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Alexis Ostapenko

Dongho Yoo

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TRIPPING OF ASYMMETRICAL STIFFENERS UNDER COMBINED LOADING

Final Report

by

Alexis Ostapenko
Dongho Yoo

Fritz Engineering Laboratory Report No. 513.3

August 1988

U. S. DEPARTMENT OF TRANSPORTATION
Maritime Administration
Office of Research and Development
TRIPPING OF ASYMMETRICAL STIFFENERS
UNDER COMBINED LOADING

Prepared by

Alexis Ostapenko
Dongho Yoo

LEHIGH UNIVERSITY
Fritz Engineering Laboratory
Bethlehem, PA

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TRIPPING OF ASYMMETRICAL STIFFENERS UNDER COMBINED LOADING

Alexis Ostapenko and Dongsu Yoo

Lehigh University
Department of Civil Engineering
Fritz Engineering Laboratory #913
Bethlehem, PA 18015

U.S. Department of Transportation
Maritime Administration
Office of Research and Development
Washington, D.C. 20590

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Presented is a study of the tripping strength of asymmetrical longitudinal plate stiffeners subjected to a combination of axial and lateral loads. The loaded edges were taken to be simply supported. The method of analysis is based on the principle of minimum total potential, and ten displacement functions were used to describe the deformations of the plate and stiffeners. First yielding was used as the criterion of ultimate strength. Instability under axial loading was investigated for the symmetrical (Tee) and asymmetrical (Angle) stiffeners. Angle stiffeners showed greater capacity than Tee stiffeners for lower values of the slenderness ratio (L/r), but lower capacity for higher values of the slenderness ratio, especially, after consideration of the deformations of the stiffener web. Under combined axial and lateral loads, there was a significant decrease in the capacity for asymmetrical (Angle) stiffeners due to the distortion of the cross section as compared to the undeformed or symmetrical sections. A modified effective width concept was introduced into the analysis to consider the effect of postbuckling plate deformations under combined loading condition. The effect of initial imperfections of stiffeners was also included. A relationship for the interaction between the axial and lateral loads was established for the ultimate condition. It shows that the ultimate strength can actually increase under axial loading when the lateral loading is applied in the direction from plate to stiffener. When the lateral loading is applied in the other direction, the interaction is almost linear. The computer program developed during the study was the main tool to provide the needed numerical results.

Analysis

Buckling
Torsion
Longitudinal Stability
Tortional Stability

Tripping
Ultimate Strength

Research
Structural Components
Plates
Stiffeners

Research Structural Components Plates Stiffeners

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ABSTRACT

Presented is a study of the tripping strength of asymmetrical longitudinal plate stiffeners subjected to a combination of axial and lateral loads. The loaded edges were taken to be simply supported. The method of analysis is based on the principle of minimum total potential, and ten displacement functions were used to describe the deformations of the plate and stiffeners. First yielding was used as the criterion of ultimate strength.

Instability under axial loading was investigated for the symmetrical (Tee) and asymmetrical (Angle) stiffeners. Angle stiffeners showed greater capacity than Tee stiffeners for lower values of the slenderness ratio (L/r), but lower capacity for higher values of the slenderness ratio, especially, after consideration of the deformations of the stiffener web.

Under combined axial and lateral loads, there was a significant decrease in the capacity for asymmetrical (Angle) stiffeners due to the distortion of the cross section as compared to the undeformed or symmetrical sections.

A modified effective width concept was introduced into the analysis to consider the effect of postbuckling plate deformations under combined loading condition. The effect of initial imperfections of stiffeners was also included.

A relationship for the interaction between the axial and lateral loads was established for the ultimate condition. It shows that the ultimate strength can actually increase under axial loading when the lateral loading is applied in the direction from plate to stiffener. When the lateral loading is applied in the other direction, the interaction is almost linear. The computer program developed during the study was the main tool to provide the needed numerical results.
1. INTRODUCTION

1.1 Introduction

Longitudinally stiffened plates are frequently used in many types of structures, such as, ship hulls, grillages, box girders, and offshore structures.

Figure 1 shows the typical sections of longitudinal stiffeners that have been used: tee-section (Tee), flat bar (Flat), bulb-flat (Bulb), angle-section (Angle), zee-section (Zee), etc. They are given in the groups of symmetrical and asymmetrical stiffeners, and the shape designation is shown for each cross section.

Some classical approaches have given solutions of torsional-flexural buckling for these stiffeners based on the assumption of undeformed stiffener cross section. [2, 4, 5, 6, 7, 19] These solutions show a higher strength for asymmetrical section stiffener (Angle) than for symmetrical section stiffener (Tee) when the stiffener flange sections for both have the same dimensions. However, there were also a few studies that have provided more rigorous solutions by considering, for example, the distortion of the stiffener cross section. These studies indicate that the classical approaches may considerably overestimate the strength of stiffeners, especially, the asymmetrical stiffeners. [1, 20]

The buckling modes of a stiffened plate under axial loading are the following:

a) Plate buckling mode
b) Tripping (torsional mode)
c) Overall buckling mode (column-like behavior)

These modes may be coupled with each other to give the critical strength which is generally lower than the critical strength for any individual mode. [8]
The maximum strength of a stiffened plate may be also controlled by yielding. This would be particularly valid for panels under lateral loading or when there are initial imperfections. Especially for asymmetrical stiffeners, the effect of section distortion becomes very important in computing the maximum stress when the section is checked for initiation of yielding.

1.2 General Description of the Problem

The behavior and capacity of stiffened plates depend on the type of loading, geometrical properties, type of section, aspect ratio and width-thickness ratio of the plate, initial imperfections, material properties, etc.

The two important representatives of the symmetrical and asymmetrical stiffeners shown in Fig. 1 are the Tee and Angle sections. Thus, all quantitative discussions in this study will be made with respect to them. Since many geometrical properties of asymmetrical (Angle) stiffeners and symmetrical (Tee) stiffeners are different from each other, all direct comparisons between Angle and Tee stiffeners will be made after assuming that the stiffener flange width and thickness are exactly the same for both as indicated in Fig. 2.

The most significant difference between the Angle and Tee stiffener sections is in the value of the warping constant. The reason is that the reference point for the rotation of these stiffener sections is approximately at the junction of plate and stiffener, not at the shear center of the stiffener section alone. This results in the Angle section to have a significantly larger value of the warping constant than the Tee section. Furthermore, the reference point of flange rotation is also different from each other. For, the Angle section, the flange has the reference point approximately at the web tip, while the Tee section has it at the center of the flange. This is especially important when the distortion of the cross section is included in the analysis.
In consequence, a classical torsional buckling analysis which assumes the cross section to be undeformable, gives a higher strength for an asymmetrical section (Angle) than for a symmetrical section (Tee). And this is reflected in the current design recommendations. [6, 7, 11] However, some studies, such as, Van der Neut’s, which consider distortion of the cross section, point out the opposite to be true, especially for the column mode of failure. [20] However, because of the non-recognition or the complexity of the problem, no specific design recommendations have yet been made for asymmetrical stiffeners.

In addition to the torsional buckling behavior (the tripping mode), the plate buckling behavior should be carefully taken into consideration, especially, when the width-thickness ratio is relatively high. In the most studies done previously, the effect of the plate on the stiffener motion was included simply by replacing the plate with a rotational spring. (Table 1, Line 8) Thus, the major concern was put on the stiffener behavior alone. However, this may be only a crude approximation. When the stiffened plate buckles, there are several buckling mode shapes that should be checked. [3, 19] The two most significant mode shapes for plate buckling are the symmetrical and antisymmetrical since they define the motion of two consecutive stiffeners as indicated in Fig. 6; the symmetrical mode means that the two stiffeners have opposite directions of motion (Fig. 6. a) and the antisymmetrical means that they have the same directions (Fig. 6. b).

