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THEORETICAL ANALYSIS
OF
RIGID FRAME KNEES

This work was done under the direct supervision of AA Topradtsoglou

By
Lewis Schneider
14 January 1950

Submitted to Professor B. G. Johnston for credit requirements in Course C.E. 213, Structural Research.
The following comprises a series of calculations to determine the strength of rigid frame connections of various designs. This problem is part of a broader program of welded rigid frame knee tests now under way in Fritz Laboratory.

The knees fall into two classifications; those having curved or straight haunches and those having ninety-degree reentrant corners. The methods of analysis are based on those offered by F. Bleich\textsuperscript{1}, and W. R. Osgood\textsuperscript{2}, and the results of tests conducted by the National Bureau of Standards\textsuperscript{3}.

\textsuperscript{1,2,3} See list of bibliography.
STRESSES IN FLANGE DUE TO CURVATURE

\( \sigma_r \) is resultant of \( r \) components of \( \sigma_t \)

TRANSVERSE LOAD PRODUCES TRANSVERSE BENDING OF FLANGES

LONGITUDINAL STRESS VARIES ALONG FLANGE

TRANSVERSE BENDING PRODUCES TRANSVERSE BENDING STRESSES \( \sigma' \)

\[
\text{max } \sigma = \frac{\sigma_i}{u} \\
\sigma' = \mu \sigma_{\text{max}}
\]

COEFFICIENTS GIVEN IN BLEICH, "DESIGN OF RIGID FRAME KNEES," P.10 TABLE II

STATE OF STRESS OF ELEMENT AT CENTER OF FLANGE

\( \text{max } \sigma_i \) and \( \sigma' \) are principle stresses since \( \tau_{xy} = 0 \)

\[
\sigma_{\text{total}}^2 = \sigma_i^2 + \sigma'^2 - \sigma_i \sigma'
\]

\( \sigma_i \) IN THIS CASE IS \( \sigma_{\text{max}} \)

*BLEICH - DESIGN OF RIGID FRAME KNEES.*

AISC 1943
ASSUMPTIONS OF STRESS

DISTRIBUTION OF FIBER STRESS, (STIFFENER AT ad and ae)

* Bleich - Design of Rigid Frame knees, P. 5.

LOADING OF KNEE

All normal stresses at arc a-a intersect at o, the intersection of t1 and t2.

SECTION WITH NON-PARALLEL FLANGES
WEDGE THEORY

The wedge theory as developed by the theory of elasticity is applied to sections of the knees having non-parallel flanges. This theory is exact when the flanges are straight, but when a flange is curved the theory is an approximation. The equations developed by Osgood are for symmetrical sections, and modification was required to make them usable for the present case.
The focused energy on development of the theory of

specifics to equation of motion of the charged

particles is essential. This means the correct

method to understand and analyze the

phenomena is required. We will consider the

radius of a spherical charge density and

distribution.

In order to use the conservation equation and
the analogies, we must develop the theory of

electromagnetic field.
\[ R \sin \beta = \frac{1}{3} R \quad \beta_1 = 20^\circ \]

\[ R \sin \beta = \frac{2}{3} R \quad \beta_2 = 40^\circ \]

\[ D + R \sin \beta = x + R \cos \beta \]

\[ x = D + R \sin \beta - R \cos \beta \]

\[ H + R = \rho \sin \beta + R \cos \beta \]

\[ \rho = \frac{H + R (1 - \cos \beta)}{\sin \beta} \]

**Proof.**
MODIFICATION OF WEDGE THEORY FOR UNSYMMETRICAL SECTIONS.

\[ \sigma_{11} = \frac{c_1}{f} \sin \theta \quad (3) \]
\[ \sigma_{22} = 0 \quad (4) \]
\[ \tau_{12} = 0 \quad (5) \]

FOR EQUILIBRIUM
\[ \int_{-a_2}^{a_1} \sigma_{11} \sin \theta \, d\theta = 0 \]
\[ \int_{-a_2}^{a_1} \sigma_{11} \cos \theta \, d\theta - P_1 = 0 \]
\[ \int_{-a_2}^{a_1} \tau_{12} \sin 2\theta \, d\theta = 0 \]
\[ c_1 \int_{-a_2}^{a_1} \tau_{12} \sin 2\theta \, d\theta - P_1 = 0 \]

FOR LOAD \( P_1 \)

\[ \sigma_{11} = \frac{c_1}{f} \sin \theta \]

but
\[ \sigma_{11} = \frac{c_1}{f} \sin \theta \]

FOR LOAD \( P_2 \)
\[ \sigma_{22} = \frac{d_1}{f} \cos \theta \quad (8) \]
\[ \sigma_{12} = 0 \quad (9) \]
\[ \tau_{12} = 0 \quad (10) \]

FOR EQUILIBRIUM
\[ \int_{-a_2}^{a_1} \tau_{12} \sin 2\theta \, d\theta = 0 \quad (11) \]
\[ a_1 \int_{-a_2}^{a_1} \tau_{12} \cos 2\theta \, d\theta + P_2 \quad (12) \]

Note: See Bleich's report for simpler procedure.
FOR MOMENT Y 
\[ \sigma_{y} = -\frac{4}{r} \frac{d'z}{r^2} \sin \theta \quad d'z = \text{const.} \]
\[ \sigma_{o} = 0 \]
\[ \sigma_{r} = -\frac{4}{r^2} \frac{d'z}{r^2} \sin \theta \]
\[ T_{t o} = \frac{4d'z}{r^2} \int_{0}^{\theta} \sin \theta \, d\theta \]
\[ 4d'z \int_{-\theta}^{\theta} \sin \theta \, d\theta + M = 0 \]

APPLICATION TO SECTION WITH THIN FLANGES

EQ. 7 
\[ A_0 = \text{Area outside flange}, \quad t = \text{thickness of web}, \quad A_i = \text{inside} \]
\[ c_i = \left( A_o \sin^2 \chi + A_i \sin^2 \chi \right) + c_i t \int_{-\partial z}^{\partial z} \sin^2 \theta \, d\theta - P_i = 0 \]
\[ c_i = \frac{P_i r}{(A_0 \sin^2 \chi + A_i \sin^2 \chi) + c_i t \int_{-\partial z}^{\partial z} \sin^2 \theta \, d\theta} \]
\[ \int_{-\partial z}^{\partial z} \sin^2 \theta \, d\theta = \int_{-\theta}^{\theta} \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \, d\theta \]
\[ \int_{-\theta}^{\theta} \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \, d\theta = \frac{1}{2} \chi (\chi + 2\partial z) - \frac{1}{4} (\sin 2\chi + \sin 2\partial z) \]
\[ c_i = \frac{P_i r}{(A_0 \sin^2 \chi + A_i \sin^2 \chi) + \frac{1}{2} \chi (\chi + 2\partial z) - \frac{1}{2} (\sin 2\chi + \sin 2\partial z)} \]
\[ = \frac{P_i r}{(A_0 \sin^2 \chi + A_i \sin^2 \chi) + \frac{1}{4} (2\chi + 2\partial z - \sin 2\chi - \sin 2\partial z)} \]
\[ \sigma_{y} = \frac{c_i \sin \theta}{r} \]

(14)

(16)

(17)

(18)

(19)
\[ r_1 = \int_0^{2\pi} d\theta \sin 2\theta \]
EQ. 12
\[ \frac{a_1}{r} \left( A_0 \cos^2 \theta + A_i \cos \theta \right) + \alpha_1 t \int_{-\delta_1}^{\delta_1} \cos^2 \theta \, d\theta + P_2 = 0 \]

\[ a_1 = \frac{-P_2 r}{(A_0 \cos^2 \theta + A_i \cos \theta) + \alpha_1 t \int_{-\delta_1}^{\delta_1} \cos^2 \theta \, d\theta} \]

\[ \int_{-\delta_2}^{\delta_2} \cos^2 \theta \, d\theta = \left[ \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right]_{-\delta_2}^{\delta_2} = \frac{1}{2}(\delta_1 + \delta_2) + \frac{1}{4}(\sin 2\delta_1 + \sin 2\delta_2) \]

\[ a_1 = \frac{-P_2 r}{(A_0 \cos^2 \theta + A_i \cos \theta) + \frac{\alpha_1 t}{4} (2\delta_1 + 2\delta_2 + \sin 2\delta_1 + \sin 2\delta_2)} \]

\[ \delta_2 = \frac{a_1}{r} \cos \theta \quad (21) \]

EQ. 17
\[ b = \text{width of outer flg.} \quad t_f = \text{thickness of flg.} \]
\[ b_i = \text{inner flg.} \quad t_i = \text{im.} \]
\[ 4d_2^2 \sin 2x_1 \frac{t_f}{r} + 4d_2^2 \sin 2x_2 \frac{t_i}{r} \]
\[ + 4d_2 \int_{-\delta_2}^{\delta_2} \sin 2\theta \, d\theta + M = 0 \]

\[ 4d_2^2 \left( x_1 A_0 \sin 2x_1 + x_2 A_i \sin 2x_2 \right) + 4d_2 t \int_{-\delta_2}^{\delta_2} \sin 2\theta \, d\theta = -M \]

\[ d_2' = \frac{-M r}{4 \left( x_1 A_0 \sin 2x_1 + x_2 A_i \sin 2x_2 \right) + 4\delta_1 t \int_{-\delta_2}^{\delta_2} \sin 2\theta \, d\theta} \]

\[ \int_{-\delta_2}^{\delta_2} \sin 2\theta \, d\theta = \left[ \sin 2\theta - \frac{\theta \cos 2\theta}{2} \right]_{-\delta_2}^{\delta_2} \]
\[
\int_{\theta} \sin 2\theta d\theta = \frac{\sin 2\theta_1}{4} - \frac{\sin(2\theta_2)}{2} - \frac{\sin(2\theta_3)}{4} - \frac{\sin(2\theta_4)}{2}
\]

\[
\int_{\theta} \sin 2\theta d\theta = \frac{1}{4} (\sin 2\theta_1 + \sin 2\theta_2) - \frac{1}{2} (\cos 2\theta_1 + \cos 2\theta_2)
\]

\[
d_2' = \frac{-\mu r}{4 (2A \cos 2\theta_1 + 2d_2 \cos \sin 2\theta_2) + \frac{\mu r}{2} \left( \left(\sin 2\theta_1 + \sin 2\theta_2 \right) - \frac{\mu r}{2} \left( \cos 2\theta_1 + \cos 2\theta_2 \right) \right)}
\]

\[
\delta r_3 = -\frac{4}{r^2} d_2' \sin 2\theta
\]

\[
T_{r_3} = \frac{4}{r^2} d_2' \int_\theta \theta' \sin 2\theta d\theta \quad \theta' = \lambda, \text{ or } -\lambda
\]

For \(\theta' = \lambda\):

\[
T_{r_3} = \frac{4}{r^2} d_2' \left[ \frac{A_0}{r} \sin 2\lambda_1 - \frac{1}{2} \left( \cos 2\lambda_1 + \cos 2\lambda_2 \right) \right]
\]

For symmetrical cases:

\[
c_1 = \frac{P_1 r}{2 A f \sin^2 \lambda + 2 r (\lambda + \frac{\sin 2\lambda}{2})}
\]

\[
a_1 = \frac{-P_2 r}{2 A f \sin^2 \lambda + 2 r (\lambda - \frac{\sin 2\lambda}{2})}
\]

\[
d_2' = \frac{-\mu r}{3 A f \sin 2\lambda + 2\mu r \left( \sin 2\lambda - 2 \cos 2\lambda \right)}
\]

\[
\delta r_1 = \frac{c_1}{r} \sin \theta
\]

\[
\delta r_2 = \frac{a_1}{r} \cos \theta
\]

\[
\delta r_3 = -\frac{4}{r^2} d_2' \sin 2\theta
\]
EFFECT OF DIAGONAL STIFFENER IN A RECTANGULAR KNEE

\[ l = \text{length of stiffener } ac = \sqrt{h_1^2 + h_2^2} \]

\[ ba = \sqrt{bf^2 + af^2} \]

\[ \Delta l = l - ba = l - \sqrt{(h_2 - h_1 y_1)^2 + (h_1 - h_2 y_2)^2} \]

\[ \Delta l = \frac{l^2 - (h_2 - h_1 y_1)^2 - (h_1 - h_2 y_2)^2}{l + \sqrt{(h_2 - h_1 y_1)^2 + (h_1 - h_2 y_2)^2}} \]

\[ \Delta l = \frac{2h_1 h_2 (y_1 + y_2) - (h_1^2 y_1^2 + h_2^2 y_2^2)}{2l} \quad \text{Eq. (G)} \]
Assumed Loading of Diagonal Stiffener

(Based on Progress Report No. 9,

\[ \Delta l = \frac{6 \max l}{2E} \quad \text{Eq. (1)} \]

\[ \delta_{\max} = \left[ \frac{2 h_1 h_2 (x_1 + x_2) - (h_1^2 x_1^2 + h_2^2 x_2^2)}{l^2} \right] \]

\[ \delta_{\max} = \left[ \frac{2 h_1 h_2 (x_1 + x_2) - (h_1^2 x_1^2 + h_2^2 x_2^2)}{l^2} \right] \in \text{Eq. (1)} \]

