac{M_0 \rho}{AE} = \frac{M_0 \rho}{AE} \]

Therefore the total rotation over the whole length of the beam is

\[ \Delta \phi_1 = \int_{0}^{L} \frac{M_0 \rho}{AE} = \frac{1}{AE} \int_{0}^{L} M_0 \rho \left[ 0.707P (1 + \cos \Theta - \sin \Theta) d\Theta \right] \]

\[ = \frac{1}{AE} \left[ \frac{M_0 \pi}{2} + 0.707P \rho \left( \Theta \cos \frac{\pi}{2} \right) \right] = \frac{1}{AE} \left( 1.57M_0 + 0.504P \rho \right) \]
but in the connection tested \( \theta_0 = 0.707 P \times \delta \) where \( \delta \) = moment arm of the section at which the knee starts.

Substituting, we have

\[
\phi_1 = \frac{1}{AE} \left[ 1.11P^2 + 0.504P \right] \tag{15}
\]

The rotation at any section due to normal force is:

\[
\phi_2 = \frac{ \frac{Nd} {2E} } { \frac{AE} {2} } = \frac{Nd} {AE} \]

\[
\phi_2 = -\frac{1}{AE} \int_0^{\pi/2} 0.707P \left( \sin \theta + \cos \theta \right) d\theta = \frac{\sqrt{2}}{2AE} P \tag{16}
\]

The rotation at any section due to shear is

\[
\phi_3 = 2\alpha \frac{V}{AE} \quad \text{for } \alpha = 2.2 \text{ and } V = V_0 = 0.707P
\]

\[
= 4.4 \frac{V_0}{AE} = 3.1 \frac{P}{AE} \tag{17}
\]

Therefore, total rotation in the knee = \( \phi_1 + \phi_2 + \phi_3 \).

To this any rotation in the rolled section should be added as \( \phi_4 = \frac{M}{EI} d \) if comparison with measured values is to be made.

The eccentricity \( e \) used in equation (16) is the distance between centroidal and neutral axes, that is \( e = \rho - \nu \) where \( \rho \) = radius of centroidal axis,

\[ \nu \] = " neutral axis.

The first is given by the specimen under investigation while the second can be obtained from the expression, (see Fig. 15(c)).
\[
p = \frac{2bt + w(h-2a)}{\log_2 \frac{a}{d} + \log_3 \frac{b}{d} + \log_2 c/e}
\]

**(Example)** To find the theoretical rotations in the knee of the connection 6. See Fig. 16. Substituting the proper values in equation (19), the radius to the neutral axis

\[
p = \frac{2 \times 4.03 \times 1/4 + 1/4(8.03 - 0.03)}{4.03(0.00733 + 0.00828) + 1/4 \times 0.21038} = 35.53 \text{ in.}
\]

and \( e = 4 - p = 56.03 - 35.53 = 0.50 \text{ in.} \)

The rotations are:

\[
\phi_1 = \frac{F \times 10^{-6}}{4.03 \times 29.4 \times 0.50} (1.21 \times 21.5 + 0.306 \times 56.03)
\]

\[
= 0.590 \times 10^{-5} \text{p}
\]

\[
\phi_2 = \frac{1.414F}{112 \times 10^6} = 0.012 \times 10^{-5} \text{p}
\]

\[
\phi_3 = \frac{3.1F}{45.2 \times 10^6} = 0.063 \times 10^{-5} \text{p}
\]

\[
\phi_4 = 1.3 \times \frac{14.65F}{42 \times 29.4 \times 10^6} = 0.019 \times 10^{-5} \text{p}
\]

Total computed rotation = \((0.590 + 0.012 + 0.063 + 0.019) \times 10^{-5} \text{p}
\]

\[
= 0.666 \times 10^{-5} \text{p}
\]

**Initial Yielding in Square Knees of Types 5, 7, 8 \((\tau = 5)\)**

It is of importance that early yielding does not occur in knees, since such a yield will cause large deflections. Therefore, the investigation of yielding in a knee web especially in types 5, 7, and 8 which are not reinforced diagonally is valuable.
Given the square knee shown in Fig. 16(a) acted on by \( V, H, \) and \( M \), to find moment \( M \) under which yielding occurs in the web, assume:

a. \( I = 2bt \times (\frac{a}{2})^2 = \frac{bt a^2}{12} \) aprr. (See Fig. 16(b)).

b. Disregard effect of normal force \( N \) and stress concentrations.

c. Shear stresses uniform along horizontal sections such as \( W \).

The bending stresses at the section DA cause shearing stresses at the web with the maximum shear force acting in the geometrical center of the knee and equal to

\[
P_1 + P_2 = f(1 - \frac{t}{a}) \cdot bt + \frac{f}{3} (1 - \frac{2t}{a})(\frac{a}{2} - t)w \quad (See \ Fig. \ 16(c))
\]

Therefore,

\[
\tau = \frac{P_1 + P_2}{dw} = f(1 - \frac{t}{a}) \cdot \frac{bt}{dw} + \frac{f}{3} (1 - \frac{2t}{a})(\frac{a}{2} - t)
\]

Substituting, \( f = \frac{Md}{E} \) where \( M \) is the moment at section DA, and simplifying we have:

\[
\tau = \frac{M}{tw} \left[ \frac{bt}{w} (1 - \frac{t}{a}) + \frac{1}{3} (1 - \frac{2t}{a})(\frac{a}{2} - t) \right] \quad (29)
\]

Since the normal stresses are negligible at the center of the knee, and a state of pure shear may be assumed,

\[
\sigma_1 = -\sigma_2 = \tau
\]
and the octahedral shear stress relation \( \sigma_y^o = \sigma_1^o + \sigma_2^o + \sigma_3^o \) is reduced to
\[
\sigma_y^o = 3 \tau^o
\]
By substituting the value of \( \tau \) given above, the moment at initial yielding can be obtained as follows:
\[
\sigma_y = \sqrt{3} \left[ \frac{h}{w} \left( 1 - \frac{t}{d} \right) + \frac{1}{2} \left( 1 - \frac{6t}{a} \right) \left( \frac{d}{d} - t \right) \right]
\]
and
\[
M_1 = \frac{2I \sigma_y}{\sqrt{3} \left[ \frac{h}{w} \left( 1 - \frac{t}{d} \right) + \frac{1}{2} \left( 1 - \frac{6t}{a} \right) \left( \frac{d}{d} - t \right) \right]}
\]
where \( M_1 \) = moment at section DA which will cause yielding at the center of the knee,
\( I \) = the moment of inertia (calculated approximate or handbook value)
\( \sigma_y \) = the lower yield point of the material
\( b, t, w \) = as shown in Fig. 16(b)

At the extreme fiber of section AD,
\[
\sigma = \frac{Mh}{2Tw} + \frac{Ma}{A(t - d/2)} = \mu \left( \frac{d}{2Tw} + \frac{1}{A(t - d/2)} \right)
\]
where
\( A \) = area of the rolled section
\( z \) = distance from the center of the knee to the external force
\( d \) = width of the knee. See Fig. 16(a).
The octahedral shear theory applied to the extreme fiber stress at section DA results in:

\[ \sigma_y^4 = 6 \]

and by substituting in equation (21)

\[ \sigma_y = \sigma = \frac{d}{EI}\left[1 + \frac{1}{A(t + d/2)}\right] \tag{22} \]

where \( M_0 \) is moment to cause yielding in the extreme fiber of the compression side.

The following cases may arise in respect to knees:

- If \( M_0 > M_1 \) Initial yielding occurs in the knee (shear yield)
- If \( M_0 < M_1 \) Initial yielding occurs outside knee (bending yield)
- If \( M_0 = M_1 \) Initial yielding occurs simultaneously inside and outside the knee.

A simpler procedure for investigation of yielding can be obtained if the bending stresses at the web at the section AD are neglected and the moment is assumed to be taken totally by the flanges. Then the shear in the web of the knee is

\[ I = \frac{V(\frac{1}{a} - \frac{1}{b})}{wI} \tag{See also Fig. 13(c)} \]

In the center of the knee \( G_1 = -G_2 = I \)

therefore,

\[ V_y = \frac{1}{\sqrt{3}} \frac{V}{Q} \frac{x w x}{(\frac{L}{a} - \frac{L}{b})} \]

and in the case of the connections tested, \( V_y = \frac{I W G_y}{\sqrt{3} Q x 0.707(\frac{L}{a} - \frac{L}{b})} \)
The extreme fiber stress due to bending

\[ G = \frac{M}{I} = \frac{\text{val}(\frac{d}{2} - \frac{d}{2})^3}{2} \]

and

\[ V_y = \frac{21G}{(\frac{d}{2} - \frac{d}{2})^2} \quad \text{and} \quad F_y = \frac{21G}{0.707d^2(\frac{d}{2} - \frac{d}{2})} \]

The smaller of the two, \( V_y \), determines the place at which yielding will start.

Some rolled sections have been investigated with the method developed above. The results of this investigation, given in Table I, show that in square knees with no diagonal stiffener yielding takes place at the web of the knee.

