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Notes on shanley's theory of inelastic columns, October 1951

C. H. Yang

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REPORT

NOTES ON SHANLEY'S THEORY OF INELASTIC COLUMNS

TO

BY

C.H. YANG

~~Discussion on Shanley's Theory of Inelastic Column~~

Assume the column reach at a load $P_r = \frac{E_t A}{L}$ without bending, and then start to bend at that load when load increases we'll see if we can get an equilibrium system.

$$\Sigma M = 0$$

$$\therefore \quad P = \frac{AE_t}{L}\left[\frac{e_1 + e_2 k}{e_1 + e_2}\right] \quad ---- (A)$$

$$\Sigma F = 0$$

(1) $\Delta P = \frac{e_1 A E_t}{2} - \frac{e_2 A E_2}{2} = \frac{AE_t}{2}(e_1 - e_2 \frac{E_2}{E_t})$, $E_2 = kE_t$, $\therefore \Delta P = \frac{AE_t}{2}(e_1 - ke_2)$

$$\therefore \quad P = P_r + \Delta P = \frac{AE_t}{L} + \frac{(e_1 - e_2 k) AE_t}{2} \quad (1)$$

$$\therefore \quad P = \frac{AE_t}{L}\left[1 + \frac{L(e_1 - e_2 k)}{2}\right] \quad --- (B)$$

$$\therefore \quad 1 + \frac{L(e_1 - e_2 k)}{2} = \frac{e_1 + e_2 k}{e_1 + e_2}$$

$e_1 + e_2 - e_1 \mp e_2 k = e_2(1-k)$

is the condition of equilibrium

$$e_2(1-k) + \frac{L}{2}(e_1 - e_2 k)(e_1 + e_2) = 0 \quad --- (C)$$

Substitute (c) to (A) we find the relation to maintain an equilibrium condition after $P = \frac{E_t A}{L}$, between P & e_1 or e_2

We find

$$P = \frac{AE_t}{L}\left(1 + \frac{2\delta(k-1)}{k-1+2\delta(1+k)}\right)$$

Suppose the column has been loaded up to a point between tangent modulus load and reduced modulus load with lateral support. Then the lateral support moved away and load increase.

Let the load at the column $= P_0$

$\sum M = 0$ $\qquad P = \frac{AE_0}{L}\left(\frac{e_1 + e_2 K}{e_1 + e_2}\right)$

where $\quad K = \frac{E_t}{E_0}$

$\sum F = 0$ $\qquad P = P_0 + \Delta P = P_0 + \left(\frac{e_1 - e_2 K}{2}\right) AE_0$

Condition of Equilibrium

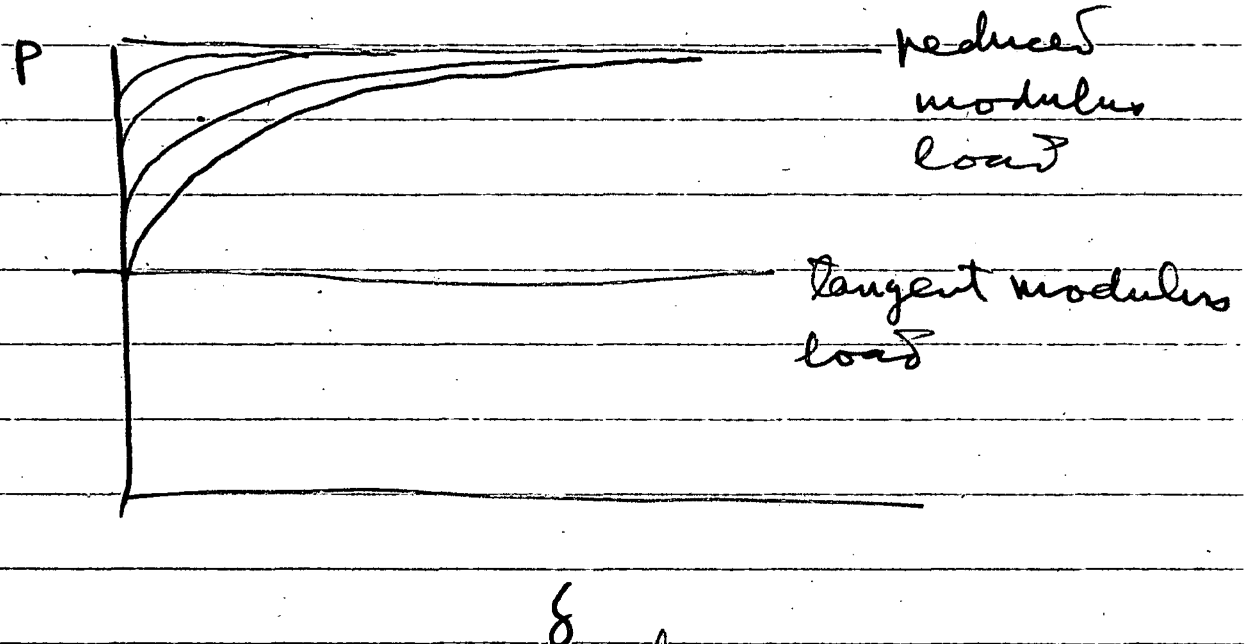
$$P_0 + \frac{AE_0}{L}\left[\frac{(e_1 - e_2 K)L}{2} - \frac{e_1 + e_2 K}{e_1 + e_2}\right] = 0$$