So far, two buckling modes, the torsional buckling mode for the stiffener and the flexural buckling mode for the plate, have been discussed. In addition to these, the overall buckling mode of the whole structure should be considered as another possibility. (It should be obvious that this mode becomes prevalent for larger values of the slenderness ratio of the whole structure.) In this mode, an asymmetrical section
would behave quite differently from a symmetrical section. One can easily see that the buckling strength of a Tee section will be given by the Euler buckling strength since the Tee section is symmetrical about its principal minor axis and there is no preference for the direction in which the section might roll over. The Angle section, on the other hand, would introduce additional deformation components into the buckling mode, such as, the sidesway bending of the flange and the consequent distortion and twisting. Thus, one can expect that the Angle section would have its buckling strength lower than the Euler column strength.

Buckling behavior of the plate alone depends on the restraint at side edges provided by the stiffeners. Depending on the rigidity of the stiffeners, the restraint may be anywhere within the range from simple support to fixed edge condition. It is also possible that the torsional buckling of the stiffener will occur first and then force the plate to buckle prematurely.

There is a need to obtain a solution which would consider complete interaction of the effects of distortion of the stiffener cross section, of the buckling and postbuckling behavior of the plate, and of the inelastic range.
2. LITERATURE REVIEW

Very few of the papers describing the behavior of longitudinal stiffeners discuss the effect of the distortion of stiffener cross section, especially for asymmetrical sections, such as angle sections. The principal references dealing with asymmetrical stiffeners and stiffener distortion are reviewed here.

Table 1 shows the references studied in this investigation and the particular topics covered by each reference, such as, the types of stiffener cross section (Tee, Bulb, Flat-Bulb, Angle, or Zee) and loading condition (axial P, lateral Q, or bending M). The method of analysis used and whether or not tests were performed or analyzed are also stated.

The controlling buckling mode of a given structure depends on the slenderness ratio of the cross section (for the column buckling mode), the aspect and width-thickness ratios of the plate (for the plate buckling mode), and the cross-sectional properties of the stiffener (for the stiffener tripping mode). Unfortunately, no study is available which considered the interaction of all these modes for asymmetrical stiffeners.

Van der Neut studied the column mode with Zee stiffeners. [20] The important contribution in his study was that pure Euler buckling mode cannot occur because lateral bending of the flange and the distortion of the web accompany the column buckling mode immediately from the start. The number of half waves of the deformation of the web was found to be the same as that of column buckling mode. Inclusion of the effect of deformability of the stiffener web resulted in a lower critical strength than the pure Euler buckling strength. Also, he indicated that the antisymmetrical mode of plate, in other words, the motion of two consecutive stiffeners by the same amount and in the same direction, is more critical than symmetrical buckling for this type of buckling mode.
Ostapenko, using a modification of the Van der Neut's approach, concluded that the angle stiffeners are stronger than the tee stiffeners in the range of small slenderness ratios (less than 20) because of the greater warping constant of the angle section but are weaker for larger slenderness ratios. [15]

Adamchak described the behavior of symmetrical stiffeners (Tee) and suggested a displacement function for the web plate in the form of a cantilever plate strip. However, his formulation has an inconsistency of the equilibrium conditions at the stiffener-plate junction. Adamchak assumed that the rotational spring constant of the plate (against the stiffener motion) interacts linearly with the axial inplane stress in the stiffeners. This approach means that, when the stress in the stiffener section reaches the plate buckling stress, the rotational constraint would disappear. [1]

Lehmann regarded the flange of an asymmetrical section as a beam on elastic foundation (foundation modulus due to the deformation of the web and plate) and as being subjected to a certain form of lateral loading. He was concerned mainly with the effective width of the flange to be used in the equivalent cross section. [14, 16]

Bijlaard dealt with various types of stiffeners (Angle, Flat, and Tee) under compressive load. He gave a classical torsional-buckling solution of the stiffened plate and also provided formulas for design approach based on his solution. Thus, neither plate buckling nor distortion of the cross section were considered. [4]

Tests on Angle section stiffeners were conducted by Horne with a particular emphasis on the effect of initial imperfections and residual stresses on the column strength. The width-thickness ratio of the plate was low to preclude plate buckling. [13]
The general conclusion from the study of the references in Table 1 is that there is a need to consider alternate modes of stiffener tripping (symmetrical and antisymmetrical), a more direct effect of the plate restraint than as a rotational spring, and an interaction of the various deformation modes. Consideration also should be given to the effects of cross-sectional distortion, initial imperfections and lateral loading.
3. SUMMARY OF CURRENT STUDY

Current study was planned to fill some needs described in the previous chapters, specifically, to formulate a method for analyzing the tripping (lateral-torsional) behavior of asymmetrical plate stiffeners subjected to axial or combined axial and lateral loading.

The Principle of Minimum Total Potential Energy and its extension, the Rayleigh-Ritz Method, were used as the basis for the method developed here. Reliability of this method depends on the displacement functions selected to duplicate real deformations as closely as possible. The displacement functions were selected for the overall deformation of stiffened plate, for the deformation of plate, and for the distortion of stiffener web. To accommodate the possibility of symmetrical and asymmetrical deformations of adjoining stiffeners, analysis was made on a typical two-stiffener unit with the tributary width of plate.

The two loading cases, the axial loading and the combined loading, were analyzed separately.

Axial loading was limited by the buckling strength. The strengths of asymmetrical (Angle) and symmetrical (Tee) stiffeners were compared and controlling modes of buckling defined. Main emphasis was put on the effect of distortion of the cross section.

Combined axial and lateral loading applied to an asymmetrical section caused continuous deformation rather than buckling. Using first yielding as the failure criterion, the interaction between the uniformly distributed lateral loading and the axial loading was studied. One of the complications was the need to locate the point of maximum stress in the cross section since, due to the distortion and sideways motion.
of the stiffener under combined loads, the stress distribution in the section would constantly change. [14, 17] (Fig. 11)

Postbuckling strength of the plate and the effects of initial imperfections were incorporated into the analysis. The effect of initial imperfections caused by the fabrication process was also included in the analysis.

The analytical formulation was then implemented into a computer program which can be used for parametric studies of the importance of the major controlling parameters.
4. METHOD OF ANALYSIS

The method of analysis in this study relies on the principle of minimum total potential energy. In this chapter, the method of applying this principle, as well as, the basic assumptions and concepts behind the application are explained.

4.1 Principle of Minimum Total Potential

Total potential energy $V$ is equal to the sum of the internal potential (Strain Energy $U$) and the external potential ($V_e$).

$$V = U + V_e$$  \hspace{1cm} (4.1)

The principle of minimum total potential states that the first variation of the total potential with respect to the displacement field must be zero when the system is in equilibrium. In other words,

$$\delta V = \frac{\partial V}{\partial w} \delta w = 0$$  \hspace{1cm} (4.2)

where $w$ is the deformation field and $\delta w$ is the variation of deformation.

The Rayleigh-Ritz method provides a practical engineering application of the principle of minimum total potential energy by approximating the unknown displacement field with a series.

$$w = \sum C_i w_i$$  \hspace{1cm} (4.3)

where each $w_i$ is a function of the independent variables and must satisfy all the geometrical boundary conditions, and $C_i$ are the unknown constants.