<table>
<thead>
<tr>
<th>Table I</th>
</tr>
</thead>
</table>

### Initial Yield in Square Knees

<table>
<thead>
<tr>
<th>Section</th>
<th>( d/y )</th>
<th>b/t</th>
<th>( I )</th>
<th>Location of Initial Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x416x1</td>
<td>55.0</td>
<td>12.0</td>
<td>7449</td>
<td>Web of knee (shear yield)</td>
</tr>
<tr>
<td>24x1410</td>
<td>47.4</td>
<td>14.1</td>
<td>8315</td>
<td>Web of knee and extreme fiber simultaneously</td>
</tr>
<tr>
<td>21x1282</td>
<td>41.0</td>
<td>11.5</td>
<td>1752</td>
<td>Web of knee (shear yield)</td>
</tr>
<tr>
<td>16x1266</td>
<td>37.0</td>
<td>11.0</td>
<td>833</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>14x1260</td>
<td>51.4</td>
<td>17.5</td>
<td>229</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>12x1252</td>
<td>41.2</td>
<td>15.5</td>
<td>55.5</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>16x1260</td>
<td>41.4</td>
<td>18.0</td>
<td>52.7</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>12x1252</td>
<td>41.6</td>
<td>16.4</td>
<td>41.7</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>16x1260</td>
<td>25.0</td>
<td>23.2</td>
<td>30.3</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>16x1260</td>
<td>25.0</td>
<td>14.3</td>
<td>21.7</td>
<td>q, a, q, a, a, b</td>
</tr>
<tr>
<td>6x416x1</td>
<td>34.8</td>
<td>20.3</td>
<td>15.0</td>
<td>q, a, q, a, a, b</td>
</tr>
</tbody>
</table>
V - TEST PROGRAM

To investigate whether or not various types of knees meet the requirement for rigid connections discussed above, a few types were considered for testing. In the following a short description is given of the specimens tested. Detail drawings for all connections are shown in Figs. 115 to 129.

Test Specimens (Connections)

The knees tested are of types 2, 2B, 4, 5A, 7, 6B, 15 and 16, illustrated in Fig. 17 and 18. Many of the selected types were proposed by various members of the Lehigh Sub-committee which supervised the investigation or by such authorities in the Structural Engineering field as A. Amirikian, T. R. Higgins, J. Jones and others.

Some further reasons for selecting these types are:
1. frequent use of the type in practice as ascertained from literature surveys,
2. appearance,
3. economy.

Connection F, type 7, was considered because it has been frequently used in practice. Furthermore it is regarded as an economical type to fabricate. See Fig. 17. The two legs of the connection consist of a 14WF30 as a beam and 6WF21 as column. The sections have been selected as typical sections used in actual construction. The 6WF21 has been tested as a simple beam and the results reported by Luxton and Johnston (40). Test of 14WF30 as a continuous beam has also been carried out at Lehigh University (41).
After the completion of the test of Connection P, Type 7, it was felt that it would be advisable to use (a) a lighter rolled section than 14WF50, (b) the same section for both legs of the connection. Such an arrangement would result in lighter specimens which could be handled without a crane and tested in testing machines. These tests would furnish data on the influence of certain variables within several connection types with the least amount of expenditure in time and money. An 8E15 rolled section was chosen because of its similarity to 14WF50 as is seen in the following table:

**Table II**

Comparison of 14WF50 and 8E15

<table>
<thead>
<tr>
<th>Section</th>
<th>Depth</th>
<th>Flange</th>
<th>Web</th>
<th>d/b</th>
<th>t/a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Width</td>
<td>Thickness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14WF50</td>
<td>13.96</td>
<td>6.73</td>
<td>0.383</td>
<td>0.270</td>
<td>2.06</td>
</tr>
<tr>
<td>8E15</td>
<td>6.00</td>
<td>4.00</td>
<td>0.254</td>
<td>0.250</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Connection A, Type 2 was considered with the purpose of comparing it to Type 8B. It is similar to the connection at the apex of gabled roof frames and furthermore seems to be economical and simple. The lengths of the legs are shown in Fig. 17. The diagonal butt plate is 1/4 in. thick.

**Connection B, Type 2B** was taken from reference (4). It is a type that is frequently used. The house was built
of 1/4 in. plate material with a 1 to 6 slope in the lower flange.

Connection C, Type 15 also was taken from reference (4). It may be considered a modification of Type 7, the haunch with greater web area permitting higher loads. It could be directly compared to Connection N, Type 16. The haunch built from 1/4 in. plate material has a slope of 1 to 3.

Connections D-E-F, Type 4 were also taken from reference (4). The diagonal bracket extends 3 in. to both column and beam. The three specimens are alike except for the two outer stiffeners. In D and E the middle stiffener of the beam is a full stiffener welded to both flanges. The outer stiffener is sniped in D and half in E. In F both intermediate and outer stiffeners are sniped. The investigation may show the influence of variations in stiffener details.

Connections G-H-I, Type 5A have been designed to study the influence of radius of curvature and thickness of inner flange. The thickness and radius were designed according to the recommendations of AISC. These connections have inner flange radii of 30 in., 28 in., and 16 in. respectively with corresponding thicknesses of 1/4 in., 3/8 in., and 1/2 in.
Connection J, Type 5A has a small radius \( R = \frac{1}{3} \) in. with \( \frac{1}{4} \) in. flange thickness. Therefore tilting brackets in the curved portion have been added. This connection has been designed to be comparable to Connections D, E, and F, Type 4.

Connections E-L-N, Type 32 are comparable to Types 7 and 2. Connection E is reinforced with diagonal stiffener only, while L and N besides the diagonal stiffener have vertical half and clipped stiffeners respectively. A further purpose of the investigation is to find the influence of the various stiffener arrangements on the connection behavior.

Connection N, Type 16 simulates the stiffened bulkheads and decks of ship structures. It is usual to assume that 30 times the thickness of the plating acts as a flange. Therefore for \( \frac{1}{4} \) in. thickness the upper flange width was made 7 1/2 in.

**Coupon and Control Beam Tests**

Any method of experimental or theoretical stress analysis is based on a stress-strain diagram for the material used. So it was thought appropriate to obtain data on the stress-strain relations of the plate and rolled (CML) material with coupon test. Results obtained from twenty-five tensile specimens and five compression specimens are reported in Appendix B.
To obtain the Moment-Rotation Characteristics and the \( E_t \) value of the \( S \) in S3, rolled section used in the tests, a 12-ft. single beam was tested to failure and the results reported in Appendix A.

**Moment Diagrams and Load Lines (Pressure)**

Since actual loading conditions in frames must be simulated in tests it seems desirable to study some typical frames. Bending stresses are the most important stresses to which a frame is subjected; therefore bending moment diagrams are of primary importance. Bending moment diagrams for various types of frames and numerous charts can be found in (3), and analysis of numerous types of frames can be found in (42); therefore there is no need of going into Moment Diagrams here. However, an idea about the moment diagrams in rigid frames can be formed by considering **Load lines**. A load or pressure line of a rigid frame is the string polygon for the loads on the frame. Moreover load lines will clearly indicate how the specimens for the various types of connections should be loaded.

Figs. 19 and 20 illustrate several typical frames and the load lines resulting from various kinds of loadings. The beam is always under "compression" for gravity loads. If the portion of the frame between inflection points is considered, the two legs extending from the corner (inset)
of the frame are generally unequal.

In Fig. 19(c) the load line due to a lateral load on the frame is illustrated. Such a load results in moments of opposite sign at the two knees; therefore the left knee is under "tension" forces. The same conditions exist in the frame shown in Fig. 19(d). Such "tension" loadings on knees may be caused by wind forces, earthquakes, blasts or explosions, and foundation movements. However, ordinary wind forces only, combined with gravity live loads, seldom result in "tension" loadings on the knees, as shown in Table III.

### TABLE III

Combined Moments in Portal Frames

<table>
<thead>
<tr>
<th>Frame and Loads</th>
<th>h</th>
<th>$M_1$ ** (at B)</th>
<th>$M_2$ *** (at B)</th>
<th>$\frac{M_2}{M_1}$</th>
<th>Load line about corner B</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>10</td>
<td>492.51</td>
<td>9.61</td>
<td>0.02</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>457.14</td>
<td>39.57</td>
<td>0.08</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>400.00</td>
<td>150.00</td>
<td>0.33</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>330.00</td>
<td>576.00</td>
<td>1.73</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

** $M_1$ = moment due to vertical load in kip-ft.

*** $M_2$ = moment due to wind load in kip-ft.
In view of the above it was decided to load the specimen in "compression." Each specimen was loaded at the end of the legs. And since the legs are equal the load line makes 45° with both legs (symmetrical loading). In actual frames, load lines do not necessarily make 45° with the horizontal. However, experimental evidence shows that no difference exists in stress distributions at the knees for symmetrical and unsymmetrical loadings (26), (29), (30), (31).

Building frames sometimes have members meeting at angles greater than 90°, i.e., gabled roof frames. The stress distribution at the knees of such frames was found to be similar to the right-angle knees with the stresses somewhat less severe (26).

In designing the leg lengths of the connections B, C, G, H, I, and J, the rolled section S313 was made of such length that the stresses in the interior flange are as uniform as possible. This was called "worst loading".

*) The same specimen has been tested in "tension" at the completion of the "compression" tests. Results of the "tension" tests will be reported in another paper.
VI - COST OF FABRICATION OF SPECIMENS

One of the objectives of this project is to obtain information in regards to the relative cost of fabrication of the various connection types. To obtain the necessary data, the welding operations were closely followed and timed. The results obtained are not absolute but give relative costs of fabrication of the connections since they are based on the net time of cutting and welding.

In considering the time for cutting stiffeners, butt plates, etc., the piece was considered as if cut from the middle of the plate. From the data taken, an average time of cutting of 3 1/2 seconds per inch was found and used for uncut edges whenever the piece was cut from an edge. The time for welding includes time elapsed from the moment the arc is started to the instant the arc is stopped for each individual weld. For the 3/16" fillet welds, an approximate rate of welding is about 8 seconds per inch of fillet weld.

Table VII gives the relative time of fabrication of the various connections. It should be noted that the time of welding is also an indication for the weld material used.

All specimens were fabricated in Pratt Engineering Laboratory by expert welders borrowed from Bethlehem Steel Company Fabricating Shops. A Lincoln Welder, D.C., straight polarity was used with E6012 class, Type 87 Airco welding electrodes.
VII - TEST APPARATUS

The general procedure and instrumentation are approximately the same in all connection tests. The test set-ups and apparatus together with the test procedures followed and any variations from specimen to specimen are discussed below. Table VII is a chart of instruments and test set-ups used during this investigation.

