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REDUNDANCY OF SIMPLE SPAN AND TWO-SPAN WELDED STEEL TWO-GIRDER BRIDGES

FINAL REPORT

Research Project 84-20
Redundancy of Welded Steel I-Girder Bridges

Prepared For
Commonwealth of Pennsylvania
Department of Transportation
Office of Research and Special Studies

LEHIGH UNIVERSITY

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1. INTRODUCTION

1.1 Background:

The design of steel bridges in Pennsylvania requires design against fatigue caused by repetitive live loads. (1,2)* The allowable stress range used in design depends on whether the bridge is a redundant or a nonredundant load path structure. (2) A redundant load path structure is defined by the AASHTO Specifications, Art. 10.3.1, as "structure types with multi-load paths where a single fracture in a member cannot lead to collapse".

In this context the term redundant does not refer to a statically indeterminate structure. Nor does it refer to the excess capacity known to exist in normally designed bridges beyond that intended in the design. The terms redundant, nonredundant and redundancy appearing in this report are used strictly in accordance with the definitions contained in Art. 10.3.1 of the AASHTO Specifications, Ref. 2. AASHTO considers nonredundant load path members as main load carrying components subject to tensile stress where failure of a single element could cause collapse.

The allowable fatigue stress ranges for redundant load path structures provided in Table 10.3.1A of Ref. 2 resulted primarily from research into the fatigue of steel structures conducted by J. W. Fisher, et al, at Lehigh University over the past 25 years.(3-6) The stress ranges

*References are presented in Chapter 11 at the end of this report.
for nonredundant load path structures provided in Table 10.3.1A of Ref. 2 are not based on research into the fatigue of steel structures.

Design against fatigue by the use of the allowable stress ranges provided in Ref. 2 does not ensure that fracture of the steel structure cannot occur. To minimize the possibility of fracture, the control of three primary variables is required: (2,7,8,9)

1. Level of Tensile Stress:
   Even though stress levels in design are limited by the allowable stresses and load factors provided by AASHTO, actual stresses are frequently at or near the yield stress level at welded details due to residual stress.

2. Flaw and Crack Size:
   Initial flaw sizes tend to be limited by the quality of fabrication and inspection requirements provided by AASHTO. (10) However, initial flaws propagate into larger cracks during the normal life of a bridge. For a given fatigue category, stress range and number of cycles, smaller initial flaws tend to propagate into smaller cracks.

3. Material Toughness:
   Material toughness is controlled by Art. 10.3,3 of Ref. 2, and Table 7.1 of Ref. 10. The required material toughness increases as the lowest expected service temperature decreases.
These together with provisions for the control of toughness, workmanship and inspection are specified in the 1978 AASHTO Guide Specification for Fracture Critical Nonredundant Steel Bridge members.\(^{(10)}\)

To minimize the probability of fracture, AASHTO requires the use of reduced allowable fatigue stress ranges for nonredundant load path structures as provided in Table 10.3.1A of Reference 2. This concept was made part of the 11th Ed. of the AASHTO Specifications\(^{(9)}\) through a 1977 Interim Specification and continues into the 13th Ed. The reduced stress ranges were not determined by rational research but by simply shifting the values for redundant load path structures one column to the left and introducing new values for over 2,000,000 cycles as can be seen by examining Table 10.3.1A of Ref. 2. Research into the appropriateness of the reduced stress ranges for nonredundant load path structures is not part of the investigation reported herein.

As a guide to design engineers, AASHTO classifies, by example, redundant and nonredundant load path structures. These examples appear in Art. 10.3.1 of Ref. 2 including the footnote to Table 10.3.1A. In this way AASHTO classifies multi-beam bridges as redundant and one or two-girder bridges as nonredundant. Such classifications are based on beliefs commonly held by bridge designers and specification writers. These beliefs, in turn, are based on the usual over-simplified assumptions used in the design of steel girder bridges.

For example, in the design of steel two-girder bridges, the two girders alone (or the two composite girders in composite construction)
constitute the primary design load path for transmitting the dead, live and impact loads to the substructure. The deck, stringers and floorbeams are considered to transmit the vertical loads to the two girders and do not relieve the girders of any of the vertical loads. No longitudinal distribution of wheel loads by the deck is considered. With live loads positioned to one side of the bridge for maximum moment effect on the girder below, the two girders must deflect different amounts. This differential displacement in the as-built bridge must result in warping of the deck and shear in the interior cross frames. Flexure of the girders must result in forces developed in the bottom lateral system. However, all these effects are ignored in design. The bridge is designed as though it consists only of the two main girders acting independently of each other in vertical 2-dimensional planes. In reality, however, the as-built bridge must behave as a 3-dimensional structure (all members and components) to resist all combinations of loads. The vertical loads are supported not only by the girders but also by warping of the deck, shear in the cross bracing and forces in the bottom lateral system, for example.

If nearly full-depth fracture of one of the two girders should occur at midspan, say, due to undetected fatigue crack growth at a welded detail, the moment capacity of this girder is effectively destroyed. In this event it is logical to assume that the remaining 3-dimensional structure consisting of the other girder plus the deck, floor beams, stringers, cross bracing and bottom lateral bracing system would be mobilized in an attempt to support the dead, live and impact loads. An
extension of this concept is that if the 3-dimensional behavior of the fractured two-girder bridge under dead, live and impact loads is understood, then guidelines can be proposed to enable the fractured bridge to be designed to safely carry these loads thus enabling the two-girder bridge to be classified by AASHTO as a redundant load path structure.

1.2 Purpose

The purpose of this report is to present the results of an 18 month research program at Lehigh University into the behavior of three real two-girder bridge spans with assumed nearly full-depth fractures at mid-span of one of the two girders. The three bridge spans selected are:

1. Simple span right with 89-ft. span and 32-ft. roadway,

2. Simple span skew with 89-ft. span, 32-ft. roadway and 45° skew,

3. Two-span right with two 87-ft. spans and 32-ft. roadway.

All three spans are from the Betzwood Bridge carrying L.R. 1046-1 over the Schuylkill River and Reading Railroad in Montgomery County, Pennsylvania.

The spans of the Betzwood Bridge are designed and constructed noncomposite but are studied in this investigation as both composite and noncomposite. Early in this research it appeared that a noncomposite span without top lateral bracing would be nonredundant and likely collapse under less than its own dead load. To ensure some
degree of redundancy either a stiff top lateral bracing system has to be included or else the deck made composite with the girders. Part of the study assumes composite behavior and part assumes a stiff top lateral bracing system.

The studies conducted with composite behavior are not significantly influenced by the composite action since the flexural strength of the girders whether composite or noncomposite does not affect the after-fracture redistribution of dead and live loading to the cross bracing and lateral bracing systems. These systems come into play after fracture occurs and are not significantly affected by composite action in the usual design sense.

Elastic-plastic analyses of the bridge spans are conducted under incremental dead, live and impact loads. The collapse capacities of the spans are estimated using the upper and lower bound techniques of plasticity. For each span the behavior of the 3-dimensional structure in resisting these loads is determined.

Simple design procedures are presented which can be used to proportion the cross frames and bottom lateral bracing of simple span right and skewed two-girder bridges to ensure redundancy against midspan fracture of one of the two girders. Design procedures to ensure redundancy and serviceability of the two-span two-girder bridge are also presented.

The study concludes with recommendations in the form of suggestions for guidelines and design procedures which should be developed by further research and suggests further research needs.
1.3 Previous Research:

In 1978, the AASHTO Guide Specification for Fracture Critical Nonredundant Steel Bridge Members was introduced. (10) Allowable stress ranges for nonredundant load path structures and examples of redundant and nonredundant load path structures were introduced into the 12 Ed. of the AASHTO Bridge Specifications (9) with the 1979 Interim Specification. Neither the allowable stress ranges for nonredundant load path structures or the examples for redundant and nonredundant load path structures were determined by rational research. Thus previous research into redundancy as presently contained in the 13th edition of AASHTO can only date from the late 1970's.

Heins and Hou studied the effects of cross bracing (diaphragms) and bottom lateral bracing on bridge redundancy. (11) The study focused on two-girder and three-girder bridges where one or both flanges of one of the girders is assumed to be cracked. For the two-girder bridge only the bottom flange is assumed cracked. The study apparently is conducted only in the elastic range. It is shown that bracing can effectively reduce the deformations in the girders. It indicates that the effect of flange cracking on the three-girder bridge is negligible but quite important for the two-girder system. The study concludes that if bracing is utilized, the two-girder bridge behaves similarly to the three-girder bridge.

Heins and Kato followed up on the above study with an investigation of load redistribution in cracked girders. (12) The study focused on two-girder bridges where one girder is assumed to be fractured near
midspan. It is concluded that the influence of the bottom lateral bracing on load redistribution is significant. Further, the study concludes that utilization of the secondary members (cross bracing and bottom lateral bracing) effectively creates redundancy in two-girder bridges. Unfortunately, specific guidelines for the design of the bracing members to ensure redundancy are absent. This study also appears to have been conducted only in the elastic range.

Sangare conducted a computer study of the redundancy of a steel deck truss bridge. \(^{(13)}\) In this investigation one of the 340 ft. suspended spans of the Newburgh-Beacon Bridge No. 2 over the Hudson River at Newburgh, New York was modeled for computer analysis. The bridge, designed by Modjeski and Masters Consulting Engineers, Harrisburg, PA., is a deck type cantilever bridge carrying four design traffic lanes supported by two steel trusses. In the redundancy investigation the tension chord of one truss is assumed to be completely fractured at midspan. The results show that although the span is considered nonredundant by most bridge engineers it carries at least full calculated dead load (load factor of one) plus full HS20 lane loading (load factor of one) plus AASHTO impact in all four lanes. All members of both main trusses remain elastic. Redundancy is provided by the cross bracing and bottom lateral bracing systems even though many members of the bracing systems have yielded in tension or buckled in compression.

Reference 14 reviews the state-of-the-art on redundant bridge systems as of 1985. Of the 51 references listed only 8 are dated since 1980.
Of these, two appear in this report as Ref's. 11 and 12. The other 6 (as well as all those prior to 1980) do not address research on redundancy as defined by Art. 10.3.1 of the AASHTO Specifications. (2)

Among the conclusions to the review are the statements:

1. Little work has been done on quantifying the degree of redundancy that is needed in bridges.

2. It is hoped that further research into structural redundancy in bridge systems will be conducted.

3. Computer speed and available software has made evaluation of redundancy more quantifiable than previously possible.

It is interesting to note in reading Ref. 14, which is generated by several individuals, that their use of the term "redundant" is not consistent. The early and latter parts of the paper use the term mainly in the context of AASHTO Art. 10.3.1, as is the use of the term throughout this report. The middle parts of the paper, those dealing with analysis of redundancy, types of analysis and modeling for analysis, appear to refer mainly to "redundancy" as the excess capacity inherent in a normally designed structure. For example, the use of the term overload must refer to the second definition of redundancy (Art. 1.1) since one would not be investigating the overload capacity of a fractured structure if the term overload is used in its normal context to mean over design load. Rather, in a fractured bridge the designer should be content to design for a specified "underload" (ie: under design load) to ensure redundancy as defined in Art. 10.3.1 of AASHTO. This "underload" concept is developed further in this report.
1.4 Research Objectives

The overall objective of the research reported herein is to develop design procedures and guidelines to ensure the redundancy of three real welded steel two-girder study bridges in the event of a nearly full depth midspan fracture of one of the two I-girders. It is also the objective of this investigation to propose procedures and guidelines which can be extended to other bridge configurations and fracture scenarios and to suggest the direction for proposed specification provisions and future research needs. Recognizing the complexity of the 3-dimensional interaction of members, achieving these objectives requires a synthesis of bridge-related expertise, knowledge of elastic and inelastic structural behavior and computer modeling to develop meaningful results.

Specific objectives include:

1. Identify and select three suitable real welded steel two-girder bridge spans for detailed study and computer analysis.

2. Develop finite element models of the three-dimensional two-girder bridges selected for study.

3. Obtain base-line stress resultants and load-deflection behavior under linear elastic conditions for selected unfractured bridges under dead loading.

4. Obtain lower bound incremental elastic-plastic finite element load-deflection curves up to near the stability limit load for selected bridges with midspan girder fractures, accounting for member instability.

