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Commercial Load Scheduling Through A Time-of-Use Schedule with Electricity Demand Charges

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COMMERCIAL LOAD SCHEDULING
THROUGH A TIME-OF-USE SCHEDULE WITH ELECTRICITY
DEMAND CHARGES

by

Benjamin A. Mizack

A Thesis

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Master of Science

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(Date)

Lawrence V. Snyder

Tamás Terlaky
(Department Chair)

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Abstract

This thesis proposes a mathematical program that models the energy consumption and energy demand values using a time-of-use electricity rate structure. Using large commercial or industrial machines with electrical motors, the model will determine which machines will be turned on and off during a certain amount of time. Through the Integer Program, we show that by letting machines run longer and staggering the starts of machines, the electricity costs for a typical day can be greatly reduced. Also, we add a parameter that specifies the total running time of each machine which prevents a loss in productivity for a business. We test our model with both random parameters as well as with an example that uses values from the refrigeration industry. We show by using an optimized running schedule for all machines that energy costs can be greatly reduced compared to a typical un-optimized machine run schedule.

Chapter 1

Introduction

1.1 General Introduction

In this thesis we are discussing the commercial load scheduling problem using a time-of-use schedule with electricity demand charges. A time-of-use schedule provides different electricity consumption charges based on the time of the day, usually defined as peak, off peak, and partial peak. Typically, peak charges will be during the times where the electricity company experiences the highest energy demand from its customers. Off-peak charges are usually the night time and early hours of the day and partial peak is generally experienced during the morning hours, i.e. 8 A.M. to 12 P.M. Our goal is to create a mathematical program that will reduce the electricity costs for a large scale commercial application. Unlike most residential

electricity rates, large commercial electricity rates take into account the electricity consumed in kilowatt hours as well as the electricity demand during the day measured in kilowatts. Electricity demand is the amperage draw associated with starting up large machines, while electricity consumption is the electricity required to keep a machine running after it has started. For utility companies, the maximum demand is typically measured in 15-minute or 30-minute intervals, although a 60-minute interval may at times be utilized [3]. The utility company charges demand rates based on the maximum amount of demand realized during a fifteen or thirty minute segment of the month. For example, suppose if a company's maximum demand for the month is 15 kW and a utility company charges \$10.00 per kW. Then, the company would be charged \$150.00 for their demand charge. In our model we will assume that the demand charge is the maximum demand realized during one time period and the demand cost is charged daily. Therefore, it would be in the firm's best interest to spread out the start up of machines in order to reduce the firm's energy demand, in turn reducing their electricity costs.

Electricity consumption rates using the schedule proposed by Pacific Gas and Electric Company utilize a time-of-use schedule [2]. With a time of use schedule, consumption charges may be time differentiated by season and/or time of day [3]. Typically, time-of-use rate structures incorporate an on-peak and off-peak pricing scheme where electricity costs more during the day time hours than it does at night.

For example, a utility company may charge \$.10 per kWh from the hours of 10 AM to 6 PM then \$.08 per kWh otherwise. Utility companies such as Pacific Gas and Electricity Company implement a peak, partial peak, off-peak rate structure where peak is from 1 PM to 7 PM, partial peak is 10 AM to 1 PM and 7 PM to 10 PM, and off peak is all other hours. Here the utility company may charge \$0.16 kWh for peak, \$0.14 for partial peak, and \$0.12 for off peak hours. Also, the time-of-use rate schedule by Pacific Gas and Electric Company incorporates seasonal rates based on summer and winter where the consumption as well as the demand charges are lower during the winter compared to summer. Summer electricity costs will be more than those electricity costs charged during the winter due to cooling loads and higher stresses on the power grid. Although our model can handle both summer and winter electricity rate schedules, we will use only the summer rates in our simulations.

We should also note that commercial rate structures vary depending upon the type of voltage delivered to the firm. A typical rate structure will list different costs for primary, secondary, and transmission voltages. For example, a house will generally use 115 volt and 230 volt which are known as secondary voltages. In a commercial setting, a firm will generally use 460 volt, if the voltage is available by the service provider, which is a secondary voltage as well. The primary and transmission voltages are much higher such as 230,000 volts for transmission and 20,780 volts for primary.

Customers that take service at the transmission voltage level will have transformers within their facility to achieve the voltage that they need for operation. That is, the utility company does not have to provide any voltage transformations for customers that utilize transmission voltages. For primary voltages, the utility company provides some voltage transformations in order to service the customers that desire primary voltages. Lastly, in order to achieve secondary voltages, a utility company must provide several voltage transformations to service households, small businesses, etc.. Therefore, different demand charges are necessary for each of the voltage levels which reflects the necessary voltage transformation costs that the utility company accrues in order to provide the different voltage levels [3]. Hence, the demand charge increases for each level of electricity transformation. The A-10 TOU rate schedule [2] also implements a fixed charge that takes into account the cost of the electricity metering device, however we will ignore this in the model.

For this problem we will be using the refrigeration industry to obtain data on energy demand and energy consumption by different refrigeration systems. In the refrigeration industry, as in most industries, they do not use kilowatts to describe how much electricity is required to start up a machine or run one. Hence we will define three new terms referred to as *rated load amps*, *maximum continuous current*, and *locked rotor amps*. The maximum continuous current, or MCC, of a compressor is the value at which the internal overload protector of a compressor's motor will trip

and turn off the compressor [1]. Manufacturers of refrigeration compressors meticulously test their machines to find out the value at which the compressor's overload protector will trip and stop the compressor from running. Hence, if the maximum continuous current of a compressor was 35 amps and compressor drew 36 amps the overload protector would trip and the compressor would shut off. The MCC is an important value used to find the rated load amperage of a compressor. The rated load amperage (RLA) is a mathematical calculation specified by Underwriters Laboratories (UL) that uses the MCC value to derive the RLA value of a compressor. Since compressor amperage draws vary based on extraneous factors such as ambient temperature, the RLA value is the accepted amp draw that a compressor will exhibit when it's running [1]. Locked rotor amperage, or LRA, is the amperage required to energize a compressor in a locked state, when it's off, to a running state. Another term is known as full load amperage, or FLA, which is used more frequently with fan motor running amperages. Since the amperage draw of compressors and machines in general change based upon conditions, we use the RLA values as amperage required to run a compressor in our model. Using the amperage found in the literature provided by Heat Transfer Products Group ([4], [5], [6], and [7]), and using our assumption that the electricity supply is 460 volt, we can transform the amperage draws into kilowatts by:

$$kW = \frac{A \times V}{1000}$$

Where A is the amperage value obtained for a machine's RLA and LRA values, and V is the voltage which we assumed is 460 volt.

Typically, in large commercial applications, the electricity draw to start a machine greatly exceeds the electricity required to keep a machine running. Hence, during typical usage, the costs for electricity demand will exceed the electricity consumption costs. Spreading out machine start ups may decrease the demand charge however, it may also reduce the productivity of the firm. Here we will model a company trying to reduce its energy costs while preventing loss of productivity.