**Loading Methods**

The fourteen specimens A to N have been tested either in the 500,000-lb. hydraulic machine or 500,000-lb. screw type machine. Figs. 21 and 22 show specimens set up in the two types of testing machines. Each specimen was held between the heads of the machine with the beam and column (two legs) at 45° to horizontal. The loads were applied through a loading fixture designed and fabricated to be used for all connections (A to N). The external load is transmitted to the flanges of 9X13 through 8-3/4" @ bolts, bolted to both specimen and loading fixture.

Specimen P, Type V was loaded by a hydraulic jack, and the forces were measured with a dynamometer. To accommodate these, special end fixtures were designed which were welded to the ends of the two members. The connection was set in place with the 14W350 beam at about
45° with the horizontal. Fig. 23 illustrates the loading method for this specimen. Fig. 24 is a photo showing the loading method.

Lateral Supports

Connection P was supported laterally on both sides. Care was taken during the test not to restrain the connection with the lateral supports. Fig. 26 shows the I-beam lateral supports which are clamped to the base beam at the lower end and tied to a cross-beam at the top. The distance between lateral supports on both sides was regulated with turn-tackles. This connection was also supported longitudinally as shown in Figs. 23 and 24.

Connections tested in the 300,000-lb. hydraulic machine (A, B, H, F, E, N, C) were laterally supported as shown in Fig. 26. The supporting angles were bolted to a base-beam at the lower end and to two pairs of diagonals at the upper end.

Connection B, tested in the 300,000-lb. machine, had I-beam lateral supports clamped to a base beam and tied to the machine columns at the upper end. Fig. 26 shows the south side I-beams and the ties.

The rest of the connections (F, G, H, I, J, L) tested in the 800,000-lb. machine were laterally supported with two pairs of bars. The bars, which were flexible in two planes, were stretched to about 300 lbs. at the beginning
of the test. Each bar of one pair (North and South) had
two 82-4 gages mounted for measuring the lateral forces.
Ruggenbergers were used on the other pair for the same
purpose. Fig. 22 shows the rear pair of bars. Fig. 37
shows how these bars were tied to a beam bearing on the
machine columns. Fig. 23 shows the 57-4 gages and the
Ruggenberger tensometers mounted on the bars. Connections
G, on the other hand, is the only one in this group which
had for lateral support four instead of two pairs of bars.
See Fig. 29.

Instrument and Measurements

The instrumentation in this investigation was more
or less the same for all specimens. Generally deflections
and rotations were measured with arms, dials, and level bars.
The following is a short account of the instruments used
and the measurement taken.

Deflections were measured with a deflection gage
located between the legs of each connection and supported
as shown in Fig. 21 and 29. The dial support was adjustable
making the resetting of the gage possible. This deflection
gage set-up was used for all the fourteen connections A to
M. To measure deflections of the Connection F, fifteen
dial gages were supported on a special rig which was in turn
supported on the 140750 beam. Fig. 30 gives the location
of the gages, and Fig. 34 shows the special support for the
gages.

Certain other deformation measurements were also taken when testing the connections. For instance in Connection F the distance between the line of loading and the knee, and also the length of the piston rod of the jack, were measured. The change in moment arm of the loads as the knee deformed was obtained from these measurements. In all other connections (A to E) a mirror gage was employed to measure the increase in the moment arm of the applied forces. Figs. 22, 25, and 29 show the mirror gage.

To detect lateral deflections or local buckling deflections at the flanges or web, deflection dials were used at certain places on some connections. Table VII shows the specimen on which such dials were used. Fig. 29 illustrates the use of three gages for the above purposes. The gage for web buckling can be seen on the left of the specimen near the top. On the left lower part of the specimen about 5 in. in the built-up part is the gage to detect local buckling in the flanges. The gage for lateral deflections can be seen on the right lower side of the specimen.

Rotations were measured with level bars. Five such levels, one at each end of the members and three near the
jacks, were used for Connection F. Supports for the three level bars near the knee are shown in Fig. 51. Two level bars were used for Connections A, B, C, D, E, G, H, and H to measure the rotations on the knee area. Fig. 51 shows the two level bars in place. Fig. 52 shows how the level supports were in turn supported from two round bars welded to the web of the SSH. An additional level bar located at the wide end of the haunch was used in Connection G. The level bar supports were changed from round bars to brackets for the rest of the connections. See Fig. 52 and 57. Care was taken not to stiffen the web with these brackets. Four level bars were used for Connections F, G, H, and G.

Electrical SR-4 gages were used to measure strains. The strains were measured with a Baldwin Strain Indicator (K-Box). Fig. 51 shows the strain indicator, the switch box, and the AC-DC converter. The location of the SR-4 gage on each specimen (A to H) is given in Figs. 59 to 70. All of the gages used on the specimens in this group are of the A 11 type. Fig. 53 gives the types of the SR-4 gages mounted on Connection F.

Whiteshade was used to observe the yield lines which develop during the test. All connections were whiteshaded near the knee.
VIII - TEST PROCEDURE

After the setting up of the instruments had been completed, and prior to the beginning of the test, trial loads of low magnitude were applied to detect any possible friction. During this "Friction test" the load was increased and decreased between 1000 and 4000 lbs., and readings were taken of one deflection gage, one level bar and two SR-4 strain gages.

**Connection F:** Up to 3000 lbs., readings were taken at every 1000 lbs., and above 3000 the increments were decreased to 500 lbs. Since there is creeping under load after part of the structure has yielded, the readings for the load in consideration are taken only after the rate of creeping has reduced to certain prescribed figure. It is then said that the rate of creeping is within the "criterion." The "criterion" in this case was adjusted to be geometrically the same as that used in the continuous beam tests, and, when the rate of deformation (or creeping) at dial gage No. 1, under constant load, had decreased to 0.05 in. per 15 minutes, yielding was considered as ceased. A complete set of readings was then taken. The load was maintained constant during the readings, simulating dead-load conditions.

**Connections A to H:** The same testing procedure was in general used for this group of connections. The increments
were usually of 1000 lbs. In the plastic region a "criterion" of 0.0015 in. per 15 minutes for the rate of deflection was used. This is in line with the criterion used in testing Connection F.

All tests were carried to failure. Loads, deflections, rotations, displacement, yield lines and modes of failures were observed and recorded. Complete data on the above is on file in Frits Engineering Laboratory.

IX - TEST RESULTS

The data obtained from the tests was studied, and the results are reported below.

M-φ Curves

In Fig. 34, the Moment-Rotation (or Curvature) curves are given for the 14WF30 beam and SWP31 column. Some typical Moment-Rotation curves for S513 rolled section used in the fourteen connections A to H are given in Fig. 35. Since yielding in most of the connections has not reached the strain gages, the points fall on a straight line even though the connection has deformed seriously.

Yielding

Connection F: At 5.2 kips the first yield lines were observed (local yielding). This yielding has no structural effect (on deflections, rotations, etc.) although it may affect strains locally as measured with SR-α gages and consequently may affect the stress distributions. At 6.3
Hips, initial yielding took place at the web in the center of the knee as predicted. Distinct lines were observed on the whitewash at the center of the knee at this load. The Moment-Rotation curve also verifies this observation since at about this load it starts deviating from a straight line.

On further loading, the yield lines in the web of the knee increased in number and thickness. Fig. 36 shows yield lines in the web when $P = 12$ kips, after general yielding took place. Fig. 37 shows the deformation of the knee, near the end of the test before deflection gages were removed. Additional flaking of the whitewash is noticeable.

Connection A to E: Table VIII gives the moments at which local and initial yielding took place in Connections A to N. Fig. 26 shows where the yield lines first started in Connection B. Fig. 27 shows the yield lines developed in Connection I. In Fig. 29 the yield lines in the curved inner flange should be noted. Fig. 32 illustrates the locations at which yielding started in Connection K.

**Strength and Rigidity**

One of the main objectives of the present investigation is to obtain data on strength (loads or moments carried) and rigidity (deflections and rotations). These can best be studied with moment-rotation and moment deflection curves.
**Connection F:** The rotations in the knee were measured with level bars attached to the connection as shown in Fig. 51. The results plotted as moment at the knee against rotations in the knee are given in Fig. 50. The curve deviates from a straight line at about 500kip-inches which corresponds to a load of 6500 lbs. The comparison between the 67FSI and 147V30 rotation curves and the rotations in the knee should be noted.

The deflections in the connection as measured by gage No. 1 (Fig. 30) are given in Fig. 39. This curve deviates from a straight line (initial yield) at approximately the same moment as the rotation curve. The final measured maximum deflections are given in Fig. 40. The shape of the connection at the end of the test is illustrated in Fig. 41.

**Connections A to N:** The rotations in the knees of Connections A to N as measured with level bars are plotted in Figs. 42, 43, 44, and 45. The deflection for the Connections A to N are illustrated in Figs. 46, 47, 48, and 49. The moment developed at the knees are compared to the moments which would cause a fiber stress of 20 ksi in the connection.

For Connections A to N, the mirror gage used gave the increase in moment are of the loads. Fig. 50 gives
some typical curves for the increase in moment arm. This increase has been considered when the moments in the knee of the connections were obtained.

In Fig. 51, the rotations in the knee divided by the equivalent length in each knee (the rotations per inch or unit rotations) are shown plotted against the moments at the knee and compared to the rotations of the rolled section SHS.

In Fig. 52, the moments developed at the end of the rolled section are plotted against the unit rotations in the knee and compared to the rotations of the SHS rolled section used for the Connections A to N.

**Stresses**

To obtain strains in various parts of the connections and compare them to computed strains, SR-4 electrical gages were installed in all specimens. The following are the results obtained.

**Connection B:** The types of SR-4 gages mounted on this connection are given in Fig. 55. The gages were installed to make a complete study of the strain distributions in the connection. At the present, longitudinal strains along the flanges and normal (fiber) strains at various sections have been obtained. The data are also useful in studying the shears transferred to the knee.