5. Obtain upper bound rigid-plastic mechanism loads for all study bridges with midspan girder fractures, accounting for member instability.

6. Identify and evaluate the alternate load paths which account for redundancy of those bridges which do not collapse and remain relatively serviceable under a
collapse and remain relatively serviceable under a specified loading condition.

7. Bring the results of the investigation into focus by explaining the redundancy of two-girder bridges with midspan girder fractures.

8. Provide redundant designs of the study bridges using the procedures and guidelines developed in the investigation.

9. Provide recommendations for further research and suggest a direction for proposed specification provisions to ensure redundancy.

1.5 Research Tasks:

The objectives of the investigation are carried out through the following tasks:

Task 1 - Literature Review: Review, evaluate and report other research conducted on redundancy as defined by Art. 10.3.1 of the AASHTO Specifications 13th Ed. specifically other research on welded steel two-girder bridges.

Task 2 - Bridge Type Identification: Identify the several types of welded steel two-girder bridges in common use in Pennsylvania.

Task 3 - Load Path Identification: With the co-operation and assistance of the Pennsylvania Department of Transportation select those bridges to be studied, obtain design calculations and bridge plans from the Department.

Note: Tasks 2 and 3 were completed during the four months prior to the May 31, 1985 contract start date under a Notice to Proceed. With the assistance of Mr. Ken L. Fullom, Project Manager, Bureau of Design, PADOT, three spans of the Betzwood Bridge were selected for study (Art. 1.2) and design calculations and bridge plans furnished to Lehigh University.
Task 4 - **Determine Capacity and Collapse Mechanisms:**
Estimate the dead, live and impact load carrying capacity of the three spans of the Betzwood Bridge after near full depth midspan fracture of a girder. Generalize collapse mechanisms and suggest design guidelines for ensuring redundancy of the two-girder bridges included in the investigation.

**Note:** This investigation went somewhat further than the scope outlined in Task 4. Specific collapse mechanisms are presented for each of the three study bridges; specific design procedures are proposed for the simple span and two-span bridges; these procedures are illustrated by redesigning each of the study bridges to ensure not only redundancy but also serviceability; guidelines are suggested for design, specification provisions and future research.

Task 5 - **Presentations and Reports:** Prepare and deliver quarterly reports, a progress presentation after Task 2, a progress presentation after Task 4, an interim report and a final report containing conclusions and recommendations.

**Note:** The first presentation was held on May 31, 1985 the first day of the contract at the conclusion of Tasks 2 and 3. The second presentation was held September 24, 1986. An interim report was prepared and presented.\(^{(15)}\).

1.6 **Analytical Approach**

Determination of the collapse load capacities of the fractured two-girder bridge spans included in this investigation necessarily involves the elastic and inelastic behavior of the 3-dimensional structure and the effect of member instability on the analytical results. There are several possible approaches to the analysis.
A reasonably "exact" collapse load capacity of each span can be found through an incremental finite element (FE) analysis, using small loading increments, accounting for second-order elastic plastic member (element) behavior,\(^{(16)}\) member instability (inclusion of the geometric stiffness matrix to reduce the elastic stiffness matrix due to axial compression),\(^{(17)}\) and the effects of large displacements.\(^{(17)}\)

Unfortunately, such an analysis of a complex 3-dimensional bridge span is quite difficult and time consuming and requires extensive pre-and post processing capabilities of state-of-the-art finite element programs. This approach is not considered feasible within the time and resources available in this study.

An alternate approach involves an estimate of the collapse load capacity of each span using the upper and lower bound theorems of plasticity. This approach is followed in this investigation.

A lower bound collapse load capacity is achieved for two of the three spans using an incremental FE analysis. In the analysis a lower bound elastic-plastic load deflection curve is obtained by requiring all stress resultants to be at or a little below the appropriate yield criterion at the end of each loading increment. In this way larger increments of loading are possible. The analysis considers first-order elastic-plastic member behavior and accounts for member instability by reduction of member stiffness to near zero at or near the buckling load or instability limit load of the member. The effect of large displacement is not accounted for. However, it was found that at the end of the last load increment, when the stiffness of each bridge is
substantially reduced and is near the stability limit load, the displacements, although significant were not considered large enough to have a significant effect on the results based on a small displacement assumption.

A lower bound finite element analysis is not performed for the simple span skew bridge. The mechanism analysis for the skew bridge provides an upper bound result that is very close to that for the simple span right bridge. It is concluded that the lower bound results are also close. Therefore, although the finite element model of the skew bridge was prepared it was decided after discussion with Mr. K. L. Fullom, Project Manager, Bureau of Design, Commonwealth of Pennsylvania Department of Transportation (PADOT), to divert project time and resources to a more detailed analysis of the two-span bridge.

An upper bound collapse load capacity and corresponding collapse mechanism are achieved for each of the three spans by examining a number of possible collapse mechanisms and selecting the mechanism corresponding to the lowest upper bound load capacity. The results of the lower bound analyses are used to suggest the best mechanisms for the upper bound analyses. Rigid-plastic member behavior is assumed with equilibrium formulated for the corresponding virtual displacement field. Member instability is considered by removing any member undergoing shortening as defined by the virtual displacement field.

Although the collapse load capacities of all three spans are estimated from the upper and lower bound analyses, of more importance is the behavior of each span and the sequence in which members reach
respective yield criteria during the incremental lower bound load-deflection analyses, and the least upper bound collapse mechanisms that are achieved. This information is of direct value in understanding the alternate load paths involved in the load distribution which led to the analytical approach taken to redesign each of the three study bridges to ensure redundancy and to suggest guidelines for the design of similar spans for redundancy and for proposed specification provisions to ensure redundancy and after-fracture serviceability.
2. DESCRIPTION OF BRIDGES

2.1 Bridge Selection

Three bridge spans were selected for investigation in this study:

1. Simple span right bridge
2. Simple span skew bridge with 45° skew
3. Two-span right bridge

The two types of right bridges are real spans taken from the Betzwood Bridge in Montgomery County, Pennsylvania. The skew bridge was developed from the simple span right bridge and doesn't occur as a skew span in the Betzwood Bridge. All three bridges have approximately 90 ft spans and 32-ft wide roadways carrying two lanes of traffic. Designed for HS-20 live loading using the 1961 AASHTO Specifications, the Betzwood bridge was built in 1964 with an A36 steel superstructure and noncomposite 8" thick reinforced concrete deck. (18)

Figure 2.1 shows the overall plan and elevation of the Betzwood Bridge. Southbound spans 2, 3, and 8 are selected for investigation. Span 8 is the simple span right bridge in this study, and spans 2 and 3 constitute the two-span right bridge. The skew study bridge was developed from span 8. The cross section shown is common to all three bridges investigated in this study.

2.2 Description of Simple Span Right Bridge

Figure 2.2 shows plan and elevation views of the simple span right bridge. Figure 2.2(a) shows a plan view of the superstructure below the deck. The stringers and girders are equally spaced transversely at
74" c.-c. Cross bracing, floor beams and outriggers are equally spaced longitudinally at 17'-10" c.-c. Bottom laterals connect to the girders at these same locations. The actual span length is 89' - 2".

Figure 2.2(b) shows the transversely and longitudinally stiffened girder. Each girder web is 92" deep and 3/8" thick. The top and bottom flanges are 17-in. wide and change thickness from 1-1/2 in. to 2-in. as shown in the figure. Three sizes of transverse attachment plates are used. The bearing stiffeners are 7-1/2" x 1" plates welded on both sides of the web. At cross bracing locations, the connection plates are 7-1/2" x 1/2", also on both sides of the web. The remaining transverse stiffeners, longitudinal stiffeners, and bottom lateral connection plates are 3/8" thick.

Figure 2.3 shows the typical cross section of all three bridges. The 32-ft. roadway is supported by the two girders which are only 18'-6" apart. The roadway overhang is supported by stringers connected to outrigger brackets as shown. The stringers and floor beams are wide flange sections W18x45 and W24x84, respectively. The cross bracing horizontals are C7x14.75 channels, and the cross bracing diagonals are 6 x 3 1/2 x 3/8 angles.

2.3 Description of the Simple Span Skew Bridge

Figure 2.4 shows a plan view of the simple span skew bridge. Although not actually part of the Betzwood bridge, the skew bridge is considered to be exactly the same as the simple span right bridge, except for a 45° skew. The skew floor beams at the ends are assumed to consist of W24x84 sections, the same as the right floor beams.
2.4 Description of Two Span Right Bridge

The 2-span right bridge consists of southbound spans 2 and 3 of the Betzwood Bridge, as shown in Fig. 2.1. Figure 2.5 shows more detailed plan and elevation views of the 2-span right bridge.

Figure 2.5(a) shows a plan view of the superstructure of the 2-span right bridge. Like the simple span bridge, each span consists of 5 bays, although the span length is slightly less at 87'-2". Fixed bearings are located at the interior support, with expansion bearings at the ends.

Figure 2.5(b) shows the elevation view of the stiffened girder. Like the simple span bridges, the stiffened girder web is 92" deep and 3/8" thick. Unlike the simple span bridges, 3 sizes of flanges are used. All are 17" wide, and the flange thicknesses range from 3/4" near the ends and dead load inflection points to 1-1/8" in the positive moment region to 1-3/4" in the negative moment region above the interior support. A bolted field splice appears in the girder in the first span near the dead load inflection point.

2.5 Connections

Figure 2.6 shows bottom lateral connections. Fig. 2.6(a) shows the connection detail where bottom laterals cross. Only one of the lateral bracing members is continuous through the connection. Figure 2.6(b) shows the bottom lateral connection plate detail. The bottom laterals and cross bracing horizontals come together near the bottom girder flange. This view is taken from Fig. 2.3. All fasteners are specified in the drawings as 7/8" rivets.
Figure 2.7 shows the bolted girder splice in the two-span right bridge. The location of this splice is shown in Fig. 2.5(b). The flange splices employ 1/2" plates, and the web splices use 3/8" plates. The splice is adequate to develop the full design shear and moment of the girder section at that point.

Figure 2.8 shows the bearings. The fixed bearing, shown in Fig. 2.8(a), is held in place by two anchor bolts of 1-1/4" diameter. The expansion bearing is shown in Fig. 2.8(b). Longitudinal girder movement is accommodated by the rocker, while transverse movement is resisted by the welded keeper plates on the two sides.

2.6 Deck

Figure 2.9 shows a longitudinal cross section of the reinforced concrete deck in the simple span right bridge. The deck of the simple span right bridge is 8" thick, with #5 reinforcing bars for flexural reinforcement in the transverse direction, as shown in the figure. The longitudinal distribution steel consists of #4 and #5 bars, as shown. The concrete in the deck is class AA, corresponding to an ultimate strength of $f'_c = 3500$ psi. The deck in the simple span skew bridge is considered to be identical.

In the positive moment regions of the two-span right bridge, the deck cross section shown in Fig. 2.9 still applies. In the two-span bridge, however, construction joints exist at the two cross bracing near the dead load inflection points. In the negative moment region, the transverse steel is spaced at 12" which was obtained from the bridge drawings. The spacing on the drawings should have been 6". It is not known if the as-built bridge has a spacing of 6" for these bars.
3. THEORETICAL BASIS FOR UPPER AND LOWER BOUND ANALYSES

3.1 Justification of Analytical Procedure

The analytical approach used in this investigation is discussed in Art. 1.6. It is pointed out there that a reasonably "exact" prediction of the collapse load and collapse mechanism of even a relatively simple 3-dimensional two-girder bridge is quite involved and time consuming. Of necessity such an analysis must consider inelastic member behavior (steel and reinforced concrete), member instability and the effects of large displacements near the collapse load.

Fortunately, such an analysis is not necessary and an alternate approach is used. This approach uses the well known and well documented upper and lower bound techniques of the theory of plasticity. (16, 19-22) In the structural engineering field, the use of plastic theory is much more prominent in steel framed building research than in steel bridge research. The AISC specification for the design of steel buildings is greatly influenced by plastic design concepts developed over three decades. Plastic design concepts are a foundation of the new AISC Load Resistance Factor (LRFD) specification now in its first edition in 1986. (23)

The full use of plastic design for steel bridge research has been retarded due to the "shakedown" or incremental collapse phenomenon which occurs due to repetitive live loading of the structure. However, the concept of plasticity forms the basis for the new Autostress-design procedure for short span steel bridges. (24)
Under normal conditions a steel bridge is expected to be serviceable for many years and to sustain a very large number of repetitive live loads. However, upon fracture of a main girder it is unreasonable to expect continued serviceability for an extended period of time even if the bridge is designed for redundancy. It is more reasonable to expect the bridge to survive for a very short period following fracture, say a few days, a week or a month at the most, during which time the fracture is detected and steps are taken to repair the bridge. During this short time it is also improbable that the bridge is subjected to extreme design loading conditions. Thus larger allowable stresses or smaller load factors are appropriate when designing for redundancy.