For symmetrical case \( h_1 = h_2 \), \( x_1 = x_2 \)

\[ \delta_{\max} = \left( \frac{4 h_1^2 x}{l^2} \right) \in = \frac{2 h_1^2 \gamma E (2 - \gamma)}{l^2} \]

\[ \delta_{\max} = \frac{4 h_1^2 \gamma E}{l^2} \quad \text{Eq. (2)} \]

\( \delta_{\max} \) can be found from the extreme fiber stress or the reentrant angle omitting stress concentration

\[ \delta_{\max} = \frac{12 \sigma}{E} \text{ @ extreme fiber} \]

Substituting in Eq. (2) the angle of rotation \( \gamma \) could be obtained for connections like \[ \text{or} \]

\[ \text{or} \]
\[ P = 6 \max A_3 \]
\[ A_3 = \text{area of stiffener} \]
\[ T_{2^*} = 6 \max A_3 \cos \alpha \]
\[ T_{1^*} = 6 \max A_3 \sin \alpha \]

\[ \gamma_1 = \frac{T_1}{A w_1 G} \quad (c) \]
\[ \gamma_2 = \frac{T_2}{A w_2 G} \]

\[ T_{1^*} = 6 \max A_3 \sin \alpha \quad (d) \]
\[ T_{2^*} = 6 \max A_3 \cos \alpha \]

\[ M = \frac{f_1}{e} (T_i + T_{i^*}) \quad f \approx \frac{1}{h_i} \]
\[ M = \frac{f_2}{e} (T_z + T_{z^*}) \quad f_2 \approx \frac{1}{h_{2z}} \]
LOCAL EFFECT OF DIAGONAL STIFFENER

\[ P = \delta_{\text{max}} A_s \]
\[ P = l t w_r T_5 (z) \]
\[ \delta_{\text{max}} = \text{shearing stress between stiffener and web.} \]

\[ T_5 = \frac{\delta_{\text{max}} A_s}{2 l t w_r} \quad (3) \]

FROM EQ. (1) \( \delta_{\text{max}} \) can be expressed in terms of \( \delta \) using (c),(d), and (e).

FOR \( h_1 = h_2 \) and \( y_1 = y_2 \)

\[ \delta_{\text{max}} = \frac{4 h_2^2 y E}{l^2} \]
\[ y_i = \frac{T_i}{A w G} \]

\[ T_5 = \frac{A_s}{2 l t w_r} \left( \frac{4 h_2^2 E}{l^2} \right) \left( \frac{T_i}{A w G} \right) \]
\[ A w = h t w_r \]
\[ \frac{E}{G} = 2.6 \quad (\mu = 0.3) \]

\[ T_5 = \frac{5.2 A_s h T_i}{l^3 t w_r} \quad (4) \]

STATE OF STRESS = FIG. A + FIG. B
CONCLUSION

(Tentative, subject to further interpretation on the basis of test results)

EFFECT OF DIAGONAL STIFFENER: A diagonal stiffener across the knee provides extra rigidity. However, the increased strength is not as great as might be expected since the stiffener itself is highly stressed by small deformations of the web and produces high local stresses.

STRAIGHT HAUNCHES: Straight haunches increase the strength of the connections, but a comparison between connections B and C indicates that it is advisable to haunch both the vertical and horizontal member. A sharp reentrant angle should be avoided.

CURVED FILLET: A curved fillet is desirable, but it is very important that stiffeners be provided to resist the effect of transverse forces on the inner flange. The greater the radius of the fillet, and the thicker the inside flange, the lower stress that will be developed.
REFERENCES

1. Bleich, F. - "Design of Rigid Frame Knees"
   AISC 1943


4. Timoshenko, S. - "Theory of Elasticity"
   McGraw-Hill, New York, Chapter 3, 1934

SUMMARY OF RESULTS

STRESSES IN PSI IN TERMS OF
LOAD P IN POUNDS.

CONNECTION

A

REMARKS

INITIAL YIELD WILL
OCUR AT C
WITH LOAD OF
18,400 psi

THE DIAGONAL
STIFFENER WILL
YIELD AT A
LOAD OF
18,900 psi

\[ 2.42P = \]

\[ \text{Leg } 59.38^\circ \]

CONNECTION

B

NOS. IN ( )
CALCULATED BY
BLEICH EQ. 9.

INITIAL YIELD AT P = 19,000 psi

INITIAL YIELD AT P = 26,400 psi

\[ \text{Leg } 48.88^\circ \]
Figs. in ( ) by Bleich Eq. 9

Conn. C

Initial yield at \( P = 17,000 \) lb

Conn. D, E, F

Initial yield at \( P = 23,900 \) lb

By Bleich theory, stiffening bracket will yield at 23,900 lb.

By wedge theory, bracket will yield at \( P = 33,600 \) lb.

Leg = 39.38"
INITIAL YIELDING WILL OCCUR AT \( P = 16,300 \) d due to longitudinal stress plus bending stress due to transverse bending of the flanges.

\( \text{Conn. H} \)

INITIAL YIELD AT \( P = 17,000 \) d due to longitudinal stress plus bending stress due to transverse bending of the flanges.

\( \text{Conn. I} \)
Connection J

Initial Yield at P = 21,200#

Leg 47.63"

Connections K & L (See Connection M)

Connection M

Initial Yield at P = 17,800#

Diagonal stiffener will yield at P = 19,800#

Leg 39.38"
HELO AT

P = 24,600 lb

CONN. N

L = 39.38
$P$ for yield at extreme fibers:

\[ 2.42P = 44,500 \quad \Rightarrow \quad P = 18,400 \text{ psi} \]
LOCATION OF N.A.  \[ y_{\text{NA}} = \frac{2.42 (8.00)}{4.48} = 4.33'' \]

\[ Q_{\text{NA}} = 4(0.254)(3.88) + 3.42(0.23)(2.04) = 3.44 + 1.66 = 5.10 \text{ in}^3 \]

\[ Q_{\text{Fillet}} = 3.94 + 0.31 (0.3)(3.60) = 4.27 \text{ in}^3 \]

\[ \tau_{\text{NA}} = \frac{VQ}{Ic} = \frac{0.707P(5.60)}{39.5(0.23)} = 0.436P \]

**FOR YIELDING AT N.A.**  \[ 0.436P = \frac{44500}{\sqrt{3}} \]

\[ P = 58900'' \]

\[ \tau_{\text{Fillet}} = \frac{0.707P(4.27)}{39.5(0.23)} = 0.332P \]

\[ \sigma_{x \text{ at Fillet}} = \frac{3.77}{4.33} (-2.42P) = -2.17P \]

**COMBINED STRESS AT FILLET**

\[ \sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} = P \sqrt{(-2.17)^2 + 3(0.332)^2} = P \sqrt{5.05} \]

\[ = 2.24P \]

**FOR YIELDING AT INSIDE FILLET**

\[ 2.24P = 44500 \]

\[ P = 19800'' \]
STABILITY OF BEAM FLANGES

ASSUME SIMPLY SUPPORTED ON INNER EDGE

FROM TIIIO. "PLATES & SHELLS" P. 317 TABLE 39 AND CORRECTION EQUATIONS.

\[ \sigma_{cr} = \frac{1330(30 \times 10^6)(0.254)^2}{30 \times 10^2} \times \frac{2.00}{2.00} = 214,000 \text{ psi} \]

BEAM FLANGES WILL NOT BUCKLE \[ \sigma_{cr} \]
ELASTIC STABILITY OF WEB USING BLEICH'S APPROXIMATE SOLUTION

\[-6_1 + 6_2 = -2.17P\]
\[c = 0.334P\]

\[\beta = \frac{0.334P}{2.17P} = 0.154\]
\[\mu = 2.16\]
\[\kappa u = 4.82 + 3.63 = 8.45\]
\[\kappa = 0.390\]

\[\frac{d}{l} = 1\]

\[\chi u = \frac{\pi}{\sqrt{\kappa u}} \sqrt{\frac{k^2 + \beta^2}{1 + 3\beta^2}} \left( \frac{d}{l} \right)\]

\[= \frac{\pi}{\sqrt{8.45}} \sqrt{\frac{(0.390)^2 + (0.154)^2}{1 + 3(0.154)^2}} \left( \frac{6.9}{0.23} \right)\]

\[= \frac{\pi}{2.91} \sqrt{\frac{0.1483 + 0.0247}{1.07}} \left( \frac{6.9}{0.23} \right) = \frac{\pi (0.646)6.9}{2.91 (0.23)}\]

\[= 20.9\]

\[\delta w^* = 44 - 0.16 \chi u \approx 44 - 0.16(20.9)\]

\[= 40,650 \text{ psi}\]

\[\delta w = \frac{\delta w^*}{\sqrt{1 + 3\beta^2}} = \frac{40,650}{1.07} = 37,700 \text{ psi}\]

LOAD \(P\) FOR BUCKLING \(2.17P = 37,700\)

\[P = 17,400^*\]

\[\tau w = \delta w \beta = 37,700 (0.154) = 5810 \text{ psi}\]

LOAD \(P\) FOR BUCKLING \(0.334P = 5810\)

\[P = 17,400^*\]

This formula for design very much on safe side. It cannot be used to predict the actual stability of the webs.
STRESSES IN KINNE WEB.

\[ T_{xy} = \frac{2T_1}{A_w} \]
\[ T_x = \frac{4.21}{4.33} (2.42P)(4)(0.254) = 2.39P \]
\[ T_y = \frac{3.05}{3.67} (2.06P)(4)(0.254) = 2.02P \]
\[ T_1 = \frac{2.39 + 2.02}{2} P = 2.20P \]

\[ \{ P_1 \text{ = load req'd to produce shear strain } \gamma \}, \{ P_2 \text{ = load req'd to deform diagonal stiffener} \} \]
\[ T_{xy} = \frac{2(2.20P)}{6.9(0.23)} = 2.77P \]

OCTAHEDRAL SHEAR STRESS THEORY USED TO DETERMINE YIELD POINT.
\[ \delta yd^2 = \delta_1^2 + \delta_2^2 - 2\delta_1 \delta_2 \]
\[ \delta_1 = \frac{\delta_x + \delta_y}{2} + \sqrt{\left(\frac{\delta_x - \delta_y}{2}\right)^2 + \tau_{xy}^2} \]

COMBINED STRESS DUE TO LOCAL ACTION OF STIFFENER.
\[ \tau_3 = \frac{6.2A_3 h T_1}{l^2 t_w^2} = \frac{5.2(1.00)(6.9)(2.77P)}{9.75^3(0.23)^2} = 2.03P \]
\[ \delta_x = 2.03 \text{ in} \]
\[ \delta_y = -2.03 \text{ in} \]
\[ \tau_{xy} = 2.77 \text{ in} \]

\[ \delta_1 = \frac{\delta_x^2 + \tau_{xy}^2}{6} \]
\[ \delta_2 = -\frac{\delta_x^2 + \tau_{xy}^2}{6} \]

\[ \delta_{yld}^2 = 3 \left( \delta_x^2 + \tau_{xy}^2 \right) = 3 \times (2.03^2 + 2.77^2) = 34.53 \text{ in}^2 \]

\[ \delta_{yld} = 5.87 \text{ in} \]

For yielding in web \[ P = \frac{44,500}{5.87} = 7580 \text{ psi} \]

\[ 2\theta = \frac{\tau_{xy}}{G} = \frac{2.77}{11.6(10)^6} = 0.00239 \text{ rad} \]

\[ \delta_{\text{max}} = \frac{4h^2\theta E}{\ell^2} = \frac{4(6.9)^2(0.00239)(30)(10)^6}{2 \left( \frac{6.9}{0.107} \right)^2} \]

\[ = 173,000 \text{ psi} \]

\[ \text{Max value of } P_1 = \frac{44,500}{173,000} = 2620 \text{ ft} \]

\[ T_2 = 44,500 (1.00)(0.707) = \delta_{\text{max}} A \sin \theta \]
\[ = 31,400 \]

\[ P_2 = \frac{T_2}{2.20} = 14,300 \]

\[ P = P_1 + P_2 = 14,300 + 4600 = 18,900 \text{ ft} \]
EXTREME FIBER STRESSES

\[ f = \frac{N}{A} + \frac{M_y}{I} \]

\[ f_x = 0.707 \frac{P}{3.83} + 0.707 \frac{P(16.13)}{9.88} = 0.185P + 1.154P = 0.969P \]

\[ f_x = -0.185P - 1.154P = -1.339P \]

FOR YIELDING AT EXTREME FIBER

\[ 1.339P = 44.500 \]

\[ P = \frac{33,200}{1.1} \]
\[ y_{na} = \frac{1339}{2.308} (3.00) = 4.64" \]

\[ Q_{na} = 4(1.254)(3.88) + 3.11(0.23)(2.20) = 394 + 1.65 = 395.25 \text{ in}^3 \]

\[ Q_{fillet} = 3.94 + 0.3(0.8)(3.60) = 3.94 + 0.33 = 4.27 \text{ in}^3 \]

\[ T_{na} = \frac{0.707P(6.16)}{39.3(0.23)} = 0.434P \]