Substitution of $w$ from Eq. (4.3) makes the total potential a function of $C_i$. Then, the first variation of the total potential becomes a series of equations.

$$\delta V = \frac{\partial V}{\partial C_i} \delta C_i = 0$$  \hspace{1cm} (4.4)
Since \( \delta C_i \) are arbitrary, the set of the simultaneous equations for the unknown \( C_i \) is given by

\[
\frac{\partial V}{\partial C_i} = \frac{\partial U}{\partial C_i} + \frac{\partial V^x}{\partial C_i} = 0
\]  

(4.5)

Displacement functions \( w_i \) for each component and their contributions to the Total Potential are described in the subsequent sections. The equations for the whole system are summarized in Appendix A.

4.2 Overall Deformation of Stiffened Plate

The overall deformation is defined by the deflection of the stiffeners (stiffener - plate junction lines) between the end supports. Since, as indicated in some references, there is a possibility that two consecutive plate-stiffener junction lines may deflect by alternate magnitudes, [3, 19] the deformation pattern of the stiffened plate is assumed to be given by the following displacement function in terms of two constants, \( C_3 \) and \( C_{10} \): (See Fig. 6. c)

\[
w_{overall} = \sin \alpha m x \left( C_3 + \frac{C_{10} - C_3}{b} y \right)
\]

\[
= \sin \alpha m x \left[ C_3 \left( 1 - \frac{y}{b} \right) + C_{10} \frac{y}{b} \right]
\]  

(4.6)

where \( y = 0 \) to \( b \), and \( \alpha_m = m \pi x / a \) (\( a/m = \) half-wave length along the stiffener-plate junction line)

The shape of overall deformation along the length is directly related to the column mode failure.
4.3 Plate Deformations

As shown in Fig. 6, several buckling modes of stiffened plates are possible. [2, 3] The likelihood of a specific mode to dominate is dependent on the aspect ratio, slenderness ratio, and other geometrical and material properties.

Generally, the critical mode occurs when the plate has one half-wave in the transverse direction between the edge boundaries (symmetrical mode), but some references indicate that the antisymmetrical mode may be more critical for plates with angle type stiffeners. [2, 3, 20] For example, Van der Neut took the antisymmetrical mode into account in his analysis of the overall column buckling.

In the formulation of total potential in the current study, both modes are considered concurrently, and the solution shows which one is dominant for a particular section.

Keeping these considerations in mind, the displacement function for each mode was assumed as follows.

- For symmetrical mode (Fig. 6. a)

\[
\mathbf{w}_{\text{plate}} = \sin \alpha_n x \left\{ C_4 \sin \frac{\pi}{\lambda} y + C_5 4 \left[ \frac{y}{h} - \left( \frac{y}{h} \right)^2 \right] \right\} \quad (4.7)
\]

Where the sinusoidal term in braces gives the general displacement shape and the second term gives the shape with constant curvature across the plate width. Constants \(C_4\) and \(C_5\) represent the amplitudes of the two displacement shapes at the mid-point, respectively.
For antisymmetrical mode (Fig. 6. b)

\[ w_{\text{plate}} = \sin \alpha_n \lambda \left\{ C_6 \sin \frac{2\pi}{b} y + C_7 64 \left[ \frac{1}{6} \left( \frac{y}{b} \right) - \frac{1}{2} \left( \frac{y}{b} \right)^2 + \frac{1}{3} \left( \frac{y}{b} \right)^3 \right] \right\} \]  \hspace{1cm} (4.8)

Where the sinusoidal term in braces gives the general displacement shape of this mode and the second term gives the shape with the linear variation of curvature (zero curvature at mid-point) across the plate width. Constants \( C_6 \) and \( C_7 \) represent the amplitudes of the two displacement shapes at the quarter point, respectively.

In addition to the above, the rotation of the plate edges is expressed in terms of the displacement parameters of the plate so that compatibility at the stiffener to plate junction is satisfied. That is,

\[ \theta = \sin \alpha_n \lambda \left[ C_4 \frac{\pi}{b} + C_5 \frac{4}{b} + C_6 \frac{2\pi}{b} + C_7 \frac{3\pi}{b} \right] \\
\hspace{1cm} + \sin \alpha_m \lambda \left[ -C_3 \frac{1}{b} + C_{10} \frac{1}{b} \right] \]  \hspace{1cm} (4.9)

where \( \alpha_n = n\pi x/a \) (a/n = half-wave length along the plate)

4.4 Stiffener Deformations

4.4.1 Deformation of Stiffener Web

For the purpose of making careful consideration of the stiffener web deformations, the stiffener web was considered as an individual plate. Two curvature configurations, longitudinal and transverse, were considered to contribute to the strain energy of web plate. However, to simplify computations, the longitudinal curvature contribution was neglected since its contribution does not exceed more than 5% of the total strain energy of the web plate. Consequently, the web plate was modeled as a series of transverse strips deformationally constrained along the flange and the stiffener-plate junction line.

The transverse web strips were assumed to deform as cantilevers fixed at one end
(flange end) and subjected to an end moment and a concentrated load at the other end (web-plate junction). This way, lateral displacement of the web across the depth of the web was defined in terms of the lateral displacement \( C_1 \) and the rotation \( C_2 \) at the junction to the flange with respect to its original position. (Fig. 4)

\[
v_w = \sin \alpha \frac{\pi x}{a} \left( C_1 \left( \frac{z}{d} \right)^2 - 2 \frac{z^3}{d^3} \right) + C_2 d \left[ -\frac{z}{d} \right]^2 + \left( \frac{z}{d} \right)^3 + \theta d \left[ -\frac{z}{d} \right] - 2 \frac{z^2}{d} + \frac{z^3}{d^3} \right]
\]

where \( \alpha = \frac{\pi}{a} \) \((a/l = \) half-wave length along the stiffener) and \( \theta \) is for the rotation of the stiffener base which is the same as the rotation of plate edges defined by Eq. (4.9).

### 4.4.2 Deflection of Stiffener Flange

During the distortion of the web, the stiffener flange resists motion by its torsional and flexural rigidity, in effect, by pulling the web back against the rotation and the sideways flexural deformation about \( z \)-axis. Behavior of the flange in stiffened plates is analogous to that of a column (or beam) on an elastic foundation with rotational and flexural constraints, especially, if the remaining portions of the structure are relatively stiff. [14, 16]

### 4.4.3 Deformation of Stiffener Section

The center of motion (rotation and side ways bending) of the stiffener section is approximately located at the junction of the plate and stiffener so that the geometrical properties are computed with respect to this point. It is important to note that the warping constant of an Angle stiffener about this point is significantly greater than of a Tee stiffener. [2, 12]

For compatibility, rotation of the stiffener base must be equal to the transverse slope of the edge of the plate. Thus, the stiffener motion is affected by the mode shape of plate buckling.
The angle of flange rotation is the same as the angle of web edge rotation (parameter $C_2$) and this is the angle which is used in computing torsional strain energy of the flange. However, torsional strain energy of the stiffener web is computed by using the angle of rotation at the web base ($\theta$) (the rotation at the plate edge) and the rest of strain energy in the stiffener is due to the flexural motion of the web strips defined by Eq. (4.10). (Fig. 4) Note that when web distortion is neglected, rotations of the flange and web base are the same.

4.5 Loading Conditions

4.5.1 Axial Loading

Under axial loading only, the problem is to find the minimum eigenvalue (buckling strength) of the system. In general, two buckling modes can be shown to control the instability.