Since little shakedown or incremental plasticity should occur from fracture until detection of the fracture it appears reasonable to apply the theory of plasticity to compute the load capacity and collapse mechanism for the fractured structure. In addition the design for redundancy can employ plastic rather than elastic principles, if necessary, since the members of the redundant load path are expected to be serviceable for a very short period of time and be subjected to very few cycles of somewhat lower live loading.

This chapter compares the basic conditions for elastic and plastic analyses, presents the theoretical basis for the upper and lower bound theorems of plasticity and discusses their application to the study of the three two-girder bridges included in this investigation.

3.2 Conditions for Correct Elastic and Plastic Analyses

Table 3.1 shows the three fundamental conditions necessary for a
correct elastic analysis as compared with those for a correct plastic analysis. In the table, the conditions are illustrated by applying them to the analysis of a fixed-ended steel beam which is subjected to a uniformly distributed load.

Evaluation of the plastic condition varies depending on the type of member considered. For example, for the steel beam in flexure shown in Table 3.1, it is the fully plastic moment, $M_p$. For a steel tension member it is the yield load, $P_y$. For a reinforced concrete slab it is the ultimate moment capacity, $M_u$, along a yield line.

With regard to continuity or compatibility the situation in plastic analysis is just the reverse of that which exists in elastic analysis: In plastic analysis attainment of the plastic condition is required at a sufficient number of locations to allow the structure (or part of it) to deform as a mechanism or linkage. Of course, equilibrium must be satisfied regardless of the analytical approach.

Two useful methods of plastic analysis take their names from the particular conditions being satisfied:

1) Mechanism Method: Satisfies Equilibrium condition.
2) Statical Method: Satisfies Plastic condition.
In the Mechanism Method a mechanism condition is assumed (such as the linkage shown in Table 3.1) and the resulting equilibrium equations are solved for the ultimate load. The ultimate load is correct only if the plastic condition also happens to be satisfied. Otherwise, this value will always be larger than the correct value (upper bound).

In the Statical Method, an equilibrium distribution of stress resultants (moments, axial tension, etc.) is assumed such that no stress resultant exceeds the corresponding plastic condition anywhere in the structure, and the ultimate load is computed from the equilibrium condition. The resulting ultimate load is correct only if the mechanism condition also happens to be satisfied. Otherwise this value will always be smaller than the correct value (lower bound).

3.3 Principle of Virtual Displacements

In the Mechanism Method the principle of virtual displacements is useful in formulating the equilibrium condition. (17,22,25)

The principle of virtual displacements states:

If a virtual displacement is applied to a structural system which is in equilibrium with a set of applied loads, the total work done by the applied loads acting through the virtual displacements plus the internal stress resultants (moments, axial forces, etc.) acting through the internal virtual distortions is equal to zero.

A virtual displacement is defined as:

Any displacement or distortion of a structure, large or small, real or imaginary, given to a structure such that all applied forces and internal stress resultants remain constant in magnitude and direction during the virtual displacement.
Since all applied forces and internal stress resultants must remain unchanged during a virtual displacement, then for real structures virtual displacements must be imaginary. Since any imaginary displacement will qualify, those that result in the simplest analytical equations are normally used. Such equations will result when formulating the equilibrium condition in the Mechanism Method if the material properties are assumed to be rigid-plastic. This accounts for the straight line or planar deflected shapes selected for a mechanism analysis.

3.4 Theorems of Plastic Analysis

3.4.1 Lower Bound Theorem

For a given structure and loading if there exists any distribution of stress resultants throughout the structure which satisfies the plastic condition and is statically admissible with the loading, then that load must be equal to or less than the correct ultimate load.

A simple proof of this theorem can be offered for a flexural member, as follows:

Let external loads, $P_u$, and internal moments, $M_j$, be associated with the correct collapse mechanism, and $\alpha P_u$ and $M_j'$ be associated with the lower bound Statical Method, and $M_{p_j}$ be the plastic moment at the $j^{th}$ plastic hinge.

The principle of virtual displacements states that:

$$\sum P_i \delta_i = M_j \delta_j$$  \hspace{1cm} (3.1) $$

where $P_i = i^{th}$ applied load
\[ \delta_1 = \text{virtual displacement of } P_1 \]
\[ M_j = j^{th} \text{ bending moment } = M_{pj} \]
\[ \theta_j = \text{virtual distortion (rotation) of } M_j \]

Let \( \theta_j \)\((j = 1, 2, \ldots, n)\) be at the true location of plastic hinges with rotations consistent with the direction of the plastic moments \( M_j \) \((j = 1, 2, \ldots, n)\) so that \( \Sigma M_j \theta_j \) is always positive. This means that \( \Sigma P_j \delta_i \) must also always be positive.

Then for the correct ultimate loads, \( P_u \), the virtual work is given by,

\[ \Sigma P_{ui} \delta_i = \Sigma M_j \theta_j \]  \hspace{1cm} (3.2)

For a lower bound to the correct ultimate load,

\[ \Sigma \alpha P_{ui} \delta_i = \Sigma M'_j \theta_j \]  \hspace{1cm} (3.3)

Subtracting Eq. 3.3 from Eq. 3.2

\[ (1-\alpha) \Sigma P_{ui} \delta_i = \Sigma (M_j - M'_j) \theta_j \]  \hspace{1cm} (3.4)

Since \( M_j = M_{pj} \) and \( M'_j \leq M_{pj} \), then \( (M_j - M'_j) \geq 0 \) and \( (1-\alpha) \geq 0 \). Thus \( \alpha \leq 1 \), confirming that the applied loads must be equal to or less than the correct ultimate load.

3.4.2 Upper Bound Theorem

For a given structure and loading, the load computed on the basis of an assumed mechanism must be equal to or greater than the correct ultimate load.

A simple proof of this theorem is again offered for a flexural member, as follows:
Let external loads, $P_u$, and internal moments, $M_j$, be associated with the bending moment diagram corresponding to the correct ultimate load, and $\beta P_u$ and $M_j'$ be associated with the upper bound Mechanism Method.

For the correct ultimate loads, $P_{ui}$, the external and internal virtual work is again given by Eq. 3.2.

For an upper bound to the correct ultimate load

$$\sum \beta P_{ui} \delta_i = \sum M_j' \Theta_j$$  \hspace{1cm} (3.5)

where both sides are again positive.

Subtracting Eq. 3.2 from Eq. 3.5

$$(\beta-1) \sum P_{ui} \delta_i = \sum (M_j' - M_j) \Theta_j$$  \hspace{1cm} (3.6)

Since $M_j' = M_{pj}$, and $M_j \leq M_{pj}$, then $(M_j' - M_j) \geq 0$

and $(\beta-1) \geq 0$. Thus $\beta \geq 1$ confirming that the applied loads must be equal to or greater than the correct ultimate load.

3.4.3 **Uniqueness Theorem**

If for a given structure and loading, the ultimate loads computed by the upper and lower bound theorems coincide then this load is the unique collapse load for the structure.

3.5 **Example**

Figure 3.1(a) shows a fixed-ended beam of length, $L$, with a midspan concentrated load, $P$, together with the stress-strain curve of the material. The resulting plastic moment capacity of the beam is $M_p$. An
upper and a lower bound solution for $P$ are required as well as the unique solution.

An upper bound solution for $P$ is obtained using the mechanism method illustrated in Fig. 3.1(b). Three plastic hinges are required to produce a mechanism. They can be located anywhere along the beam provided the load $P$ is located between two of the plastic hinges. Figure 3.1(b) shows assumed locations of the three plastic hinges together with the assumed virtual displacement $\Theta$.

The external virtual work, $W_e$, is

$$W_e = \frac{PL\Theta}{6} \quad (3.7)$$

The internal virtual work, $W_i$, is

$$W_i = \frac{8M_\Theta}{3P} \quad (3.8)$$

The resulting equilibrium equation is given by $W_e = W_i$, or

$$\frac{PL\Theta}{6} = \frac{8M_\Theta}{3P} \quad (3.9)$$

from which an upper bound solution, $P_u$, is

$$P_u = \frac{16M}{L^2} P \quad (3.10)$$

A lower bound solution for $P$ is obtained using the statical method illustrated in Fig. 3.1(c). An equilibrium bending moment diagram can be found from midspan loading of a simple span beam of span $L$, giving a bending moment at midspan of $\frac{PL}{4}$. The resulting triangular moment diagram can be positioned anywhere within the boundaries, indicated
by plus and minus $M_p$ as shown in the figure. The location shown in Fig. 3.1(c) is one possible choice.

Equilibrium therefore requires that, at midspan

\[
\frac{PL}{4} = 1.5 M_p \tag{3.11}
\]

from which a lower bound solution, $P_{\ell}$, is

\[
P_{\ell} = \frac{6}{L} M_p \tag{3.12}
\]

The unique solution is obtained when the upper and lower bound solutions coincide. This will occur when plastic hinges occur at the two ends of the beam and at midspan. An upper bound solution, $P_p$, is obtained from

\[
\frac{PL^2}{2} = 4M_p \theta
\]

or

\[
P_p = \frac{8}{L} M_p \tag{3.13}
\]

The corresponding lower bound solution, $P_p$, is obtained from

\[
\frac{PL}{4} = 2M_p \tag{3.14}
\]

or

\[
P_p = \frac{8}{L} M_p
\]

The unique or correct solution is therefore $P = \frac{8}{L} M_p$

3.6 Finite Element Method as a Lower Bound

As previously mentioned, the finite element technique is a numerical procedure for solving complex mechanics problems with an accuracy acceptable to engineers. In practice, most problems are too
complicated for a closed-form mathematical solution. A numerical solution is required, and the most versatile method available today that provides it is the finite element method.

Since this method has become widely used and accepted during the last three decades, the details of its underpinnings, formulation, and computational procedures are not reproduced in this report. Some fundamentals regarding its basis are discussed however, such as Rayleigh-Ritz. This is a method that reduces a continuum problem to one with a finite number of degrees of freedom. Its use of total potential energy suggests tactics that, when supplemented with physical insight, leads to a theoretical understanding of the conditions under which the finite element method may be used as an acceptable "lower bound" for the purpose of analyzing the study bridges.

Finite elements appeal to structural engineers because they resemble pieces of an actual structure. Often physical intuition, more than mathematical intuition, guides the modeling and treatment of boundary conditions. A central question, however, occurring repeatedly in any approximate method such as this relates to how good is the approximate solution. Also, where does the computed solution lie, above or below the exact solution? To answer these questions, at least in part, both mathematical and physical insight are needed.

When dealing with finite elements, it is generally accepted that the following conditions must hold for monotonic convergence to the exact solution. First, the displacement field within an element must be
continuous. Second, the element must be able to assume a state of constant strain. These two conditions are satisfied in the limit with increasing mesh refinement. Third, rigid body modes must be present. Fourth, elements must be compatible. Fifth, an element should have no preferred directions (geometrically isotropic or spatially isotropic). If some of these conditions are violated, convergence to the correct results may be slowed or even precluded. Also, the computed results may be less than or greater than the exact results.

A brief review of the result of using a Rayleigh-Ritz technique is warranted in order to estimate whether the finite element solutions produced in this study actually serve as the intended lower bound (the computed loading must be less than or equal to the correct load) or as an upper bound (the computed loading must be greater than or equal to the correct load).