1.2 Typical Refrigeration System

Later in this thesis we provide an example using the integer program with refrigeration equipment electrical data. We have modeled refrigeration systems that would be typical for a medium sized supermarket and recorded the corresponding electrical data for each of the refrigeration systems. Before we run the model, it is imperative to discuss refrigeration systems and the industry's methods for making refrigeration systems more energy efficient.

To begin, a refrigeration cycle is a "sequence of thermodynamic processes through

which a refrigerant passes, in a closed or open system, to absorb heat at a relatively low temperature level” [9]. In order to explain this definition in greater detail, we provide the diagram in Figure 1.1.

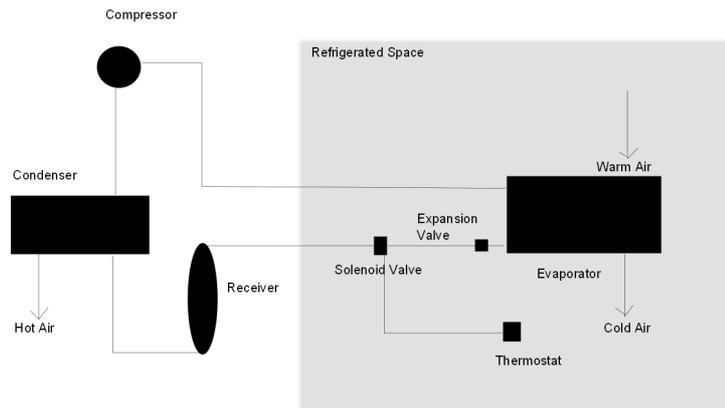


Figure 1.1: Traditional Refrigeration System

From our definition, the refrigeration cycle is an open or closed system (Figure 1.1 is a closed system), where a refrigerant passes through. There are a myriad of different types of refrigerants that are used in refrigeration systems based on the application. Some trade names that may sound familiar include Freon and Puron which are typical for air conditioning and high temperature refrigeration applications. The refrigeration cycle begins at the compressor which compresses the refrigerant from a low temperature, low pressure vapor, into a high pressure, high temperature gas. The refrigerant then travels to the condenser where it is cooled from a high temperature gas into a high temperature liquid. As seen in the

diagram, the high pressure, high temperature liquid flows through the liquid line until it reaches the “metering device”. At the metering device, also known as an expansion valve, the refrigerant is injected into the evaporator at a low pressure. The refrigerant then “boils off” inside the evaporator coil, creating a cooling effect as it evaporates into a low pressure, low temperature vapor. The evaporator coil, inside the refrigerated space, provides the cooling necessary to reach the temperature desired. The low temperature, low pressure vapor then travels back through the suction line to the compressor where the refrigeration cycle begins once again. From the Figure 1.1, the refrigerant absorbs the heat in the refrigerated space, travels through the compressor, then the heat is rejected through the condenser, which is consistent with the definition of a refrigeration cycle.

The refrigeration cycle continues until the thermostat inside the refrigerated space is satisfied, i.e. it reached the temperature set point. When the thermostat is satisfied, a solenoid valve on the liquid line closes which cuts off the refrigerant flow to the evaporator. The compressor continues to operate, or pump down, until most of the refrigerant in the suction line has been compressed which creates low pressure in the suction line. The drop in pressure trips a switch in the compressor that turns it off. From now on we refer to the compressor turning on and off as compressor cycling. As discussed previously, every time a refrigeration cycle begins, energy demand is realized due to energy required to start the compressor,

i.e. changing it from a rotor locked state to a running state. Once the compressor is running, it utilizes significantly less energy than the energy required to start the compressor. An increase in compressor cycles also reduces the longevity of the refrigeration compressor. Therefore, in order to reduce the number of cycles, a mechanical method known as hot gas bypass for compressor capacity control is utilized. By reducing the number of cycles, the compressor will demand less energy to start during a time period as well as prevent excessive wear on the compressor. Next we discuss that by controlling the capacity of the compressor using hot gas bypass, the cycling of the compressor is also reduced.

1.2.1 Hot Gas Bypass Capacity Control

Hot gas bypass, or HGBP for short, utilizes a mechanical method for capacity control that prevents the refrigeration compressor from cycling. In a given twenty-four hour period, a compressor may cycle twenty times or even more given the cooling demands of the refrigerated space. The cycling of the compressor leads to large temperature deviations in the refrigerated structure which is unacceptable in some applications such as blood and plasma banks. The HGBP method essentially hinders the refrigeration system from turning off even after the cooling requirements of the refrigerated structure have been satisfied.

Hot gas bypass begins to influence the closed system once the cooling demands

of the refrigerated structure have been satisfied. Once the cooling load has been satisfied, a solenoid closes which restricts flow of refrigerant to the evaporator as discussed in Section 1.2.1. Instead of the compressor pumping down, the hot gas line solenoid valve opens and feeds the evaporator with hot gas. By feeding the evaporator hot gas a “false load” is created in the room which eventually leads to the thermostat calling for cooling. Once the thermostat calls for cooling in the room the normal refrigeration cycle begins again.

Energy savings are realized here by preventing the compressor from cycling. As discussed in the introduction, energy demand is realized every time the compressor starts. HGBP prevents the compressor from turning off therefore the number of starts are greatly reduced or nonexistent in some instances. Although HGBP prevents compressor cycling, energy is still being consumed by the compressor. More recent electrical and mechanical methods not only prevent the compressor from cycling, but also address the issue of the compressor constantly consuming energy.

1.2.2 Cylinder Unloading

Several types of refrigeration compressors, much like an automobile, utilize pistons or cylinders to compress the refrigerant gas. In the industry today, these compressors have two, three, four, six, or eight pistons typically. Cylinder unloading is used for capacity control by “[interrupting] the gas flow, and the corresponding pistons

operate in the ‘idle mode’ without gas pressure” [8]. During full load operation, the compressor will operate with all cylinders. However, during part load operation, a mechanical mechanism prevents gas flow to certain cylinders putting them into an idle state. For example, if we were to have a four cylinder compressor, cylinder unloading would prevent gas flow to a pair of those cylinders when refrigeration conditions allow for part load operation. Cylinder unloading is a relatively simple idea which keeps the compressor running through part load operation instead of turning off. Not only does it prevent the compressor shutting off, but it also cuts down on the compressor energy consumption.

A single compressor has a power consumption factor of 1 during full load operation i.e. all cylinders are being utilized. If a compressor is unloaded, its power consumption factor will decrease based on how many cylinders of a compressor are unloaded. For instance with a three cylinder compressor, with two cylinders unloaded, it achieves a power consumption factor of approximately 0.4 at ten degrees Celsius. Note that the power consumption factor is dependent upon temperature since the compressor requires more energy to compress lower temperature refrigerant gas. Therefore, cylinder unloading not only cuts down the amount of compressor cycles, but also the power consumption of the compressor during part load conditions. Hence, cylinder unloading is more efficient than the hot gas bypass method. Unfortunately, cylinder unloading can only be applied to large compressors, therefore it

is not suitable for every application. However, note also that there are other ways to “unload” compressors without cylinders using different methods that will not be discussed in this master’s thesis. A new electronic method has been developed recently which provides a larger application range than cylinder unloading.