- plate buckling mode (edges remains in the original position)
- column-like buckling mode (overall buckling mode, when the whole structure deflects, not just the plate)

The buckling mode of a given structure will depend on a number of parameters, such as, the width-thickness ratio of the plate ($b/t$), overall slenderness ratio ($a/r$), the type (Angle or Tee) and proportions of the stiffener section, the depth-breath ratio ($d/b$) of the stiffener web, and so on.

The external potential of axial loading is expressed by

$$V_p = -P\delta$$
$$= -\int_A \sigma \delta dA$$

(4.11)

where $A$ represents the cross-sectional area of structure, and $\delta$ is the relative axial displacement of the loaded cross section from the original position. This relative displace-
ment of the two ends of the structure at a fiber location \((x,y)\) is caused by curvature (neglecting axial strains).

\[
\delta(x,y) = \frac{1}{2} \int_0^a \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \, dz \tag{4.12}
\]

The external work for the whole system is presented in detail in Appendix A.

4.5.2 Lateral Loading

Lateral loading is taken as a uniform line load \(Q\) over the full length of the structure and applied at the junction line of the stiffener and plate. The positive direction of this loading is upward as shown in Fig. 5.

The external potential due to lateral loading for a two-stiffener unit is given by

\[
V_Q = -Q \int_0^a \left[ C_3 \sin \alpha z + C_{10} \sin \alpha z \right] \, dx \tag{4.13}
\]

where \(C_3\) and \(C_{10}\) represent the amplitudes of the displacement functions.

Analysis of asymmetrical sections (Angle) shows that, under lateral loading, additional stresses develop in the flange due to the lateral bending and rotation of the stiffener section. These stresses are not uniform as for Tee stiffeners; they vary linearly across the flange width. Accordingly, the maximum stress occurs at either of the flange edges or in the plate depending on the direction of lateral loading and geometrical proportions. The stress in the flange is then

\[
\sigma_Q = -\frac{Qa^2}{8} \frac{1}{I_y} z + Ev^* y \tag{4.14}
\]

with the compressive stress being positive. Note that the stress in the plate is given by the first term of Eq. (4.14) alone used with the corresponding value of \(z\).
4.5.3 Combined Loading

When the stiffened plate is subjected to a combination of axial and lateral loading, the stress distribution becomes more complex, in particular, the location of the maximum stress becomes more indefinite. As an illustration, the stiffener deformation under lateral loading, such as, sidesways bending of the flange section, will be affected if there is also axial loading. In this case, axial loading can have a beneficial effect for a flange section under positive lateral loading or accelerate the flange deformation under negative lateral loading. Both axial and lateral loading interact with each other to meet the First Yield Criterion at the location of the maximum stress.

\[ \sigma = \frac{P}{A} - \frac{P_e}{I_y} z - \frac{Q a^2}{8 I_y} z + E v^* y \]  

where \( e \) is the vertical deflection of the whole structure due to loading applied, and the stress in the plate is given by the first three terms of Eq. (4.15) with the corresponding value of \( z \).

Analysis which included all these effects was compared with the normal beam analysis which does not consider cross-sectional distortion.

4.6 Summary of Displacement Functions

So far, the assumptions with respect to individual components have been described. However, it can be concluded that one stiffener with its tributary plate cannot successfully represent the behavior of the full structure (series of stiffeners) because the consecutive stiffeners may deform in opposite directions. This happens in the case of alternating overall deformations and in the case of the symmetrical mode of plate buckling. The different types of motion of the consecutive stiffeners can be adequately described by having a unit with two stiffeners and the tributary plate width as the basic unit to be analyzed. (Figs. 5 and 7) The minimum number of displacement func-
tions selected to approximate deformations of such a unit was ten; two for the column (overall) mode (one for each stiffener-plate junction), four for the tripping mode (two for each stiffener), four for the plate (two for each mode, SINUSOIDAL AND CURVATURE). The following equations summarize the displacement functions of the plate and of the stiffener web, respectively:

\[
w_0 \sim b, b \sim 2b = \sin \alpha_n x \left\{ \pm C_4 \sin \frac{\pi}{b} y \pm C_5 4 \left( \frac{y}{b} \right) - \left( \frac{y}{b} \right)^2 + C_6 \sin \frac{2\pi}{b} y \right\}
+ C_7 64 \left[ \frac{1}{6} \left( \frac{y}{b} \right) - \frac{1}{2} \left( \frac{y}{b} \right)^2 + \frac{1}{3} \left( \frac{y}{b} \right)^3 \right]
+ \sin \alpha_m x \left[ C_{3,10} \left( 1 - \frac{y}{b} \right) + C_{10,3} \frac{y}{b} \right] \quad (4.16)
\]

\[
v_{wL,wR} = C_{1,8} \sin \alpha_l x \left[ -2 \left( \frac{z}{d} \right)^3 + 3 \left( \frac{z}{d} \right)^2 \right] + C_{2,9} d \sin \alpha_l x \left( \frac{z}{d} \right)^3 - \left( \frac{z}{d} \right)^2
+ d\theta_{L,R} \left[ \left( \frac{z}{d} \right)^3 - 2 \left( \frac{z}{d} \right)^2 + \left( \frac{z}{d} \right) \right] \quad (4.17)
\]

\[
\theta_{L,R} = \frac{\partial w}{\partial y} (y = 0 \text{ or } b)
= \sin \alpha_n x \left\{ \pm C_{4} \frac{\pi}{b} \pm C_{5} 4 \frac{\pi}{b} \right\}
+ \sin \alpha_m x \left\{ -C_{3} \frac{1}{b} + C_{10} \frac{1}{b} \right\} \quad (4.18)
\]

4.7 Plate in Postbuckling Range

4.7.1 Consideration of Effective Width --- General Formula

Under axail load, a plate component may have a significant amount of postbuckling strength. Then, the stress distribution is no longer uniform over the plate width. The postbuckling strength depends mainly on the width-thickness ratio, and it directly affects the ultimate strength of the structure.

To incorporate the postbuckling strength into a design procedure, the effective width concept has been often used. Several formulas have been proposed for computing the effective width as a function of the width-thickness ratio and the average or edge stress.
The following effective width formula was selected for use in this study. [9]

\[
\frac{b_{\text{eff}}}{b} = \sqrt{\frac{\sigma_{\text{cr}}}{\sigma_e}} \left( 1 - 0.22 \sqrt{\frac{\sigma_{\text{cr}}}{\sigma_e}} \right) \leq 1.0
\]

(4.19)

where \( \sigma_e \) = stress at the edge of plate (here, at the stiffener-plate junction)

\( \sigma_{\text{cr}} \) = critical buckling stress of a simply supported plate

Equation (4.19) is based on experimental results and incorporates the effects of residual stresses and initial imperfections. Thus, depending on the width-thickness ratio \( (b/t) \), reduction of the actual width may take place before the buckling stress of the plate is reached. The limits of the effective width are the actual width \( (b_e/b = 1.0) \) and the minimum width at the ultimate plate capacity assumed to be reached when the edge stress \( (\sigma_e) \) equals the yield stress of the material.

Since under axial loading, the stress in the plate is constant over the length, the effective width is also constant.

### 4.7.2 Effective Width for Combined Loading

As stated previously, ultimate strength of the structure is assumed to be reached when the maximum stress in the cross section is equal to the yield stress. To find the maximum stress and its location, it is necessary to consider the effect of sidesway bending of the flange section as well as of the overall beam deformations.