From a mathematical perspective, the Raleigh-Ritz model of a structure is either exact or it is too stiff. This occurs because the structure is permitted to displace only into shapes that can be described by superposing terms of the assumed displacement field. Therefore, the correct shape is excluded, unless the assumed displacement field happens to contain it. Effectively the assumed field imposes constraints that prevent the structure from deforming the way it wants to. Constraints obviously stiffen a structure. In effect the method, generally but not always, creates a structure that is stiffer than the real one.
If all convergence criteria are met, and a linear elastic structure is properly constrained to match reality, then a structure carrying a single load $P$ has the single corresponding displacement computed as a lower bound. That is, the approximate solution yields a displacement field such that the work done by the load is less than the exact value. Note that not all displacements are necessarily underestimated though. In other words, when an element representation is too stiff, then the displacement of the model is less than the displacement of the real structure if the model and the real structure are subjected to the same load. Conversely, to obtain the same displacement in the model as in the real structure, a larger force must be applied to the model since the stiffness of the model is greater than the stiffness of the real structure.

Note that in the above discussion the strain energy $U$ is equal to the external work $W$, so the approximate solution underestimates $U$ when loads are prescribed. If displacements are prescribed, $U$ is overestimated because extra force is needed to deform an overly stiff structure. It is important to realize that when loads and displacements are prescribed, $U$ may be too high or too low.

As stated, not every displacement is underestimated in the classical Rayleigh-Ritz technique. Also stresses, as calculated from displacements may be low in one location of a real structure and high in another. Additionally, when elements exist that generate a discontinuous field by overlapping or by separating from one another,
as in the case of modeling a fracture, one cannot state categorically that the structure of elements is too stiff.

Thus when realistic structures are assembled from a variety of different element types, some of them incompatible, and where both loads and displacements are prescribed, it cannot be rigorously proven that the finite element models are too stiff or too flexible. That is, the finite element analysis of most real structures probably is between an upper and lower bound solution. The question then is what can be done as far as the analysis is concerned to "move" the solution toward a lower bound for obtaining the load-deflection curves for the study bridges.

There are several things that can be done regarding behavioral assumptions and analysis procedures to accomplish the above goal. First, in the study bridges, the stiffness of the overall structure is reduced to produce increments of loads less than those the real bridge can carry by requiring all stress resultants to be at or below the appropriate yield criteria at the end of each loading increment. Second, the analysis considers elastic-perfectly plastic behavior instead of including any strain hardening, and is based on small deflection considerations. Third, it accounts for member instability by the reduction of member stiffness to near zero at or near the buckling load or instability limit load. These factors, taken together with the previous discussion, will result in a reduction of the load carrying capacity of the finite element models of the study bridges.
Considering the complexity of behavior of the real bridge, the finite element modeling techniques and behavioral assumptions used in this study cannot be mathematically proven to produce an absolute lower bound to the true load-deflection curve at all points. However, based on the use of some non-conforming elements, the physical assumptions of behavior, and the analysis procedures used herein, (refer to Chapter 4), it is reasonable to assume that an approximate overall lower bound is achieved.
4. FINITE ELEMENT MODELING OF TWO-GIRDER BRIDGES

4.1 Introduction

4.1.1 Rationale for Using the Finite Element Technique

Since its popularization approximately thirty years ago, beginning with the landmark paper by Clough, Turner, Martin, and Topp, the finite element technique, coupled with digital computers, has been amply demonstrated to handle complex structural systems with a degree of accuracy previously unattainable in structural analysis. This does not imply, however, that it is considered to be a panacea in any sense. The technique can be misapplied or the results misinterpreted if the engineer is not well-grounded in the fundamentals of structural behavior. It is important to realize though that this technique has had these three decades to be refined and enhanced to a level where it is considered to be a viable, practical tool to be applied to a wide variety of structural problems.

The finite element methodology today serves as a widely accepted sophisticated, accurate, and reliable framework to study the broad range of behavior exhibited in bridge structures, for example, static or dynamic, linear or nonlinear, and elastic or inelastic behavior. Interactions among structural components can be studied to a level of detail that is appropriate to the needs of the analyst and to the complexity of the physical situation under investigation. This capability to vary the complexity of the mathematical model of the physical structure as well as vary the level of the discretization of
the model yields the necessary flexibility in technique that is required in this investigation.

With the broad spectrum of capabilities available today in terms of both hardware and software, more realistic, three-dimensional interaction models can be constructed and analyzed. This has permitted the researchers to examine bridge behavior to a more detailed level of accuracy from which more reliable estimates of overall system behavior can be gleaned.

In order to accomplish these detailed analyses in a reasonable and expeditious manner, a true computer-integrated engineering system is required. By this the researchers mean one which integrates the finite element analysis, computer graphics, and the voluminous databases that may be produced from the structural analysis. Such systems have only recently become available for use on mini-computers and desktop computers. As of today there are a number of analysis programs available on a wide variety of 32-bit hardware systems. These programs offer the graphics pre- and post-processors which provide the structural engineer with the ability to readily change the complexity of the model as desired. They also permit the analyst/designer to have control over system facilities for the production of graphical output so that the response of these three-dimensional structures can be readily visualized and interpreted from the patterns which can be observed in the displays.
4.1.2 Need for Three-Dimensional Analysis

As mentioned earlier in this report, this investigation employs the finite element technique as the theoretical foundation for the lower bound analyses. It is an accurate and efficient analytical method for determining the response of bridges including displacements, stresses, and reactions.

The actual bridge structures investigated in this project are interconnected, three-dimensional assemblages of structural components or elements. In attempting to model these real bridge structures as accurately as feasible, a library of various finite element types is utilized in the three dimensions. Based on the experience of the researchers this choice, in contrast to a two-dimensional grillage type of analysis, affords the opportunity to study interactions among the various bridge components. It also permits the investigation of displacements and the load redistribution in each component, for example the deck elements, bottom laterals, flanges and webs of the girders (fractured or unfractured).

For the scope of problems mentioned in the preceding paragraph, a number of finite element based, computer-integrated systems such as ADINA, ANSYS, GTSTRUDL, and NASTRAN are widely available for production use. The next section discusses some of the particular features of the system, GTSTRUDL, that is used in this project.

4.1.3 Overview of GTSTRUDL

GTSTRUDL, as representative of state-of-the-art finite element analysis systems, was chosen to satisfy three basic purposes for the bridge
structures examined in this study: (1) a research and development tool to explore the complex nature of the response of the bridges; (2) a commonly used tool that exists at a large number of offices and installations on a variety of hardware devices both large and small; and (3) a general purpose practical tool for a design engineer. The tool chosen for the lower bound analyses is relatively flexible in scope and is widely available to other practitioners and researchers.

GTSTRUDL is a computer-aided structural engineering software system for assisting an engineer in the structural analysis and design process. It integrates automatic mesh and data generation, finite element analysis, interactive graphics, and structural database management. Some of the key features and reasons for its use in the investigation include the following:

a) Broad range of member and finite element types. GTSTRUDL contains 5 member types (constant or variable cross-section) and 35 conventional, isoparametric, and hybrid formulation finite element types. Special transition elements are also provided.

b) Automatic mesh generation. Parts or all of a structure mesh may be generated using objects consisting of collections of joints, members, finite elements and other objects. These objects may be moved or repeatedly copied to any position in space.

c) Efficient equation solvers. A variable band, variable partition, sparse equation solver is used in conjunction with memory and disk management procedures.

d) Structural engineering terminology. A command structured Problem-Oriented Language (POL) is provided in the man-machine interface.
e) Selective results processing. All or any portion of the problem data and results may be selectively displayed, tabular or graphical, for loading conditions, joints, members, and finite elements.

f) Structural database management. The structural problem database may be saved for future use, review, or modification. The storage, retrieval, and updating are controlled by the user.

g) Graphical display. A wide variety of graphical display options are available to view structural geometry, topology, and response. Capabilities include 2-D and 3-D color plots (with rotation), hidden lines removal, selective annotation, automatic or user controlled scaling, windowing, contour plots, and overlay plots.

h) System support. The software is validated and supported by a professional staff at the Georgia Institute of Technology.

The analytical capabilities of the software provide an environment within which the iterative process of modeling, discretization, analysis, and interpretation of results can be performed to meet the basic needs of the lower bound analyses of this project.

4.2 Finite Element Discretization of Simple Span Right Bridge

Figure 4.1 shows the simple span right bridge viewed from underneath. This 3-dimensional view shows the interconnections of the various structural components. The figure also shows the identification scheme for the various portions of the bridge. The 5 bays are numbered consecutively starting from the expansion bearing end. The 6 cross bracing locations are similarly numbered based on the bays they lie between. For clarity, stiffeners and some other details are not shown.
Figure 4.2 shows the finite element model of the simple span right bridge. This view is in the same orientation as that of Fig. 4.1. The discretization employed is significantly more complex than those used for the overall structure models shown in Refs. 11 and 12. The initial model with the girder fracture has 1646 nodes and 9136 degrees of freedom. This discretization is relatively fine, in order to accomplish the following:

1. In addition to serving as a lower bound analytical approach, finite element analysis also serves as an equivalent laboratory test, providing data to which a simpler theoretical approach must compare.

2. With such complex structures as the study bridges and not knowing ahead of time what elastic-plastic behavior and alternate load paths will develop, the discretization must be sufficiently fine throughout to capture the real bridge behavior.

3. Substructuring is not part of this study. The primary interest is in global behavior. Thus, the global discretization must be sufficiently fine to capture all behavior of potential interest.

Table 4.1 summarizes the finite elements used in the model. Four types of elements are used: 3-D truss elements, 3-D beam elements, plane stress elements, and plate (flat shell elements). As listed in the table, 3 GTSTRUDL planar elements are used:

1. The CSTG triangular element assumes constant stresses and strains within the element and along the boundaries.

2. The PSHQ hybrid quadrilateral element assumes a quadratic field for stresses within the element and linear variation of displacements on the element boundaries.
3. The SBHQ6 hybrid stretching and bending quadrilateral combines:

- PSHQ in-plane

- BPHQ bending. The BPHQ hybrid quadrilateral is a compatible element with quadratic stress field within the element, cubic transverse displacement along the boundaries, and linear normal rotations along the boundaries.

- A fictitious rotational stiffness for suppressing instabilities in shell problems.

The in-plane and bending stiffness are uncoupled. The modeling of specific structural components is described in greater detail in the following articles.

4.2.1 Main Girders and Stringers

Figure 4.3 shows the finite element discretizations employed for the stringers and main girders in a typical bay. The flanges of the main girders and stringers are modeled with 3-D beam elements. Plane stress elements are used to model the webs. There are 5 such elements through the depth of the girder web. Although the out-of-plane degrees of freedom of the web elements are undefined, the girders and stringers in the model are still able to move freely in 3-D space. The use of plane stress elements in the web neglects the small bending stiffness contributed by the webs when out-of-plane movement does occur. Plate elements can be used instead for modeling the webs, but the additional expense is not warranted.

Transverse and longitudinal stiffeners in the girders are also modeled using 3-D beam elements. Since the focus of this finite element study is the 3-dimensional behavior of the structure as a whole, the
discretization neglects the gaps at the ends of the transverse stiffeners and floor beam connection plates. These stiffeners are modeled as being fully attached to the girder flange. This approach assumes that the local web gap detail has a negligible effect on the global stiffness of the bridge.

Each of the 5 bays of the girders are modeled the same, except where the fracture is imposed. Figure 4.4 shows the modeling of the main girder fracture, imposed at midspan in bay 3 of the west girder. The fracture is assumed to pass through the bottom flange and the full depth of the web, but not through the top flange.

4.2.2 Cross Section at Floor Beam Location

Figure 4.5 shows the finite element discretization employed at a cross section at a floor beam location. There are six such floor beam locations along the bridge, as shown in Figs. 4.1 and 4.2. The modeling considerations for the floor beams and outriggers are similar to those for the girders and stringers. Flanges and stiffeners are modeled using 3-D beam elements. The floor beam flanges are considered not to be coped where the floor beam is attached to the girders. Webs are modeled using plane stress elements.

4.2.3 Cross Bracing and Bottom Laterals

The discretization of cross bracing is shown in Fig. 4.5. The plane of the bottom lateral bracing system is indicated in Fig. 4.5 and shown in Fig. 4.6. The members in the cross bracing and bottom laterals can be considered to have negligible depth, unlike the floor beams, stringers and girders. The cross bracing and bottom laterals can thus be modeled.
with 3-D beam elements and truss elements. Beam elements are used for the horizontal cross bracing members and the bottom laterals. Truss elements are used to model the cross bracing diagonals. All bottom laterals are assumed to be continuous from one girder to another, as shown in Fig. 4.6(a). The assumption that both members are continuous, instead of only one, has little effect on overall structural stiffness. In order to ensure that the bottom laterals behave primarily as axial force members, while maintaining the connection where they cross, moment hinges are imposed at the ends of the bottom laterals. This is shown in Fig. 4.6(b).