For yielding at \( \text{N.A.} \)

\[ 0.434P = \frac{44500}{\sqrt{3}} \]

\[ P = 59,300 \# \]

Combined Stress at Inside Fillet

\[ T_{nf} = \frac{0.707P(4.27)}{39.3(0.23)} = 0.333P \]

\[ \sigma_x = \frac{4.08(-1.339P)}{4.64} = -1.18P \]

\[ \sigma_{cong} = \sqrt{\sigma_x^2 + 3T_{nf}^2} = P\sqrt{(1.18)^2 + 3(0.333)^2} = P\sqrt{1.738} = 1.32P \]

For yielding at fillet

\[ 1.32P = 44,500 \]

\[ P = 33,700 \# \]

Stresses at Z-Z

Radius of arc \( Z-Z = 12(\frac{20}{\pi}) = 60.00" \)

\[ \tan 2\alpha = \frac{1}{3} \]

\[ 2\alpha = 11.32^\circ \]

\[ \alpha = 5.66^\circ \]

Length arc \( Z-Z = 60.00 \left(\frac{11.32}{57.3}\right) = 11.86" \]

\[ N = P_{con}(45 - 5.66) = 0.774P \]
\[ \alpha = 60 \text{ dim} \times 5.66^\circ = 5.92 \text{ in} \\
\beta = 1.92^\circ \\
\epsilon = 60 \text{ cos} 5.66^\circ - 23.87 = 35.84 \\
\mu = 0.707P(1.92 - 35.84) = -24.0P \\
\rho = 70^\circ \text{ (BY APPROXIMATION)} \\
A = 2(4)(\frac{1}{4}) + 11.36(\frac{1}{4}) = 2.00 + 2.84 = 4.84 \text{ in}^2 \\
I = 2(1.00) \times (5.81)^2 + \frac{1}{12}(\frac{1}{4}) (11.36)^3 = 67.5 + 30.4 = 97.9 \text{ in}^4 \\
\\n\text{Bleich Eq. 9} \\
\sigma = \frac{P}{\cos \alpha} \left[ \frac{N}{A} - \frac{M}{pA} - \frac{Mc}{E} \left( \frac{\rho}{p+c} \right) \right] \\
\sigma_0 = \frac{1}{0.995} \left[ -0.774P + \frac{24.0P}{70(4.84)} + \frac{24.0P(5.93)(70)}{97.9(75.93)} \right] \\
= 1.005 (-0.160P + 0.071P + 1.342P) \\
= 1.005 (1.253P) = 1.260P \\
\sigma_1 = 1.005 \left[ -0.160P + 0.071P - \frac{24.0P(593)(70)}{97.9(64.1)} \right] \\
= 1.005 (-0.160P + 0.071P - 1.587P) \\
= -1.682P \\
\text{FOR YIELDING AT 2-2} \\
1.682P = 44,500 \\
P = 26,400 \text{ lb}
STRESSES AT Z-2 BY ORDINARY BEAM THEORY

\[ N = 0.707P \]

\[ M = 0.707P (2.00 - 36.13) = -24.1P \]

\[ A = 4\left(\frac{1}{4}\right)2 + 11.50\left(\frac{1}{4}\right) = 2.00 + 2.875 = 4.88 \text{ in}^2 \]

\[ I = 2 \left(\frac{1.00}{2}\right)(5.88)^2 + \frac{1}{12} \left(\frac{1}{4}\right)(11.5)^3 = 69.3 + 31.7 = 101.0 \text{ in}^4 \]

\[ f_o = - \frac{0.707P}{4.88} + \frac{24.1P(6.00)}{101.0} = -0.145P + 1.427P \]

\[ = 1.282P \]

\[ f_i = -0.145P - 1.427P = -1.572P \]

\[ \text{ARC 3-3} = 50 \left(\frac{11.32}{57.3}\right) = 9.89'' \]

\[ a = 50 \sin 5.66'' = 4.93'' \quad e_v = 0.93'' \]

\[ e_4 = 50 \cos 5.66'' - 23.87 = 25.89'' \]

\[ N = 0.774P \quad M = 0.707P(0.93 - 25.89) = -17.7P \]

\[ A = 2(1.00) + 9.39\left(\frac{1}{4}\right) = 2.00 + 2.35 = 4.35 \text{ in}^2 \]

\[ I = 2(1.00)(4.82)^2 + \frac{1}{12}\left(\frac{1}{4}\right)(9.39)^3 = 46.4 + 17.2 = 63.6 \text{ in}^4 \]

\[ \phi = 46'' \]
\[
\delta_0 = \frac{1}{0.995} \left[ -\frac{0.774P}{4.35} + \frac{17.7P}{46(4.35)} + \frac{17.7P(4.94)(46)}{63.6(50.9)} \right] \\
= 1.005 \left( -0.178P + 0.088P + 1.242P \right) \\
= 1.160P \\
\delta_0' = \frac{1}{0.995} \left( -0.178P + 0.088P - \frac{17.7P(4.94)(46)}{63.6(41.1)} \right) \\
= 1.005 \left( -0.178P + 0.088P - 1.540P \right) \\
= -1.64P \\
\text{FOR YIELDING AT 3-3, } 1.64P = 44,500 \\
P = 27,100 \text{ kN}
\]

**Stresses at 3-3 by Ordinary Beam Theory**

\[N = 0.707P\]  
\[M = 0.707P(100 - 26.13) = -17.7P\]  
\[A = 2.00 + 9.50 \left( -\frac{1}{4} \right) = 2.00 + 2.375 = 4.375 \text{ in}^2\]  
\[I = 2(100)(4.88)^2 + \frac{1}{12} \left( \frac{1}{4} \right)(9.50)^3 = 47.4 + 17.9 = 65.3 \text{ in}^4\]  
\[f_0 = \frac{0.707P}{4.38} + \frac{17.7P(5.00)}{65.3} = -0.161P + 1.355P \]
\[= 1.194P\]  
\[f_i = -0.161P - 1.355P = -1.516P\]
**EXTREME FIBER STRESSES, OSGOOD THEORY**

\[
P_r = 620P \\
P_e = 0.785P \\
\alpha = 56.6^\circ = 0.9987 \text{ rad.} \\
\sin \alpha = 0.9987 \\
\cos \alpha = 0.9951 \\
\theta \text{ in } \circ = \theta \text{ in } \text{rad.}
\]

\[\delta_r = \frac{c_1}{r} \sin \theta = 24.4P(0.0987) = 240P \left| \theta = \alpha \right. \\
\delta_r = -240P \left| \theta = -\alpha \right.
\]

\[\delta_r = \frac{q_1}{r} \sin \theta = -0.263P \cos \theta = -0.262P \left| \theta = \pm \alpha \right.
\]
\[ \bar{d}_2 = \frac{-Mr}{8Af \sin 2\theta + 2r(\sin 2\theta - 2x \cos 2\theta)} \]

\[ \bar{d}_2 = \frac{14.06 Pr}{0.155} = 91.2 Pr \]

\[ \delta r_3 = -\frac{4}{r^2} d_2 \sin 2\theta \]

\[ \delta r_3 = -\frac{4}{60} P(91.2) \sin 11.32^\circ = -0.668 P \]

\[ P = 19,000 \text{#} \]

**Max. Fiber Stresses**

**Outside Fiber**

\[ \sigma_0 = P \left( \frac{240}{240} - 0.62 - 0.67 \right) = 1.11P \]

**Inside Fiber**

\[ \sigma_0 = P \left( -\frac{240}{240} - 0.62 + 0.67 \right) = -2.35P \]

**For Yielding at z-2**

\[ 2.35P = 44,500 \]

**Fiber Stresses at z-3**

\[ P_1 = 0.620P \]

\[ P_2 = 0.785P \]

\[ M_0 = -14.06P \]

\[ \theta = 50^\circ \]

\[ c_1 = \frac{P_1}{2Af \sin 2\theta + tr (\theta - \frac{\pi}{2})} = \frac{0.620P_1}{0.0195 + 0.25(50)(0.004)} \]

\[ c_1 = \frac{0.620P_1}{0.0245} = 25.3Pr \]

\[ \delta H_1 = \frac{c_1}{r} \sin \theta = 25.3P (0.0987) = 2.49P | \theta = \alpha \]

\[ -2.49P | \theta = -\alpha \]
\[ q_1 = \frac{-P_e r}{2 A_f \sin^2 \alpha + t_r (\alpha + \sin \omega \frac{2 \alpha}{2})} \]
\[ = \frac{-0.785 \Pr}{0.0195 + 0.25(50)(0.1970)} = \frac{-0.785 \Pr}{2.47} \]
\[ = -0.315 \Pr \]

\[ \delta r_2 = q_1 \frac{\rho \Theta}{\rho} = -0.315 \Pr \cos \alpha \]
\[ = -0.314 \Pr \theta = \pm \alpha \]

\[ d_2 = \frac{-Hr}{8 A_f \sin 2 \alpha + 2 t_r (\sin 2 \alpha - 2 \alpha \cos 2 \alpha)} \]
\[ = \frac{14.06 \Pr}{0.155 + 2(0.25)(50)(0.004)} = 55.0 \Pr \]
\[ \delta r_3 = \frac{4}{r^2} d_2 \sin 2 \theta \]
\[ = \frac{4}{50} (55.0 \Pr \sin 2 \alpha) = -0.719 \Pr | \theta = \alpha \]
\[ = 0.719 \Pr | \theta = -\alpha \]

**Extreme Fiber Stresses**

Outside Flange \[ \delta_0 = P (2.49 - 0.31 - 0.72) = 1.46 P \]

Inside Flange \[ \delta'_w = P (-2.49 - 0.31 + 0.72) = -2.08 P \]

For Yielding at 3-3 \[ 2.08 P = 44,500 \]
\[ P = 21,400 \]
STRESSES AT 1-1

\[
f = \frac{N}{A} \pm \frac{M_c}{I}
\]

\[
f_o = -\frac{0.707 P}{3.83} + \frac{0.707 P (34.10)}{9.88} = -0.185P + 2.44P
\]

\[
f_o = 2.26P
\]

\[
f_{xi} = -0.185P - 2.44P = -2.62P
\]

FOR YIELDING AT EXTREME FIBERS AT 1-1

\[
2.62P = 44,500
\]

\[P = 17,000\]
\[ Y_{N_A} = \frac{2.62}{4.88} (8.00) = 4.28'' \]

\[ Q_{N_A} = 4(0.254)(3.88) + 3.47(0.23) = 5.76 \text{ in}^3 \]

\[ Q_{	ext{fillet}} = 4.40 \text{ in}^3 \]

\[ T_{N_A} = \frac{0.707P(5.96)}{39.5(0.23)} = 0.463 P \]

For yielding at N.A., \[ 0.463P = \frac{44,500}{\sqrt{3}} \]

\[ P = 55,500 \text{ lb} \]

\[ T_{	ext{fillet}} = \frac{0.707P(4.40)}{39.5(0.23)} = 0.342 P \]

\[ \sigma_y \text{ at fillet} = \frac{3.72}{4.28} (-2.62P) = -2.28 P \]

Combined stress at inside fillet:

\[ \sigma = \sqrt{\sigma_y^2 + 3T_{	ext{fillet}}^2} = P\sqrt{(-2.28)^2 + 3(0.342)^2} \]

\[ = P\sqrt{5.01 + 0.35} = 2.31 P \]

For yielding, \[ 2.31P = 44,500 \]

\[ P = 19,300 \text{ lb} \]
STRESSES AT 3.3
\[ f_0 = -0.185P + \frac{0.707P(21.85)}{9.88} = -0.185P + 1.498P \]
\[ = 1.313P \]
\[ f_i = -0.185P - 1.498P = -1.683P \]

For yielding at extreme fibers
\[ 1.683P = 44,500 \]
\[ P = 26,500 \text{#} \]

\[ q_{na} = \frac{1.683}{2.996}(8.00) = 4.48'' \]

\[ q_{na} = 4.06 + 3.27(0.23)(2.12) = 4.06 + 1.60 = 5.66'' \]

\[ q_{fillet} = 4.40'' \]

SHEAR STRESSES
\[ t_{na} = \frac{0.707P(5.66)}{39.5(0.23)} = 0.440P \]

For yielding at NA
\[ 0.440P = 23,700 \]
\[ P = 53,300 \text{#} \]

COMBINED STRESS AT INSIDE FILLET
\[ t_y = \frac{0.707P(4.40)}{39.5(0.23)} = 0.342P \]
\[ b_x = \frac{3.92}{4.48}(-1.683P) = -1.47P \]

\[ b_{comb} = P\sqrt{(-1.47)^2 + 3(0.342)^2} = P\sqrt{2.51} = 1.58P \]