Under combined loading \( (P \text{ and } Q) \), postbuckling deformation of the plate between stiffeners would accelerate failure of the structure. With the ends simply supported, the stress varies parabolically along the structure and the maximum stress would occur at mid-span. Since the effective width is a function of the stress in plate, it would also vary parabolically with the smallest value at mid-span. In fact, depending on the stress variation, the effective width may have a full value near the ends and a parabolic reduction over the middle portion as shown in Fig. 12.
Nonlinear interdependence among the effective width, the edge stress, overall deformations, and the stress at other points in the cross section for a given loading presents considerable difficulties in obtaining a solution. Numerical integration and trial-and-error iteration computations would be involved. In order to simply the computational procedure and eliminate the need for numerical integration, the effective width was conservatively assumed to be constant over the full length and equal to the value at mid-span. Then, iterations were needed only to bring the effective width and the edge stress at mid-span into agreement. Generally, five to ten cycles were sufficient to obtain a tolerance of 0.1 percent for the effective width.

At each load increment, a check of the stresses in the cross section (flange edges and plate-stiffener junction) was made against the yield stress to pinpoint the reaching of the ultimate load capacity. The maximum stress at each location considering the reduced section (effective section) is, then,

\[
\sigma = \frac{P}{A_{\text{eff}}} - \frac{P_e}{(I_y)_{\text{eff}}} z - \frac{Q a^2 / 8}{(I_y)_{\text{eff}}} z - \frac{P_{e_{\text{eff}}}}{(I_y)_{\text{eff}}} z + E v^y
\]  

(4.20)

where the subscript "eff" generally means the effective section and its related properties, while \( e_{\text{eff}} \) represents the additional eccentricity due to the change of the section. The stress in the plate is given by the first four terms of Eq. (4.20) with the corresponding value of \( z \).

Rotational interaction between the plate and the stiffener web was assumed not to be affected by the postbuckling behavior of the plate and, thus, was based on the full plate width.
4.8 Consideration of Initial Imperfections

Effect of initial imperfections on the behavior and strength of asymmetrical (Angle) stiffeners was also considered under the combined axial and lateral loading condition. Two types of imperfection were considered to be most important; the overall deflection of the stiffener in the vertical direction (perpendicular to the plate) and the lateral deflection of the stiffener flange (parallel to the plate). The shape of the stiffener was assumed not to change, and thus, the stiffener flange rotated through the angle equal to the initial lateral deflection divided by the stiffener depth.

The pattern of initial imperfections was assumed to correspond to the displacement parameters and functions of the general formulation. Initial imperfections of the plate were expected to have very minor effect and were indirectly considered by the use of the effective width concept. The computer program included the effect of initial imperfection.

4.9 Computer Program

A computer program was written in FORTRAN-77 to implement the method of analysis described above. The program uses some outside routines for matrix operations, such as, the solution of the eigenvalue problem (buckling under axial loading) and the solution of simultaneous equations (behavior under combined loading). A self-explanatory outline of the program is given by the flow chart in Fig. 16 for the case of combined loading case. The direct consideration of the postbuckling behavior of the plate (effective width) is incorporated through an interactive procedure for a specified degree of tolerance (e.g., 0.001). In each cycle, the (10x10) coefficient matrix is correspondingly adjusted, and the ultimate strength condition checked (first yield).

For the case of axial loading alone, (lateral loading = 0), the program is automatically adjusted and the buckling load is found as an eigenvalue.
5. RESULTS OF ANALYSIS

A computer program written to implement the method of analysis described in the previous chapter was used to analyze some sample cases. The results obtained are discussed here.

5.1 Axial Loading

5.1.1 Buckling Modes

Results of a study of the predominance of a particular buckling mode in the elastic range for asymmetrical stiffeners, the plate buckling or the overall buckling, are shown for three sample sections in Fig. 9. (The dimensions of these sections are listed in Table 3) It can be seen that the controlling mode depends on the width-thickness ratio of the plate \( b/t \), the slenderness ratio \( a/r \), and the stiffener depth to plate width ratio \( d/b \).

The overall buckling mode computed for Fig. 9 incorporates the interaction of the tripping and the column modes since for an asymmetrical section these two modes cannot be independent. Although, as the structure becomes longer and longer (larger slenderness ratio), the column mode becomes more dominant, the tripping mode of the stiffener always accompanies it mainly due to the distortion of the stiffener cross section. As indicated by Van der Neut, the corresponding plate deformations are likely to be antisymmetrical.* [20] The effect of the coupling between the column and tripping modes is illustrated in Table 2. Three specimens with the same angle stiffener but different plate widths (different \( b/t \) values) were analyzed using the proposed method, and the buckling values were compared with the pure column (Euler) buckling load. Due

* For symmetrical stiffeners (Tee), the plate buckling mode shape is expected to be symmetrical and to accompany the tripping mode of the stiffener. The column buckling mode of Tee stiffeners is independent of the other modes, especially for larger slenderness ratios.
to the interaction of the modes and the distortion of the cross section, the first two specimens reached only 77% and 74% of the Euler load, respectively. The third specimen failed with the plate deformation of the symmetrical mode at 68% of the Euler load. The results of applying a computer program based on the method by Van der Neut are also listed in Table 2. [15] The values are essentially the same as by the proposed method, except that the possibility of symmetrical mode of plate deformation in the third specimen could not be detected since Van der Neut considered that the antisymmetrical mode of the plate deformation was more critical for the overall buckling mode and simply used the corresponding rotational constraint in the analysis.

5.1.2 Comparison of Buckling Strengths of Angle and Tee Stiffeners

Figure 10 shows a comparison of the buckling strength between Angle and Tee stiffeners (the flanges of both are of the same proportions). The ratio of the Angle strength to the Tee strength ($\sigma_A/\sigma_T$) is plotted against the slenderness ratio $a/r$.

For lower slenderness, the Angle stiffener has greater capacity than the Tee stiffener mainly because of a greater warping constant of the angle to resist tripping. In this region, the curve in Fig. 10 is above the $\sigma_A/\sigma_T = 1.0$ line up to 1.42. This agrees with the results of classical analysis which does not consider the effect of section distortion. However, with an increasing slenderness, the Angle section gradually becomes weaker as indicated by the curve dropping below 1.0.

Although not shown directly, the computations made for the figure indicate that Tee sections, especially for larger values of slenderness ratio (greater than 60 in Fig. 10), have the buckling strength actually equal to the Euler buckling strength. On the other hand, the Angle sections buckle in a coupled mode of the column and tripping modes and the buckling strength is less than the Euler strength or that of a Tee section ($\sigma_A/\sigma_T = 0.92$ for $a/r = 70$). Yet, for very slender structures, $\sigma_A/\sigma_T$ moves closer to 1.0.
5.2 Combined Loading

Only asymmetrical sections were studied under the combination of axial and lateral loads because, under lateral loading, they are subject to a significant effect of section distortion while symmetrical sections do not distort till after tripping of the stiffener.

5.2.1 Stiffener Deformations and Stress Distributions

Under lateral loading, an asymmetrical (Angle) section starts to rotate about the toe as well as to deflect vertically from the beginning of load application. The sidesway motion (rotation and distortion) makes the stress distribution vary over the flange width. This variation of stresses over the cross section also changes along the stiffener. The resultant stresses at mid-length can be expressed in terms of the curvature of sidesway deformation of the flange ($C_1 \sin \frac{x}{a}$).