4.2.4 Reinforced Concrete Deck

Figure 4.7 shows the finite element discretization of the bridge deck. Heavy lines on the figure indicate the location of the cross bracing and girders underneath the deck.

Of all the structural components in this bridge, the deck is the most difficult to model. The modeling considerations for the deck can become quite complex if an attempt is made to simulate its structural behavior exactly. There are several limit states to consider, such as crushing and cracking, as well as a significant range of nonlinear load-deformation behavior. These considerations are discussed in greater detail in Art. 4.5.1.4.

Since the bridge is constructed with a non-composite deck another question is the degree of composite interaction. Analytical and experimental experience has indicated that for load levels up to the elastic limit, one can assume complete interaction between the girders
and the deck.\textsuperscript{(14)} As discussed in Chapter 1, it was decided to assume for modeling purposes that the deck is composite with the girders and stringers. Complete interaction is assumed through the full range of behavior.

It is evident that the element employed must account for in-plane stresses as well as bending stresses, since the deck functions as a top flange when it is assumed to act compositely with the girder. Since it was decided not to monitor the progression of concrete cracking through the depth of the deck, the use of layered elements was ruled out.\textsuperscript{(26,27)} Thus, a flat thin-shell element is employed to model the deck.

Complete composite interaction is modeled by having the deck elements share nodes with the top flange of the girders and stringers. A side-effect of this approach is to lower the center of gravity of the deck. This modeling approximation is conservative, consistent with the lower-bound approach underlying this implementation of finite element analysis. Another side-effect is to make the stringers continuous where they cross over the floor beam.

Since the deck is not heavily reinforced, the reinforcing steel has a negligible effect on the stiffness of the deck in the uncracked condition. The presence of the reinforcing steel is therefore neglected in the computation of element properties. As a byproduct of this assumption, concerns about modeling such things as bond degradation, dowel action, and tension stiffening can be neglected.
Cracking due to creep and shrinkage is also considered not to affect deck stiffness.

4.2.5 Bearings

A significant modeling issue is the number of degrees of freedom to specify at the supports. Modeling of supports is known to have a significant influence on stress resultants for horizontally curved girder bridges.\(^{28}\) Modeling of supports has less sensitivity on straight girder bridges. Once a midspan girder crack is imposed, however, the sensitivity to boundary condition idealization is not well known. The bridge becomes asymmetrical, and the stress resultants may be significantly affected by the support conditions.

In order to investigate the sensitivity of this bridge to support modeling assumptions, a comparative study was performed. Figure 4.8 shows the 3 support modeling alternatives considered. Case (b) just constrains rigid body motion in the horizontal plane, and cases (a) and (c) are overconstrained. Elastic finite element analyses were conducted for all 3 cases.

The results are summarized in Fig. 4.9. The support reactions were found to be significantly affected by the choice of boundary conditions. Fig. 4.9(b) shows that in order to avoid overconstraining the horizontal boundary conditions, a lateral deflection of 0.67" occurs at the expansion bearing. This is more movement than would be permitted by the actual expansion bearing used (shown in Fig. 2.8(b)), due to the keeper plates. Thus, the constraint condition shown in Fig. 4.8(a) is used in the initial models, recognizing the lateral restraint
contributed by the keeper plates. Early in the loading (Chapter 5) the keeper plates and anchor bolts fail thus reverting to condition 4.8(b).

4.3 Finite Element Discretization of Two-Span Right Bridge

Figure 4.10 shows the finite element model of the two-span right bridge. This view is in the same orientation as that of Figs. 4.1 and 4.2. The model is symmetrical about the fixed bearing support at the center. The 10 bays are numbered consecutively starting from the north expansion bearing end. The 11 cross bracing locations are numbered based on the bays they lie between, as shown, for example in Fig. 4.7.

The discretization employed is somewhat coarser than that used for the simple span right bridge. The initial unloaded model with the girder fracture has 2276 nodes and approximately 12000 degrees of freedom, which is about 30% more than the simple span right bridge model.

The modeling of specific structural components is described in the following articles, emphasizing the aspects that are different from the simple span right bridge model.

4.3.1 Main Girders and Stringers

Figure 4.11 shows the finite element discretization of stringers and girders in bay 4 of the two-span right bridge. The discretization of stringer in Fig. 4.11(a) is identical to that for the simple span right bridge shown in Fig. 4.3.
Figure 4.11(b) shows the discretization of girder flanges and web, transverse stiffener, floor beam connection plate, and longitudinal stiffener in bay 4. There are three transverse stiffeners on the girder in a typical bay. In bays 2, 3, 8, and 9, however, there are only two transverse stiffeners.

Longitudinal stiffeners are placed near the bottom flange in bay 4 through bay 7, which is the negative moment region, while longitudinal stiffeners are placed near the top flange in the other bays.

4.3.2 Cross Section at Floor Beam Location
Figure 4.12 shows the finite element discretization of a cross section at a floor beam location. The cross section is identical to that of the simple span right bridge, except for girder flange thickness. The floor beams and outriggers are discretized more coarsely compared to those of the simple span bridge.

4.3.3 Cross Bracing and Bottom Laterals
Figure 4.12 also shows the cross bracing and bottom laterals. The configuration of the bottom lateral bracing system shown in Fig. 4.13 is identical to that for the simple span right bridge.

4.3.4 Reinforced Concrete Deck
Figure 4.14 shows the finite element discretization of the deck for the two-span right bridge. The deck in bays 2, 3, 8, and 9 is divided transversely into six lines of elements to coincide with the arrangement of the transverse web stiffeners on the girders, as
described in Art. 4.3.1. The deck is discretized more coarsely than
the single span right bridge in the longitudinal direction as well.

It is anticipated that the tensile stresses in the deck in the negative
moment region will result in cracking. Complete composite interaction
is modeled in the positive moment region of the two-span bridge as is
done for the simple span right bridge. Bays 5 and 6 in the negative
moment region are non-composite.

Figure 4.15 shows the finite element discretization of the non-
composite deck in bays 5 and 6. As shown in Fig. 4.15(a), the deck is
separated from the top flanges of the girders and/or stringers, but
linked by very stiff truss elements to carry loads vertically to the
girders and stringers. This is shown in Fig. 4.15(b).

4.3.5 Bearings
The comparative study performed for the simple span right bridge
indicates the effects of the choice of constraint conditions on the
results of the analysis. Figure 4.16 shows the initial boundary
conditions assumed for the two-span right bridge. Eight horizontal
restraints are imposed including six transverse restraints, and two
longitudinal restraints at the fixed bearings.

4.4 Loading of the Simple Span and Two-Span Bridges
Only static finite element analysis is performed on the finite element
models representing the study bridges. Thus only static dead and live
loads are applied. Although the models of the bridges are subjected to
a mid-span fracture of one of the two main girders, crack propagation
and its driving force are not the main focus of this study. Dynamic effects at the instant of girder fracture are neglected, as well.

Dead load consists of dead weight computed by GTSTRUDL in addition to applied loads due to curb, parapet, railing and future wearing surface. The level of live load chosen is the AASHTO HS-20 truck (rather than HS-25), for both lanes because the bridge is designed for the HS-20. Also, the HS-20 provides a convenient reference load.

4.4.1 Impact and Load Factors

Although no dynamic analyses are performed in the finite element analyses described in this report, the dynamic effects represented by the AASHTO impact factor are accounted for. The question arises about what value of impact factor to use. The span length of 89 ft. for the simple span bridges suggests initially that an impact factor of less than 30% could be used. An impact factor of 30% is chosen for all three bridges because of increased bridge deflections and resulting dynamic effects expected to occur after the fracture of a main girder.

A second question arises about what values of load factors to use. Should the dead load factor be 1.3, 1.0 or something else? What about the live load factor? These are not trivial questions; the analytical results will differ substantially depending on the choice of load factors. The answers to these kinds of questions depend on the purpose of the finite element analyses. The purpose of the finite element analyses is to provide a lower bound estimate of the load-deflection behavior and capacity of the existing bridges, not to provide a "go/no
go" assessment of a fracture-damaged design bridge for some factored design load. Design for redundancy is addressed in Chapter 7.

In lieu of code provisions for such matters, it was decided to use a load factor of 1.0 for dead load for purposes of the lower bound analyses. The first question to be answered by the finite element analyses is whether or not the bridge is redundant under its own dead load. It is inconsistent to use a load factor other than 1.0 if this is indeed the question to be answered.

If the bridge can carry its own dead load, then the real variable of interest becomes the number of live loads that it can carry. Thus, it was decided to use a load factor of 1.0 for the live (L+I) load as well, and let the finite element analysis results indicate how many live loads the bridge can sustain.

4.4.2 Simple Span Right Bridge

The total dead load computed by and input to GTSTRUDL for this bridge is 615 kips. Since the finite element model is discretized to model the as-designed conditions closely, this load can be expected to be more accurate than that used in the original design calculations.

Figure 4.17 shows the application of the HS-20 wheel loads to the deck of the bridge. The AASHTO traffic lanes and wheel positions are located to the right as shown in order to maximize the live load applied to the fractured girder. Longitudinally, the trucks are placed where in conventional design they would induce the highest moment in the fractured girder had it not been fractured. The wheel loads are
distributed as equivalent concentrated loads to the deck slab nodes through the use of simple statics. The heavy lines in Fig. 4.17 indicate the locations of the underlying girders and cross frames. The total live load amounts to 187 kips, resulting in a live load/dead load ratio for this bridge of approximately 0.3.

4.4.3 Two-Span Right Bridge

The total dead load for the two-span right bridge computed by and input to GTSTRUDL is 1124 kips.

Figure 4.18 shows the application of the HS-20 wheel loads to the deck of one span only. Placing wheel loads on both spans may cause more damage in the negative moment region but will decrease the damage in the vicinity of the fracture. The wheel load position that maximizes the bending moment of the fractured girder would be different from that of the single span right bridge, if the only concern were elastic behavior. Since it is difficult to anticipate inelastic behavior and to decide on the appropriate load position, the wheel loads are arranged identically to the simple span right bridge.

4.5 Limit State Criteria Employed

Table 4.2 summarizes the limit state criteria employed for the various components of the finite element models for the simple span and 2-span bridges. Although the bridges are designed in accordance with the 1961 AASHTO Specifications, the limit states were formulated wherever possible according to the intent of the 1983 AASHTO Load Factor Design provisions.
4.5.1 Cross Bracing

Typical cross bracing is shown in Fig. 2.3 and modeled as shown in Fig. 4.5. The compression limit state for the diagonals (modeled as truss elements) is the inelastic column buckling strength, as specified by AASHTO Formula 10-151. The tension limit state is taken to be the yield strength. For the horizontal members, both the beam-column stability and strength are checked. The horizontal member is considered to be braced by the presence of the catwalk between the diagonal members which is shown in Ref. 18 but not shown on Fig. 2.3. End connections are assumed to be strong enough to develop the full limit values.

4.5.2 Bottom Laterals

Both tension and compression limit states must be defined for the bottom laterals. The only bending that they are considered to carry is due to their own weight. In tension, the limit state is taken to be the yield strength. In compression, the column buckling limit state takes into account the influence of the other diagonal member crossing at mid-length on buckling in the vertical plane. With both bottom lateral diagonals in a bay assumed to be continuous, as shown in Fig. 4.6(a), the effective length of the compression member is reduced by 50%, increasing the elastic buckling load fourfold. This shorter effective length accounts for the buckling load of 188 kips for the compressive bottom laterals shown in Table 4.2. End connections are assumed to be strong enough not to fail before the member itself fails.
4.5.3 Flexural Members

In the stringers and floor beams, the plastic moment $M_p$, or the reduced plastic moment $M_{plc}$, reduced due to the presence of axial force, would normally be taken as the limit criterion. In components having finite depth and constructed of several elements through the depth, however, tracking the moment in the overall component is far from straightforward. The axial forces in the flanges are monitored instead and compared against the yield force, as an indicator of plastic moment. The plastic moment, similarly monitored via the flange forces, is also taken to be the limit criterion in the top flange of the west girder above the full-depth girder fracture. The axial force in the bottom flange of the unfractured plate girder is similarly monitored.