1.2.3 Electronic Compressor Speed Control

A compressor with a frequency inverter or a variable speed compressor utilizes an electronic control that varies the speed of the compressor based on the cooling demands of the refrigerated space. The electronic control varies the speed of the compressor motor which provides step-less capacity control of the refrigeration compressor. This differs from cylinder unloading where capacity control is dependent upon the compressor’s quantity of cylinders. Therefore, cylinder unloading provides stepped capacity control. Note with variable speed compressors, the definitions of full load operation and part load operation vary from the definitions in Section 1.2.3. The compressor operates at full load when it is running at its maximum designed speed. Part load operation occurs when the compressor is running at some speed less than its maximum.

Some manufacturers incorporate a soft starter into their electronic control which prevents the compressor from drawing large amperage to start which relates to a

large spike in energy demand. In addition, some manufacturers claim that by utilizing a variable speed control, the capacity of the compressor can be reduced to ten percent of the published full load capacity. Therefore, when a typical refrigeration system would complete its cycle, a system utilizing a variable speed compressor would run at reduced load until higher cooling demands were realized. Similar to compressor unloading, electronic speed control prevents cycling of a compressor as well as reduces the compressor's energy consumption through part load operation. Unfortunately, compressors with variable speed controls or frequency inverters are expensive compared to their mechanical counterparts. Therefore there is some debate as to whether or not the costs inhibit the benefits of variable speed compressors.

1.2.4 Refrigeration Conclusion

Through this section we discussed several ways to promote cost savings using electronic or mechanical means to control the cycling of the compressor. HGBP utilizes a mechanical method that kept the compressor cycling. The two latter methods utilize mechanics and electronics to not only prevent the cycling of the compressor but reduce the energy consumption as well. All of these methods have their own limitations due to application and cost. However, these methods illustrate the importance of using capacity control to reduce the amount of compressor cycles, leading to possible cost savings which we investigate later.

Chapter 2

Literature Review

There is a large amount of scholarly articles dealing with the management of electricity and trying to optimize costs or minimize the demand on a power grid. Two topics of particular interest are Multi-Agent Home Automation Systems (MAHAS) and Electricity Management Controllers (EMC). The purpose of these devices is to allow the agents, i.e. devices or appliances, to cooperate and coordinate their actions in order to find an acceptable near optimal solution for power management [10]. In their paper they propose an algorithm to reduce energy costs by postponing or delaying starts of appliances all while taking into account the comfort of the inhabitant that lives in the residence. Cohen [11] demonstrates that it may be beneficial to a utility company to control the running of appliances to reduce the strain on the electrical grid during times of peak load. Using a dynamic programming approach,

he shows how a utility company could level out the energy load throughout the day by controlling a residential area's usage of appliances, i.e. air conditioners and hot water heaters.

There is a large body of literature on load management or load control. Hu, Chen and Bak-Jensen [12] discusses optimizing energy loads by managing consumer energy demand in Denmark. Denmark uses a time-of-use electricity rate schedule where electricity prices are set the day before through market trading and then those prices are implemented the following day [12]. Therefore, consumers know the time-of-use rate schedule beforehand. Using a linear program, they model the energy costs of a consumer during the day with an objective to minimize those costs. Since the consumers know the rate schedule, they can “reduce the consumption near the price peaks in order to reduce the energy costs” [12]. The authors show a price curve plotted against time with large price spikes. By using the LP, the authors show that program shifts energy consumption away from the peaks in order to reduce costs. Therefore, by shifting consumption, the end user will experience energy costs savings.

Luo, Kumar, Sottile, and Yingling [13], discuss a MILP formulation for load side demand control. Here the authors focus on the demand component of electricity costs instead of consumption costs. Their MILP considers loads that are on at time t then decides whether or not to shed those loads, when it needs to be shed, and

when operation of that load shall resume [13]. The MILP also looks at loads that are off and schedules when those loads are to resume again. The authors implement the model using constraints that set bounds on the minimum and maximum downtime for each of the loads, likewise for the up time. The authors also utilize three variables for each of the loads, one binary variable for off and on, another that tracks the time on, and lastly one that tracks the time off. By tracking the time off and time on, the authors can figure out the production of each of the loads over the time horizon. The model also utilizes a maximum demand constraint which enforces that the system demand must not exceed the max demand value. The authors apply their model to a coal mine case study where they model the electricity demand costs. Using demand control by shedding loads can reduce demand costs all while reducing any loss in productivity associated with load shedding [13]. Also, by preventing machines from constantly stopping and starting, the wear on the machines can be reduced [13].

Mohsenian-Rad, Wong, Jatskevich, and Schober [15] propose a model similar to that proposed by Hu, Chen, and Bak-Jensen. Their model proposes the shifting of energy consumption as in [12], however they propose that consumers will not change their consumption habits without incentive. They explain that most energy consumption in the United States occurs in buildings and “there are two general approaches for energy consumption management in buildings: *reducing consumption* and *shifting consumption*” [15]. In their paper, they propose the use of energy

consumption devices similar to those proposed in [10] and [11] where there is an electronic controller in each household and in all households in the neighborhood are connected to one local controller. The algorithm they propose solves the energy consumption schedule for each household in the neighborhood, then communicates it back to the local controller. This process continues until each household achieves its own maximum payoff, or reduction in energy costs. The authors explain that the energy consumption schedule changes by shifting soft appliances such as dishwashers, clothes driers, etc. [15]. By shifting these appliances, households are able to achieve the maximum payoff or cost savings. Hard appliances such as lighting, air conditioners, refrigerators, etc. are not allowed to be rescheduled in their model.

A model proposed by Middelberg, Zhang, and Xia [14] also utilizes the idea of load shifting to reduce energy consumption costs. The authors model a series of conveyor belts from a South African Colliery by using a binary integer programming method. Their objective is to reduce the operational electricity costs. In order to reduce electricity costs, the model shifts electricity demand from peak TOU periods to those periods that are less expensive or the off-peak periods [14] similar to [12] and [15]. By utilizing this approach, they show that with the South African case study, there was a 49% reduction in the cumulative energy costs during 5 weekdays in a high-demand season [14]. However, they also showed that the total energy that was consumed during peak TOU periods over the five days was reduced from 25%

to 8% compared to the non-optimal data from the case study.

Ashok and Banerjee [16], take the shifting principles proposed in [12] and [15] and applies that idea to an industrial setting. They propose a mixed integer linear program to reduce energy costs based on a case study for a typical flour mill. The authors use a myriad of constraints such as production, storage, process flow, sequential, maximum demand, downtime of machines, and electrical load to properly model a flour mill's production. The objective of the model is to reduce energy consumption costs while the constraints ensure that the flour production is optimized as well. They claim "the proposed model is capable of analyzing the industry response to different tariffs, operational strategies like two or three shift operation, variation of equipment size or storage capacity and adoption of new technologies" [16]. By implementing the model, the authors claim that in the case study the plant would experience an energy cost reduction of 29% by implementing their model. The cost savings is a result of spreading out the peak energy consumption times to take advantage of the part-peak and off peak electricity costs proposed by the time-of-use energy schedule they used for their case study.