**Normal strain distributions at three cross-sections**
of both beam and column and for five loads are given in Figs. 53 and 56. The figures show that although the strain distribution is uniform away from the knee, it is irregular in sections nearest the knee.

The transverse stress distribution in both compression and tension flanges at the 145/800 beam and 5W31 column are given in Figs. 55 and 56 for \( P = 3.2, 9.4, 11.45, 15.6, \) and 17.8 kips. Theoretical strains for \( P = 6.2 \) and 17.8 are shown in the same figures dotted. In most cases the distribution is regular.

The strain variation along the flanges of the beam and column were drawn for external loads of 5.2, 9.4 and 11.45 kips. Fig. 57 gives the strain distribution in the upper and lower beam flange. It is notable that maximum stress for a load occurred above the stiffener. The assumption that the moment (or stress) is maximum at the face of the column is confirmed by these results. The second assumption, that the stress falls uniformly to zero, does not seem to be true for higher loads. Nevertheless, the results show that it can be used with no serious error.

Fig. 58 gives the strain distributions in the column flanges. The distribution is as expected. These lines also give the stresses in the stiffeners.

A carefully made inspection of Fig. 57 and 58 shown that in the elastic part the stresses are proportional to
loads, but this is not the case with 11.45 kips load. At this load there is extensive yield in the knee which causes a redistribution of the stresses.

Connections A to H: Measured strains and their comparison to theoretical strains for Connections A to H are given in Figs. 59 to 70. In general there is a good agreement between calculated and measured strains.

The ratio of the measured to computed strains about an inch from the knee (at the re-entrant corner) for the various connections is given in the following table.

**TABLE IV**

**Stress Concentration Factors**

<table>
<thead>
<tr>
<th>Connections</th>
<th>Measured Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1.70</td>
</tr>
<tr>
<td>E</td>
<td>1.27</td>
</tr>
<tr>
<td>H</td>
<td>2.16 &amp; 2.20</td>
</tr>
<tr>
<td>C</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Stress concentration factors, therefore, in certain cases can be as high as 2.2, and probably higher for points nearer to the knee. The data taken show that such stress concentrations are completely avoided by the use of curved inner flanges or gently sloping haunches.

The Wedge Theory which has been used as applied to curved flanges to compute strains gave satisfactory results.
Instability and Modes of Failure

In general the connections tested failed by instability after considerable yielding. The following are short accounts of failures of the various connections tested.

Connection P: Local buckling of the lower beam flange was first observed at \( P = 17,000 \) lbs. \( (N = 1050 \text{ kip-inches}) \). However, the local deformations were not excessive, both sides of the lower beam flange behaved in similar fashion. On further loading, the flanges of the column also buckled, the deformations of this flange being smaller than those of the beam. See Fig. 72.

Such a connection is apparently able to carry a significant increase in load after complete yielding in the knee web. General yielding of the knee started at 637.5 in-kips. The ultimate moment being 1150 in-kips is about 70 per cent over the moment at general yield.

After considerable additional deformation in the knee had occurred and at about the maximum load, a crack started at the root of the lower beam fillet (Fig. 73). With further deformation, the crack opened, extended about .2 in. into the knee, and initiated cracks in the end stiffener plates.

The maximum load, with fluctuations, was maintained during most of the rest of the test and was starting to
decrease when the limit of the straining mechanism was reached. Fig. 41 shows the final shape of the connection at the end of the test. Fig. 40 gives the deflected contour at maximum load.

Yielding was observed in both flanges of the column close to the knee. In the compression flange, this extended about 14 in. down the column from the corner.

Connections, Type 2 and 68: Figs. 73 and 74 show the mode of failure for type 2 and 68. After considerable yielding, the compression flanges tend to buckle locally in a form of a wave. As the load is further increased, one side of the flange deflects (or buckles) more, thus forcing the web to buckle and bringing about the failure of the connection. Local buckling occurs simultaneously with lateral buckling.

Connection L shows higher strength probably due to better lateral support offered by the bars used as lateral supports. See Figs. 27, 38, and 59. The lateral deflections that took place were measured in some connections with a gage as shown in Fig. 75. The measured lateral deflections for Connection L are given in Fig. 76. Local buckling was detected as shown in Fig. 76. Fig. 77 gives the local buckling curve obtained. This curve deviates from a straight line exactly at the load at which the lateral buckling curve does. This shows that the two instabilities occur simultaneously.
The lateral forces measured with SE-6 gages are plotted in Fig. 78. The maximum lateral force, at the end of the test of Connection L, is about 2750 lbs, a small percentage of the applied load. Fig. 79 is a photo which shows the local buckling and the yielding that took place in Connection L. Note on each end of the diagonal stiffener the cut ends of the lateral support bars.

Connections D, E, and F, Type 4: The failures of this type are similar to those of Type 2b described above. Fig. 80 shows the permanent lateral deflections at the end of the test of Connection D. Connection F which was supported laterally with flexible bars as shown in Fig. 81 failed in the same way, i.e. by plastic local instability. Fig. 82 shows the connection at the end of the test.

The lateral forces as measured by the strains in the bars are given in Fig. 83. The maximum lateral force at the last loading is 1015 lbs.

Connections C and N, Types 15 and 16: Connections C and N failed in the same way as did connections of Type 2b and 4. Figs. 85 and 86 show the place where local buckling of the flanges and web took place in C and N.

Connection A, Type 2D: Connection A also failed the same way. The built-up flanges buckled in a form of waves after extensive yielding. See Fig. 87. The connection
as a whole buckled laterally as shown in Fig. 90.

**Connection 1, Type 5A**: Fig. 90 shows the S-shape permanent distortions due to lateral deflections in the connection 1. The extensive yielding that took place is shown in Fig. 90. A close up of the flange which has buckled locally is given in Fig. 91. Some local buckling occurred in the flanges of the rolled section too as shown in Fig. 96.

**Connection 3, Type 5A**: Figs. 92, 93, and 94 show the progress of local buckling in the curved inner flange of connection 3, Type 5A. This connection, having a small radius of curvature and 1/4 in. curved flange, is subject to larger crossbending stresses. Further local buckling developed at the lower end of the knee is shown in Fig. 95. The final lateral deformations are shown in Fig. 96.

**Connection II and C, Type 5A**: Figs. 97 and 98 show the permanent lateral deflection at the end of the test of connections II and C. The local buckling in the flange of connection C is shown in Fig. 99.
X = DISCUSSION OF TEST RESULTS

A summary of test results is given in Table VIII. In the following the test results are discussed for Connection P and Connections A to H. See Figs. 17 and 18.*

Connection P

The parts of the Connection P outside the knee behaved according to the ordinary beam theory. Inside the knee the stress distribution does not follow the beam theory. The maximum tensile stresses in the flanges "pass" from the beam to the column through the web of the knee thereby causing high shear stresses. These stresses are several times larger than the shear stresses at the critical section for bending.

The high shear stresses caused an early shear yield in the web of the knee resulting in large rotations and deflections. The $N_p$ for the ENPD, the smaller of the two sections, could not be developed. Obviously, such a connection is inadequate not only from the "limit design" point of view but also from the Elastic design point of view. See Fig. 39.

If in certain specific cases such a connection has to be used because of other considerations, the $N_p$ for analysis must be smaller that the $N_p$ for the weaker of the two members framing in the knee. This seems to be the most
obvious disadvantage of this connection if aesthetic considerations are not taken into consideration. It is true that there is large stress concentration at the re-entrant angle. This is not critical for static loads (fatigue not considered). This connection is unsatisfactory, since it meets neither the strength nor the rigidity requirements.

Connections A to H

Influence of Fabrication Details: An examination of Figs. 46 and 47 will show that there is no considerable change in behavior in the various type 6B and 4 connections. The variations in stiffener details did not alter appreciably the behavior typical for each type. The small variations detected may be caused by the influence of residual stresses, lateral supports and others.

In Type 6A, the influence of the radius of curvature and thickness of the inner curved flange is more apparent (Fig. 49). An inspection of Fig. 51 will show that connection C with the largest radius of curvature developed higher strength (moment) than any other connection. However, Connection C did not carry the maximum load through large rotations having failed rather suddenly. Fig. 47 shows that Connection J, which has been designed for comparison with Type 4 connections, is stronger and more rigid than Connections D, E, and F. Connection J, on the other hand,
is about twice as expensive for fabrication as Type 4 connections.

**Instability and Modes of Failure:** The study of the data on connections shows the various stages in the behavior of connections in general. The initial elastic behavior is followed by the elasto-plastic stage in which a deviation from the elastic straight-line relationship starts; and, as the loads increase, the deflections, rotations or stresses become larger for the same increments of load.

After considerable yielding, plastic instabilities start in the flanges simultaneously with a tendency to buckle or deflect laterally. With increasing loads the local buckling becomes more pronounced, and the connection fails immediately after the web starts buckling.

Measurements seem to indicate that lateral deflections start simultaneously with local flange buckling. See Figs. 77 and 78. The forces required for keeping some of the connections from buckling laterally have been measured and found to be as follows:
TABLE V

Measured Lateral Forces

<table>
<thead>
<tr>
<th>Connection</th>
<th>Max. Supported Load, Kips</th>
<th>Lateral Force at Max. Load, Kips</th>
<th>Max. Lateral Force Measured, Kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>48.5</td>
<td>0.96</td>
<td>3.98</td>
</tr>
<tr>
<td>F</td>
<td>32.5</td>
<td>1.26</td>
<td>1.88</td>
</tr>
<tr>
<td>L</td>
<td>25.8</td>
<td>1.50</td>
<td>2.75</td>
</tr>
</tbody>
</table>

The lateral forces, therefore, are only a small percentage of the maximum supported force (external).

Suitability of Various Types of Connections: Strength, rigidity, cost are the three main items which indicate the suitability of various connections.