4.5.4 Reinforced Concrete Deck

As with modeling of the deck, the selection of limit state criteria can become quite complex but must remain fairly simple. The focus of this study is the global behavior of a two-girder bridge with a girder fracture, not a rigorous analysis of progressive failure in reinforced concrete slabs.

Ideally, post-elastic modeling of the reinforced concrete deck would at least account for the nonlinear nature of the load-deformation behavior in compression, the concrete crushing in compression, the concrete cracking in tension, and the reinforcing steel yielding in tension.

In bending, the behavior of reinforced concrete slabs can be approximated as trilinear in nature as shown in Fig. 4.19. The moment-curvature relationship can be further idealized as elastic-plastic,
with the elastic slope corresponding to the cracked or uncracked section. This is consistent with the elastic-plastic behavior assumed for the steel components in this bridge model.

A basic question surfaces on how to determine the moment-curvature relationship for the situation arising in the finite element model of this bridge. The problem is that the use of moment-curvature relations for the bridge deck requires the use of moment-thrust-curvature relations due to the presence of axial forces in the bridge superstructure. In addition, the biaxial bending of the deck slab requires the adoption of two-dimensional moment-curvature relationships.\(^{27}\) Needless to say, these are not applicable for general usage. Thus, a limit state criterion based on some simplified moment-curvature relationship is not available.

The approach taken for the finite element analyses of the bridges in this study is to use a simple lower bound limit state criterion to deal with this complex situation, yet not err too much on the side of nonredundancy. Therefore concrete tension cracking is defined as the limit state. When the surface tensile stress exceeds \(7.5\sqrt{f'_c}\), discrete cracks are imposed in the finite element model for subsequent analyses. This approach conservatively neglects the post-cracking stiffness of the cracked deck elements and the contribution of the steel reinforcing bars. It also assumes a constant value for the limit state. This is consistent with the view of the finite element results as lower bound analyses.
Discrete cracks are imposed in the model when the limit state exceedance is confined to a line. Alternately, elements are softened when the surface tensile stress limit value is exceeded in a wider region of the deck. Softening is accomplished by reducing the modulus of elasticity of the deck elements by a factor of 1,000.

4.5.5 Bearing

Limit state criteria are needed for the lateral load capacity of the bearing. As suggested by Fig. 4.9(a), imposing the through-depth girder fracture causes very high support reactions in the horizontal plane. Conservatively, ignoring the restraining effects of friction, the only resistance to horizontal forces at the fixed bearings is provided by two 1-1/4" diameter anchor bolts as shown in Fig. 4.8(a). The shear capacity of these bolts, shown in Table 4.2, dictates the fixed bearing capacity. At the expansion bearings (again ignoring friction), the only resistance to lateral forces is provided by the keeper plates at the top of the rocker, as shown in Fig. 4.8(b). The keeper plate capacity shown in Table 4.2 is determined from a yield line analysis.

4.6 Analysis Methodology: Simple Span Right Bridge

4.6.1 Construction of the Load-Deflection Curve: Concepts

The concepts involved in constructing the overall load-deflection curve for the bridge are illustrated in Fig. 4.20 in constructing the load-deflection curve for a fixed-ended beam. This discussion is based on a similar one appearing in Ref. 16.
Consider the response under load of the fixed-ended beam shown in Fig. 4.20(a). In the elastic range, before any limit states are reached, the load-deflection curve follows the line O-a. As the applied load increases, the moment at the right support reaches the limit state moment, $M_p$. This point is designated by A on the solid curve. As load is further increased, the moment at the right support cannot increase, although the moments can increase at every other point in the beam. The subsequent increment of deflection of the beam will then be the same as for the propped cantilever shown in Fig. 4.20(b). The elastic load-deflection behavior of this structure by itself is given by the line O-b. The increment of deflection of the original beam is constructed as the line AB drawn parallel to O-b. The magnitude of the increment of load resulting in the vertical location of B is determined by considering the moment under the applied load. A certain moment corresponding to point A exists under the load in the original structure. The difference between this moment and the limit state moment, $M_p$, is made up by the moment under the load of structure (b). Designate this difference as D. The increment of load is the amount required to raise the moment of structure (b) from zero to D. The increment of deflection of structure (b) is also caused by this increment of load.

At this stage, plastic hinges exist both at the load and at the right support. The structure shown in Fig. 4.20(c) is used to determine the next increment of load and deflection in the same manner as was done for the first two load increments.
The last portion of the load-deflection curve, CD, corresponds to structure (d). In a first order analysis, catenary effects are neglected, and the structure deflects further without further increase in load. Thus, CD is horizontal.

The load-deflection curve of the structure loaded to ultimate load is thus obtained by superimposing on the elastic load-deflection curve of the virgin structure portions of the elastic load-deflection curves of auxiliary structures. The load-deflection behavior is considered to be linear between the formation of successive plastic hinges.

4.6.2 Construction of the Load-Deflection Curve: Applied

The basic incremental approach illustrated in the preceding article is applied to the analysis of the simple-span bridge finite element model. The basic assumptions are the same:

1. Superposition is valid.
2. Structural behavior is considered to be linear between the attainment of successive limit states.
3. Once limit states are exceeded, components continue to sustain the stress resultants that caused the limit state exceedance.

An incremental approach is required because the global stiffness matrix, to reflect the nonlinear material behavior, depends on the state of stress existing in the structure. Since the state of stress changes during the loading process, the global stiffness matrix also changes during the loading process.

Applied to the full 3-dimensional bridge model, this approach must account for some complexities. In the example presented in Fig. 4.20,
only the bending moment is of interest. In the full bridge, a myriad of load effects are of interest, e.g., stresses, moments, beam-column interaction. These must be checked, not just against \( M_p \), but against the various limit state conditions discussed in Art. 4.5.

Another complication involves the number of load increments. In the example presented in Fig. 4.20, a single \( M_p \) exceedance dictates the need for a hinge formation, i.e., a modified structure model along with a new load increment and analysis. In the analysis of the full bridge model, resource limitations preclude such a precise "hinge by hinge" tracking of exceedances. Instead, some exceedances are anticipated slightly before they are actually reached. This is consistent with the lower bound philosophy underlying the finite element studies. The number of load increments are thus reduced to a more manageable number.

A different kind of complication concerns the definition of collapse. Collapse, although obvious in Fig. 4.20(d), is not so clear-cut in the analysis of the full bridge model. Collapse in the full bridge model does not simply correspond to a mechanism condition, as discussed in Chapter 1. Long before that occurs the span may be totally unserviceable, which from the point of view of traffic on the bridge, is tantamount to collapse. It was therefore decided to identify collapse by excessive deflection, say, three to five feet or so, at midspan over the fractured girder. A five foot deflection, for example, results in a deck slope of about 15° which is likely enough to overturn vehicles.
The approach taken utilizes "small" strain, "small" displacement finite element analysis to construct a piecewise linear tangent stiffness solution. This approach is summarized as follows:

1. Impose the through-depth fracture in the west girder at midspan (Fig. 4.1). Perform elastic finite element analysis.

2. If instability or excessive deflections result,
   a) If not yet sustaining the full dead (and live) loads, consider the model to be non-redundant.
   b) Otherwise, terminate analyses.

3. Compare the stress resultants produced by the preceding analysis against the remaining capacities of the not-yet-failed elements of the model. Identify in which elements the limit state values are exceeded.

4. Scale down the applied load increment such that a state of incipient limit state exceedance exists in the elements flagged in step 3. Add this scaled down load increment to the cumulative total.

5. If the new cumulative total is greater than the full dead (and live) load, consider the model to be redundant under that load.

6. Modify the finite element model by substantially reducing the stiffness, by a factor of 1,000, of components expected to fail soon as well as components at incipient limit state exceedance.

7. Perform elastic finite element analysis of the revised model. This is the unscaled next load increment.

8. Go to step 2.

The described approach implicitly assumes shored construction. In effect, the bridge is considered to be constructed with a girder fracture in it. It is as if the entire bridge were to be shored up and the girder sawn through. Then the shoring is removed gradually and
slowly, in a uniform manner. Then, once the full dead load is sustained, two HS-20 trucks are gently lowered by a crane onto the deck, at midspan, one in each lane.

This approach, artificial and contrived though it may be, is a consequence of the decision to avoid dealing with dynamic effects of the dead load at the instant of fracture as well as to avoid dealing with the effects of a moving live load on the fractured bridge.

4.7 Analysis Methodology: Two-Span Right Bridge

4.7.1 Construction of the Load-Deflection Curve: Concepts

The concepts involved in constructing the overall load-deflection curve for the two-span right bridge are illustrated in Fig. 4.21. The curve O-A-B-C-D represents the load-deflection curve considering inelastic behavior for a fixed-ended beam shown in Fig. 4.21(a) as loading gradually increases to the full plastic load $P_F$.

The first line O-A-F$_a$ is the elastic load-deflection curve for the beam shown in Fig. 4.21(a). Point A represents the attainment of the first plastic hinge at the right support. Since the plastic moment $M_{pl}$ occurs at a load of $P_A$, only segment O-A of line O-A-F$_a$ will be valid.

The deflection O-O$_b$ corresponds to the beam in Fig. 4.21(b) subjected only to the plastic moment $M_{pl}$ at the right support. The line O$_b$-A-B-F$_b$ is the elastic load-deflection curve for the beam loaded as shown in (b). Point B represents the attainment of the next plastic hinge, this time under the load. Since the plastic moment $M_{p2}$ occurs
under the load $P_B$, and since the structure is already sustaining $P_A$, only segment A-B of line $O_b-A-B-F_b$ is valid.

Similarly the deflection $O-O_c$ corresponds to applying the plastic moments $M_{p1}$ and $M_{p2}$, at the right support and at the loading point respectively, for the unloaded beam in Fig. 4.21(c). The line $O_c-B-C$ is the elastic load-deflection curve for the beam loaded as shown in (c). Only the segment B-C is valid.

The last portion of the load-deflection curve, segment C-D, corresponds to the attainment of the mechanism condition shown in Fig. 4.21(d). The structure deflects further without an increase in load. Therefore, the full plastic load, $P_F$, has been reached.

The load-deflection curve of the structure loaded to the ultimate load is thus obtained by connecting all the appropriate segments.

Figure 4.22 shows how limit states such as buckling are considered in constructing the load-deflection curve. Components reaching buckling limit states are assumed not to sustain these limit states. The same fixed-ended beam and the same full plastic load $P_F$, as demonstrated in Fig. 4.21, are examined. Assume that when $M_{p2}$ is attained under the load point, the beam fractures. Fracture here is similar to buckling in that the joint or member will not sustain the limit state forces. Then Fig. 4.22(c) containing the plastic moment $M_{p2}$, is replaced by Fig. 4.22(c') with plastic moment $M_{p2}$.

The deflection $O_c-O_c'$ results from releasing the plastic moment $M_{p2}$ of the unloaded beam in Fig. 4.22(c). Since there is no difference in
beam stiffness between Figs. 4.22(c) and (c'), the line $O_c'B'C'$ is constructed parallel to the line $O_cB'C$ obtained from (c). The line connecting points $B$ and $B'$ is horizontal as long as the applied load $P_F$ remains constant.

The next plastic hinge corresponding to point $C'$ in Fig. 4.22 occurs under the load $P_F'$, which is less than $P_F$, because the cross section at the load point has lost the stresses developed prior to releasing $M_{p2}$.

The load-deflection curve $O-A-B-B'-C'-D'$ is constructed by connecting all the appropriate segments as shown in the figure. This curve demonstrates the influence of buckling or fracture type limit states.

4.7.2. **Construction of the Load-Deflection Curve: Applied**

The basic nonincremental approach illustrated in the preceding article is applied to the analysis of the two-span right bridge finite element model. The basic assumptions are as follows:

1. Superposition is valid.

2. Structural behavior is considered to be linear between the attainment of successive limit states.

3. Once their limit states are exceeded, components lose their corresponding stiffness, but sustain the limit states by applying the limit state loadings.

   a) Components whose limit state is buckling or fracture do not sustain these limit state loadings. This is analogous to the situation described in Fig. 4.22(c'). Such components include bottom laterals, cross frame horizontals, and girder flanges under compression as well as the bearing keeper plates. Some components such as cross frame horizontals and keeper plates have negligible effect on the load-deflection curve.
b) Other components sustain the limit state. Limit state loads are represented as applied loads in subsequent analyses. In Fig. 4.22, \( M_{p1} \) is an example of a limit state which is sustained and \( M_{p2} \) is a limit which is not sustained.