Roos and Lane [17] propose a linear program that is applied to the industrial setting as in [16]. They propose that "the purpose of this paper is to add more insight into the electricity cost saving potential of real time pricing (RTP) through intelligent demand management" [17]. Instead of using a time-of-use schedule, the

electricity prices are variable for specific time periods throughout the day. Since the utility company provides the consumer with the pricing information beforehand, Roos and Lane propose an intelligent demand system similar to the EMC devices proposed in [10] and [11]. Through linear programming optimization, the authors propose a “load scheduling strategy that may result in minimum electricity costs to the end user” [17]. The objective is to reduce the electricity costs to the end user under real-time pricing electricity rates. Through intelligent demand management which describes the optimal load scheduling, an end user could experience substantial electricity costs savings.

Mohseninan-Rad and Leon Garcia [18] propose a model for residential consumers that also uses real-time-pricing (RTP) as discussed in [17]. The authors claim, “the lack of knowledge among users about how to respond to time-varying prices and the lack of effective home automation systems are two major barriers for fully utilizing the benefits of real-time pricing tariffs” [18]. Although the RTP schedule allows consumers to shift their higher energy demand appliances to times where energy rates are lower, these shifts are done manually by the consumer. The authors propose a model that will optimally schedule consumer appliances in order to minimize the consumer’s total electricity costs. The authors make the assumption that each residential consumer is equipped with a smart meter with an energy scheduling unit similar to the EMC devices discussed in [10], [11], and [17]. The authors devise

a linear program that reduces consumer electricity costs while also minimizing the time between when a device is called to turn on and when the linear program schedules the appliance to turn on. Since the electricity costs may not be known for the entire day to the end user, the authors propose an equation that predicts the electricity costs during the day based on past electricity cost data. By utilizing the schedule proposed by the linear program, an end user can experience reduced electricity costs.

A model for demand response using real-time-pricing was proposed by Conejo, Morales, and Baringo which applies to a household or small business. Similar to the pricing scheme in Denmark [12], the consumer knows beforehand the electricity rate they will receive for the following hour therefore, they can adjust their consumption pattern accordingly [19]. Using a linear program algorithm, the authors were able to provide electricity consumption results based on a typical 24 hour day for a household or small business. They claim that by implementing their linear program algorithm into an EMS as discussed by [10], [11], [17], and [18], consumers would be able to reduce their energy consumption costs by optimizing their energy consumption patterns.

In [15], the authors stated that most of the energy consumption of the United States occurs in buildings. A paper by Braun [20] explains that “the use of a building’s thermal storage for load shifting can significantly reduce operational costs,

even though the total zone loads may increase” [20]. As discussed in Section 1.2, the author proposes using efficient cooling systems that utilize part load operation. Also, the model he proposes takes advantage of the thermal mass of the building, i.e. how well insulated it is, as well as a time-of-use electricity rate structure. Braun proposes a mathematical program where the objective is to reduce both the energy and demand costs of cooling a building by precooling the building during pre-workday hours. Essentially, the model precools the building before it will be occupied by workers during the working hours, i.e. cooling the building to some temperature lower than the normal thermostat set point such as 72 degrees. Therefore, for some hours of the working day, the building will not need to be cooled since the program relies on the thermal properties of the building structure. Also, by precooling the building in the early morning hours, the air conditioning system does not work as hard since there is no one in the building and the outside ambient temperature will not put a large thermal load on the building. Therefore, the model shifts the demand and energy charges by shifting the load to the early morning hours to precool the building and to take advantage of the off peak energy rate. Hence, energy savings are realized by using efficient equipment utilizing part load operation as well as precooling the building during off-peak hours.

Our model will deal with commercial load scheduling similar to the papers proposed by the authors we’ve discussed. Our model will shift the electrical loads from

peak times to partial peak or low peak times similar to [12], [15], and [16], . However the models proposed by most of the authors in this review neglect to take into account the demand charges associated with running large electrical motors. In [14] they discuss reducing energy demand and energy consumption simultaneously similar to our model. The main difference is the application of the model proposed in [14] and the model we propose. The model for the commercial load scheduling problem is much more general and takes into account both the demand costs and consumption costs through the scope of a time-of-use schedule. As shown in many of the papers discussed previously, consumers can take advantage of the time-of-use schedule by shifting energy loads from peak periods to periods where electricity is cheaper. Also, similar to the model in [14], our model staggers starts preventing large energy demand charges which can be quite substantial at times. By taking into account both the energy consumption costs and energy demand costs, we provide an integer program that can greatly reduce energy costs for a commercial or industrial firm that uses large machines with electrical motors.

Chapter 3

Problem Definition

For the commercial load scheduling problem using a time-of-use schedule with electricity demand charges we are given a set of N machines that have energy consumption and demand parameters p_n and a_n , respectively. We assume that each machine n is independent of the others, i.e. one machine does not need another machine to be running in order to operate. The energy consumption parameter p_n will take on the value of the running load amps of machine n . As described earlier, we will transform the running load amp values into kilowatt hours. The a_n values, or energy demand, will use the locked rotor amperages of machine n . The values of a_n will be converted into kilowatts. We assume that the initial start up amperage a_n for machine n is not included as part of the machine's power consumption p_n . That is an appliance starts up instantly and its start up amperage is not observed as part

of the machine's energy consumption.

In our model we are using a time horizon with T discrete time periods, hence we define the rate schedule c_t which specifies the cost of electricity per kilowatt hour at time t . If the time periods are not hourly, we simply convert c_t to correspond to the time periods we specify, i.e. if the time periods are 15 minutes we will divide the values of c_t by 4. Since we are using a time of day rate schedule, the prices will fluctuate with different values of t . We also define the parameter P_{LOAD} which is the price per kilowatt of energy demanded. Hence, if we have an energy demand of 10 kilowatts, and P_{LOAD} is \$10.00 per kilowatt, our energy demand cost equals \$100.00.

We define our decision variable as x_{nt} where x_{nt} is a binary variable. The variable x_{nt} takes on the value 1 when machine n runs at time t and zero otherwise. Now we specify our maximum demand variable D_{LOAD} which is the maximum value of the sum of n machines specified to run at time t over all values of $t \in T$ or:

$$D_{LOAD} \equiv \max_{t \in T} \left(\sum_{n \in N} a_n \cdot x_{nt} \right)$$

Utility companies have different ways of calculating D_{LOAD} such as averaging the daily demand loads over a month's time to derive D_{LOAD} [3]. However, we assume that D_{LOAD} is the maximum over the time horizon T .

The parameter β_n specifies the desired number of periods of operation for each

machine n . β_n takes on values between 0 and T for all $n \in N$. However, in our model we allow β_n to be violated using a variable y_n where $0 \leq y_n \leq \beta_n$ for all $n \in N$. The variable y_n allows the model to choose if a machine can run less than the desired number of periods specified by β_n , but we assign an associated cost ζ_n for violating β_n . ζ_n can be thought of as the cost to the firm for not running machine n the desired number of periods as specified by β_n . We assume that the penalty cost ζ_n is a constant term, however it may be different for each n . We also assume that a machine n can run for consecutive time periods. In other words, the machine may start, then run for several time periods before turning off. Hence, we do not observe the starting amperage draw for each time period that the machine is on. Therefore, we define the binary variable ψ_{nt} as follows:

$$\psi_{nt} = \begin{cases} 1 & \text{if appliance } n \text{ is on at time } t \text{ and was on at time } t - 1; \\ 0 & \text{otherwise} \end{cases}$$

For example if machine n is running during time periods $t, t + 1, \dots, t + 5$, ψ_{nt} constrains the model to only count the starting amperage draw during time period t instead of during each time period $t, t + 1, \dots, t + 5$.