For yielding at fillet
\[ 1.58P = 44,500 \]
\[ P = 28,200 \text{#} \]
STRESSES AT Z-2

\[ \tan \beta = \frac{1}{3}, \quad \beta = 18.42^\circ \]

Radius of arc Z-2 = \( 18.25 + \frac{24.00}{\tan 18.42^\circ} = 42.25^\circ \)

Length of arc Z-2 = \( 42.25 \times \left( \frac{18.42}{57.3} \right) = 13.58^\circ \)

Area outer flange = 1.00 in²
Area inner flange = 1.016 in²
Section assumed symmetrical about centerline

\[ \alpha = 9.21^\circ \]

\[ N = P \cos (45^\circ - 9.21^\circ) = 0.811 P \]

\[ y = 42.25 \cos 9.21^\circ = 6.76'' \]
\[ e = 1.24'' \]

\[ b = 42.25 \cos 9.21 - 1.15 = 41.69 - 1.15 = 40.54'' \]

\[ M = 0.707 P (1.24 - 40.54) = -27.8 P \text{ in} \cdot \text{lb} \]

\[ \rho (\text{BY APPROXIMATION}) = 53'' \]

Since \( \rho > 2d \), I of section can be used instead of Z.
\[ I_{eq} = 1.00 (6.67)^2 + 1.016 (6.67)^2 + \frac{0.233}{12} (3.03)^3 \]
\[ = 44.4 + 45.1 + 42.7 = 132.2 \text{ in}^4 \]
\[ A = 1.00 + 1.016 + 13.08 (0.23) = 4.90 \text{ in}^2 \]

Bleich Eq. 9

\[ \sigma = \frac{1}{\cos \alpha} \left[ \frac{N}{A} - \frac{M \cos \beta}{I} \left( \frac{P}{P+C} \right) \right] \]

\[ f_0 = \frac{1}{\cos 9.21} \left[ -\frac{0.811 P + 27.8 P}{4.90} + \frac{27.8 P (6.79)}{132.2} \left( \frac{53}{59.79} \right) \right] \]
\[ = 1.013 \left[ -0.1657 P + 0.1072 P + 1.268 P \right] \]
\[ = 1.013 \left( 1.209 P \right) = 1.225 P \]

\[ f_c = 1.013 \left[ -0.1657 P + 0.1072 P - \frac{27.8 P (6.79) (53)}{132.2 (46.21)} \right] \]
\[ = 1.013 \left( -0.1657 P + 0.1072 P - 1.640 P \right) = 1.013(-1.699 P) \]
\[ = -1.720 P \]

For yielding at 2-2

\[ 1.720 P = 44,500 \]

\[ P = 25,900 \text{ lb} \]

\[ P \text{ for yield at 2-2} \]
STRESSES AT Z-Z. BY WOFE THEORY

\[ P_1 = P \cos(45 + 9.21) = 0.586P \]
\[ P_2 = P \cos(45 - 9.21) = 0.811P \]

\[ M_0 = 0.707P(4.00 - 23.00) = -13.42P \]

\[ c_1 = \frac{P \gamma}{2 Af \sin^2 \alpha + \tau r (\alpha - \sin \omega \alpha)} \]
\[ = \frac{0.586Pr}{2(1.00)(0.0256) + 0.25(42.25)(0.161 - 0.158)} = \frac{0.586Pr}{0.0829} \]
\[ = 7.07Pr \]

\[ \sigma_1 = \frac{c_1 \sin \theta}{r} = 7.07P \sin 9.21 = 1.03P \quad \theta = \omega \]
\[ = -1.03P \quad \theta = -\omega \]

\[ a_1 = \frac{-P \gamma r}{2 Af \sin \omega \alpha + \tau r (\omega + \sin \omega \alpha)} \]
\[ = \frac{-0.811Pr}{0.0512 + 0.25(42.25)(0.319)} = \frac{-0.811Pr}{3.42} \]
\[ = -0.237Pr \]

\[ \sigma_2 = \frac{a_1 \cos \theta}{r} = -0.237P \cos 9.21 = -0.234P \theta = \pm \omega \]
\[ d_2' = \frac{-\text{Mr}}{8 \pi \lambda \sin 2\theta + 2 \pi r (\sin 2\theta - 2 \lambda c \cos 2\theta)} \]

\[ = \frac{13.42 \pi \rho r}{8(100)(0.161)(0.316) + 2(0.25)(42.25)(0.316 - 0.322 + 0.949)} \]

\[ = \frac{13.42 \pi \rho r}{0.407 + 0.211} \]

\[ = 21.7 \pi \rho r \]

\[ \sigma_{1y} = -\frac{4}{r^2} d_2' \sin 2\theta \]

\[ = -\frac{4}{r^2} P (21.7) \sin 18.4^\circ \]

\[ = -0.650 P \quad \theta = \lambda \]

\[ = 0.650 P \quad \theta = -\lambda \]

**Extreme Fiber Stresses**

**Outside Fiber**  \( \sigma_o = P (1.03 - 0.23 - 0.65) = 0.15P \)

**Inside Fiber**  \( \sigma_i = P (-1.03 - 0.23 + 0.65) = -0.61P \)

**For Yielding at 2-2**  
\[ 0.61P = 44,500 \]

\[ P = 73,200 \text{ kN} \]

For Yielding at 2-2
Properties of \( \text{BUFF13} \)

\[
\begin{align*}
A &= 3.83 \text{ in}^2 \\
I &= 39.5 \text{ in}^4 \\
S &= 9.89 \text{ in}^3
\end{align*}
\]

\[
\begin{align*}
Q_{09} &= 7.68 \text{ in}^3 \\
Q_{filler} &= 4.30 \text{ in}^3
\end{align*}
\]

(From Conn. A, Sht. 2)

**STRESSES AT 1-1**

\[
f_o' = \frac{0.707P}{3.83} + \frac{0.707P(23.38)}{9.88} = -0.185P + 1.675P = 1.490P
\]

\[
f_i' = -0.185P - 1.675P = -1.860P
\]

For Yielding at Extreme Fiber

\[
P = \frac{44,500}{1.860} = 23,900 \text{ lb}
\]

\[
y_{na} = \frac{1.860}{3.350} (8.00) = 4.44''
\]
COMBINED STRESS AT INSIDE FILLET

\[ \delta_x = \frac{3.88}{4.44} (-1.860P) = -1.63P \]

\[ T_{xy} = \frac{VQ}{It} = \frac{0.707P (4.40)}{39.5 (0.23)} = 0.342P \]

\[ \sigma_d = \sqrt{\delta_x^2 + 3T_{xy}^2} = P \sqrt{1.63^2 + 3(0.342)^2} = 1.73P \]

FOR YIELD AT FILLET,

\[ P = \frac{44,500}{1.73} = 25,700 \text{ lb} \]

\[ \text{length of arc } Z-Z = 16.0 \left( \frac{45}{57.3} \right) = 12.58'' \]

\[ a = 16 \sin 22.5^\circ = 6.12'' \]

\[ e = 4 - 1.88 = 2.12'' \]

\[ b = 16 \cos 22.5 = 14.78 \]

\[ N = P \cos 22.5 = 0.924P \text{ lb} \]

\[ M = 0.707P \left( 2.12 + 30.16 \right) = -19.8P \text{ in} \times \text{lb} \]

\[ A = 2.4(0.254) + 12.07(0.23) = 2.03 + 2.78 = 4.81 \text{ in}^2 \]

\[ \rho = 13'' \text{ (approximated)} \]

\[ P = \text{radius of centerline.} \]
\[ P = 13" \] \[ A = 4.81 \text{ in}^2 \]

\[ Z = \rho^2 \left[ 2.303 \rho \sum b \log \frac{u_i}{w_2} - A \right] \]

\[
\begin{align*}
4.00 \log \frac{19.29}{19.04} &= 0.0226 \\
0.23 \log \frac{19.04}{6.96} &= 0.1005 \\
4.00 \log \frac{6.96}{6.71} &= 0.0635 \\
Z &= 0.1866
\end{align*}
\]

\[ \text{B.E.E. Design of Rigid Frames (Kues P.8)} \]

\[ Z = 130 \text{ in}^4 \]

**Fiber Stresses at \( Z-z \)**

\[
\sigma = \frac{1}{\cos \theta} \left[ \frac{N}{A} - \frac{M}{RA} - \frac{M_0}{2} \left( \frac{P}{P + \sigma} \right) \right]
\]

\[
f_0 = \frac{1}{0.225} \left[ -0.924P + \frac{19.8P}{4.81} + \frac{19.8P(6.29)(13)}{130(19.29)} \right]
\]

\[ = 1.082 \left[ -0.1923P + 0.317P + 0.645P \right] \]

\[ = 1.082(0.770) = 0.832P \]

\[ f_1 = 1.082 \left[ -0.1923P + 0.317P + \frac{19.8(6.29)(13)}{130(6.71)} \right] \]

\[ = 1.082\left[ -0.1923P + 0.317P - 1.855P \right] \]

\[ = 1.082(-1.730P) = -1.870P \]

For yielding in extreme fibers,

\[ 1.870P = 44,500 \]

\[ P = 23,800 \text{ lb} \]
STRESSES AT 2-2 BY WOECE THEORY

\[ P_h = P \cos 67.5^\circ = 0.383P \]
\[ P_2 = P \cos 22.5^\circ = 0.924P \]
\[ H_0 = 0.707P (4 + 15.38) = 13.70P \]

\[ \theta = 67.5^\circ = 0.393 \text{ rad.} \]
\[ 2\theta = 45^\circ = 0.786 \text{ rad.} \]
\[ A_f = 1.02 \text{ in}^2 \]
\[ t_w = 0.230 \text{ in} \]

\[ C_1 = \frac{P_r}{2Af \sin \theta + r \left( \theta - \frac{1}{2} \sin 2\theta \right)} \]
\[ = \frac{0.383Pr}{2(102)(0.146) + 0.23(16)(0.393 - 0.353 - 0.040)} = \frac{0.383Pr}{0.445} \]
\[ = 0.860Pr \]
\[ \delta_{r1} = \frac{C_1}{r} \sin \theta = 0.860Pr \sin 22.5^\circ = 0.329 P_e = -x \]
\[ = -0.329 P_e = -x \]

\[ a_1 = \frac{P_{r1}}{2Af \sin \theta + r \left( \theta + \frac{1}{2} \sin 2\theta \right)} \]
\[ = \frac{-0.924Pr}{0.298 + 0.23(16)(0.746)} = \frac{-0.924Pr}{3.04} \]
\[ = -0.304Pr \]
\[ \delta_{r2} = \frac{a_1}{r} \sin \theta = -0.304Pr \cos 22.5^\circ = -0.281 P_e = -x \]
\[
\begin{align*}
\delta_2'^{1} &= \frac{-M_r}{8A_{4}x \mu \sin 2\alpha + 2\pi r (\mu \sin 2\alpha - 2x \cos 2\alpha)} \\
&= \frac{-13.70 \Pr}{8(1.02)(0.373)(0.707) + 2(0.23)(16)(0.707 - 0.786)(0.707)_{0.152}} \\
&= \frac{-13.70 \Pr}{2.27} = -4.05 \Pr \\
\delta_3' &= -\frac{4}{r_e} d_2' \sin 2\theta \\
&= -\frac{4}{16}(-4.05) \sin 45^\circ = 0.715 \Pr \quad \theta = \alpha \\
&= -0.715 \Pr \quad \theta = -\alpha
\end{align*}
\]

**EXTREME FIBER STRESSES**

OUTSIDE FIBER \( \delta_o = P \left(0.329 - 0.251 + 0.715\right) = 0.763P \)

INSIDE FIBER \( \delta_i = P \left(-0.329 - 0.251 - 0.715\right) = -1.325P \)

FOR YIELDING AT 2 - 2 \( 1.325P = 44,500 \)

\( P = 33,600 \)
DISTRIBUTION OF STRESS ON TENSION FLANGE

Analytical Solution - straight line
Web Plate is 4" thick
All welds 1/4"

**Extreme Fiber Stresses Section 1-1**

\[ A = 3.83 \text{ in}^2 \quad I = 39.5 \text{ in}^4 \quad S = 9.88 \text{ in}^3 \]

\[ f_o = -0.707 \frac{P}{3.83} + 0.707 \frac{P(17.38)}{9.88} = (-0.186 + 1.243)P \]

\[ = 1.063P \]

\[ f_i = -0.186P - 1.243P = -1.423P \]

For Yielding at 1-1

\[ 1.423P = 44,500 \]

\[ P = 31,300 \text{ in}^2 \]

\[ P \text{ for 404. at 1-1} \]
\[ A = 1.00 + 1.50 + \frac{7.37}{4} = 2.50 + 1.84 = 4.34 \]

\[ A_{y} = 1.00(7.88) + 1.50(1.875) + 1.84(4.06) \]

\[ j = 7.88 + 0.28 + 7.46 = 15.62 \]

\[ \bar{y} = 3.60'' \]

\[ I = 1.00(4.28)^2 + 1.50(3.41)^2 + \frac{0.25(7.37)^3}{12} + 1.84(0.46)^2 \]

\[ = 18.3 + 17.4 + 833 + 3.89 \]

\[ = 44.4 \]

\[ M = 0.707P(-17.38 + 0.46) = -0.707P(16.92) = -11.95P \]