A nonincremental approach is required for the two-span right bridge because lateral buckling of main girder bottom flange is anticipated, which will significantly affect the load-deflection curve.

Applied to the full 3-dimensional bridge model, this approach must account for the same complexities as the incremental approach used for the simple-span bridge, such as the myriad of load effects, the number of load increments, and definition of collapse. Instead of precise "hinge by hinge" tracking of exceedances, some exceedances are anticipated slightly before they are really reached. This approach is summarized as follows:

1. Impose the through-depth fracture in the west girder at midspan of the north span. Perform elastic finite element analysis, applying the full load.

2. If instability or excessive deflections result,
   a) If not yet sustaining the full dead (and live) loads, consider the model to be nonredundant.
   b) Otherwise, terminate analyses.

3. Compare the stress resultants produced by the preceding analysis against the limit state values of the not-yet-failed elements of the model. Identify the element or group of elements whose limit state values are most greatly exceeded.

4. Determine the appropriate fraction of the full load to construct the next point and segment in the load-deflection curve. In Fig. 4.22, this corresponds to locating point B and constructing segment A-B after the second analysis.
5. Modify the finite element model by substantially reducing the stiffness of the components whose limit states are exceeded or nearly exceeded at the fraction of load determined in step 4.

6. Superimpose the limit state loads on the modified components.

7. Perform elastic finite element analysis of the revised model carrying the superimposed limit state loads as well as the full dead (and live) loads.

8. Go to step 2.

4.8 Comparison of Analysis Methodologies

Articles 4.6 and 4.7 indicate that different analysis methodologies are used for the simple span and two-span bridges.

The approach discussed in Art. 4.6 is analytically simpler but is valid only if member buckling or member stability limits are not expected to occur since these forces cannot be sustained as required (implicitly) by the method. Although buckling of small cross bracing members of the simple span bridge does occur, they have negligible influence on the load-deflection curve. Thus the incremental approach is valid.

The approach discussed in Art. 4.7 is analytically more complex but is required for the analysis of the two-span bridge where lateral torsional buckling of the fractured girder in the negative moment region is anticipated and in fact, does occur. It is shown in Chapter 5 that this has a major effect on the load-deflection curve for the two-span bridge.
5. LOWER BOUND ANALYTICAL RESULTS

5.1 Arrangement of this Chapter
This chapter presents the lower bound analytical results for the simple span right bridge and the two-span right bridge. The use of finite element techniques for the lower bound analysis is described in Chapter 4.

First, the base-line results (undamaged bridge, no girder fracture) are presented for each of the two bridges. Then, results are presented for the simple span right bridge with a midspan full-depth girder fracture, at several stages of loading. Discussion of these results focuses on explaining the 3-dimensional behavior indicated by the finite element analyses. Next, results are presented for the two-span right bridge with full-depth girder fracture in the middle of the first span. Finally, the discussion explains the observed behavior and contrasts the behavior of the two-span bridge to the simple span bridge.

5.2 Base Line Results, Unfractured Bridges
5.2.1 Simple Span Right Bridge
Table 5.1 summarizes the major results for the undamaged simple span right bridge subjected to dead load. The midspan deflection is 0.61", and the tensile stress in the bottom flange of the main girder is 9.4 ksi. The results compare favorably with calculations based on treating the entire bridge as a simple beam subjected to a uniform dead load due to its self-weight plus the future wearing surface. The results of these calculations are shown in the last column of the table.
Even without the girder fracture, the bottom laterals and some cross bracing members carry significant forces under dead load alone. Figure 5.1(a) shows a plan view of the forces in the bottom laterals due to the applied dead load. All the bottom laterals are in tension. As the girders deflect downward, the bottom flange elongates and in turn elongates the bottom lateral system. Figure 5.1(b) shows the resulting deflected shape of the bottom lateral system. The dashed lines indicate the deflected position, while the solid lines indicate the undeflected position.

Figure 5.2 shows stress resultants in some of the cross bracing. In this figure, the contour lines represent the bending stresses in the floor beams and outriggers. The zero contour line (marked by a "0") indicates the position of the neutral axis, and "+" and "-" indicate tension and compression, respectively. Thus, in Fig. 5.2(a) the maximum bending stress in the floor beam is approximately 2.6 ksi compression, at the point where it joins the girder. In the cross bracing horizontals and diagonals, negative forces indicate axial compression and positive forces indicate axial tension. The moment arrows drawn at the joints are shown acting on the ends of the members connected to those joints.

Temporarily accounting for connection rigidity by modeling the cross bracing diagonals as SPACE FRAME members, small moments develop in them and in the horizontals. The only forces of significant magnitude, however, are the compressive forces in the cross bracing horizontals. These forces are highest in cross bracing 2-3, as shown in Fig. 5.2(a).
This and cross frame 3-4 are the ones nearest midspan. The compression is in equilibrium with the tension in the bottom laterals as shown in Fig. 5.1(b).

5.2.2 Two-Span Right Bridge

Table 5.2 summarizes the base-line results for the undamaged two-span right bridge subjected to dead load. The midspan vertical deflection is 0.39", and the longitudinal displacement at the expansion bearing is 0.09". The tensile stress in the bottom flange of the main girder is 7.69 ksi, and the compressive stress in the bottom flange of the main girder at the center support is 10.35 ksi. The maximum axial forces in cross bracing horizontal, 15.0 kips compression, occurs in cross bracing 2-3 and 7-8. The maximum axial tensile force in bottom lateral, 10.7 kips, occurs in bays 2 and 9, and the maximum compressive force in bottom lateral, 17.7 kips, occurs in bays 5 and 6.

Figure 5.3(a) shows a plan view of the forces in the bottom laterals due to the applied dead load. The maximum tensile and compressive forces occur in bay 2 and 5 respectively, as mentioned. The force pattern in sign and magnitude reflects the bottom flange force in the main girder. This 3-D analysis result demonstrates that bottom laterals substantially help the bottom flanges of the main girder in carrying gravity loads.

The corresponding deflections in the bottom laterals due to dead load are shown in Fig. 5.3(b). The dashed lines indicating the deflected position again demonstrate the consistent deformations of the bottom laterals and bottom flanges of the main girder.
Figure 5.4 shows stress resultants in some of the cross bracing. The contour lines representing the bending stresses at the floor beams and outriggers shown in Fig. 5.4(a), (b), and (c) are similar to those of the simple span right bridge shown in Fig. 5.2. But the bending stresses in the floor beams at cross bracing 5-6 are higher than the others, because the non-composite deck over the outriggers at cross bracing 5-6 does not contribute to resisting cantilever bending moment. Consequently the largest moment at base of outrigger occurs in the bearing stiffeners at cross bracing 5-6.

The largest force in cross bracing horizontals is compression and occurs at cross bracing 2-3, where the largest tensile bottom lateral forces occur. Tensile force exists in cross bracing horizontals at cross bracing 4-5, equilibrating the compression in the bottom laterals in bay 5. The force in the cross bracing horizontal at cross bracing 5-6 is not significant because the keeper plates at the fixed supports are reacting as well against the higher compression of the bottom laterals in bays 5 and 6.

5.2.3 Comparing the Two Bridges
Although the two-span right bridge result shows much smaller deflection than the simple span right bridge at midspan, the forces mentioned above are as large as those of the simple span bridge, perhaps because the thickness of the girder flanges are smaller than those of the simple span bridge.
Since there is significant shortening of bottom laterals in bay 5, the longitudinal displacement at the expansion bearing of the two-span bridge is much smaller than for the simple span bridge.

The analyses of both undamaged bridges show significant participation of secondary members such as bottom laterals, cross bracing horizontals, and floor beam connection plates under dead load alone.

5.3 Lower Bound Results, Simple Span Right Bridge

Figure 5.5 shows the load-displacement curve generated by the finite element technique for the lower bound analysis of the simple span right bridge. The dashed line indicates, for reference, the load-deflection curve of the unfractured bridge discussed in Art. 5.2.1. The solid curve shows the global result of gradually applying load to the bridge with a midspan girder fracture. Deflection is measured at the midspan of the fractured girder. Once the full dead load is applied to and carried by the structure, increments of live load are further applied. The curve has a distinct almost-bilinear shape reminiscent of tests performed on composite girders. Analyses were terminated when load and deflection reached point E shown in the figure. The calculation of the upper bound live load limit of HS-72 shown in the figure is given in Chapter 6.

The load-deflection curve in Fig. 5.5 is constructed as discussed in Art. 4.6 using 10 load increments. Table 5.3 summarizes the load increments employed. Points A, B, C, D, and E in Figure 5.5 and Table 5.3 mark stages for which results are summarized in Articles 5.3.1 through 5.3.5. Discussion there will focus on incremental results— the
forces and deflections in the preceding load increment, not the cumulative forces and deflections. The cumulative damage in the superstructure and deck is discussed, however, at the end of each article.

5.3.1 Results at Point A
Point A in Fig. 5.5 is examined because it is the end of linear elastic behavior and the first load increment listed in Table 5.3. During the first load increment, the finite element model incorporates the girder fracture but does not impose any other component failures. The first departure from linear elastic behavior occurs at point A. The load applied at point A is 0.29 times the dead load.

Figure 5.6 shows the reactions acting on the girders at point A. The boundary conditions in the horizontal plane correspond to those shown in Fig. 4.8(a), with 6 horizontal restraints. Initially surprising in Fig. 5.6 are the high longitudinal reactions acting at the fixed bearings. Lateral restraint at the expansion bearings induces these longitudinal reactions at the fixed bearings to balance the moments about a vertical axis.

At this point, a fixed bearing must fail, since Table 4.2 indicates that the anchor bolt capacity has been reached. It was arbitrarily decided to fail the fixed bearing on the unfractured girder. This is accomplished by removing the two horizontal restraints at that node in the finite element model. In addition, a keeper plate is considered to fail on one of the expansion bearings, removing another horizontal
restraint. The 6 horizontal restraints have thus been reduced to 3, just sufficient to constrain rigid body motion in the horizontal plane. The results of subsequent load increments will therefore no longer be affected by the support conditions.

Figure 5.7 shows a view of the deflected simple span right bridge model at load level A. Viewed from above the bridge, the figure shows the fracture in the near (west) girder. The figure suggests a differential vertical deflection of the girders at midspan and a differential elongation at the expansion bearings.

Figure 5.8 shows the forces (without parentheses) and deflections in the bottom lateral bracing system at load level A. In Fig. 5.8(a), the numbers shown in parentheses are those that would be obtained if the applied load were 1.0 times the dead load and the model remained elastic. Thus, the numbers in parentheses may be contrasted directly against those in Fig. 5.1(a) to assess the difference between the baseline (unfractured) results shown there and the results for the bridge with the midspan girder fracture. The bottom lateral forces in the fractured bridge are much higher. Both laterals are in tension in bay 3, the bay where the girder fracture is located. In each of the other bays, one lateral is in tension and the other in compression.

Figure 5.8(b) shows the deflections, amplified 345 times, of the bottom lateral system at load level A in the fractured bridge. This would correspond to an amplification of 100 times for a load level of 1.0 times the dead load. Thus, Fig. 5.8(b) may be contrasted directly with Fig. 5.1(b). In the fractured bridge, the bottom lateral system
elongates more, and there is a differential elongation between the fractured girder and the unfractured girder.

Figure 5.9 shows amplified deflections in three of the cross bracing at load level A. The other three cross bracing deflect very similarly; that is, cross bracing 2-3 behaves like 3-4, 1-2 like 4-5, and 0-1 like 5-6 (refer to Figs. 4.1 and 4.2). The second interior cross bracing (3-4, nearest the girder fracture) exhibits a slight movement of the bottom lateral system towards the fractured girder as well as a slight differential vertical deflection of the girders. The first interior cross bracing (4-5) exhibits little more than a rigid body deflection. The end cross bracing (5-6) exhibits a slight clockwise "shearing" of the cross section. All indicate a slight movement of the deck towards the fractured girder.