Let $P_0 = K' \frac{AE_0}{L}$ $\qquad K' > 1$

we have

$$\left[K' + \frac{e_1 - e_2}{2}\right] = \frac{e_1 + e_2 K}{e_1 + e_2}$$

$$P = \frac{AE_0}{L}\left[f(K', K, \delta)\right]$$



Millions of paths the column can be deviated from straight between these two limits

Stability of the system

From reduced modulus theorem we know after the column past its tangent modulus load but before its reduced modulus load the system is stable if the load has been kept const while the infinitisimal displacement is given.

But if "p" varies while the displacement is given we can investigate the stability of the system in the following way

$$p = \frac{AE_t}{L}\left(1 + \frac{2\delta(k-1)}{k-1+2\delta(1+k)}\right)$$

$$\frac{dp}{d\delta} = \frac{AE_t}{L}\left(\frac{2(1-k)^2}{\left((k-1)+2\delta(1+k)\right)^2}\right)$$

$$\frac{dp}{d\delta} = \text{Positive whatever the value } \delta \text{ is}$$

$$\lim_{\delta\to\infty} \frac{dp}{d\delta} = 0 \qquad \text{Neutral stability}$$

$$\lim_{\delta\to\infty} \frac{2\delta(k-1)}{k-1+2\delta(1+k)} = \lim_{\delta\to\infty} \frac{2(k-1)}{\frac{(k-1)}{\delta}+2(1+k)}$$

$$= \frac{k-1}{k+1}$$

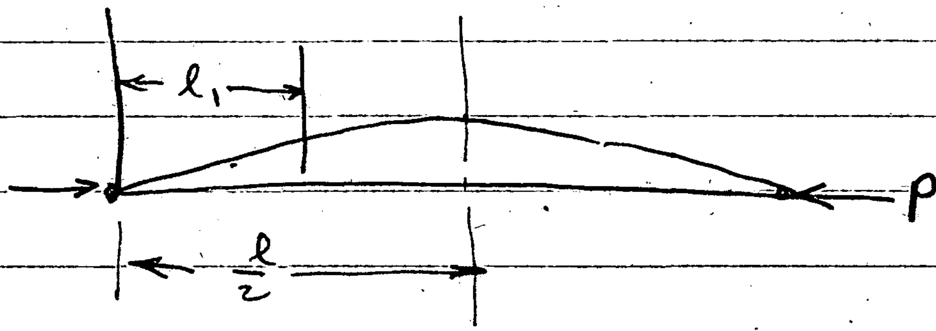
$$\therefore \quad P = \frac{AE x}{L}\left(1 + \frac{K-1}{K+1}\right)$$

reduced modulus load

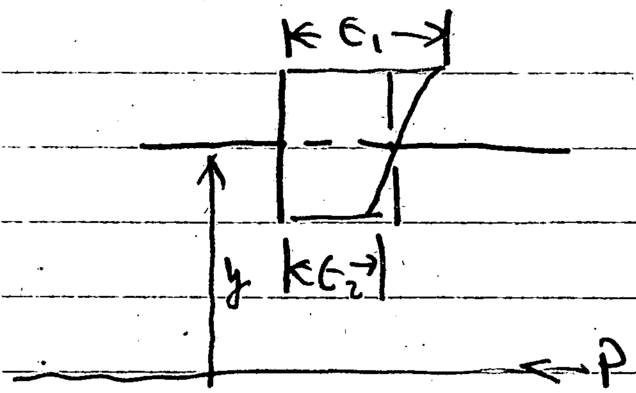
This proves the system is stable all through
to the end till P reaches reduced modulus load
the system become neutral stable

Generalized Formulation (Rectangular Cross section)

(1)



From $0 \rightarrow l_1$



$$P = \frac{E_t I \pi^2}{l^2} + \Delta P$$

$$\Delta P = \frac{\epsilon_1 + \epsilon_2}{2} \times A \times E_t$$

$$E_t I \frac{d^2 y_1}{dx^2} + P y_1 = 0$$

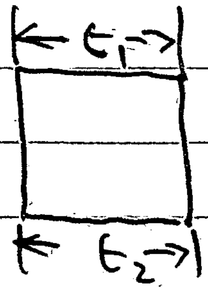
$$y_1 = C_1 \cos \sqrt{\frac{P}{E_t I}}\, x + C_2 \sin \sqrt{\frac{P}{E_t I}}\, x$$