**Extreme Fiber Stresses**

\[ \sigma_0 = \frac{-0.707P}{4.34} + \frac{11.95P(4.44)}{44.4} = \frac{-0.163P + 11.95P}{44.4} = 1.122P \]

\[ \sigma_i = \frac{-0.163P - 11.95P(3.60)}{44.4} = \frac{-0.163P - 0.971P}{44.4} = -1.134P \]

\[ P = \frac{H + R(1 - \cos \beta)}{\sin \beta} \]

\[ = \frac{8.00 + 22.0(1 - 0.9397)}{0.3420} = \frac{9.325}{0.3420} = 27.24'' \]

\[ x = D + R \sin \beta - r \cos \beta \]

\[ x = 17.63 + 22.00(0.3420) - 27.24(0.9397) = 17.63 + 7.52 - 25.50 \]

\[ = -0.35'' \]
\[ 4EC \ 3.3 = 27.24 \left( \frac{20}{57.3} \right) = 9.51'' \]

\[
\begin{array}{c|cccc}
A & 1.00 & 1.00 & y & A_y \\
8.89 (0.25) & 2.22 & 4.82 & 10.70 & \\
1.50 & 1.50 & 0.1875 & 0.28 & \\
2 & 4.72 & 20.37 & & \\
\end{array}
\]

\[ \bar{y} = 4.32'' \]

\[ \alpha_1 = 12.90^\circ = 0.2193 \text{ rad} \]

\[ \alpha_2 = \frac{4.32}{9.51} \left( 20 \right) = 9.10^\circ = 0.1599 \text{ rad} \]

\[ P_1 = P_{150} (45 + 10.9) = 0.562 \text{Pr} \]

\[ P_2 = P_{150} (45 - 10.9) = 0.858 \text{Pr} \]

\[ M = 0.707 \text{Pr} (4 - 0.35) = 2.15 \text{Pr} \]

\[ P_{150} = 0.100 \text{ in}^2 \]

\[ A_0 = 1.00 \text{ in}^2 \]

\[ A_1 = 1.50 \text{ in}^2 \]

\[ \ell_w = 0.25'' \]

\[ r = 27.24'' \]

\[
C_i = \frac{Pr}{(A_0 \sin ^2 x_1 + A_i \sin ^2 x_2) + \frac{h}{2} \left[ x_1 + x_2 - \frac{1}{2} (\sin 2x_1 + \sin 2x_2) \right]} \\
= \frac{0.562 \text{Pr}}{0.0358 + 0.0405} \\
= \frac{0.562 \text{Pr}}{0.0763} = 5.54 \frac{\text{Pr}}{r} \\
\delta_r = \frac{C_i \sin \theta}{r} = 5.54 \frac{\text{Pr}}{r} \sin x_1 = 1.028 \text{Pr} \left| \theta = x_1 \right. \\
= 5.54 \frac{\text{Pr}}{r} \sin (-x_2) = -0.863 \text{Pr} \left| \theta = -x_2 \right. \\
\]

\[ A_1 = \frac{-P_{150}}{(A_0 \sin ^2 x_1 + A_i \sin ^2 x_2) + \frac{h}{2} \left[ x_1 + x_2 + \frac{1}{2} (\sin 2x_1 + \sin 2x_2) \right]} \\
= \frac{-0.828 \text{Pr}}{0.0763 + 3.27 (0.1903 + 0.1589 + 0.1585 + 0.1563)} \\
= \frac{-0.828 \text{Pr}}{0.0763 + 3.27} = -0.334 \frac{\text{Pr}}{r} \\
\]
\[
\sigma_{r_2} = \frac{q_i}{r} \cos \theta = -0.354P \cos \theta = -0.350P \bigg| \theta = \theta_2
\]

\[
\sigma' = \frac{-M_r}{4[(x_1 \sin \theta_1 - x_2 \sin \theta_2) + 2(r_1)_{\theta}^{\frac{1}{2}}(\sin \theta_1 + \sin \theta_2) - (x_1 \cos \theta_1 + x_2 \cos \theta_2)]}
\]

\[
\sigma_{r_3} = -\frac{4}{r^2} \sigma' \sin \theta
\]

**EXTREME FIBER STRESSES**

Outside Fiber \( \sigma_0 = P(1.028 - 0.348 + 0.185) = 0.865P \)

Inside Fiber \( \sigma_i = P(-0.863 - 0.350 - 0.156) = -1.369P \)

**MAX \( \sigma \) FOR EFFECT OF CURVATURE ON INNER FLG.**
\[ \rho = H + R(1 - \cos \theta) \]

\[ \begin{align*}
\rho &= 8.00 + 22.00 (1 - 0.7660) \\
&= 13.15 \\
&= 21.44'' \\
\end{align*} \]

\[ X = D + R \sin \beta - \rho \cos \theta \]

\[ \begin{align*}
X &= 17.63 + 22.00 (0.6428) - 21.44 (0.7660) \\
&= 17.63 + 14.16 - 16.43 \\
&= 15.26'' \\
\end{align*} \]

\[ \text{ARC} \ 4-4 = 21.44'' (\frac{40}{20.5}) = 14.98'' \]

\[ \theta = \frac{40}{67.3} = 0.6\ldots \]

\[ \alpha_1 = 21.4^\circ = 0.374 \text{ rad.} \]

\[ \alpha_2 = \frac{6.95''}{14.98''} (40) = 18.6^\circ = 0.324 \text{ rad.} \]

\[ P_1 = P \cos (45 + 21.4) = 0.400P \]

\[ P_2 = P \cos (45 - 21.4) = 0.916P \]

\[ M_0 = 0.757P (4 + 15.36) = 13.67P \]

\[ A_0 = 1.00 \]

\[ A_{w} = 1.50 \]

\[ f_{aw} = 0.25 \]

\[ r = 21.44'' \]
\[
\sigma_1 = \frac{Pr}{(4A_0 + A_1 + A_2) + \frac{1}{4} (2a_1 + 2a_2 - \sin 2d_1 - \sin 2d_2)} = 0.400 Pr
\]

\[
= \frac{0.400 Pr}{0.437} = 0.916 Pr
\]

\[
\sigma_2 = \frac{Pr}{(4A_0 + A_1 + A_2) + \frac{1}{4} (2a_1 + 2a_2 + \sin 2d_1 + \sin 2d_2)} = -\frac{0.916 Pr}{2.965} = -0.309 Pr
\]

\[
\sigma_3 = \frac{Pr}{(4A_0 + A_1 + A_2) + \frac{1}{4} (2a_1 + 2a_2 + \sin 2d_1 + \sin 2d_2)} = -\frac{0.916 Pr}{2.965} = -0.309 Pr
\]

\[
\sigma_4 = \frac{Pr}{(4A_0 + A_1 + A_2) + \frac{1}{4} (2a_1 + 2a_2 - \sin 2d_1 - \sin 2d_2)} = -\frac{0.916 Pr}{2.965} = -0.309 Pr
\]

\[
\sigma_5 = \frac{Pr}{(4A_0 + A_1 + A_2) + \frac{1}{4} (2a_1 + 2a_2 + \sin 2d_1 + \sin 2d_2)} = -\frac{0.916 Pr}{2.965} = -0.309 Pr
\]

\[
\sigma_6 = \frac{Pr}{(4A_0 + A_1 + A_2) + \frac{1}{4} (2a_1 + 2a_2 - \sin 2d_1 - \sin 2d_2)} = -\frac{0.916 Pr}{2.965} = -0.309 Pr
\]
\[ \delta_{r3} = -\frac{4}{r^2} \Delta \sin \theta \]
\[ = -\frac{4}{21.4} (-4.06 \Delta) \sin \theta = \frac{4}{21.4} (-4.06 \Delta) \sin \theta = 0.571 \Delta \]
\[ = -\frac{4}{21.4} (-4.06 \Delta) \sin \theta = -0.60 \Delta \]

**Extreme Fiber Stresses**

**Outside Fiber**

\[ \sigma_0 = \rho (0.334 - 0.278 + 0.515) = 0.571 \rho \]

**Inside Fiber**

\[ \sigma_i = \rho (-0.292 - 0.293 - 0.459) = -1.044 \rho \]

**Max Stresses at 3-3 Due to Curvature of Inner Flange**

\[ \frac{b^2}{22 (0.375)} = 0.425 \]

\[ \Delta = 0.906 \quad u = 1.10 \]

\[ \delta_{i, \text{max}} = \delta_i = \frac{\sigma_i}{\rho} = \frac{-1.369 \rho}{0.906} = -1.51 \rho \]

\[ \delta' = \mu \delta_{i, \text{max}} = 1.10 (1.51 \rho) = 1.66 \rho = \delta_2 \]

\[ \delta_{yld}^2 = \delta_i^2 + \delta_2^2 - \delta_i \delta_2 \]

\[ = \rho^2 (2.28 + 2.75 + 2.50) = 7.53 \rho^2 \]

\[ \delta_{yld} = 2.74 \rho \]

For yield at 3-3

\[ 2.74 \rho = 44,500 \]

\[ \rho = 16,300 \]

\[ P_{\text{for 3-3}} = 44,500 \]
Web of knee is \( \frac{1}{4} \)" plate.

All welds \( \frac{1}{4} \)".
\[ N = 0.707P \]
\[ M = -0.707P \times 23.33 = -16.5P \]

For \( A = 3.83 \text{ in}^2 \), \( I = 39.6 \text{ in}^4 \), \( S = 9.88 \text{ in}^3 \)

\[ f_0 = -\frac{0.707P + 16.5P}{3.83} = \frac{-0.185 + 1.670}{9.88}P = 1.485P \]

\[ f_i = -0.185P - 1.670P = -1.855P \]

For yielding in extreme fibers

\[ 1.855P = 44,500 \]

\[ P = 24,000 \text{ in}^2 \]

From previous calculations, this can be taken as critical value for section 1-1.

---

\[ N = 0.707P \]
\[ M = -0.707P \times 23.63 = -16.7P \]

\[ A = 4 \left( \frac{1}{4} \right) + 7.25 \left( \frac{1}{4} \right) + 4 \left( \frac{1}{2} \right) = 1.00 + 1.81 + 2.00 = 4.81 \text{ in}^2 \]

\[ A_y = 1.00 \left( 7.88 \right) + 1.81 \left( 4.12 \right) + 2.00 \left( 0.25 \right) = 15.83 \]

\[ J = 7.88 + 7.45 + 0.50 = 15.83 \]

\[ \bar{y} = \frac{15.83}{4.81} = 3.29 \text{ in} \]

\[ I = 1.00 \left( 4.59 \right)^2 + 2.00 \left( 3.04 \right)^2 + \frac{1}{12} \left( \frac{1}{4} \right) \left( 7.25 \right)^3 \]  

\[ + 1.81 \left( 0.83 \right)^2 \]

\[ = 21.0 + 18.5 + 7.9 + 1.2 = 48.5 \text{ in}^4 \]