The results of the analysis by the computer program show that the direction of sidesway deformation of the stiffener depends on the direction of loading; for upward (positive z direction in Fig. 4) lateral loading, the stiffener section tends to rotate counterclockwise (that is, the flange moves into the negative y direction), and for downward (negative z direction) lateral loading, the stiffener section tends to rotate clockwise (the flange moves into the positive y direction). Accordingly, the deformation patterns for the two directions of lateral loading result in the stress distributions across the flange width which are reversed from each other. An example of such stress distributions is shown in Fig. 11; each stress distribution is due to a particular effect: vertical deflection of the whole structure ($\sigma_e$), sidesway bending of the stiffener section ($\sigma_v$), axial loading ($\sigma_p$), and the upward (positive) lateral loading ($\sigma_Q$), respectively.
5.2.2 Maximum Stress

Since first yielding has been accepted as the criterion of failure, the location and magnitude of the maximum stress in the cross section must be determined. While, depending on the direction, lateral loading may cause tension or compression at the same location along the structure, the axial thrust always causes compression over the whole cross section. Thus, the location and magnitude of the maximum stress depends on the given loading condition.

For downward (negative) lateral loading, the maximum stress occurs in the flange. This is expected to be so since both, the axial thrust and lateral loading, cause compression in the flange. A somewhat more complex situation exists in the case of an upward (positive) lateral loading. If the axial thrust is kept constant and lateral loading gradually increased, then the location of the maximum stress (compression) jumps from its earlier position in the plate to the free edge of the flange (tension). This shifting is attributed to the fact that there is a certain combination of loads which makes the value of the tensile stress in the flange to be greater than the compression stress in the plate.

Under a heavy axial thrust, the compression stress in the plate would reach the yield stress with only a small amount of positive lateral loading before the tension stress in the flange becomes significant. However, one must be very careful in analyzing such a case. Since the plate could be significantly deflected in the post-buckling range, the stress distribution over the whole section would be affected. As indicated in Chapter 4, the concept of effective width is introduced to take this effect into account.
5.2.3 Interaction Behavior

An interaction diagram between the axial (P) and lateral (Q) loads for the first yield condition is shown in Fig. 13. The plots are made for three different values of the slenderness ratio. The vertical axis is the axial load normalized by the yield load, and the horizontal axis is the given lateral loading Q normalized by Q_o. Q_o is the reference lateral loading defined as the loading which would cause yield stress at the extreme fiber in the given cross section of a simply supported beam with the length arbitrarily set to be 40 times of the radius of gyration of the cross section.

Under upward lateral load (right side of the plot), the interaction curve shows a considerable change in the shape. This corresponds to the jumping of the location of maximum stress which reaches first yield, from tension in the flange to compression in the plate. On the other hand, under downward lateral loading (left side of the plot), the interaction curve is almost linear. This behavior was anticipated in previous Section since the maximum compressive stress would remain in the flange throughout the loading history.

For the purpose of comparison, the stress from regular beam analysis (assuming undeformed section) was also calculated. The results are shown in Fig. 14 where the ratio of maximum stresses in a deformed and undeformed sections is plotted against P/P_o and Q/Q_o for a sample structure (see Table 3). As one would expect, the analysis of a deformed section gives a higher maximum stress than the analysis of an undeformed section under the same loading condition. The increase in this case is up to 41% at P/P_o = 0.45 and Q/Q_o = 1.45. Thus, the failure (first yielding) of a deformed section will correspondingly occur under a lower load than of an undeformed section.

Figure 15 shows the reduction of the ultimate strength of a stiffened plate when
the effect of initial imperfection is taken into account. The imperfections were defined as the initial lateral deflection of the flange in the (-y) direction and the (+z) deflection of the stiffener-plate junction divided by the length of the stiffener for the upward (+z) lateral load. The amounts of initial imperfection used in this figure were 1/1000, 1/500, and 1/250 of the length. The solid line is for the case without any imperfections. As shown, for a combination of a heavy lateral and small axial loads, the effect of initial imperfection is not very significant. However, for a greater axial loading, the reduction of capacity becomes more important.

The ultimate capacity of the structure gradually decreases with the larger amount of initial imperfection. For example, for the initial imperfection of 1/250, the reduction of the capacity at $Q/Q_o = -0.25$ is approximately 15%.
6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and Conclusions

A method was developed for analyzing the tripping (lateral-torsional) behavior of asymmetrical stiffeners in longitudinally stiffened plates subjected to axial or combined axial and lateral loads. The effects of cross-sectional distortion, postbuckling behavior of the plate (effective width), and initial imperfections were included. First yielding was used as the criterion for ultimate loading condition.

A number of sample stiffened plates were analyzed to study various effects. The cross section was assumed to distort or not to distort. Also, for the purpose of comparison, in addition to asymmetrical (Angle) sections, symmetrical sections (Tee) with the same flange dimensions were analyzed.

The following observations and conclusions can be drawn from the study:

Axial Loading

- Coupling of the buckling modes of plate buckling, tripping and column buckling was found to give a significantly lower capacity of asymmetrical stiffeners, especially, when the effect of distortion of the cross section was considered.

- The effect of distortion depends on the slenderness ratio \( \frac{a}{r} \). In comparison with a symmetrical section (Tee), an asymmetrical section (Angle) has
  - a higher capacity for lower values of \( \frac{a}{r} \) (up to 142% for the section analyzed),
  - a rapid reduction below the capacity of the Tee section (down to about 92%) with an increase in \( \frac{a}{r} \) (for a classical solution, without distortion, the capacity would remain above that of Tee),
  - a very gradual increase toward the capacity of the Tee section for very slender (long) stiffeners.
Combined Axial and Lateral Loading

- A comparison of the maximum stresses of deformed and undeformed sections shows that the relative increase in stress for the deformed section depends on the particular combination of the axial (P) and lateral (Q) loads. For the sample stiffened plate analyzed, the increase was up to 41%.

- Correspondingly, the ultimate strength (for first yielding) is detrimentally affected by cross-sectional deformations.

- The pattern of interaction between the axial and lateral loads for the ultimate strength condition (first yielding) strongly depends on the direction of lateral loading (away from or toward the plate). For lateral loading away from the plate, the interaction is almost linear, and it is bulging out for the loading toward the plate.

- The reducing effect on the ultimate strength by the initial deflections of stiffeners is most pronounced when lateral loading is relatively low and away from the plate. (The reduction with respect to an initially perfect sample structure was approximately 15% for an initial deflection equal to 1/250 of the length.)

The general conclusion of this study is that the effect of cross-sectional distortions must be taken into consideration in the analysis and design of plates with asymmetrical longitudinal stiffeners.

The method developed here is suitable for this purpose although it is very impractical for engineering application.

6.2 Recommendations

Recommendations for future work on the basis of the completed study can be put into the following three groups: (1) Utilization of the method and the computer program developed here for formulating a practical design procedure; (2) Experimental study to provide a measure on the accuracy of the assumptions and simplifications used in this and/or future methods of analysis; (3) Further development of the method of analysis in order to more accurately consider various effects and to extend the method to more general geometries and conditions of loading.
1. Utilization of the method for formulating a practical design procedure

- Generation of a data base by using the computer program with extended ranges of various dimensions and loading combinations.
- Parametric study of the functional influence of the principal parameters, such as, $a/r$, $b/t$, $d/b$, $b_f/d$, $Q/P$, $\sigma_{yd}$, initial imperfections, etc.
- Formulation of a practical, yet sufficiently accurate, design procedure.