The strength requirements depend on the point of view of the designer and on what he is looking for. A connection may be satisfactory depending on design considerations if:

a. It develops the \( M_p \) value of the rolled section or higher moment in the knee itself. (Fig. 3).

b. It develops the \( M_p \) of the rolled section at the end of the rolled section (where the knee begins, Section AA, Fig. 3).

All of the connections tested meet the first strength requirements. Fig. 51 in which the unit moment-rotations
curves of the various connections are compared to the moment-rotation curve of SE13 shows that all connections developed at the knee higher moments, higher than the $M_p$ of SE13.

However, if the second strength criterion is used, connections B, C, I, J, and N must be regarded as unsuitable in spite of the fact that if the length of the rolled section had been longer, they might have developed the $M_p$ of the rolled section. See Table VIII. In Fig. 52 the moments developed by Connections B, F, H, and L at the end of the rolled section are compared to the moments of SE13. This figure shows that Connections L and F are adequate. Connection H is adequate from the point of view of strength and rigidity but does not carry the maximum moment through large rotations. Connection B is inadequate in all respects.

Fig. 51 shows that all of the connections are more rigid than the rolled section SE13 both in the elastic and plastic region. Only Type B and BB connections in the elastic and in part of the plastic region are less rigid. The excess rotations in these knees may increase the deflections in a frame by as much as 10 per cent. See Appendix D.

Table VI gives the relative cost of fabrication of the different connections. The most economical connection is A, Type 2. Next comes the Type BB connections which
are 1.25 to 1.79 as expensive as Type 2. Connections with built-up knees are very costly. For instance, Connections B and C are 5 and 6.32 times as costly as Connection A.

Another requirement which is of importance is the rotation characteristics of the connection after it reaches its maximum. Fig. 51 shows that Connections G, H, B, and C do not carry the maximum moment through large rotations as the other connections do. The failure of these four connections is rather sudden when compared to that of the other connections. This is so, probably, because failure (local buckling) occurs in the built-up knee and not outside in the rolled section.

From the above discussion it is apparent that Type 2, 3B and 4 connections are the best to use. They meet the requirements of strength and economy, and Type 4 the requirements of rigidity completely.

However, design of knees may sometimes be controlled by aesthetics and fatigue considerations. In such a case, a Type 5A knee is recommended. It has the best appearance and no stress concentrations.

Of the Type 5A connections tested, I and J meet all the requirements (strength, rigidity and partly rotation after maximum moment has been reached). Their better performance over Connections G and H is most likely the result of the thicker flange \( t = 1/2 \) in. \( \) in I and the tilting brackets in J.
XI - DESIGN RECOMMENDATIONS

The following recommendations for the design of knees are of a general character and rather conservative since further tests would be required to obtain complementary information on the behavior of certain types of knees.

In general the knee must be so designed as to develop the $M_p$ of the rolled section either at the end of the knee or in the knee itself depending on the design. The ability of the knee to develop the plastic moment $M_p$, or higher moments, depends on the arrangement of design details such as stiffeners, brackets, etc. The recommendations that follow are based on the experience obtained from the tests and the observed behavior of the connections.

1. Square Knees (Fig. 5)

   a. Use a diagonal stiffener, as in Type 2 or 2B, unless the thickness, $w$, of the web is such that bending yield takes place outside the knee before shear yield develops in the knee.

   b. The thickness of diagonal stiffeners should be:

      1. About twice the thickness of the thinnest flanges of the two rolled sections for type 2 connections.

      2. About 1.5 times thicker than the flange of the column for knees similar to Connection K, Type 2B.
3. About the same thickness as the thinnest of the flanges for knees similar to Connections I, or M, Type 9E.

II. Bracketed or Haunched Knees (Fig. 5)

Economies may be achieved if the span of the beam in a portal frame is shortened by the introduction in the knee of a bracket or haunch. The haunch may be in the beam or column or in both depending on the shape of the moment diagram of the frame, and on the requirements laid down by the designer.

In general the knee is so designed that the plastic hinges will be developed at the rolled section, unless the designer uses the haunch solely for the purpose of strengthening his section at the knee.

a. The slope of the haunch must be such that when the ultimate load is reached the moments in various sections of the haunch are equal or smaller than the \( W_y \)'s for the sections.

b. The \( W_y \)'s for ordinary slopes at the haunch may be calculated with the ordinary beam theory.

c. The thickness of the flanges and web of the built-up knee must be at least equal to the respective thickness in the rolled sections.

d. Use of stiffeners must be made at points of internal thrust (See f below).
e. In bracketed cases the depth of the bracket should be equal to the depth of the section to which it is connected.

f. Use vertical stiffeners, as in type 4, to:
   1. Distribute direct forces such as external loads or internal thrusts.
   2. Support web against buckling or crippling.

g. The vertical stiffeners must be in general, of the same thickness as the web. In Type 4 connections, the thickness of the outer stiffener which distributes the internal thrust of the bracket must be

\[
\begin{align*}
    t_s \geq w \\
    \text{or} \\
    t_s \geq 0.7t_f
\end{align*}
\]

whichever is larger

where \( t_s \) = thickness of stiffener

\( t_f \) = " " bracket flange

\( w \) = " " web of rolled section

h. The length \( L \) of vertical stiffeners which should be enough to transfer the thrust to the web, may be obtained from the relation

\[
L = \frac{t_s b}{w}
\]

where \( b \) = width of the rolled section flange.

i. Consistent with good design the size of the fillet weld should be obtained from the relation
\[ h = \frac{v}{1.4} \]

where \( h \) = size of fillet weld.

III. Curved Knees (Fig. 5)

Sometimes aesthetics is the main factor in choosing the type of knee to be used in a frame. In such a case curved knees are used because of their pleasing appearance.

a. Shape of the Knee: Curved knees have straight flanges on the outside and circular or parabolic on the inside of the frame.

b. Radius of Inner Flange: Appearance, economy, and stability are the factors to be considered when curved knees are designed. The point on the frame at which the built-up knee will start or end depends on the moment diagram of the frame. In general the knee will be started at such a location as to achieve the best combination of appearance, economy, and stability. Cutting the rolled section at the improper location may result either in an expensive built-up knee or in a heavy rolled section. It is recommended that the curvature, \( R \), of the inner flange of the knees be such that:

\[ 2d \leq R \leq 3d \]

where \( d \) = depth of rolled section.

c. Thickness of Flanges: The thickness of the flanges of curved knees should be in general approximately equal
to the thicknesses of the rolled sections used as beam and columns.

In the case of the inner curved flanges the ratio \( \frac{b^o}{R} \) should be equal to not more than 1.5, where \( b \) is the width, \( t \) is the thickness and \( R \) the radius of curvature of the inner flange, all dimensions being in inches.

In case \( \frac{b^o}{R} \) is bigger than 1.5 four tilting brackets (stiffeners) must be used. See (e) below. However in no case should the above relationship be more than 3.

d. The thickness of the built-up web should be comparable to the thicknesses of the web of the rolled sections used as beam and columns.

e. At least one stiffener at 45° (diagonal) must be used in a curved knee. This stiffener should extend from the exterior corner of the knee to the center of the curved flange. In case tilting brackets are needed (see e above) they must be of a length equal to half the depth of the rolled section and welded both to the web and the compression flange. Two pairs of tilting brackets (stiffeners) should be used on either side of the diagonal stiffener. The thickness of the diagonal stiffener and of the tilting stiffeners should be about that of the web.

f. In designing the welds connecting the outside flanges to the web of the built-up knee, the total flange
load at the beginning of the knee should be considered. This load, which can be assumed as \( \frac{L}{d} \), divided by the distance to the exterior corner of the knee will give the shear per inch. The minimum size of the fillet weld, \( h \), which connects the curved inner flange to the web may be obtained from the relation:

\[
\text{min. } h = \frac{bt}{0.7H}
\]

where \( b \), \( R \), and \( t \) are the width, radius and thickness of the flange. The size of the fillet weld for the stiffeners may be calculated from (II-1).

IV. Types of Welds

Use fillet welds in frames and knees except where fitting of parts, or fatigue loads make it necessary to use bevel welds.

V. Lateral Bracing

Rigid portal frames should be braced to prevent lateral movements. Since lateral forces are small any purlin or gusset frame (save truss) built for the purpose of providing lateral support may be of light construction.

The larger of the two reactions at the supports of the frame may be used to obtain the approximate magnitude of the force required to resist lateral buckling at the knee. Bracing at the knee may be designed to resist about 10 per cent of the total reaction at the supports.
XII - RECOMMENDATIONS FOR FURTHER RESEARCH

The investigations carried out by the author and reported above are the first of their kind, and they must be regarded only as preliminary investigations. The subject of rigid knees is a vast one, and further tests must be carried out before a satisfactory stage of knowledge in the behavior of knees is reached.

The problem of behavior of knees can be approached empirically only if:

a. A small number of typical connections from practice are selected for investigation.

b. A number of nominally identical connections are tested to destruction.

Rigid frames are not only subjected to static loads ("Compression" and "Tension"), but also varying or pulsating loads, and to dynamic or shock loads. Therefore, it would be of interest to investigate the behavior of knees under these types of loads.

These suggested variations in types of loads are for future research. For immediate work that would supplement the investigations reported in this dissertation, the following suggestions are made:

1. To find the effect of thickness of the inner curved flange in the Type 5A connections, test the following knees with:
t = 1/4 in. and R = 22 in. similar to (R)
t = 1/4 in. " R = 16 in. " (I)
t = 1/4 in. " R = 13-3/8 in. no tilting in brackets
where t = thickness of curved flange
R = radius

2. Further types of investigation are shown in Fig. 100 (a) to (h).
(a) This is Type 6A with no diagonal stiffener, shown as 6B in Fig. 5.
(b), (c), and (h) These types are modifications of Type 2B.
(d) This connection is used in industrial buildings.
(e) Type 4 connections may be further simplified, thus cutting down cost of fabrication. This connection is obtained by omitting the interior stiffeners which do not seem to be critical.
(f) To be used where stress concentrations must be reduced.
(g) In this connection, the web of the beam and column are not in the same plane.