Fiber stresses

\[ f_0 = -\frac{0.707P + 16.7P \times 4.71}{48.5} = -0.147P + 1.621P \]

\[ = 1.474P \]

\[ f_i = -0.147P - \frac{16.7P \times 3.29}{48.5} = -0.147P - 1.133P \]

\[ = -1.280P \]
STRESSES AT 3-3

\[ \rho = \frac{H + R (1 - \cos \beta)}{\sin \beta} = \frac{8.00 + 16.0 (1 - 0.940)}{0.3420} = \frac{8.960}{0.3420} = 26.20'' \]

\[ x = D + R \sin \beta - \rho \cos \beta = 23.63 + 16.00 (0.3420) - 26.20 (0.9397) \]
\[ = 23.63 + 5.46 - 24.62 = 4.47'' \]

\[ \text{ARC 3-3} = 26.20 (0.3491) = 9.14'' \]

\[ \begin{array}{ccc}
\text{Area} & y & A_y \\
1.00 & 9.02 & 9.02 \\
2.00 & 0.25 & 0.50 \\
8.39 (25) & 2.10 & 4.70 \\
\end{array} \]

\[ 2 = 5.10 \]

\[ y' = 3.30'' \]

\[ \alpha_1 = 12.78 = 0.222 \text{ rad.} \]

\[ \alpha_2 = \frac{3.30}{9.14} (20.10) = 7.22 = 0.126 \text{ rad.} \]

\[ A_6 = 1.00 \quad \varepsilon_w = 0.25'' \quad P_1 = P \cos(45 + 12.78) = 0.532 P \]

\[ A_6 = 2.00 \]

\[ P_2 = P \cos(45 - 12.78) = 0.816 P \]

\[ \rho = 26.20'' \quad M_0 = 0.707P(44.47) = 6.26P \]
\[ c_1 = \frac{Pr}{(A_0 \sin^2 \alpha_1 + A_1 \sin^2 \alpha_2) + \frac{\pi}{4} (2 \alpha_1 + 2 \alpha_2 + \sin 2 \alpha_1 - \sin 2 \alpha_2)} \]

\[ = \frac{0.532 Pr}{0.0802} \]

\[ \delta r = \frac{c_1}{r} \sin \theta = 5.07 Pr \sin \alpha_1 = 0.648 P \bigg| \theta = \alpha_1 \]

\[ = -5.07 Pr \sin \alpha_2 = -0.637 P \bigg| \theta = -\alpha_2 \]

\[ a_1 = \frac{-Pr}{(A_0 \sin^2 \alpha_1 + A_1 \sin^2 \alpha_2) + \frac{\pi}{4} (2 \alpha_1 + 2 \alpha_2 + \sin 2 \alpha_1 - \sin 2 \alpha_2)} \]

\[ = \frac{-0.846 Pr}{0.0802 + 1.64 (1.377)} = -\frac{0.846 Pr}{2.34} \]

\[ = -0.362 Pr \]

\[ \delta r_2 = \frac{a_1}{r} \cos \theta = -0.362 Pr \cos \alpha_1 = -0.353 \bigg| \theta = \alpha_1 \]

\[ = -0.362 Pr \cos \alpha_2 = -0.360 \bigg| \theta = -\alpha_2 \]

\[ d_1' = \frac{-Mr}{4 [A_0 \sin^2 \alpha_1 + A_1 \sin^2 \alpha_2] + \frac{4 \pi}{2} \left( \frac{1}{2} (\sin 2 \alpha_1 + \sin 2 \alpha_2) - (\alpha_1 \cos 2 \alpha_1 + \alpha_2 \cos 2 \alpha_2) \right)} \]

\[ = -6.26 Pr \]

\[ d_2' = \frac{-6.26 Pr}{0.870} = -7.20 Pr \]
\[
\begin{align*}
\sigma_3 &= -\frac{4}{r^2} d^2 \sin 2\theta \\
&= -\frac{4}{26.20} (-7.20P) \sin 2\alpha_1 = 0.475P \quad \theta = \alpha_1 \\
&= -\frac{4}{26.20} (-7.20P) \sin(-2\alpha_2) = -0.274P \quad \theta = -\alpha_2
\end{align*}
\]

**Extreme Fiber Stresses**

*Outside Fiber* \( \sigma_0 = P (0.648 - 0.353 + 0.475) = 0.770P \)

*Inside Fiber* \( \sigma_i = P (-0.637 - 0.353 - 0.274) = -1.264P \)

For yielding at \( 3 - 3 \) \[1.264P = 44,500 \]

\[ P = 35,200 \text{ ksi} \] see sheet 7

**Stresses at 4-4**

\[
\begin{align*}
\rho &= \frac{H + R (1 - \cos \beta)}{\sin \beta} \\
&= \frac{8.00 + 16.00 (1 - 0.766)}{0.6428} = \frac{11.74}{0.6428} = 18.26'' \\
\theta &= D + R \sin \beta - \rho \cos \beta = 23.63 + 16.00 \left(\frac{0.6428}{0.766}\right) - 18.26 \left(\frac{0.766}{0.6428}\right) \\
&= 23.63 + 10.28 - 13.99 \\
\theta &= 19.92''
\end{align*}
\]
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Length of 4.4 = 18.26 \frac{40}{57.3} = 12.75''

Area = 1.00 + 3.00 + 2.00 = 6.00 in^2

\begin{align*}
A_y &= 1.00(12.63) + 3.00(6.5) + 2.00(0.25) \\
&= 12.63 + 19.5 + 0.5 = 32.63
\end{align*}

\( y = 5.43'' \)

\( \alpha_1 = 23.0^\circ = 0.401 \text{ rad} \)

\( \alpha_2 = \frac{5.43}{12.75} (40) = 17.0^\circ = 0.297 \text{ rad} \)

\begin{align*}
P &= P \cos (45 + 23) = 0.208 P & A_0 &= 1.00 \\
P_1 &= P \cos (45 - 23) = 0.927 P & A_1 &= 2.00 \\
M_0 &= 0.707 P (4 + 19.92) = 16.9 P & \theta_0 &= 0.25 \\
\end{align*}

\( c_1 = \frac{0.208 P}{0.457 P} \)

\( \delta v_1 = \frac{c_1 \sin \theta}{r} = 0.457 P \cos \alpha_1 = 0.219 P \quad | \theta = \alpha_1 \\
= 0.457 P (\cos \alpha_1) = 0.134 P \quad | \theta = \alpha_2 \\
\)

\begin{align*}
\delta v_2 &= \frac{c_1 \cos \theta}{\ell} = -0.275 P \cos \alpha_1 = -0.254 P \quad | \theta = \alpha_1 \\
&= -0.275 P (\cos \alpha_2) = -0.263 P \quad | \theta = \alpha_2 \\
\end{align*}
\[
\begin{align*}
\delta_2 &= \frac{-16.9 P}{4[0.401(1)(0.719) + 0.297(2)(0.557)] + 2(0.25)(18.26)[0.359 + 0.280 - 0.401(0.0285) - 0.297(0.02)]} \\
&\quad \times \left[ 4(0.288 + 0.332) + 4(0.620) \right] \bigg[ 2.48 + 1.05 \bigg] \\
&= \frac{-16.9 P}{3.53} = -4.78 P
\end{align*}
\]

\[
\begin{align*}
\delta_{v3} &= -\frac{4}{P} \delta_2 \sin 2\Theta = -\frac{4}{18.26} (-4.78 P) \sin 2\theta_1 = 0.752 P \Theta_1 \\
&= -\frac{4}{18.26} (-4.78 P) \sin 2\theta_2 = -0.586 P \Theta_2
\end{align*}
\]

**EXTREME FIBER STRESSES**

**OUTSIDE FIBER** \( \sigma_0 = P(0.178 - 0.254 + 0.752) = 0.676 P \)

**INSIDE FIBER** \( \sigma_i = P(-0.178 - 0.254 - 0.752) = -0.983 P \)

**INCREASE IN STRESS ON INNER FIBER DUE TO CURVATURE OF THE FLANGE**

\[
\begin{align*}
&\text{FOR SECTION } 3-3 \\
&\frac{b^2}{h} = \frac{1.88^2}{16(\frac{1}{2})} = 0.44 \\
&n = 0.900 \quad \mu = 1.14
\end{align*}
\]

\[
\begin{align*}
\sigma_2 &= \delta_{max} = \frac{\delta_2}{\nu} = \frac{1.264 P}{0.900} = -1.40 P \\
\sigma_i &= \delta' = 4 \delta_{max} = 1.14(-1.40 P) = -1.60 P
\end{align*}
\]

\[
\delta y dx^2 = \delta_i^2 + \delta_2^2 - \delta_1^2
\]

\[
\begin{align*}
&= P^2 (1.96 + 2.56 + 2.24) = 6.76 P \\
&44,500 = 2.60 P \quad P = 17,100 \text{ lb}
\end{align*}
\]
EXTREME FIBER STRESSES AT 1-1

\[ \delta_0 = -0.707 \frac{P}{3.83} + 0.707 \frac{P(25.75)}{9.88} = -0.185P + 1.846P \]

\[ = 1.661P \]

\[ \delta_c = -0.185P - 1.846P = -2.031P \]

FOR YIELDING AT 7-1

\[ 2.08P = 44,500 \text{ lb} \]

\[ P = 21,900 \text{ lb} \]
**EXTREME FIBER STRESSES AT 2-2**

\[
\sigma_0 = -0.185P + 0.707P(26.00) = (-0.185 + 1.862)P
\]

\[
= 1.677P
\]

\[
\sigma_i = -0.185P - 1.862P = -2.047P
\]

*For yielding at 2-2, \(2.047P = 44,500\)

\[
P = 21,700 \text{ kip}
\]

**For yielding at 2-2**

\[\rho = \frac{h + R(1 - \cos \beta)}{\sin \beta} = \frac{8.00 + 13.68(1 - 0.9397)}{0.3420} = \frac{8.824}{0.3420} = 25.80''\]

\[
x = D + R \sin \beta - \rho \cos \beta = 26.00 + 13.68(0.3420) - 25.80(0.9397) = 26.00 + 4.67 - 24.24 = 6.43''
\]

\[
\alpha_1 = \alpha_2 = 10^\circ = 0.1746 \text{ rad}, \quad \Delta_i = A_0 = 1.00 \text{ in}^2
\]

\[
P_1 = P \cos 55^\circ = 0.574P \quad \text{tw} = 0.25''
\]

\[
P_2 = P \cos 35^\circ = 0.820P \quad r = 25.80''
\]

\[
M_0 = P(0.707)(4 + 6.43) = 7.37P
\]
\[ c_1 = \frac{ Pr}{ 2 \pi \sin \alpha + \pi (\sin \alpha - \sin 2\alpha) \frac{Z}{2}} \]

\[ = \frac{0.574 Pr}{ 2(1.00)(0.0301) + 0.25(25.80)(0.1746 - 0.1711) \frac{0.0602}{0.0226} } = \frac{0.574 Pr}{0.0828} = 6.93 Pr \]

\[ \delta r_1 = \frac{c_1}{r} \sin \theta = 6.93 Pr \sin \alpha = -1.203 Pr \left| \theta = \alpha \right. \]

\[ a_1 = \frac{-Pr}{ 2 \pi \sin \alpha + \pi (\sin \alpha + \sin 2\alpha) \frac{Z}{2}} \]

\[ = \frac{-0.820 Pr}{ 0.0602 + 0.25(25.80)(0.1746 + 0.1711) \frac{0.3457}{0.3457} } = \frac{-0.820 Pr}{2.23} = -0.358 Pr \]

\[ = \frac{-0.820 Pr}{2.29} = -0.356 Pr \]

\[ \delta r_2 = \frac{a_1}{r} \cos \theta = -0.356 Pr \cos \alpha = -0.356 Pr \]

\[ = 0.197 Pr \times 0.915 = 0.194 Pr \]

\[ d_2 = \frac{-Mr}{ 8 \pi \sin \alpha + \pi (\sin \alpha - \sin 2\alpha \cos \alpha) \frac{Z}{2}} \]

\[ = \frac{-7.37 Pr}{8(1.00)(0.1746)(0.342) + 2(0.25)(25.80)(0.342 - 0.3402 \cos \alpha) \frac{0.477}{0.3282} } = \frac{-7.37 Pr}{0.655} = -10.27 Pr \]

\[ \delta r_3 = \frac{-d_2 \sin \theta}{r^2} = \frac{-4}{25.80} (-10.27 Pr) \sin 2\alpha = 0.544 Pr \left| \theta = \alpha \right. \]

\[ = -0.544 Pr \left| \theta = -\alpha \right. \]
EXTREME FIBER STRESS

OUTSIDE FIBER

\[ \sigma_0 = P\left(1.203 - 0.353 + 0.544\right) = \frac{1244 P}{194} \]

INSIDE FIBER

\[ \sigma_i = P\left(-1.203 - 0.353 - 0.544\right) = -\frac{2160 P}{194} \]

FOR YIELDING AT 3-3

\[ 2100 = 44,500 \]

\[ P = 21,200^\circ \]

EFFECT OF FLANGE CURVATURE CAN BE NEGLECTED SINCE STIFFENING BRACKETS ARE PROVIDED TO RESIST TRANSVERSE BENDING.

\[ \rho = \frac{H + R(1 - \cos \beta)}{\sin \beta} = \frac{8.00 + 13.68(1 - 0.7660)}{0.643} = \frac{11.20}{0.643} = 17.41'' \]

\[ x = D + R \sin \beta - \rho \cos \beta = 26.00 + 13.68 \sin 40 - 17.41 \cos 40 = 26.00 + 8.79 - 13.37 = 21.42'' \]

\[ \alpha = 20^\circ = 0.3491 \text{rad.} \]

\[ P = P \cos 65^\circ = 0.423 P \]

\[ P_2 = P \cos 25^\circ = 0.906 P \]

\[ M_o = 0.707P(4 + 21.42) = 17.94P \]

\[ A_f = 1.00 \text{in}^2 \]

\[ t = 0.25'' \]

\[ r = 17.41'' \]
\[ c_1 = \frac{Pr}{2Af \sin \alpha + 2r ( \alpha - \sin \alpha \frac{\pi}{2})} \]
\[ = \frac{0.423 Pr}{2 \left(0.117 + 0.25(17.41)(0.349 - 0.3214)\right) \left(0.117 + 0.1121\right) + 0.0277} \]
\[ = \frac{0.423 Pr}{0.298} = 1.776 Pr \]
\[ \delta_r = \frac{2 \sin \theta}{r} = 1.776 Pr \sin \theta = 0.606 P \quad \theta = \alpha \]
\[ = -0.606 P \quad \theta = \theta \]

\[ a_1 = \frac{-Pr}{2Af \sin \alpha + 2r ( \alpha + \sin \alpha \frac{\pi}{2})} \]
\[ = \frac{-0.906 Pr}{3 \times 14.69 - 1.93 Pr} \]
\[ \delta_r = \frac{2 \sin \theta}{r} = -0.288 P \cos \alpha = -0.274 P \quad \theta = \pm \alpha \]
\[ = -0.182 P \]

\[ d_2 = \frac{-Mr}{8Af \sin \alpha + 2r ( \sin \alpha - 2 \alpha \cos \alpha)} \]
\[ = \frac{-17.94 Pr}{8 \left(0.100(0.349)(0.6428) + 2 \times 0.25(17.41)(0.6428 - 0.5982 \cos 2 \alpha)\right) \left(0.193 + 0.140\right) - 0.535} \]
\[ = \frac{-17.94 Pr}{2.736} = -6.80 Pr \]
\[ \delta_{r_2} = \frac{-4}{r_2} \frac{d_2 \sin \theta}{\sin \alpha} = -6.80 P \left(0.643 \left(-0.604 \right) \sin 2 \alpha = 1.004 P \right) \]
\[ = -1.004 P \quad \theta = \pm \alpha \]