2. Experimental study

- Since there are essentially no test results available on the tripping behavior or strength of asymmetrical stiffeners, tests are recommended for verifying the validity of the assumptions used in analysis and the accuracy of the method(s).
- In particular, test specimens should be designed to have Angle stiffeners with the same depth and flange width as some specimens with Tee stiffeners tested in the past.
- Tests are needed on multi-span stiffened plates with or without lateral loading.
- Tests on specimens with controlled initial imperfections.

3. Further development of the method of analysis to consider:

- Initial imperfections in the plate,
- Residual stresses,
- Different yield stresses in the plate and stiffeners,
- General inelastic range,
- Moments applied at the ends,
- Continuity of stiffened plates over several spans.
7. ACKNOWLEDGEMENTS

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A. TOTAL POTENTIAL ENERGY

A.1 Internal Potential (Strain Energy)

The following summarizes the internal potential energy for the whole system (two stiffeners and the tributary plate):

\[
U = \frac{D_{pl}}{2} \int_0^b \int_0^a \left\{ \left( \frac{\partial^2 w_i}{\partial x^2} + \frac{\partial^2 w_i}{\partial y^2} \right)^2 - 2(1 - \nu) \left[ \frac{\partial^2 w_i}{\partial x^2} \frac{\partial^2 w_i}{\partial y^2} - \left( \frac{\partial^2 w_i}{\partial x \partial y} \right)^2 \right] \right\} \, dx \, dy \\
+ \frac{D_{pl}}{2} \int_0^b \int_0^a \left\{ \left( \frac{\partial^2 w_r}{\partial x^2} + \frac{\partial^2 w_r}{\partial y^2} \right)^2 - 2(1 - \nu) \left[ \frac{\partial^2 w_r}{\partial x^2} \frac{\partial^2 w_r}{\partial y^2} - \left( \frac{\partial^2 w_r}{\partial x \partial y} \right)^2 \right] \right\} \, dx \, dy \\
+ \frac{D_w}{2} \int_0^a \int_0^a \left[ \left( \frac{\partial^2 v_i}{\partial x^2} \right)^2 + \left( \frac{\partial^2 v_r}{\partial x^2} \right)^2 \right] \, dx \, dy \\
+ \frac{E_{l,vw+wp}}{2} \int_0^a \left[ \left( \frac{\partial^2 w_{st,i}}{\partial x^2} \right)^2 + \left( \frac{\partial^2 w_{st,r}}{\partial x^2} \right)^2 \right] \, dx \\
+ \frac{E_{l,zf}}{2} \int_0^a \left[ \left( \frac{\partial^2 (v_i - y\beta_i)}{\partial x^2} \right)^2 + \left( \frac{\partial^2 (v_r - y\beta_r)}{\partial x^2} \right)^2 \right] \, dx \\
+ \frac{E_{l,yf}}{2} \int_0^b \int_0^a \left[ \left( \frac{\partial^2 (w_{st,i} - y\beta_i)}{\partial x^2} \right)^2 + \left( \frac{\partial^2 (w_{st,r} - y\beta_r)}{\partial x^2} \right)^2 \right] \, dy \, dx \\
+ \frac{E_{l,af}}{2} \int_0^a \left[ \left( \frac{\partial^2 \beta_i}{\partial x^2} \right)^2 + \left( \frac{\partial^2 \beta_r}{\partial x^2} \right)^2 \right] \, dx \\
+ \frac{E_{l,wf}}{2} \int_0^a \left[ \left( \frac{\partial^2 \beta_i}{\partial x^2} \right)^2 + \left( \frac{\partial^2 \beta_r}{\partial x^2} \right)^2 \right] \, dx \\
+ \frac{GJ_f}{2} \int_0^a \left[ \left( \frac{\partial \beta_i}{\partial x} \right)^2 + \left( \frac{\partial \beta_r}{\partial x} \right)^2 \right] \, dx \\
\text{(A.1)}
\]
Each line in Eq. (A.1) expresses the contribution to the total strain energy of each component as follows:

- lines 1, 2: deformation of two tributary plates
- line 3: distortion of transverse strips of the web
- line 4: overall deformation of the stiffener web and of plate
- line 5: sidesway bending of stiffener flange
- line 6: overall deformation of stiffener flange
  (note that, neglecting the slight deformation of the flange in the y-z plane, its displacement along z-axis is $w_{u} - y\theta$)
- line 7: warping deformation with respect to the center of twist of stiffener flange
- line 8: warping deformation with respect to the center of twist of stiffener web
- line 9: St. Venant torsion of stiffener flange
A.2 External Potential of Loading at Ends

The following summarizes the external potential by axial loading for the whole system. Note that the external potential under combined loading should include the contribution by the lateral loading given by Eq. (4.13).

\[
V_p = -\frac{\sigma}{2} \int_{a}^{b} \int_{0}^{l} \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] dy \, dx
\]

\[
-\frac{g}{2} \int_{a}^{b} \int_{0}^{l} \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] dy \, dx
\]

\[
-\frac{g}{2} \int_{a}^{b} \int_{0}^{l} \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] dy \, dx
\]

\[
-\frac{g}{2} \int_{a}^{b} \int_{0}^{l} \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] dy \, dx
\]

\[
-\frac{g}{2} \int_{a}^{b} \int_{0}^{l} \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] dy \, dx
\]

(A.2)

Each line expresses the contribution to total potential due to the relative axial shortening by the deformation of each component as follows;

- line 1: deformation of plate
- line 2: deformation of web transverse strip
- line 3: overall deformation of stiffener web
- line 4: sidesway bending of stiffener flange
- line 5: overall deformation of stiffener flange
### Table 1: Literature Summary

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<td></td>
<td>Z</td>
<td>T</td>
<td>A,F,B</td>
<td>T</td>
</tr>
<tr>
<td>2. Loading</td>
<td></td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P,M,Q</td>
</tr>
<tr>
<td>3. Method Used</td>
<td></td>
<td>Differential Equation</td>
<td>(N/A)</td>
<td>(N/A)</td>
<td>Total potential</td>
</tr>
<tr>
<td>4. Test</td>
<td></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>5. Web Distortion</td>
<td></td>
<td>No</td>
<td>(N/A)</td>
<td>(N/A)</td>
<td>Yes</td>
</tr>
<tr>
<td>6. Overall Buckling</td>
<td></td>
<td>No</td>
<td>(N/A)</td>
<td>(N/A)</td>
<td>No</td>
</tr>
<tr>
<td>7. Initial Imperfections</td>
<td></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>8. Rotational Constraint</td>
<td></td>
<td>Plate</td>
<td>(N/A)</td>
<td>(N/A)</td>
<td>Spring</td>
</tr>
<tr>
<td>9. Inelastic</td>
<td></td>
<td>No</td>
<td>(N/A)</td>
<td>(N/A)</td>
<td>Yes</td>
</tr>
<tr>
<td>10. Residual Stresses</td>
<td></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>11. Effective Width (Plate)</td>
<td></td>
<td>Yes</td>
<td>(N/A)</td>
<td>(N/A)</td>
<td>Yes</td>
</tr>
<tr>
<td>12. Comparison with others</td>
<td></td>
<td>No</td>
<td>No</td>
<td>Merrison's Rule</td>
<td>Smith's Tests FEM Results</td>
</tr>
</tbody>
</table>