We specify the objective function as follows:

$$C = D_{LOAD}P_{LOAD} + \sum_{t \in T} \sum_{n \in N} c_t p_n x_{nt} + \zeta_n \sum_{n \in N} y_n$$

Therefore the IP can be written as:

$$\min D_{LOAD} P_{LOAD} + \sum_{t \in T} \sum_{n \in N} c_t p_n x_{nt} + \zeta_n \sum_{n \in N} y_n \quad (3.1)$$

$$s.t. \quad D_{LOAD} \geq \sum_{n \in N} a_n \cdot x_{nt} \quad \forall n \in N, t = 1 \quad (3.2)$$

$$D_{LOAD} \geq \sum_{n \in N} a_n \cdot (x_{nt} - \psi_{nt}) \quad \forall t \in \{2 \dots T\} \quad (3.3)$$

$$0 \leq y_n \leq \beta_n \quad \forall n \in N \quad (3.4)$$

$$\sum_{t \in T} x_{nt} = \beta_n - y_n \quad \forall n \in N \quad (3.5)$$

$$\psi_{nt} \leq x_{nt} \quad \forall n \in N, \forall t \in \{2 \dots T\} \quad (3.6)$$

$$\psi_{nt} \leq x_{n,t-1} \quad \forall n \in N, \forall t \in \{2 \dots T\} \quad (3.7)$$

$$\psi_{nt} \geq x_{nt} + x_{n,t-1} - 1 \quad \forall n \in N, \forall t \in \{2 \dots T\} \quad (3.8)$$

$$x_{nt} \in \{0, 1\} \quad \forall n \in N, \forall t \in T \quad (3.9)$$

$$\psi_{nt} \in \{0, 1\} \quad \forall n \in N, \forall t \in T \quad (3.10)$$

(3.1) is the sum of the demand cost, consumption cost, and the penalty cost for all machines $n \in N$ over the time horizon T . The consumption costs are only accounted for when x_{nt} is equal to 1 or machine n runs at time t . Otherwise, x_{nt} will equal zero and the consumption charge for machine n at time t will be zero. Combined, (3.2) and (3.3) is the maximum demand observed over the time horizon

T . Note with (3.3) that the right hand term utilizes the variable ψ_{nt} . As stated previously, ψ_{nt} equals 1 if machine n is running at time t and was also running at time $t - 1$. This forces (3.3) to not account for starting amperage draws, or locked rotor amperages, if a machine is running during consecutive time periods. Lastly, we have (3.5) enforcing the amount of periods a machine n will run over the time horizon. Again, we allow (3.5) to be violated using the variable y_n which can take on any value between β_n and 0. This constraint allows the program to decide if we can decrease a machines desired periods of operation in order to reduce the total costs. However, we introduce the penalty term ζ_n which may represent lost productivity as a result of not running a machine the specified number of periods β_n .

Chapter 4

Numerical Results

4.1 Experimental Design

First we run the model using randomly generated data for the terms p_n , a_n , β_n and ζ_n . The uniform distribution is used to randomly generate these terms. Then T is set to 24 which would correspond to 60 minute time intervals if the time horizon was a single day and the demand charge $P_{LOAD} = 10.88$ per kW which is specified by the A-10 schedule [2]. Then we use the values of c_n in Table 4.1 for the consumption costs.

Note however that we put bounds on the terms p_n , a_n , β_n and ζ_n as described in the Table 4.2.

We set the values of a_n to be randomly distributed over the interval $[1, 20]$ which

Time Period		Cost
$t = 1 \dots 9$	Off-Peak	0.12687
$t = 10 \dots 12$	Part Peak	0.14301
$t = 13 \dots 19$	Peak	0.15257
$t = 20 \dots 24$	Off-Peak	0.12687

Table 4.1: Actual Time-of-Use Costs [2]

Paramter	Distribution
p_n	Uniform(1, 20)
a_n	Uniform($p_n + 1$, 130)
β_n	Uniform(1, 24)
ζ_n	Uniform(a_n , 200)

Table 4.2: Variable Distributions

corresponds to small to medium machine rated load amp draws. To insure that the a_n values were not less than the p_n values, we set a_n to be randomly generated within the interval $[p_n + 1, 2(p_n)]$. This is true since the locked rotor amperages are never less than the rated load amperages of a machine. Thirdly, we set the values of ζ_n to be randomly distributed over the interval $[a_n, 200]$. The interval ensures that we will not get a solution where the model decides that it's less expensive to not run any of the machines. Lastly we set the values of β_n to be randomly distributed over the interval $[1, 24]$ rounded to the next integer.

N	LRA (Amp)	RLA (Amp)	ζ_n (\$)	β_n
1	16.301	102.62	132.14	16
2	84.5712	12.8262	190.734	23
3	55.4378	16.1431	194.013	17
4	81.011	16.1431	113.378	8
5	84.1908	8.11649	132.704	6
6	72.0503	16.4512	170.089	4
7	110.196	3.34176	119.898	10
8	49.6543	8.48442	165.985	16
9	117.576	16.8367	149.52	5
10	45.8443	18.0657	48.1491	23

Table 4.3: List of Parameters

4.2 Representative Instance

Using a 2.4 GHz Intel Xeon machine with 4000 MB of memory, we use the solver CPLEX 12.2 in AMPL to solve the model. We test the results of the Integer Program solution against a result randomly generated in Excel. In order to obtain the Excel solution, we use the Bernoulli distribution with $p = \frac{\beta_n}{24}$. That is, for each machine n , we derive a set of random Bernoulli variables. We chose to use $p = \frac{\beta_n}{24}$ for all n since we can expect that we will get β_n desired periods of operation for each n . Running the model in AMPL we derive Table 4.2.

After running the solver in AMPL we find it took 220.95 seconds for the solver to derive the optimal solution. Table 4.4 gives the result of the optimal solution as well as the solution that was derived in Excel. The table consists of cost values as well as the percentage of the total cost.

Solution	Total Cost	Demand	Consumption	Penalty
Excel	\$2863.56	\$2,171.26 %75.82	\$117.39 %4.1	\$574.19 %20.05
Optimal	\$678.36	\$588.44 %86.74	\$89.92 %13.26	\$0.00 %0
% Difference	%76.32	%72.90	%23.40	%100

Table 4.4: Description of Costs

From Table 4.4 we see that the largest cost component for both the randomly generated solution and optimized results is the demand costs. These are the costs that are associated with D_{LOAD} which reflects the maximum energy demand realized over time. Figure 4.1 shows the optimal and randomly generated demand costs over time.