**Extreme Fiber Stresses**

**Outside Fiber**
\[ \sigma = P \left(0.506 - 0.271 + 1.004\right) = 1.339 P \]

**Inside Fiber**
\[ \sigma = P \left(-0.606 - 0.271 - 1.004\right) = -1.881 P \]

**For Yielding**
\[ \sigma = 1.881 P = 44,500 \]
\[ P = 23,700 \]
STRESSES AT 1-1

FIBER STRESSES

\[ f = \frac{N}{A} \pm \frac{M}{S} \]

\[ f_0 = \frac{-0.707}{3.83} + \frac{0.707 \cdot (31.38)}{9.88} = -0.185P + 2.24P \]

\[ = 2.06P \]

\[ f_i = -0.185P - 2.24P = -2.42P \]

FOR YIELDING AT EXTREME FIBERS

\[ 2.42P = 44,500 \]

\[ P = 18,400 \text{ lb} \]

\[ y_{ma} = \frac{2.42}{4.48} (8.00) = 4.32'' \]
\[
T_{10} = \frac{3.56}{3.68} (2.06P)(4)(0.254) = 2.02P
\]
\[
T_{1i} = \frac{4.20}{4.32} (2.42P)(4)(0.254) = 2.40P
\]
\[
V_{aa} = 2.02P + 1.94P(0.23)(3.43) - 2.28P(0.23)(4.07) = 2.02P + 0.763P - 1.07P = 1.71P > 0.707P
\]

**FIBER STRESSES AT 2-2**

**SECTION 2-2**

\[
\begin{align*}
\text{Web} & : 1.78 & 4.13 & 7.36 \\
\text{Outer fillet} & : 0.812 & 8.12 & 6.58 \\
\text{Inner fillet} & : 1.00 & 0.125 & 0.125 \\
\end{align*}
\]

\[
X_{oo} = 3.92''
\]

\[
I = 0.812(4.21)^2 + 1.00(3.80)^2 + \frac{1}{12}(0.23)(7.75)^3 + 0.78(0.21)^2 = 14.4 + 14.5 + 8.84 + 0.08 = 37.8 \text{in}^4
\]

\[
S_o = \frac{37.8}{4.33} = 8.73 \text{in}^3
\]

\[
S_i = \frac{37.8}{3.92} = 9.64 \text{in}^3
\]

\[
\begin{align*}
f_o &= \frac{-0.707P}{3.59} + \frac{0.707P(31.38)}{8.73} = -0.197P + 2.54P = 2.34P \\
f_i &= \frac{-0.197P - 0.707P(31.38)}{9.64} = 0.197P - 2.30P = -2.50P
\end{align*}
\]
LOAD P TO PRODUCE YIELDING IN EXTREME FIBERS AT 2-2:

\[ 2.50 \times P = 44,500 \]

\[ P = 17,800 \text{ lb} \]

\[ y_{na} = \frac{2.50 (8.25)}{4.84} = 4.26'' \]

\[ T_{2i} = \frac{4.14}{4.26} (2.50P)(0.90) = 2.43P \]

\[ T_{20} = \frac{3.87}{3.99} (2.34P)(0.812) = 1.84P \]

\[ V_{bb} = 1.84P + 2.20P \left( \frac{3.14}{2} \right)^{0.23} - 2.36 \left( \frac{4.01}{2} \right)^{0.23} \]

\[ V_{bb} = 1.84P + 0.94P - 1.09P 
= 1.69P > 0.707 \]

Combined Stresses

Since the diagonal stiffener takes part of the shear in the knee, the critical section will be taken as outside the vertical web stiffeners.

\[ V = 0.707P \]

For 1-1

\[ Q_{c7g} = 3.87 (1.02) + 3.44 \left( \frac{0.23}{2} \right) 
= 3.93 + 1.36 = 5.29 \]

At c.g.,

\[ T_{xy} = \frac{0.707P(5.29)}{39.5(0.23)} = 0.412P \]

\[ \delta_x = \frac{0.32}{4.32} (2.42P) = 0.179P \]

\[ \sigma_{com} = P \sqrt{0.179^2 + 3(0.412)^2} = P \sqrt{0.032 + 0.507} 
= 0.734P \]

For Yielding at C.G.

\[ P = \frac{44,500}{0.734} = 60,700 \text{ lb} \]
AT FILLET

\[ Q = 3.93 + 0.31(3.59)(0.23) \]
\[ = 3.93 + 0.79 = 4.72 \]

\[ T_{xy} = \frac{0.707P(4.20)}{39.5(0.23)} = 0.327P \]

\[ \delta_x = \frac{3.76 (-2.92P)}{4.32} = -2.11P \]

\[ \delta_{con8} = P \sqrt{2.11^2 + 3(0.327)^2} = P \sqrt{4.45 + 0.32} = 2.18P \]

FOR YIELDING AT FILLET

\[ P = \frac{44,500}{2.18} = 20,400 \text{ } \# \]

FOR SECTION 2-2

\[ Q_{NA} = 0.812(4.21) + 3.74(2.21)(0.23) \]
\[ = 3.42 + 1.90 = 4.32 \]

\[ T_{xy} = \frac{0.707P(4.32)}{37.8(0.23)} = 0.351P \]

\[ \delta_{yld} = \sqrt{3T_{xy}^2} = P \sqrt{3(0.351)^2} = 0.608P \]

FOR YIELDING AT N.A.

\[ P = \frac{44,500}{0.608} = 73,100 \text{ } \# \]

AT FILLET

\[ Q = 3.42 + 0.31 (3.60)(0.23) \]
\[ = 3.68 \]

\[ T_{xy} = \frac{3.68 (0.608P)}{4.32} = 0.518P \]

\[ \delta_x = \frac{3.76 (-2.50P)}{4.26} = -2.17P \]

\[ \delta_{con8} = P \sqrt{(-2.17)^2 + 3(0.518)^2} = P \sqrt{4.71 + 0.80} = 2.35P \]

FOR YIELDING AT FILLET

\[ P = \frac{44,500}{2.35} = 18,900 \text{ } \# \]
STRESSES IN KNEE

\[ T_1 = \frac{2.02 + 2.40}{2} P = 2.21 P \]

\[ T_2 = \frac{2.43 + 1.84}{2} P = 2.14 P \]

\[ \gamma_1 = \frac{2.21 P}{6.88(0.23)(11.6)(10)^6} = 12.05 \times 10^{-8} P \]

\[ \gamma_2 = \frac{2.14 P}{7.37(0.23)(11.6)(10)^6} = 10.90 \times 10^{-8} P \]

\[ \delta_{\text{max}} = 2 \left[ \frac{h_1 h_2 (\gamma_1 + 2 \gamma_2)}{(2,25^2) E} \right] \]

\[ = 2 \left[ \frac{6.88 \times 7.37 \times (10.90 + 12.05) \times (10^8)}{(2,25^2) \times (10^8) P^2} \right] \]

\[ = 2 \left[ \frac{10^{-6} (1165 P_i)}{97.4} \right] 30 (10)^6 = 718 P_i \]

\[ T_3 = \frac{7.18 P_i (0.938)}{2 (9.87)(0.23)} = 1.49 P_i \text{ sol. of Eq. (3)} \]

\[ T_{\text{max}} = \gamma_1 G + \gamma_2 G = 11.6 (10)^6 (22.95)(10^8) P_i = 264 (10^{-2}) P_i \]

\[ = 2.64 P_i \]
STATE OF STRESS AT C

\[ \sigma_1 = \sqrt{(1.49P)^2 + (2.64P)^2} = 3.03P \]
\[ \sigma_2 = -3.03P \]
\[ \sigma_{yld}^2 = 3 (3.03P)^2 = 27.5P^2 \]
\[ \sigma_{yld} = 5.25P \]

FOR YIELDING AT C
\[ P = \frac{44,500}{5.25} = 8460^* \]

STATE OF STRESS AT POINT A:

\[ \sigma_x = -2.11P \left( \frac{7.81}{8.25} \right) + 1.49P = (-2.00 + 1.49)P = 0.51P \]
\[ \sigma_y = -2.17P \left( \frac{7.44}{8.00} \right) - 1.49P = (-2.02 - 1.49)P = -3.51P \]
\[ T_{xy} = 2.64P \]

\[ \sigma_1 = \frac{0.51 - 3.51}{2} P + \sqrt{\left( \frac{0.51 + 3.51}{2} P \right)^2 + (2.64P)^2} \]
\[ = -1.50P + P \sqrt{4.03 + 6.97} = -1.50P + 332P \]
\[ \sigma_1 = 1.82P, \quad \sigma_2 = -4.82P \]
\[ \sigma_{yld}^2 = P^2 (3.31 + 24.2 + 8.76) = 36.27P^2 \]
\[ \sigma_{yld} = 6.03P \]

FOR YIELDING AT A, \[ P = \frac{44,500}{6.03} = 7380^* \]
At B, it is assumed $\gamma_3 = 0$, since it is a local effect.

\[
\delta_x = -2.11P \left( \frac{0.56}{8.25} \right) = -0.144P,
\]

\[
\delta_y = \frac{3.55}{3.99} \left( \frac{7.44}{8.00} \right) 2.34P = 1.94P,
\]

\[
T_{xy} = 2.64P,
\]

\[
\delta_1 = \frac{-0.144 + 1.94}{2} P + \sqrt{\left(\frac{-0.144 + 1.94}{2} P\right)^2 + (2.64P)^2}
\]

\[
= 0.90P_1 + P_1 \sqrt{(-2.08)^2 + 2.64^2}
\]

\[
= 0.90P_1 + 3.36P
\]

\[
\delta_1 = 4.26P, \quad \delta_2 = -2.46P
\]

\[
\delta_{yld} = P^2 (18.1 + 6.05 + 10.5) = 34.65P^2
\]

\[
\delta_{yld} = 5.88P
\]

For yielding at B

\[
P = \frac{44,500}{5.88} = 7560
\]

At D, assume $\gamma_3 = 0$

\[
\delta_x = \frac{3.12}{3.68} (2.06 P_1) \frac{7.81}{8.25} = 1.65P,
\]

\[
\delta_y = -\frac{3.82}{4.26} (2.50 P_1) \left( \frac{0.56}{8.00} \right) = -0.157P,
\]

\[
T_{xy} = 2.64P,
\]

\[
\delta_1 = \frac{1.65 - 0.16}{2} P_1 + \sqrt{\left(\frac{1.65 + 0.16}{2} P_1^2 + (2.64)^2 P_1^2\right)}
\]

\[
= 0.745P_1 + P_1 \sqrt{(0.905)^2 + (2.64)^2}
\]

\[
= 0.745P_1 + P_1 \sqrt{7.78} = 0.745P_1 + 2.79P
\]

\[
\delta_1 = 3.53P, \quad \delta_2 = -2.05P,
\]
\[ \sigma_{\text{yld}}^2 = P_i^2 (12.8 + 4.20 + 7.24) \]
\[ \sigma_{\text{yld}} = 4.92 P_i \]

For yielding at D
\[ P_i = \frac{44,500}{4.92} = 9040 \text{ N} \]

Therefore point A is the controlling location
\[ P_i = 7380 \text{ N} \]

From sol. of Eq. (3) \( \sigma_{\text{max}} \) in stiffener
\[ \sigma_{\text{max}} = 7.18 P_i \]
\[ \sigma_{\text{max}} = 7.18(7380) = 53,000 \text{ p.s.i.} \]