1. $x = 0 \quad y_1 = 0 \quad \therefore \; C_1 = 0$

2. $x = l_1$ a. $\left|\frac{dy_1}{dx}\right| = \left|\frac{dy_2}{dx}\right|$ determine l_1

   b. $y_1 = y_0$ determine C_1

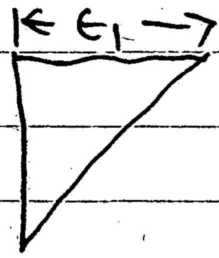
at $x = 0$



$$\epsilon_1 = \epsilon_2$$

$$\Delta p = \epsilon_1 E_t A$$

at $x = l_1$ $\qquad y_1 = y_0$



$$\epsilon_2 = 0$$

$$\Delta p = \frac{\epsilon_1}{2} E_t A$$

$$\text{Moment (internal)}_1 = E_t I \frac{d^2 y_1}{dx^2}$$

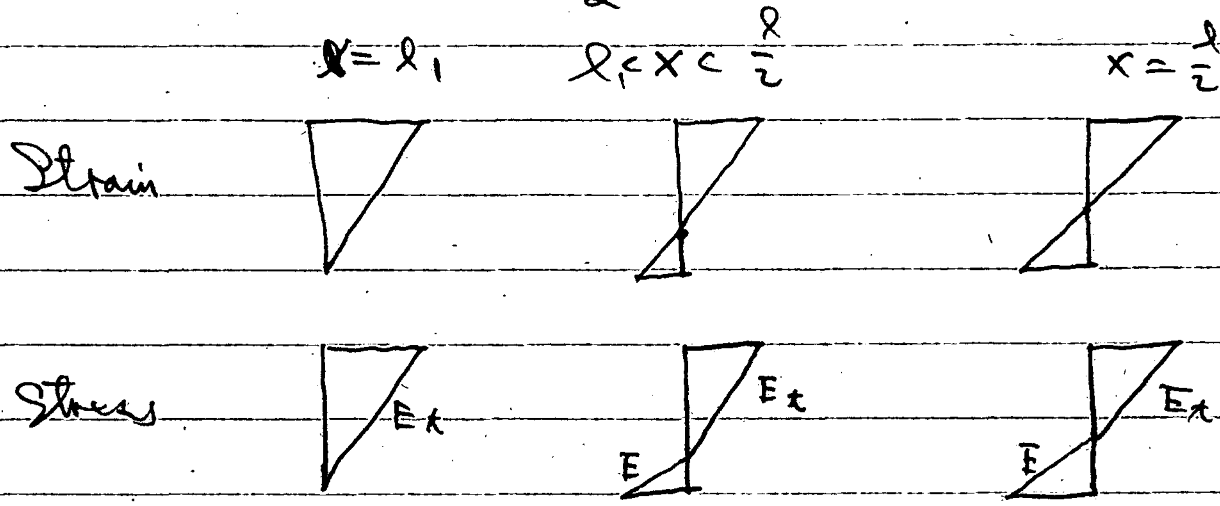
$$\frac{d^2 y_1}{dx^2} = \frac{1}{R} = \frac{\epsilon_1}{h} = \frac{2 \Delta p}{E_t A h}$$

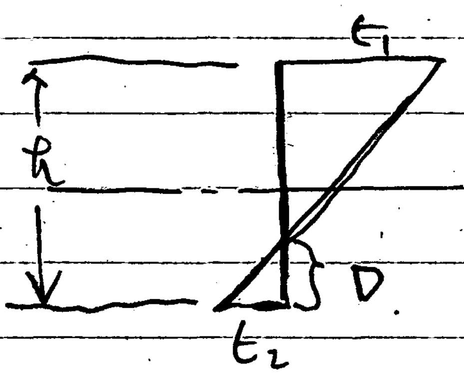
A = crossection area
h = depth of the beam

$$M_{(\text{external})} = P y_0 = \left(\frac{E_t I \pi^2}{l^2} + \Delta p \right) y_0$$

$$\therefore \quad y_0 = \frac{2 \Delta P I l^2}{A h \left(E_t I \pi^2 + \Delta P \right)}$$

From l_1 to $\frac{l}{2}$





① $\frac{1}{R} = \frac{\epsilon_1+\epsilon_2}{h} = \frac{d^2y}{dx^2}$

② $D = \frac{\epsilon_2}{\epsilon_1+\epsilon_2}$ $\quad h - D = \frac{\epsilon_1}{\epsilon_1+\epsilon_2}$

③ $E_t h t \left[\epsilon_1 \times \frac{\epsilon_1}{\epsilon_1+\epsilon_2} - \epsilon_2 \times \frac{K\epsilon_2}{\epsilon_1+\epsilon_2}\right] = 2\Delta P$