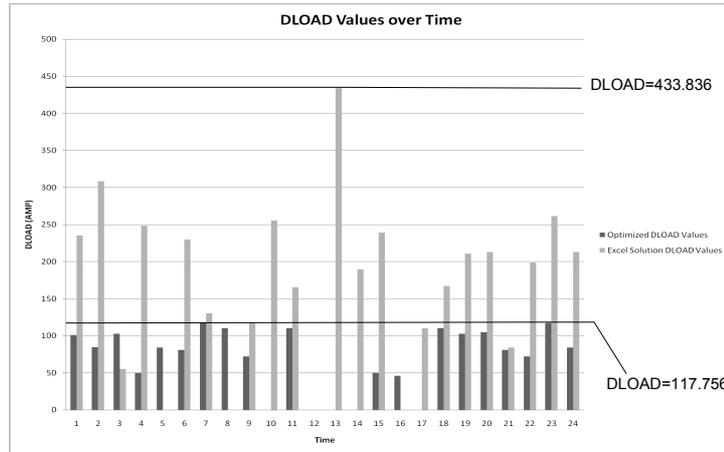


Figure 4.1: Excel and Optimal Solution DLOAD Values

From Figure 4.1 one can see that the demand, in amperages, is much greater in the randomly generated values than that of the optimal values. The drastic

difference in energy demand is directly related to the amount of times each machine n starts. In Figure 4.2 we show the amount of starts each machine n exhibits.

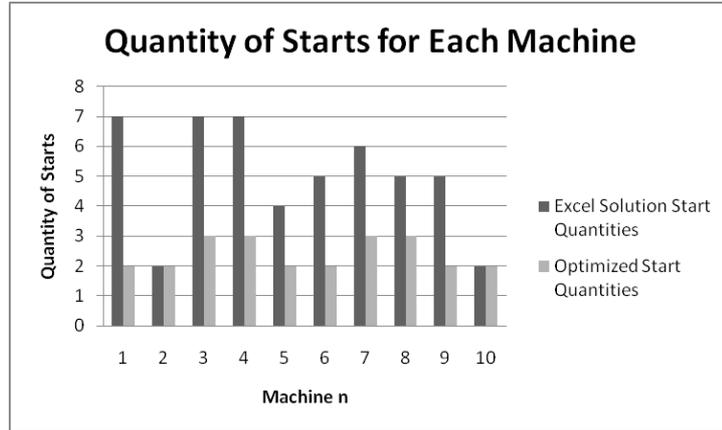


Figure 4.2: Excel Solution and Optimized Quantity of Machine Starts

As one can see, the randomly generated start quantities are greater than or equal to the optimized number of starts for each machine n . This translates to an increase in the amount of energy demanded since every time a machine starts demand is realized. Since the randomly generated start quantities are so large, the simulated D_{LOAD} values are much greater than that of the optimized values.

Another interesting component of the total cost is the consumption cost. Table 4.4 shows that the optimized consumption cost is less than that of the randomly generated consumption cost. In Figure 4.3 we show the consumption costs of both the optimized solution and the randomly generated cost per hour.

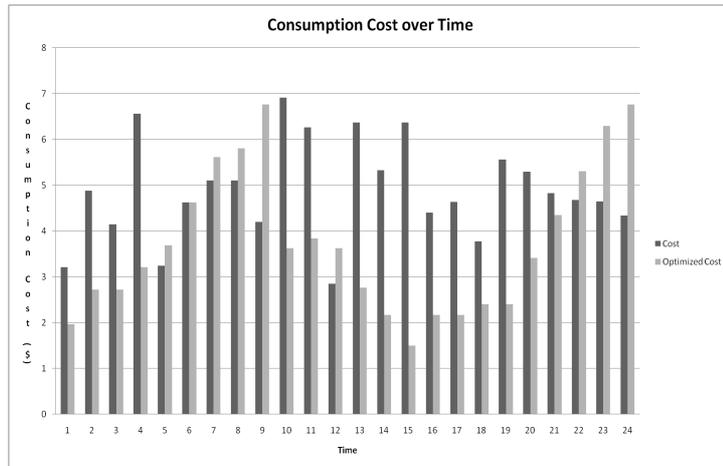


Figure 4.3: Randomly Generated and Optimized Consumption Costs

From Figure 4.3 one can see that the optimized costs are higher during the non-peak hours and are much smaller during the peak hours between periods 13 and 19. Also note that although the IP costs are optimized, there are times in which the optimized consumption costs are greater than the randomly generated consumption costs. However, from Table 4.4 we see that consumption cost is a relatively small component of the total cost compared to the demand costs that the two solutions exhibit. Therefore, although the consumption costs for the Excel solution may be lower than the optimal solution in some instances, the lower randomly generated consumption costs do not largely affect the total cost of the Excel solution.

Solution	Avg. Total Cost	Avg. Demand	Avg. Consumption	Avg. Penalty
Excel	\$3,151.67	\$2,054.78	\$72.31	\$1,024.57
Optimal	\$691.83	\$612.90	\$75.62	\$3.31

Table 4.5: Description of Average Costs

4.3 Extended Numerical Study

Now we generate 110 data sets and solve each in AMPL and compare the results to a randomly generated solution in Excel using the methods described previously. In Table 4.5 we provide a chart illustrating the average total cost, demand, consumption, and penalty costs for the random and optimal solutions.

From Figure 4.2 we see that the average total cost for the randomly generated solutions are much greater than that of the optimized solutions. The large difference in the average total costs is reflected in the large demand and penalty costs that the randomly generated solutions exhibit. In Figure 4.4 we provide a comparison of the average penalty, consumption, and demand costs as a percentage of the average total costs.

From Figure 4.4 we see that largest cost components of the average total cost for the optimized and randomly generated solutions are the demand cost. In fact, we see that the second largest cost components for the randomly generated solutions are the average penalty costs while average penalty costs for the optimal solutions are minimal. If we were to adjust the average total costs so that it doesn't include

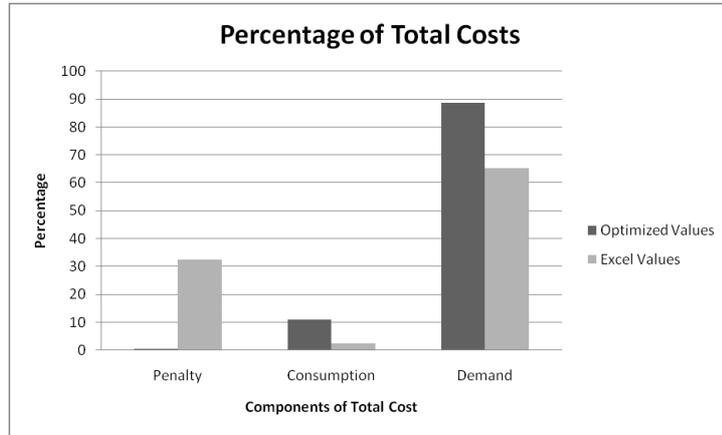


Figure 4.4: Percentages of Average Total Cost

penalty costs in the Excel solutions, the adjusted average total cost is still far greater than that of the optimal solutions due to the large demand costs. Interestingly, the average consumption costs for the randomly generated solutions are less than those of the optimal solutions. However, we conjecture that the average consumption costs are less for the Excel solutions due to the randomness of the Excel solution. Therefore, machines were not run as long as β_n specified. By using $p = \frac{\beta_n}{24}$ in the Excel solutions we can only expect that each machine will run as long as β_n specifies. By not running machine n the specified number of periods, there is a reduction in the consumption costs but an increase in the penalty costs which is reflected in Table 4.5.