Max. value of \( D_i = \frac{44,500}{53,000} \times (7380) = 6200 \text{ N} \]

\[ T'_i = 44,500(0.938) \cos 44.2^\circ = 29,900 \text{ N} \]

\[ T'_i = 2.21 P'_i \]
\[ P'_i = \frac{29,900}{2.21} = 13,600 \text{ N} \]

\[ P_i = 6,200 \text{ N} \]

\[ P = P_i + P'_i = 19,800 \text{ N} \]
Properties of Section 1-1

<table>
<thead>
<tr>
<th>Item</th>
<th>A</th>
<th>%</th>
<th>Ax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner flg.</td>
<td>1.02</td>
<td>0.127</td>
<td>0.13</td>
</tr>
<tr>
<td>Web</td>
<td>1.59</td>
<td>3.87</td>
<td>6.15</td>
</tr>
<tr>
<td>Outer flg.</td>
<td>1.88</td>
<td>7.63</td>
<td>14.36</td>
</tr>
</tbody>
</table>

\[ \bar{A} = 4.49 \text{ in}^2 \]

\[ \bar{y} = 4.58'' \]

\[ I = 1.02(4.42)^2 + \frac{1}{12}(0.23)(7.50)^3 + 1.59(0.71)^2 + 1.88(3.00)^2 \]

\[ = 19.91 + 8.06 + 0.80 + 17.48 \]

\[ = 46.25 \text{ in}^4 \]

\[ S_0 = \frac{46.25}{3.17} = 14.60 \text{ in}^3 \]

\[ S_{11} = \frac{46.25}{4.58} = 10.11 \text{ in}^3 \]

Fiber Stresses at 1-1

\[ f_0 = \frac{0.707P}{4.49} + \frac{0.707P(23.63)}{14.60} = -0.1575P + 1.144P \]

\[ = 0.986P \]

\[ f_i = -0.1575P - \frac{0.707P(23.63)}{10.11} = -0.158P - 1.65P \]

\[ = -1.81P. \]

Load to cause yielding in extreme fibers at 1-1

\[ P = \frac{44,500}{1.81} = 24,600 \]

Shear Stresses

\[ Q_{NA} = \frac{1.81(7.75)}{2.80} = 5.01'' \]

\[ Q_{NA} = 7.30 \left(\frac{4}{3}\right)(3.68) + 2.49(0.23)(1.67) = 5.72 + 0.96 \]

\[ = 6.68 \text{ in}^2. \]

\[ \tau = \frac{0.707P(6.68)}{46.25(0.23)} = 0.444P \]

For yielding at NA, \[ 0.444P = \frac{44,500}{\sqrt{3}} \]

\[ P = 58,900 \text{ in}^2 \]
INSIDE FILLET

\[ Q_{bb} = 4(0.254)(4.46) + 0.31(0.3)(4.18) = 4.53 + 0.39 = 4.92 \text{ in}^3 \]

\[ \sigma_x = \frac{4.45}{5.01} (-1.81P) = -1.61P \]

Combined Stress \( \sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \)

\[ \tau_{xy} = \frac{0.707P(4.46)}{46.25(0.23)} = 0.326P \]

\[ \sigma = P\sqrt{(-1.61)^2 + 3(0.326)^2} = P\sqrt{2.91} = 1.70P \]

For Yielding at Inside Fillet,

\[ 1.70P = 44,500 \]

\[ P = 26,200 \text{ lb} \]

OUTSIDE FILLET

\[ Q_{aa} = 7.5(\frac{1}{4})(3.05) + 0.31(0.3)(2.77) = 5.72 + 0.26 = 5.98 \text{ in}^3 \]

\[ \tau_{xy} = \frac{0.707P(5.98)}{46.25(0.23)} = 0.398P \]

\[ \sigma_x = \frac{2.30}{2.74} (0.986P) = 0.827P \]

Combined Stress \( \sigma = P\sqrt{(0.827)^2 + 3(0.398)^2} \)

\[ \sigma = P\sqrt{0.663 + 0.475} = P\sqrt{1.158} = 1.08P \]

For Yielding at Outside Fillet

\[ 1.08P = 44,500 \]

\[ P = 41,200 \text{ lb} \]
Fiber Stresses at 3-3

\[ f_0 = -0.158P + \frac{0.707P(15.63)}{14.60} = -0.158P + 0.756P = 0.598P \]

\[ f_i = -0.158P - \frac{0.707P(15.63)}{10.11} = -0.158P - 1.092P = -1.25P \]

For yielding at extreme fiber

\[ P = \frac{44,500}{1.250} = 35,500 \text{#} \]

Combined Stresses at 66

\[ \tau_{xy} = 0.326P \]

\[ y_{na} = \frac{1.250}{1.848} \]

\[ \delta_y = \frac{4.68}{5.24} \]

\[ \delta_{ya} = \sqrt{\delta_y^2 + 3\tau_{xy}^2} = P\sqrt{(-1.117)^2 + 3(0.326)^2} = 1.25P \]

For yielding at 66, \( P = 35,500 \text{#} \)

Yielding at 3-3 will begin at \( P = 35,500 \text{#} \)

Radius of arc 2-2 = 16 + 7.75(2) = 31.5"

Length of 2-2 = 31.5 \( \frac{26.5}{57.3} \) = 14.60"

\[ P = 24' \text{ (by approximation)} \]
Location of C.G.

<table>
<thead>
<tr>
<th>Item</th>
<th>( x )</th>
<th>( A_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner flg.</td>
<td>1.02</td>
<td>0.127</td>
</tr>
<tr>
<td>Webb</td>
<td>3.24</td>
<td>7.30</td>
</tr>
<tr>
<td>Outer flg.</td>
<td>1.88</td>
<td>14.48</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
I &= 1.02(8.40)^2 + 1.88(5.96)^2 + 3.24(1.47)^2 \\
&\quad + \frac{1}{12} (0.23)(14.10)^3 \\
&= 71.9 + 56.8 + 6.99 + 53.4 \\
&= 189.1\text{ in}^4 \\
S_0 &= \frac{189.1}{6.08} = 31.1\text{ in}^3 \\
S_1 &= \frac{189.1}{8.52} = 22.2\text{ in}^3
\end{align*}
\]

Determination of Moments and Direct Stress

\[\theta = 31.50\text{ cm} \cdot 11.07'' + 0.13 = 30.80 + 0.13 = 30.93''\]
\[\alpha = 31.50\text{ cm} \cdot 11.07'' = 36.04''\]

\[N = \text{Direct Stress Component} = P \cos 33.9^\circ = 0.829 P\]
\[M = 0.707 P (2.17 - 30.93) = -20.4 P\]

Take radius of curvature of centerline

\[= 24'' = \rho\]

Use Bleich Eq. (9) - "Design of Rigid Frame Knees"

\[
\delta = \frac{1}{\cos x} \left[ \frac{N}{A} - \frac{M}{\rho A} - \frac{Mc}{2 \left( \frac{\rho}{\rho + c} \right)} \right]
\]

\[z = \rho^2 (2.303 \rho \delta \log \frac{\rho}{\rho z} - A),\]

\[\rho_0 = \text{radius centerline} \]
\[ P = 25'' \]
\[ A = 6.14 \text{ in}^2 \]
\[ \frac{7.50 \times 31.08}{30.83} = 0.0264 \]
\[ \frac{0.230 \times 30.83}{16.73} = 0.0611 \]
\[ \frac{4.00 \times 16.73}{16.48} = 0.0380 \]
\[ \varepsilon = 0.1255 \]

\[ 7.20 \]
\[ Z = 625 \left( 2.303 \times 25 \times 0.1255 - 6.14 \right) = 662 \text{ in}^4 \]

\[ \sigma = \frac{1}{c \times d} \left[ \frac{N}{A} - \frac{M}{\rho A} - \frac{Mc}{Z \left( \frac{\rho}{\rho + c} \right)} \right] \]

\[ \alpha = 11.07^\circ \quad \alpha_z = 15.01^\circ \]
\[ N = -0.829P \]
\[ M = -204P \]
\[ \rho = 25'' \]
\[ C = +6.08 \]
\[ -8.52 \]

\[ f_0 = \frac{1}{\cos 11.07} \left[ \frac{-0.829P}{6.14} + \frac{20.4P}{25 \times 6.14} + \frac{20.4(6.08)P(25)}{622 \times 31.08} \right] \]
\[ = 1.020 \left[ -0.1352P + 0.1329P + 0.1603P \right] \]
\[ = 1.020(0.1580P) = 0.161P \]

\[ f_z = \frac{1}{\cos 15.01} \left[ -0.1352P + 0.1329P - \frac{20.4(8.52)P(25)}{622 \times 16.48} \right] \]
\[ = 1.036 \left[ -0.1352P + 0.1329P - 0.424P \right] \]
\[ f_i = 1.036 (-0.426P) = -0.441P \]

For yielding at extreme fibers,
\[ 0.441P = 44,500 \]
\[ P = 100,700 \text{ lb} \]

This value probably erroneous as it is very difficult to get a good value of \( P \).
\( p \) APPROXIMATED BY ALTERNATE METHOD

\[ p = 47" > 2d \implies I = 2 \]

FROM EQ 9 (BLEICH)

\[
\begin{align*}
fo &= \frac{1}{\cos 11.67} \left[ -0.1352P + \frac{20.4P}{47(6.08)} + \frac{20.4P(6.08)(47)}{189.1(53.08)} \right] \\
&= 1.020 (-0.1352P + 0.0707P - 0.581P) \\
&= 1.020 (0.517P) = 0.527P
\end{align*}
\]

\[
\begin{align*}
f_i &= \frac{1}{\cos 15.01} \left[ -0.1352P + 0.0707P - \frac{20.4P(8.52)(47)}{189.1(38.48)} \right] \\
&= 1.036 (-0.1352P + 0.0707P - 1.122P) \\
&= 1.036 (-1.186P) = -1.228P
\end{align*}
\]

FOR YIELING AT \( z = 2 \)

\[ 1.228P = 44,500 \]

\[ p = 36,300 \text{ lb} \]

\( \text{Distribution of stress on tension fiber} \)
STRESSES AT Z-Z BY WEDGE THEORY

\[ r = 31.50'' \]

\[ P = P \cos (45^\circ + 11.07^\circ) = 0.557 P \]
\[ P_2 = P \cos (45^\circ - 11.07^\circ) = 0.830 P \]
\[ M_0 = 0.707 P (3.88 + 0.13) = 2.84 P \]

\[ \alpha_1 = 11.07^\circ = 0.193 \text{ rad.} \]
\[ \alpha_2 = 15.51^\circ = 0.270 \text{ rad.} \]

**Eq. 18**

\[ c_1 = \frac{P r}{(A_0 \sin^2 \alpha_1 + A_1 \sin^2 \alpha_2) + \frac{r}{4} (2\alpha_1 + 2\alpha_2 - \sin 2\alpha_1 - \sin 2\alpha_2)} \]

\[ = \frac{0.557 P r}{0.0696 [1.02 (0.0717)] + 0.23 (31.50) [0.386 + 0.540 - 0.376 - 0.515]} \]
\[ = 0.1428 + 0.0634 \]
\[ = 0.557 \frac{P}{0.206} = 2.71 \frac{P}{r} \]

\[ \delta_r = c_1 \frac{P \sin \theta}{r} = 2.71 P \sin 11.07^\circ = 0.519 P \bigg|_{\theta = x_1} \]
\[ = 2.71 P \sin (-15.51^\circ) = -0.724 P \bigg|_{\theta = -x_2} \]

**Eq. 20**

\[ a_1 = \frac{-P_2 r}{(A_0 \sin^2 \alpha_1 + A_1 \sin^2 \alpha_2) + \frac{r}{4} (2\alpha_1 + 2\alpha_2 + \sin 2\alpha_1 + \sin 2\alpha_2)} \]

\[ = \frac{-0.830 P r}{0.1428 + 0.23 (31.50) [0.386 + 0.540 + 0.376 + 0.515]} \]
\[ = 0.1428 + 3.29 \]
\[ a = - \frac{0.830 \Pr}{3.43} = -0.242 \Pr \]
\[ \delta r_2 = \frac{a_i}{r} \sin \theta = -0.242 (P) \cos 11.07^\circ = -0.238 P \theta = x_1 \]
\[ = -0.242 P \cos (15.51^\circ) = -0.233 P \theta = -x_2 \]

EQUATION 22

\[ \delta r_3 = \frac{-N_r}{4(x_{A0} \sin 2x_i + x_{A2} \sin 2x_i)} + \frac{4 \pi r}{2} \left[ \frac{1}{2} \left( \sin 2x_i + \sin 2x_i \right) - (x_{A0} 2x_i + x_{A2} 2x_i) \right] \]

\[ = \frac{-2.84 \Pr}{1.187} = -2.40 \Pr \]
\[ \delta r_3 = - \frac{4}{r^2} \sin 2\theta \]
\[ = - \frac{4}{31.50} (-2.40 \Pr) \sin 2x_i = 0.115 P \left| \theta = x_1 \right. \]
\[ = - \frac{4}{31.50} (-2.40 \Pr) \sin 2x_2 = -0.158 P \left| \theta = -x_2 \right. \]

EXTREME FIBER STRESSES

OUTSIDE FIBERS \[ \sigma_0 = P (0.319 - 0.238 + 0.115) \]
\[ = 0.396 P \]

INSIDE FIBERS \[ \sigma_i = P (-0.724 - 0.238 - 0.158) \]
\[ = -1.115 P \]

FOR YIELDING AT 2-2 \[ 1.115 P = 44,500 \]
\[ P = 39,900 \text{ psi} \]