3. At the completion of the experimental investigation of connections for portal frames, tests of interior connections for multi-span frames should be carried out. Information from such tests would supplement the knowledge gained from investigations of portal-frame connections.
The above are some inexpensive connections that may be tested. Other problems related to the subject of the portal frame knees would be investigations of fatigue and residual stresses in knees. The subject of encased knees could also be of interest and use.
REFERENCES


43. Progress Reports to Lehigh Project Subcommittees (Not for Publication)


Additional References


Campus, F., "Nouveaux essais sur modes de noeuds rigides", L'Essaiature Metallique, Novembre Année, No. 5, Mars 1940.

Campus, F., "Nouveaux essais sur modes de noeuds rigides", L'Essaiature Metallique, Novembre Année, No. 6, Avril 1940.


Morrison, D. E., "The Effect of Non-Uniform Distribution of Stress on the Yield Strength of Steel", Bulletin No. 578, Engineering Experiment Station, University of Illinois.


<table>
<thead>
<tr>
<th>Connection</th>
<th>Type</th>
<th>Cutting of Rolled Section</th>
<th>Cutting Stiffeners &amp; Butt Plates</th>
<th>Beveling</th>
<th>Total Combined Burning</th>
<th>Welding</th>
<th>Total Fabrication Time</th>
<th>Ratio based on Conn. A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>2.00</td>
<td>1.52</td>
<td>1.06</td>
<td>6.00</td>
<td>12.53</td>
<td>17.53</td>
<td>1</td>
</tr>
<tr>
<td>K</td>
<td>2B</td>
<td>1.40</td>
<td>5.06</td>
<td>-</td>
<td>6.46</td>
<td>15.11</td>
<td>21.57</td>
<td>1.23</td>
</tr>
<tr>
<td>H</td>
<td>2B</td>
<td>1.40</td>
<td>7.14</td>
<td>-</td>
<td>6.54</td>
<td>16.45</td>
<td>23.59</td>
<td>1.43</td>
</tr>
<tr>
<td>L</td>
<td>2B</td>
<td>2.28</td>
<td>6.35</td>
<td>-</td>
<td>9.03</td>
<td>22.53</td>
<td>31.56</td>
<td>1.79</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>1.40</td>
<td>8.33</td>
<td>1.09</td>
<td>11.21</td>
<td>25.59</td>
<td>37.19</td>
<td>2.03</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>1.40</td>
<td>9.41</td>
<td>1.08</td>
<td>12.29</td>
<td>31.09</td>
<td>43.37</td>
<td>2.44</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>1.40</td>
<td>8.41</td>
<td>0.53</td>
<td>11.28</td>
<td>33.29</td>
<td>44.56</td>
<td>2.61</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>1.40</td>
<td>13.45</td>
<td>1.42</td>
<td>17.08</td>
<td>31.11</td>
<td>48.18</td>
<td>2.70</td>
</tr>
<tr>
<td>I</td>
<td>5A</td>
<td>1.50</td>
<td>17.19</td>
<td>1.10</td>
<td>20.19</td>
<td>45.55</td>
<td>66.14</td>
<td>3.71</td>
</tr>
<tr>
<td>H</td>
<td>5A</td>
<td>3.45</td>
<td>16.36</td>
<td>1.15</td>
<td>21.24</td>
<td>52.22</td>
<td>73.46</td>
<td>4.15</td>
</tr>
<tr>
<td>N</td>
<td>16</td>
<td>1.40</td>
<td>29.45</td>
<td>0.54</td>
<td>31.59</td>
<td>51.53</td>
<td>83.57</td>
<td>4.70</td>
</tr>
<tr>
<td>J</td>
<td>5A</td>
<td>2.00</td>
<td>21.27</td>
<td>1.43</td>
<td>25.10</td>
<td>59.54</td>
<td>85.07</td>
<td>4.76</td>
</tr>
<tr>
<td>B</td>
<td>2B</td>
<td>1.40</td>
<td>26.19</td>
<td>2.16</td>
<td>30.15</td>
<td>59.30</td>
<td>89.35</td>
<td>5.00</td>
</tr>
<tr>
<td>O</td>
<td>5A</td>
<td>2.00</td>
<td>19.19</td>
<td>2.15</td>
<td>23.34</td>
<td>71.37</td>
<td>95.11</td>
<td>5.32</td>
</tr>
</tbody>
</table>
### TABLE VII - CHART OF INSTRUMENTS

<table>
<thead>
<tr>
<th>Connection</th>
<th>Type</th>
<th>Sketch</th>
<th>No. of Dials</th>
<th>No. of Levels</th>
<th>Mirror Gage</th>
<th>No. of SR-4 Testing Machine</th>
<th>Gages on Lateral Support</th>
<th>Connection</th>
<th>Type</th>
<th>Sketch</th>
<th>No. of Dials</th>
<th>No. of Levels</th>
<th>Mirror Gage</th>
<th>No. of SR-4 Testing Machine</th>
<th>Gages on Lateral Support</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>300,000</td>
<td>I</td>
<td>5A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>2B</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>800,000</td>
<td>J</td>
<td>5A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td></td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>10</td>
<td>300,000</td>
<td>K</td>
<td>8B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>300,000</td>
<td>L</td>
<td>8B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>300,000</td>
<td>M</td>
<td>8B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>300,000</td>
<td>N</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5A</td>
<td></td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>800,000</td>
<td>P</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>5A</td>
<td></td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>800,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- Connection types: A, B, C, D, E, F, G, H.
- Sketches represent different configurations.
- No. of Dials indicates the number of gages on the instrument.
- No. of Levels indicates the number of levels on the instrument.
- Mirror Gage indicates the type of mirror gage used.
- No. of SR-4 Testing Machine indicates the number of SR-4 machines used for testing.
- Gages on Lateral Support indicate the type of gages used on the lateral support.
### Table VIII: Tabulated Results

<table>
<thead>
<tr>
<th>Connection</th>
<th>Type</th>
<th>Observed 1st. Yield Line (Local Yielding) Moment at the Knee (in.-kips)</th>
<th>Observed Initial Yield Moment at the Knee (in.-kips)</th>
<th>Calculated Initial Yield Moment at the Knee (in.-kips)</th>
<th>Observed Initial Yield Calculated Initial Yield</th>
<th>Max. Moment in Rolled Section (in.-kips)</th>
<th>Max. Moment at the Knee (in.-kips)</th>
<th>Max. Moment in Rolled Section (M_p) 487 (for 8B 13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>163</td>
<td>369</td>
<td>433</td>
<td>0.85</td>
<td>486</td>
<td>548</td>
<td>0.996</td>
</tr>
<tr>
<td>B</td>
<td>2B</td>
<td>305</td>
<td>380</td>
<td>533</td>
<td>0.71</td>
<td>346</td>
<td>728</td>
<td>0.710</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>402</td>
<td>563</td>
<td>524</td>
<td>1.07</td>
<td>492</td>
<td>751</td>
<td>0.725 **</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>477</td>
<td>477</td>
<td>587</td>
<td>0.81</td>
<td>526</td>
<td>804</td>
<td>1.136 **</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>364</td>
<td>472</td>
<td>587</td>
<td>0.80</td>
<td>526</td>
<td>804</td>
<td>1.136 **</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>264</td>
<td>276</td>
<td>587</td>
<td>0.47</td>
<td>552</td>
<td>844</td>
<td>1.133</td>
</tr>
<tr>
<td>G</td>
<td>5A</td>
<td>436</td>
<td>654</td>
<td>920</td>
<td>0.71</td>
<td>482</td>
<td>1296</td>
<td>1.080</td>
</tr>
<tr>
<td>H</td>
<td>5A</td>
<td>263</td>
<td>681</td>
<td>840</td>
<td>0.81</td>
<td>490</td>
<td>1228</td>
<td>1.004</td>
</tr>
<tr>
<td>I</td>
<td>5A</td>
<td>526</td>
<td>516</td>
<td>883</td>
<td>0.58</td>
<td>400</td>
<td>1180</td>
<td>0.820</td>
</tr>
<tr>
<td>J</td>
<td>5A</td>
<td>300</td>
<td>475</td>
<td>620</td>
<td>0.77</td>
<td>452</td>
<td>915</td>
<td>0.820</td>
</tr>
<tr>
<td>K</td>
<td>8B</td>
<td>251</td>
<td>316</td>
<td>433</td>
<td>0.73</td>
<td>496</td>
<td>560</td>
<td>1.017</td>
</tr>
<tr>
<td>L</td>
<td>8B</td>
<td>276</td>
<td>176</td>
<td>433</td>
<td>0.41</td>
<td>553</td>
<td>624</td>
<td>1.135</td>
</tr>
<tr>
<td>M</td>
<td>8B</td>
<td>226</td>
<td>216</td>
<td>433</td>
<td>0.50</td>
<td>511</td>
<td>576</td>
<td>1.048</td>
</tr>
<tr>
<td>N</td>
<td>16</td>
<td>402</td>
<td>577</td>
<td>606</td>
<td>0.95</td>
<td>1100</td>
<td>1150</td>
<td>1.210 **</td>
</tr>
<tr>
<td>P</td>
<td>7</td>
<td>311</td>
<td>493</td>
<td>753</td>
<td>0.65</td>
<td>1100</td>
<td>1150</td>
<td>0.920 **</td>
</tr>
</tbody>
</table>

*Point at which moment-rotation curve deviates from straight line.

**See Table VII for section a-a and b-b.

† 0.920 is ratio based on 1190 in.-kips, the M_p of 8W 3i.
APPENDIX A

Control Beam Test

One of the objects of the present investigation is to compare the moment-rotation characteristics (M-\( \phi \) curve) of the rolled section to that of the knee. The M-\( \phi \) curve for the rolled section can very easily be calculated from the tensile and compressive stress-strain diagrams of the same material. General procedures for calculating M-\( \phi \) curves for any shape section from the stress-strain diagrams, or the reverse, are well known (15). These procedures are based on the assumptions that the longitudinal strain in all stages of bending varies linearly across the beam section and that the strain in the most stressed fiber is uniformly the same in regions of constant moment in both elastic and plastic regions.