Where t = thickness of section
h = depth
$k = \frac{E}{E_t}$

$$\therefore \quad E_t R t \left[\frac{\epsilon_1^2}{\epsilon_1+\epsilon_2} - \frac{K\epsilon_2^2}{\epsilon_1+\epsilon_2} \right] = 2\Delta P$$

From ①, ②, ③, we have

Solve pour
$$\begin{cases} \frac{h}{R} = \epsilon_1 + \epsilon_2 \\ \frac{R}{h} E_t A t \left[\epsilon_1^2 - K\epsilon_2^2 \right] = \Delta P \\ \frac{R\epsilon_2}{h} = D \end{cases}$$

$$\cancel{R = f(D)} \qquad p = f\left(\frac{d^2y}{dx^2}\right)$$

then $\quad P y_2 + E_t I \dfrac{d^2y}{dx^2} + F(D, E, E_t) = 0$

$$P y_2 + E_t I \frac{d^2y}{dx^2} + F\left(\frac{d^2y}{dx^2}, E, E_t\right) = 0$$

$$y_2 = \varphi(\Delta P, E_t, E, t, h, x)$$

① $x = l_1 \qquad y_2 = y_0$

$x = \frac{l}{2} \qquad \frac{dy_2}{dx} = 0$

$x = l_1 \qquad \left(\frac{dy_2}{dx}\right) = \left(\frac{dy_1}{dx}\right)$

$$\frac{h}{R} = \epsilon_1 + \epsilon_2 \quad - - - \quad (1)$$

$$\frac{R}{h} E_t A t \left[\epsilon_1^2 - K \epsilon_2^2 \right] = \Delta p \quad \cdots \quad (2)$$

$$\frac{R \epsilon_2}{h} = D \quad - - - - - \quad (3)$$

Take (2) $K = 1$

$$\epsilon_1 = \frac{h}{R} - \epsilon_2$$

$$\therefore \frac{R}{h} E_t A t \left[\frac{h^2}{R^2} - 2 \frac{h}{R} \epsilon_2 \right] = \Delta p$$

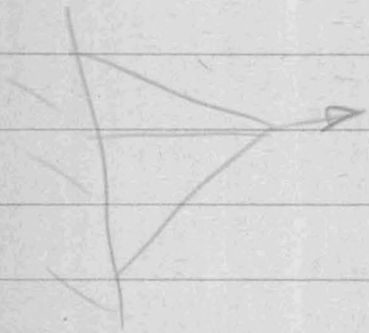
$$\frac{R}{2h}\left(E_t A t \frac{h}{R} - \Delta p \right) = \epsilon_2$$

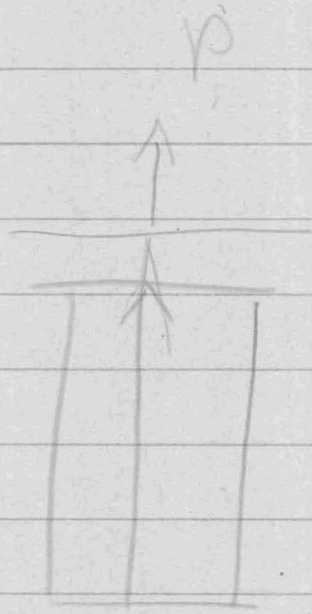
$$\therefore \epsilon_2 = \frac{E_t A t}{2} - \frac{R \Delta p}{2 h}$$

$$\therefore D = \frac{R}{h}\left(\frac{E_t A t}{2} - \frac{R \Delta P}{2 h} \right)$$

$$\therefore D = \frac{d^2 y}{h\, dx^2}\left(\frac{E_t A t}{2} - \frac{\frac{d^2 y}{dx^2} \Delta p}{2 h} \right)$$

Non linear Equation

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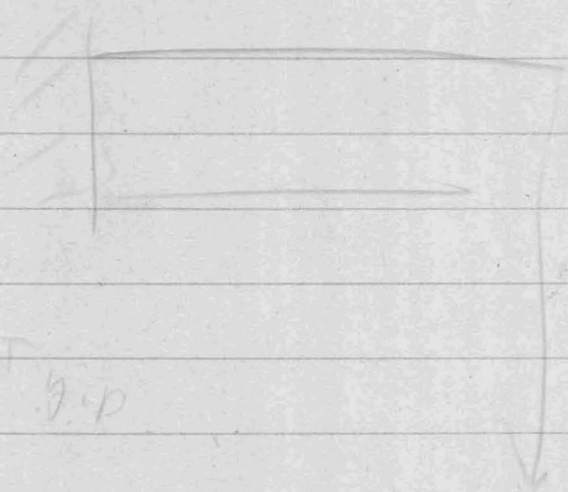
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