**Reference sources are tabulated in chronological order of publication**

**Stiffener Section:** A - Angle  F - Flat  T - Tee  B - Bulb Flat  Z - Zee  C - Channel

**Loading:** P - Axial  M - End Moment  Q - Lateral
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>A,T,F</td>
<td>Z</td>
<td>A,T</td>
<td>Z,T,C</td>
<td>A</td>
</tr>
<tr>
<td>2.</td>
<td>P</td>
<td>P</td>
<td>P,M,Q</td>
<td>P,M</td>
<td>Q</td>
</tr>
<tr>
<td>4.</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>5.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>6.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>7.</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>8.</td>
<td>Spring</td>
<td>Spring</td>
<td>Spring</td>
<td>Spring</td>
<td>Spring</td>
</tr>
<tr>
<td>9.</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>10.</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>11.</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

**Stiffener Section**: A - Angle  F - Flat  T - Tee  B - Bulb  Z - Zee  C - Channel

**Loading**: P - Axial  M - End Moment  Q - Lateral

*** Finite Difference Method for web element
Table 2: Comparison of Computed Results with Van der Neut’s Solution in Elastic Range

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$a/r$</th>
<th>$b/t$</th>
<th>$d/b$</th>
<th>Current Method</th>
<th>V. d. Neut (Ostapenko)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.0</td>
<td>29.0</td>
<td>0.54</td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td>2</td>
<td>32.5</td>
<td>33.0</td>
<td>0.54</td>
<td>0.74</td>
<td>0.74</td>
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<tr>
<td>3</td>
<td>32.0</td>
<td>39.0</td>
<td>0.54</td>
<td>0.68****</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Note: The section is modified from the example in Van der Neut’s paper.

Table 3: Section Properties of Specimens Used in Figures
(All dimensions are in inches)

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$a$</th>
<th>$b$</th>
<th>$t$</th>
<th>$d$</th>
<th>$t_w$</th>
<th>$b_f$</th>
<th>$t_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 9</td>
<td>48.0 to 220.0</td>
<td>24.0</td>
<td>0.3 to 0.8</td>
<td>4.0 to 8.0</td>
<td>0.3</td>
<td>3.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Fig. 10</td>
<td>72.0 to 192.0</td>
<td>24.0</td>
<td>0.55</td>
<td>6.0</td>
<td>0.3</td>
<td>3.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Fig. 13</td>
<td>50.0 to 150.0</td>
<td>25.0</td>
<td>0.33</td>
<td>5.0</td>
<td>0.25</td>
<td>3.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Fig. 15</td>
<td>100</td>
<td>25.0</td>
<td>0.33</td>
<td>5.0</td>
<td>0.25</td>
<td>3.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Fig. 14</td>
<td>72.0</td>
<td>24.0</td>
<td>0.4</td>
<td>6.0</td>
<td>0.3</td>
<td>3.5</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note: Parameters, such as, $a/r$, $b/t$, and $d/b$, are to be adjusted as shown in each figure.

**** Plate in symmetrical mode
FIGURES

Figure 1: Various Types of Stiffener Section

Figure 2: Angle and Tee Stiffeners (Other Dimensions in Fig. 3)
Figure 3: Geometry of Angle Section

Figure 4: Deformation of Stiffener Web
Figure 5: Unit to be Analyzed and Loading Applied (in positive direction)
a. Symmetrical Mode

b. Antisymmetrical Mode

c. Overall Mode

Figure 6: Deformation (Buckling) Modes
Left Stiffener

- \( C_1 \): Lateral displacement at the tip of the flange
- \( C_2 \): Angle of rotation at the tip of the stiffener
- \( C_3 \): Overall displacement of the stiffener-plate junction

Plate

Symmetrical Mode

- \( C_4 \): Amplitude of the sinusoidal form of deflection
- \( C_5 \): Amplitude of the polynomial form of deflection

Antisymmetrical Mode

- \( C_6 \): Amplitude of the sinusoidal form of deflection
- \( C_7 \): Amplitude of the polynomial form of deflection

Right Stiffener

- \( C_8 \): Lateral displacement at the tip of the flange
- \( C_9 \): Angle of rotation at the tip of the stiffener
- \( C_{10} \): Overall displacement of the stiffener-plate junction

Figure 7: Definitions of Constants in Assumed Displacement Functions
Figure 8: Buckling of Stiffened Plates
Figure 9: Buckling Mode Transition
Figure 10: General Comparison of Buckling Strength for Angle and Tee Stiffener

Figure 11: Stress Distribution under Combined Loading
Figure 12: Modified Effective Width Concept under Combined Loading
Figure 13: Interaction between P and Q Considering the Effect of Slenderness Ratio
Figure 14: Ratio of maximum stresses between deformed and undeformed sections for variable axial and lateral loads
Figure 15: Interaction between P and Q with Initial Imperfection (a/r = 58.24)
calculate the section properties

increase loading (P or Q)

generate matrix A

generate matrix B

solve the set of simultaneous equations \( Ax = B \)

calculate stresses in the plate and at both edges of the flange at mid-span

repeat until tolerance satisfied

repeat until the first yield criterion is met

check tolerance (0.001) for the \( b_{\text{eff}} \) and \( \sigma_{\text{edge}} \) (stress in the plate)

choose the largest among the three stresses

next input

\[
A = \frac{\partial U}{\partial C_i}
\]
\[
B = \frac{\partial V}{\partial C_i}
\]
\[
x = \{ C_i \}
\]

Figure 16: Flow Chart for Computer Program
NOMENCLATURE

A  Cross-sectional area of stiffend plate
a  Length of stiffened plate
b  Plate width between stiffeners
b_{eff}  Effective width of plate
b_f  Width of stiffener flange
\beta_l  or \beta_r  Angle of rotation of stiffener flange (left or right) from its original position
C_{1,10}  Displacement constants
d  Depth of stiffener
D_{pl}  Flexural rigidity of plate
D_w  Flexural plate rigidity of stiffener web
E  Modulus of elasticity
G  Shearing modulus of material
I_{wf}  Warping constant of stiffener flange about the center of twist
I_{ww}  Warping constant of stiffener web about the center of twist
I_y  Moment of inertia of the whole section about the centroidal y-axis
I_{yf}  Moment of inertia of stiffener flange about minor principal axis of total section
\( I_{yw+yp} \)  
Moment of inertia of plate and stiffener web about minor principal axis of total section

\( I_{sf} \)  
Moment of inertia of stiffener flange about the centroidal z-axis of stiffener section

\( J_f \)  
St. Venant torsional constant of stiffener flange

\( r \)  
Radius of gyration of total section about y-axis

\( \sigma \)  
Applied axial stress (P divided by total area)

\( \sigma_{cr} \)  
Buckling stress of simply supported plate

\( \sigma_e \)  
Edge stress (stress in plate at stiffener-plate junction)

\( \sigma_{Euler} \)  
Euler buckling stress

\( t_f \)  
Thickness of flange

\( t \)  
Thickness of plate

\( t_w \)  
Thickness of web

\( U \)  
Internal potential (strain energy)

\( V \)  
Total potential energy

\( V_e \)  
External potential

\( V_p \)  
External potential due to axial loading P

\( V_Q \)  
External potential due to lateral loading Q

\( v_l \) or \( r \)  
Lateral displacement in y-direction at the stiffener tip measured from its original position (left or right stiffener in unit)
$v_{w_l}$ or $v_{w_r}$ Displacement in y-direction along the stiffener web (left or right)

$w_p$, $w_{l}$ or $r$ Displacement of plate in z-direction (left or right)

$w_{st,l}$ or $w_{st,r}$ Overall displacement of structure at stiffener location in z-direction

$\theta$, $\theta_{l}$ or $r$ Angle of rotation at the stiffener-plate junction (left or right)