However, past experiments with sections tested as rolled showed that the observed plastic moment on simple beams is usually smaller than the one calculated from the stress-strain diagram. The observed moment usually is 0.90 to 0.95 of the calculated moment (40). Therefore, it was considered appropriate to obtain an M-\( \phi \) curve for the SE13 section from an actual test of a simple beam and to base the comparisons between the various knees and the SE13 on the results of this test.
Specimen and Apparatus

The beam tested was 18'-0" in span loaded at third points. Fig. 101 gives diagrammatically the test set-up, and shows the loading arrangement and the supports. Four 3R-6 electrical gages were used in the web at the center section of the beam, each pair 1 1/2" from the neutral axis of the beam. These gages were used to obtain the M-\phi curve for the section. Five deflection dials were placed in the central portion of the beam to measure deflections. A sixth dial was used to detect lateral buckling. The beam was laterally supported in four places, two in and two outside the middle portion.

Results

Fig. 102 gives the moment-rotation curve for the section which follows a straight line up to about 450 kips-inches. The curvature beyond this load is quite sharp as it should be for I-sections. This curve is compared to the theoretical M-\phi curve based on the stress-strain properties of the flange and web material. The graphs show that the measured \( M_p \) is about 6 per cent less than the theoretical \( M_p \).

Fig. 103 shows the compression flange and the amount of lateral deflection. It is clear that lateral support was deficient and that the center of the beam should have
been supported laterally. Fig. 104 is a close-up showing yield lines in the compression and tension parts.

The maximum load carried by the beam is:

Max. live load = 20.00 kips
Dead load (fixtures) = 0.21 kips
" " (beams) = 0.10 kips

\[ P_{\text{max}} = 20.51 \text{ kips} \]

Maximum Moment = 247 in.-kips.

Therefore, \( M_p = 24 \times P_{\text{max}} = 24 \times 20.51 = 497.44 \text{ in.-kips} \)

With \( G_y = 41.8 \text{ ksi} \) and \( G_y = 47.1 \text{ ksi} \) for the flange and web material respectively the calculated \( M_p \) is 519.91 in.-kips.

**TABLE IX**

Tabulated Results

<table>
<thead>
<tr>
<th>Moment at Obs. Initial Yield ( M_y )</th>
<th>Calc. Moment at Initial Yield ( M_y )</th>
<th>Obs. ( M_p )</th>
<th>Observed ( M_p )</th>
<th>Calculated ( M_p )</th>
<th>Obs. ( M_p ) / Calc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>444.00</td>
<td>435.56</td>
<td>1.02</td>
<td>497.44</td>
<td>519.91</td>
<td>0.957</td>
</tr>
</tbody>
</table>
APPENDIX E

Coupon Tests.

In order to design and analyze properly, and to predict the behavior of a structure, the mechanical properties of the material of which the structure is made must be known.

To obtain the mechanical properties of a material the most suitable tests are the tensile and the compression tests. A careful tensile test of structural steel (low carbon content) will give the Modulus of Elasticity $E$, the Upper Yield Point $G_y$, the Lower Yield Point $G_y$, the length of the lower yield point range (10 to 20 times the total elastic strain), and the slope of the strain-hardening range.

The most significant property in connection with the present work is the lower yield point $G_y$, which is more stable and well defined. Both tensile and compression tests of the material used in the connection test program have been made and are reported herewith.

Tensile Tests

Figs. 105 and 106 give the kind of specimens used and their location with respect to the rolled section or plate and the results obtained. Twenty-five specimens were tested in tension. Eleven of them were taken from the rolled section and the rest from the plate material.
Certain precautions must be taken and a definite procedure, as described below, must be followed to obtain uniform or comparable results. Since lower yield point is affected by speed, control of rate of straining is important. There are several methods of speed control, e.g. control of rate of applying the load, or control of crosshead speed, or control of the rate of straining in the specimen. In the present tests the rate of loading was controlled in the elastic range by adjusting the opening of the valve of the hydraulic machine. The same opening was left in the plastic range. The speeds used are indicated in each case.

Procedure: 1. The specimen must be placed properly to secure uniform stress distribution.

2. Complete records must be obtained of original average area, loads, elongations, temperature during test, speed of test, kind of instrument, size and shape of specimen.

3. The valve opening for a predetermined speed must be found. This opening must be kept in the plastic range.

4. While in the plastic range any resetting must be done as fast as possible and the resulting error (specimen elongates during resetting) corrected by noting the rate of strain and time elapsed for resetting.
5. Obtain about 10 points in the strain-hardening range.

Discussion of Results: Test results obtained clearly indicate that in the given rolled section the average $G_y$ for the web is about 10 to 12 per cent higher than the $G_y$ for the flange material. Other investigators arrived at the same conclusion (40).

It is of importance therefore to use the proper $G_y$ when analytical results are sought. The $G_y$ for the plate material is the lowest, averaging about 39.1 ksi. It must be noted, however, that there is variation in speed in these tests which may have influenced the results. The slopes of strain-hardening ranges in the different specimens show quite a large scatter.

Compression Tests

The types of compression specimens are shown in Fig. 107. Of the five specimens, three are small and two are sections of 5B13. These last two are to be regarded as pilot tests for a new technique in obtaining stress-strain relations of rolled sections. The technique has many advantages since it gives a more accurate picture of the behavior in compression and an average of the mechanical properties in rolled sections. The behavior of rolled sections is obviously influenced by residual stresses due to rolling and non-uniform cooling.
To design the height-to-thickness ratio, h/t, of a specimen, the factors to be considered are: (1) bending (buckling), (2) instrumentation (gages), and (3) friction between head and specimen.

The first gives the upper limit of this ratio, since, if this limit is exceeded, the specimen will bend and the stress will not be uniform. The other two factors give the lower limit since the specimen must be long enough to accommodate the gages and also to have a length of specimen bigger than the gage length by 1 or 2 times the thickness. It is recommended that: For round specimens, if t = diam., then $2 \frac{h}{t} < 10$. The flat specimens tested (A, C, D) have the ratio $\frac{h}{t} = 4.5$. The heights of specimens E and F are 1 inch and 6 inches respectively.

Test of Small Specimens: The three small specimens, A, C, and D were tested in the 60,000 hydraulic machine with the approximate speed of 1 micro-inch per second, and at room temperature. Two type A-5, SR-4 strain gages were used to measure strain, and, since these gages go out of range before strain hardening starts, an Ames dial was used to obtain additional points. One strain-indicator was used for each SR-4 strain gage because at the plastic range the strains vary at a great speed. The specimens were centered in the
machine and eccentricities were avoided either by the use of a wedge or by shimming up with cigarette papers. Both methods were tried. Fig. 108 shows specimen D, in place compressed between two special fixtures. This specimen was centered with cigarette papers. The wedges used for specimens A and C are shown on the right in front of the strain indicator. The Huggenberger tensometers seen on the specimen were used for centering only.

Test of Large Specimens: Fig. 109 shows specimen E at the completion of the test. Four A-5 electrical gages were used to obtain an average value for strain, and two deflection dials to obtain strains when SR-4's are out of range. Fig. 110 shows specimen F at the completion of the test. The instrumentation is exactly as for Specimen E.

Results: The number of compression tests made is too small to obtain any representative results. Plate P3 along its direction of rolling gave \( \sigma_y = 46.1 \text{ ksi} \) in tension (specimen P3-1) and \( \sigma_y = 48.0 \text{ ksi} \) in compression (specimen A). Specimens C and D are cut from the 1/4 in. thick inner curved flange for connection Model C. This flange was cold bend, and the low \( \sigma_y \) obtained cannot be explained. The results
from specimens E and F are most gratifying. See Fig. 107. 
The lower yield point obtained from specimens E and F is a little lower than the average $G_y = 47.1$ ksi obtained from the tensile tests of the web material of the rolled section.

The results obtained with the deflection dials show that they are erroneous only in the elastic range (E could not be obtained). For any other property like the upper or lower yield point, the length of the plastic range and the slope of the strain hardening region, the deflection dials can be used without any recourse to electrical strain gages. This, of course, will make investigations of this kind very economical.

It should be added that specimen F has buckled both in the web and flanges soon after plastic deformation started indicating that the height should have been smaller.

The yield lines observed were in general horizontal. Inclined yield lines developed later due to pronounced buckling. See Fig. 110. At 160,000 lbs, diagonal yield lines appeared, and at 190,000 lbs, the specimen flanges and web buckled.
APPENDIX C
Welding Sequences

Assembly and welding sequences were studied for each connection specimen to avoid shrinkage distortions. A diagonal brace was used for some connections to facilitate the welding operations. Usually most of the pieces forming the knee were tack welded properly and welding followed in predetermined sequences. The sequences used for connection Model G, are a typical example of welding sequences followed. See Fig. 111.

1. Tack weld web "a" to the plates "c" to "f" in the alphabetical order given. (In all cases a 90° angle is to be maintained between web and flanges).

2. Start welding from middle of each plate, i.e., point 0, o', and o", Fig. 111(a).

3. Weld about 4 in. at a time. Follow the general sequences shown in the sketch. After all welds of one number are completed on one side turn the connection and weld the same members on the other side, Fig. 111(b).

4. Weld butt plates.

5. Weld diagonal stiffener on both sides of the web.

6. Weld rolled sections, Fig. 111(c)

Some of the welding sequences followed while fabricating the other connections are described in (43) D and I.
APPENDIX D

Influence of Connection Behavior on Frame Deflections

The results obtained from the tests show that Type 8B knees develop the $N_p$ of the rolled section. However, Fig. 61 shows that in the elastic and early elasto-plastic stage their unit rotations are larger than the rotations in the rolled section. The additional rotations may increase the deflections in the frame. The following is an example in which the effect of the rotations in the knees on the frame are investigated.