The reason for the average demand charges being so large for the randomly generated solutions is due again to the number of times each machine starts over

time. For the 110 instances, on average each machine started 4.35 times in the Excel solutions with a minimum of 2.9 starts and a maximum of 6.4 starts. Compared to the optimal solutions, on average each machine started 2.12 times with a minimum of 1.6 starts and a maximum of 2.9 starts. On average, machines in the Excel solutions started and stopped more frequently, leading to a large average energy demand cost. Also, we know that machines on average were not run the specified number of periods due to the large average penalty costs exhibited in Table 4.5. Hence, although the machines were not run the specified number of periods, the increased number of starts leads to a large demand cost. For the optimal solutions, a reduction in the number of starts and an increase in running times led to a much smaller demand cost. We know that the machines in the optimal solutions ran longer from the very small average penalty cost exhibited in Table 4.5 and the small number of starts. Hence, machines ran the specified number of hours as specified by β_n for all n instead of violating the β_n constraint. Therefore, by reducing the number of starts and running the amount of hours specified by β_n , the optimal solutions exhibit smaller demand costs than that of the randomly generated solutions.

On average, the MILP took 197.8297 seconds to solve over the 110 instances. The program exhibited a minimum run time of 1.95 seconds and a maximum run time of 2,380.85 seconds. In Figure 4.5, we show a histogram with the solve time frequencies.

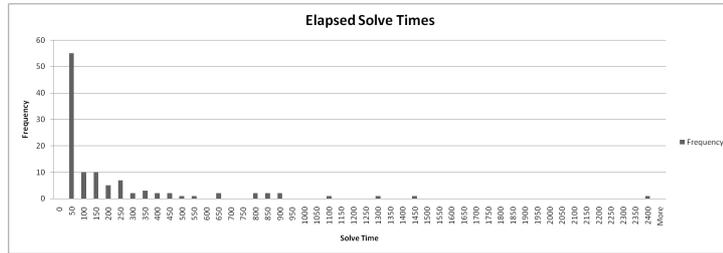


Figure 4.5: Frequency of Solve Times for 110 Instances

From Figure 4.5 we see that 55 of the solve times were in the 0 to 50 second range encompassing 50% of the iterations. In fact 96.4% of the solve times were between 0 and 1000 seconds and 88.2% of the solve times were between 0 and 500 seconds. From the data it appears long run times resulted when the values of a_n and p_n were in a small range for all n .

4.4 Refrigeration Example

In this section we use the model to determine the run schedule of refrigeration systems in order to minimize the electricity costs. We derive the electrical information from several systems using the literature from Witt Refrigeration a HTPG USA LLC company ([4], [5], [6], and [7]). Assuming that the equipment is all 460 volt, we can use the rated load amperage and locked rotor amperage values from the refrigeration literature to derive the corresponding electricity demand and consumption. Table 4.6 gives the system numbers as well as the corresponding amperage values

System	RLA Value	LRA Value	RLA Conversion	LRA Conversion
1	5.8	41	2.668	18.86
2	10.4	60	4.784	27.6
3	10.2	60	4.692	27.6
4	10.2	60	4.692	27.6
5	14.1	85	6.486	39.1
6	16.1	83	7.406	38.18
7	8.1	52	3.726	23.92
8	10.2	60	4.692	27.6
9	3.4	23	1.564	10.58
10	2.1	15	0.966	6.9

Table 4.6: Amperage to Kilowatt Conversions for Refrigeration Systems

and amperage to kilowatt conversions.

For the refrigeration example, we set $T = 96$, or each time period is equal to 15 minutes. We use the original model, however we add an additional constraint that will enforce that each machine n must run at least 30 minutes per hour. We call this parameter λ which specifies how many periods a machine must run in a certain time interval. Then we add an additional parameter θ which will be the number of periods per hour. Hence, in this example, $\lambda = 2$ and $\theta = 4$. The additional constraint for all $t \in T$ and $n \in N$ is written as follows:

$$\sum_{i=t}^{t+\theta-1} x_{ni} \geq \lambda$$

The additional constraint is consistent with how actual refrigeration systems work since it is impractical to allow a refrigeration system to lie idle for a long time

Solution	Total Cost	Demand	Consumption	Penalty
Excel	\$3,390.96	\$2,322.23	\$96.71	\$972.02
Optimal	\$997.38	\$905.869	\$91.49	\$0.00

Table 4.7: Description of Costs for Refrigeration Example

period since it will create a large rise in temperature in the cooler or freezer. Also in the refrigeration industry, it is assumed that low temperature systems run for 18 hours per day and medium and high temperature systems run for 16 hours per day. In our data, systems 1, 2, 3, 7, and 8 are medium temperature systems and systems 2, 5, 6, 9 and 10 are low temperature. So we set the values of $\beta_n = 64$ for systems 1, 3, 4, 7, and 8 then set the values of $\beta_n = 72$ for systems 2, 5, 6, 9, and 10.

After running the model in AMPL we find it took the solver 17,946.7 seconds to derive the optimal solution. Table 4.7 provides the results to the optimal solution as well as the results derived in Excel.

From Table 4.7 we see again that the major cost contributor to the total cost is the demand cost for both the optimal solution and the randomly generated solution. The D_{LOAD} value for the Excel solution is 464 amps compared to 181 amps for the optimized solution. Figure 4.6 shows the energy demands for both the optimal and randomly generated solution over time.

In Figure 4.6 we see that there are large energy demand peaks with the Excel values especially at $t = 1$ and $t = 96$. On the other hand, the energy demand values

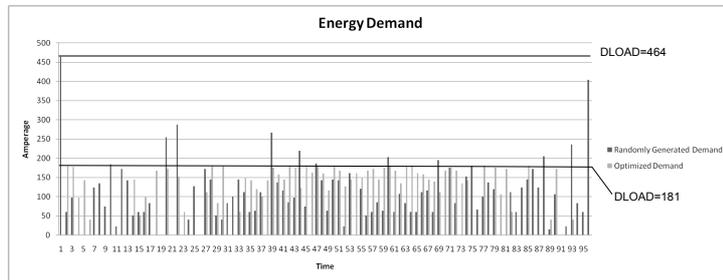


Figure 4.6: Randomly Generated and Optimal Energy Demand

for the optimal solutions are steady throughout the time period with some points where there is no energy demand i.e. no systems started at those time periods. In Figure 4.7 we look at the number of starts each system exhibits.