Consider the portal frame shown in Fig. 112(a) with both beam and columns of 8E13 section. The knees at B and D are like the Connection N, Type 8B. The frame is loaded at the third points with concentrated loads. The bending moment diagram is shown in Fig. 112(b).

The moment at which yielding starts at the rolled section is $M_y = 444$ kip-inches (See Table IX). With increasing loads, $M_y$ will be reached at B and D when $P = 8.9$ kips, and the moment diagram will be as is shown in Fig. 112(c). Assuming rigid connection the deflection at C, the midpoint of the frame, when $P = 8.9$ kips has been calculated and found to be 2.45 in.

Connection N, however, at 444 in.-kips rotates more than the 8E13 by $325 \times 10^{-6}$ radians per inch. See Fig. 51.
The total additional rotation in the knees, therefore, is
3.3 x 8 x 10^{-6} = 0.0026 radians. The additional rotation
at the knees will cause further deflections at the midpoint C.

The additional deflections may be found in two ways:

1. With the beam ED in Fig. 112(a) considered as
simply supported, a load of P = 0.54 kips would produce an
angle \( \theta = 0.0026 \) radians at the supports. Fig. 112(d).
The deflection at point C then is 0.24 in., that is 0.3 per
cent more than the deflection computed on the assumption
of a rigid knee at A and D.

2. Again the beam ED is assumed simply supported.
The moment at the ends B and D which would produce an angle
of 0.0026 radians at the ends has been computed (M = 21.7
in.-kips). Fig. 112(e). The deflection at C due to this
moment, is 0.187 in., that is 7.6 per cent more than the
computed on the assumption of rigid knees.

It can be concluded, therefore, that use of Type 2B
connections may result in 10 per cent higher deflections
than the calculated deflections based on the assumption of
rigid connections.

**Strength Requirements in Knees**

Strength requirements for knees were first stated
on p. 16. Then they were restated when the test results
were discussed on p. 62. The following is a further
explanation of what is meant by strength of knees.
Consider the frame AEDD loaded as shown in Fig. 115(a) with nP concentrated loads. In designing this frame according to "Plastic Design" method, as a first approximation, plastic hinges are assumed at E, C and at E and P (segment EP completely plastic). The moment diagram is shown full in Fig. 115(b). The plastic hinge value required as obtained from statics is \( M_p = \frac{nPa}{6} \). A section which would develop the calculated \( M_p \) can be picked out by use of the relation
\[
Z = \frac{M_p}{G y}
\]
where \( Z = \) "the Plastic Modulus" or the static moment of the whole section,
\( G y = \) lower yield point of the material used.

All of the tested connections, except Type 7, would be adequate, if the design is not controlled by aesthetics and fatigue (stress concentration), and the preliminary design is accepted as a final.

Some economies, however, may result if the length of the rolled section is reduced by inserting a properly designed knee. Since the external loads are fixed, from equilibrium considerations the sum of the moments under the load and the corner must always be \( \frac{nPa}{6} \). If a second section is chosen with plastic hinge value \( M_p2 \), then the connection must be good at least for a moment.
\[ M_{P3} = \frac{nF_l}{3} - M_{P2} \]

In such a case, the rolled section can be stopped at section XX' as shown in Fig. 113(b).

If the knee can not develop the required moment \( M_{P3} \), the frame will collapse before the load \( nP \) is reached. Actually the knee should be designed to develop slightly higher moments than \( M_{P3} \). In such a case the \( M_{P2} \) will be developed at XX', and factors which reduce the capacity of knees will not be effective.

Therefore, a well-designed knee must be strong enough to:

a. Develop the \( M_p \) of the rolled section at the end of the rolled section, and consequently higher moments at the knee, or,

b. Develop the \( M_p \) of the rolled section at the knee if additional strength is not required (Example: the first or preliminary case above).

**Example:** Design a frame of 24-ft. span carrying concentrated loads at third points, Fig. 114. The frame is to reach its carrying capacity when the \( nP = 11.3 \) kips.

It is assumed that three plastic hinges at the beam are required for collapse. For simplicity's sake the column and its connection to the knee are assumed adequate; also the kind of supports for the frame is not considered in the problem.
Preliminary Design: When the maximum load $nF = 11.5$ kips is reached it is assumed that plastic hinges develop at $E, E$, and at the segment $CD$ between the loads. From the statical equilibrium of the segment $BC$ the required moment is found to be

$$M_P = 543 \text{ in.-kips}$$

For $g_y = 40 \text{ kips/in.}^2$ and shape factor 1.15 a S815 section would develop $M_P = 543 \text{ in.-kips}$. $M'H$ is the moment diagram in this case. The designer can stop here and consider the S815 as the final design section. In such a case any of the connections $A$ to $N$ would be adequate if not economical.

Revised Design: A haunched knee may be considered at this stage of design either because of doubt as to the adequacy of a square knee or for economy. In such a case the rolled section may be reduced to S813 for which $M_P = 455 \text{ in.-kips}$. If S813 is used, the moment diagram takes the shape shown by $N'H$ (dotted). The connection C, Type 15 which has been tested will be considered for a knee. When the experimental moment diagram $0O'$ at the maximum load is drawn it is seen that the connection is adequate.

Fig. 114 shows that collapse in the knee occurred for moment values less than the calculated $M_y$'s at the various sections. In the design recommendations it has been proposed that in designing bracketed knees, the moment curve of the frame never to exceed the $M_y$ curve at the bracket or haunch.
APPENDIX B

Connection Design Details and Further Test Results

To supplement the information on specimens tested and on test results presented earlier in this paper, some quantitative test results and the design details of the various connections are given below.

Connection P

Fig. 115 gives the design details of Connection P. The stiffeners were designed with double-bevel joints for the welds to the flanges. This design would be probably followed for mass production because of difficulties in fitting stiffeners due to mill variations in rolled sections. The column flanges also were welded to the flanges of the beam with bevel welds.

Table X gives results of Connection P test.

<table>
<thead>
<tr>
<th></th>
<th>Load in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed 1st yield lines</td>
<td>3.2</td>
</tr>
<tr>
<td>Observed initial yield</td>
<td>9.3</td>
</tr>
<tr>
<td>Predicted initial yield</td>
<td>12.7</td>
</tr>
<tr>
<td>Observed start of general yield</td>
<td>11.0</td>
</tr>
<tr>
<td>Local buckling of flanges</td>
<td>16.0</td>
</tr>
<tr>
<td>Maximum load</td>
<td>17.8</td>
</tr>
</tbody>
</table>

Connection A to N

The design details for Connection A to N are given in Figs. 116 to 129. From these figures it is seen that
1/4 in. or 3/16 in. fillet welds were mainly used in the built-up knees. The rolled section was welded to the built-up knee usually with bevel welds. All bevelling and cutting was done manually.

Table XI gives test results of Connections A to N.
### TABLE XI

<table>
<thead>
<tr>
<th>Connection</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed 1st Yield lines (Kips)</td>
<td>6.5</td>
<td>8.0</td>
<td>13.0</td>
<td>19.0</td>
<td>14.5</td>
<td>10.5</td>
<td>10.0</td>
<td>8.5</td>
<td>22.0</td>
<td>12.0</td>
<td>10.0</td>
<td>11.0</td>
<td>9.0</td>
<td>16.0</td>
</tr>
<tr>
<td>Obs. Initial Yield (Kips)</td>
<td>15.5</td>
<td>10.0</td>
<td>18.0</td>
<td>19.0</td>
<td>19.0</td>
<td>11.6</td>
<td>15.0</td>
<td>22.0</td>
<td>21.6</td>
<td>19.0</td>
<td>12.0</td>
<td>7.0</td>
<td>8.5</td>
<td>25.0</td>
</tr>
<tr>
<td>Local Buckling of Flanges (Kips)</td>
<td>21.0</td>
<td>18.0</td>
<td>21.5</td>
<td>23.0</td>
<td>31.0</td>
<td>31.0</td>
<td>29.0</td>
<td>35.0</td>
<td>43.5</td>
<td>34.5</td>
<td>21.5</td>
<td>20.5</td>
<td>21.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Maximum load (Kips)</td>
<td>21.5</td>
<td>19.0</td>
<td>22.5</td>
<td>29.4</td>
<td>31.5</td>
<td>32.5</td>
<td>29.3</td>
<td>39.0</td>
<td>48.6</td>
<td>35.8</td>
<td>22.0</td>
<td>23.8</td>
<td>22.5</td>
<td>35.0</td>
</tr>
</tbody>
</table>
V I T A

The author, Mr. Anastasios Anthony Topractsoglou, was born on February 17, 1920 in Istanbul, Turkey. He is the second son of Anthony and Despina Topractsoglou.

He entered Robert College after his graduation from the Greek Gymnasium of Pera, Istanbul in 1939, and received his Bachelor of Science degree in Civil Engineering in June, 1942. Mr. Topractsoglou worked for Braithwaite Engineering Company from July to October 1942.

In February 1943 he joined the Allied Middle-East Forces and was attached to H.Q.-SHAF. He served until August 1946 in the Middle-East and Greece.

Mr. Topractsoglou was admitted as a graduate student to the University of Minnesota in September 1946. He received the degree of Master of Science in Civil Engineering from the University of Minnesota in August 1947. During the summer of 1947 he worked for the State Highway Department of Minnesota. For the last three years he was employed by Lehigh University as a Graduate Assistant to teach courses in the Civil Engineering Department. He was employed by Bethlehem Steel Company during the summer of 1948 and 1949.

Mr. Topractsoglou, who is a student member of American Welding Society and American Concrete Institute, is co-author of the paper "Curved Beam Deflections with Celluloid Models" Product Engineering, September 1950, and author of several discussions and reports.