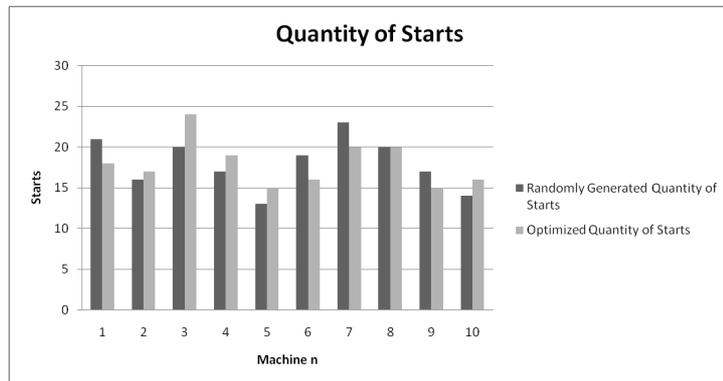


Figure 4.7: Quantity of Starts for Each Machine

Compared to previous figures of machine starts, in this example, there are no large deviations between the optimal and randomly generated number of starts. The small deviation in total start quantities for each machine is a result of the additional

constraint we imposed in this example. We can also look at the average run time for each machine.

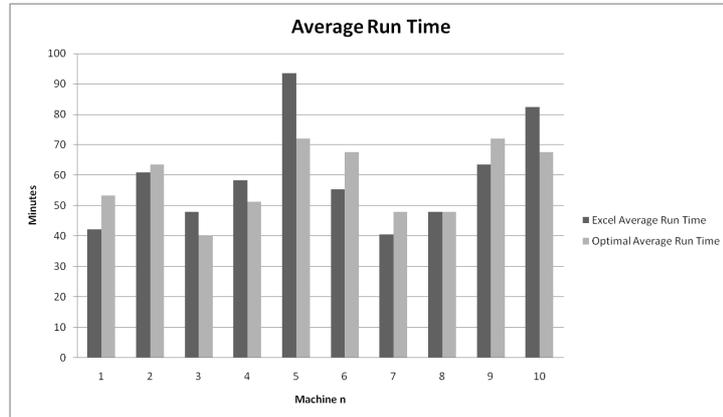


Figure 4.8: Average Run Times for Each Machine

In Figure 4.8 we see that the average run times are similar for the optimal solution and randomly generated solution. The reason for the large difference with system 5 is due to the Excel solution running system 5 for 81 time periods or 1,215 minutes compared to the 72 time periods for the optimal solution. The averages for the optimal run times are again due to the additional constraint imposed. Each system will run for a long time, 5-20 time periods, during the off peak hours then during part and off peak hours each machine will run 1 to 2 time periods at a time in order to satisfy the additional constraint. Therefore, the additional constraint causes the reduced average run times for the optimal solution. Now we investigate the consumption cost of the solutions over the time period in Figure 4.9.

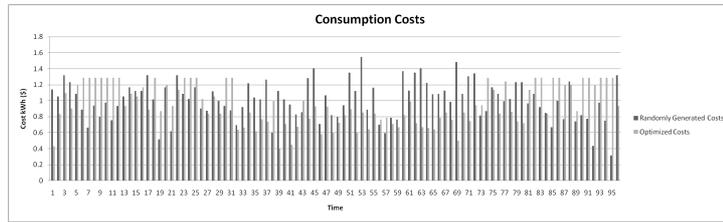


Figure 4.9: Randomly Generated and Optimal Consumption Costs

For the consumption costs we see that the optimal values are larger especially between time periods $t = 1 \dots 31$ and $t = 73 \dots 95$. The reason that the optimal consumption costs are higher than those of the randomly generated consumption costs is that the time periods $t = 1 \dots 39$ and $t = 80 \dots 96$ are off-peak costs so it would make sense to run more machines during these time periods. Also, we see that the randomly generated consumption values are greater than that of the optimal during the peak hours where the cost is the highest. Although the Excel solution has higher consumption costs during the peak hours, the total consumption costs is close to the total consumption cost for the optimal solution as we see in Table 4.7. The close consumption costs may be due to the fact that the Excel solution does not run each system for the amount of time as specified by β_n which is reflected in the penalty cost. Another reason that the consumption costs are more for the randomly generated version in this example is that the Excel solution runs some systems much longer than specified by β_n . For example system 5 ran for 81 time periods and system 10 ran for 77 time periods when β_n specified a total run time of

72 for each of those systems.

We see in this example that the total cost is affected mostly by the costs associated with D_{LOAD} . Upon investigating the average run times of each system and the number of starts for each system, we find that there is no large difference between the randomly generated solution and optimal solution. We find that the reason there is no large difference in Figures 4.7 and 4.8 is due to the additional constraint imposed. However, the IP schedules the starts of each system n in order to minimize D_{LOAD} while the Excel solution is randomized using Bernoulli random numbers. Therefore, due to the randomness of the Excel solution the D_{LOAD} value is very large compared to the optimal D_{LOAD} value. This example stresses the importance of staggering startups of refrigeration systems in order to reduce the demand costs associated with starting them. Using the methods described in the refrigeration introduction, such as cylinder unloading and variable speed compressors, we could run the systems 24 hours a day which would reduce the D_{LOAD} cost, but would inflate the consumption costs associated with running those systems.

Chapter 5

Conclusion

In this thesis we proposed an IP that reduces the energy and energy demand costs associated with running machines a specified number of hours. We have shown that by using the Integer Program, we reduce the energy demand realized over the time period which significantly reduces the total costs. By shifting the starts of each machine and letting them run for periods of time instead of shutting them on and off, the IP greatly reduced the total costs compared to the randomly generated values. By scheduling equipment to run according to the IP, a firm could expect significant cost savings compared to conventional control methods without a loss in productivity. Unfortunately, our model does not take into account external factors such as thermal loads and unexpected loads that could be seen in real world applications. For example, the refrigeration example does not take into account any of

the thermal properties of cooling a refrigerated structure.

Some possible extensions to the IP we proposed would be to add additional constraints to account for thermal loads of air conditioning and refrigeration equipment. Another extension would be to make the machines dependent upon each other such as seen in large industrial applications. There are other possible extensions that we did not take into account in our mathematical program which could be explored in future research such as applications to different industries. Also, other rate structures could be explored, especially for how the demand rate is structured. Although we did not take into account these possible extensions, we have confirmed through randomly generated solutions and optimization, that the IP reduces energy costs greatly by reducing the energy demand costs associated with running large machines without a significant loss in productivity.

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Vita

Benjamin A. Mizack, the son of Virginia and Joseph Barry Mizack of Bethlehem, Pennsylvania, was born on November 16, 1985 in Easton, Pennsylvania. He attended Notre Dame High School in Easton, Pennsylvania where he graduated in the top 15% of his class in 2004. In high school, he excelled at academics especially mathematics as well as athletics. He was a member of the National Honors Society, SADD, and the Spanish Honors Society and a varsity football player and track athlete. He attended Moravian College in 2004 where he double majored in mathematics and economics. In 2008 he graduated with a Bachelors of Science degree in Mathematics with a focus in Statistics. In the Fall of 2008, he began pursuing a Masters of Science degree in Applied Mathematics at Lehigh University. He transferred to the Industrial & Systems Engineering department at Lehigh University during the Fall of 2009 where he began pursuing his Masters of Science degree in Industrial Engineering. He currently works at Refrigeration Specialists Company NE LLC as a senior applications and specifying engineer.