Figure 5.10 shows the cross bracing stress resultants (forces, moments, and finite element stresses) in the same 3 cross bracing. High forces and moments are developing in the cross bracing horizontals and diagonals. Fig. 5.10(a) shows high compressive forces developing in the cross bracing horizontal where it connects to the unfractured girder as well as high bending moments in the middle portion of the horizontal. The left portion of the horizontal is considered to be buckled for subsequent load increments. Buckled members are modeled so that additional moments and additional axial forces are not sustained as loading is increased.

Also in Fig. 5.10(a), moments are developing in the floor beam connection plates at the base of the outrigger. These resist the
tendency of the bottom lateral system to move towards the fractured girder.

The pattern of contour lines in the floor beam and outriggers in Fig. 5.10(a) indicates double curvature bending in the floor beam. Figure 5.10(c) also shows high bending moments in the cross bracing horizontal and bearing stiffeners at the base of the outriggers as well as double curvature bending in the floor beam. But the forces, moments and stresses in this, the end cross bracing are all in directions opposite to those in the second interior cross bracing shown in Fig. 5.10(a). The forces and moments are considerably less in the first interior cross bracing shown in Fig.5.10(b). The overall pattern of stress resultants in the cross bracing differs markedly from those in the unfractured bridge shown in Fig. 5.2 and are consistent with the deflections shown in Fig. 5.9.

After point A is reached on the load-deflection curve (Fig. 5.5), the cross bracing diagonals are changed from SPACE FRAME to SPACE TRUSS elements so that they develop no moments in addition to those shown in them in Fig. 5.10. This was done to simplify the analyses since the moments were found to be small.

Figure 5.11 shows the failures imposed in the steel superstructure at load level A. The bridge is viewed in the figure from the same viewpoint as Figs. 4.1 and 4.2. In addition to the bearing failures (not shown), two cross bracing horizontals are considered buckled. These are shown by the heavy lines in the figure. The vertical arrows indicate the cumulative vertical reactions at the bearings.
Figure 5.12 shows the deck damage imposed at load level A. Two discrete deck cracks are imposed, shown by heavy lines on the figure. The transverse crack above the fractured girder is due to longitudinal tensile stresses on the bottom surface of the deck. The longitudinal crack along the unfractured girder in bay 3 is due to transverse tensile stresses on the top surface of the deck.

5.3.2 Results at Point B
At point B in Fig. 5.5, the third load increment which is 0.43 times the dead load has just been applied. The cumulative applied load is now the full dead load. This article presents the results for the third load increment.

Figure 5.13 shows the amplified deflections in the bridge resulting only from the third load increment. Warping in the deck and differential girder deflection are more pronounced here than in Fig. 5.7. It is evident also at this stage of loading how the near stringer is helping to bridge the transverse deck crack above the girder fracture.

Figure 5.14 shows stress resultants and deflections in the fractured girder resulting from load increment 3. Figure 5.14(a) shows that, except for local "hot spots" where the bottom laterals connect to the girder, the longitudinal (bending) stresses are not changed much during this load increment. The deflection increment shown in Fig. 5.14(b) therefore results essentially from a rigid-body movement.
Figure 5.14(c) shows the bending moment about a vertical axis in the bottom flange of the fractured girder. The purpose of this view is to assess the significance to the girder flange of the bottom lateral forces and cross bracing horizontal forces being transmitted to it. Even the largest such moment, 63 k-in. at the first interior cross bracing is not yet a cause for concern.

Figure 5.15 shows stress resultants and amplified deflections in the unfractured girder resulting from load increment 3. Note the location of the neutral axis indicated by the "0" contour line: the assumption of composite behavior moves the girder neutral axis up nearly to the top flange.

Contrasting Fig. 5.15 with Fig. 5.14 reveals some significant differences between the two girders. The stress contours in Fig. 5.15(a) show that bending stresses in the unfractured girder increase approximately 7 ksi. The "hot spots" due to bottom lateral forces are most pronounced at the first interior cross bracing, in contrast to Fig. 5.14(a), where they occur at the second interior cross bracing indicating that the bottom lateral diagonal members are involved as alternate load paths. The resulting deflection increment is shown in Fig. 5.15(b). There is some movement of the left support in Fig. 5.15(b) since the "fixed" bearing at that point previously failed, at the end of load increment 1.

Figure 5.15(c) shows the bending moment about a vertical axis in the bottom flange of the unfractured girder. These moments are much higher...
than those in the fractured girder shown in Fig. 5.14(c). Although not yet nearly high enough to fail the flange, these moments are an indicator that the high bottom lateral forces induced by the girder fracture are not resisted only by the cross bracing horizontals: once those horizontals buckle, the lateral rigidity of the girder flange contributes to resisting the forces in the bottom laterals. Another lesser contributor, not shown in this figure, are the floor beam connection plates.

Figure 5.16 shows the forces without parentheses and amplified deflections in the bottom lateral bracing system resulting from load increment 3, the load increment culminating in load level B in Fig. 5.5. The numbers shown in parentheses are those that would be obtained if the applied load were 1.0 times the dead load. The numbers in parentheses may thus be contrasted directly against those in parentheses in Fig. 5.8(a) to assess the change in load path as load increases from load increment 1 to load increment 3. Tension forces in the middle bay bottom laterals are less, since the load paths provided by the cross bracing horizontals are in effect no longer there since the horizontals in the middle cross bracing are buckled. Some middle bay bottom lateral tension forces are still maintained, however, because the bottom flange of the unfractured girder is still there to provide a path for these forces. The highest bottom lateral forces are compressive forces in members in bays 2 and 4, adding to compressive forces in those members generated by the preceding load increments.
Figure 5.16(b) shows the deflections, amplified 235 times, of the bottom lateral system resulting from load increment 3. This would correspond to an amplification of 100 times for a load level of 1.0 times the dead load. Thus, Fig. 5.16(b) may be contrasted directly with Fig. 5.8(b). The buckled cross bracing horizontals allow for more movement of the lateral bracing system towards the fractured girder as well as for more elongation of the lateral bracing system.

Figure 5.17 shows the amplified deflections in three of the cross bracing resulting from load increment 3. This figure may be contrasted with Fig. 5.9. Figure 5.17(a) shows the deflection at the second interior cross bracing nearest the girder fracture. In addition to moving downward, the deck is moving laterally towards the fractured girder, but not nearly so much as the plane of the bottom laterals. As a result, moment hinges have already formed in the floor beam connection plates as shown in the figure. The opposite behavior is apparent at the end cross bracing shown in Fig. 5.17(c). Here, the plane of the bottom laterals is restrained from deflecting towards the fractured girder by the bearings. In this end cross bracing the cross bracing horizontal is buckled at the end of load increment 2. In each of Figs. 5.17(a), (b), and (c), the lateral deflection at deck level is approximately the same, confirming the fact that the deck exhibits high in-plane (horizontal) rigidity.

Figure 5.18 shows the cross bracing stress resultants in the same three cross bracing. Except for the already failed members, the behavior is similar to that observed in Fig. 5.10. The pattern of stress contours
in Fig. 5.18(a) shows double curvature bending in the floor beam of cross bracing 2-3. Double curvature bending in the opposite direction is evident in the floor beam of cross bracing 0-1. An additional moment hinge is about to be introduced in the second interior cross bracing at the 202 k-in. moment shown in Fig. 5.18(a).

Figure 5.19 shows the failures imposed so far in the steel superstructure at load level B, which corresponds to full dead load. In addition to the failures shown in Fig. 5.11, cross bracing horizontals are buckled in the end cross bracing and plastic moment hinges have developed in the girder flange above the fracture and in floor beam connection plates as well as in cross bracing horizontals. These plastic hinge limit states are confined to the middle bay containing the fracture and the two cross bracing nearest the fracture. The vertical arrows indicate the cumulative vertical reactions at the bearings, corresponding to full dead load.

Figure 5.20 shows the cumulative deck damage imposed in the model at load level B (full dead load). In addition to the discrete cracks shown in Fig. 5.12, several other discrete cracks have been imposed, as shown by additional heavy lines in the figure. Also, the elements shown cross-hatched in the figure are "softened" to model the effects of cracking in those elements.

Figures 5.19 and 5.20 cumulatively indicate the level of damage in the bridge at dead load, and Fig. 5.5 indicates a midspan deflection of approximately three inches. The bridge is unquestionably redundant under dead load.
5.3.3 Results at Point C

At point C in Fig. 5.5, the sixth load increment which is 0.40 times HS-20 live load plus impact has just been applied. The cumulative applied load is now 1.0 times dead load plus 1.03 times HS-20 live load plus 30% impact. This article presents the results for the sixth load increment.

Figure 5.21 shows the deflections in the bridge resulting only from the sixth load increment. In contrast to Fig. 5.13, there is some uplift at midspan of the overhang on the side of the unfractured girder caused by the stiff floor beam outriggers or brackets. The discontinuity of the deck at midspan is more noticeable as well, since the deck cracking is more extensive and since the near stringer has developed a plastic hinge at midspan.

The trends observed in the previous two articles continue during the application of this increment of live load. Plastic hinges such as those shown in Fig. 5.17 continue to develop in cross bracing horizontal and floor beam connection plates. Moments in the bottom flange of the unfractured girder such as those shown in Fig. 5.15(c) increase, and forces in the bottom laterals in bays 1, 2, 4, and 5 increase with the same magnitude and sense as the forces shown in Fig. 5.16(a). The cumulative results of all these developments are shown in Figs. 5.22 and 5.23.

Figure 5.22 shows the cumulative damage in the steel superstructure at load level C. In addition to the failures shown in Fig. 5.19, compressive bottom laterals have buckled in bays 2 and 4. More plastic
moment hinges have developed, most notably in the bottom flange of the unfractured girder at the second interior cross bracing. Even transverse stiffeners have developed plastic hinges in attempting to resist the movement of the plane of the bottom laterals towards the fractured girder. The additional length of the vertical arrows indicates the distribution of live load to the bearings. Note that statics and symmetry require the reactions of the fractured and unfractured bridges to be identical, regardless of the nonlinear behavior of the fractured bridge, all the way to ultimate load capacity.

Figure 5.23 shows the cumulative deck damage imposed in the model at load level C. In addition to the damage shown in Fig. 5.20, more deck elements are shown cross-hatched to indicate cracking caused by positive bending in the vicinity of the near stringer.

Although the level of damage is significantly more extensive than at load level B, it can be concluded that the simple span right bridge is still redundant under dead and live load with deflection of the fractured girder a little more than 6 inches as shown in Fig. 5.5.

5.3.4 Results at Point D
At point D in Fig. 5.5, the ninth load increment has been applied. Although the reduced slope of the curve suggests that the ultimate capacity is not far off, the curve is almost straight along that slope for a significant distance. The cumulative applied load at point D is 1.0 times dead load plus 2.11 times HS-20 live load plus 30% impact.
Figure 5.24 shows the failures imposed in the steel superstructure at load level D. Additional hinges have formed in the stringers in bay 3.

Figure 5.25 shows the cumulative deck damage imposed in the model at load level D. Additional damage is indicated, primarily along longitudinal members. Outside the unfractured girder, the deck is cracking due to negative or upward bending. Along the fractured girder additional deck cracking is indicated near the ends of the span as well as in bays 2 and 4 due to transverse bending.

5.3.5 Results at Point E

At point e in Fig. 5.5, the tenth load increment has been applied. The cumulative applied load is now dead load plus 2.79 times HS-20 live load plus 30% impact. This is equivalent to HS-56 truck loading plus 30% impact.

Figure 5.26 shows the failures imposed in the steel superstructure at load level E. All stringers have developed plastic hinges at midspan. as the bottom flange and web have yielded at midspan of the unfractured girder, very little of the structure remains to carry additional load and the analyses are terminated. Deflection is approaching three feet as shown in Fig. 5.5, which is near the serviceability limit for this span also.

Figure 5.27 shows the cumulative deck damage imposed in the model at load level E. Significant regions of the deck have developed cracking. Most of this cracking is along stringers and girders except in bay 3, the bay containing the girder fracture.
5.4 Discussion of Lower Bound Analysis Results, Simple Span

The lower bound analyses show that the simple span right bridge model can carry a load of 1.0 times dead load plus HS-56 truck loading in two lanes with 30% impact. The capability of the structure to sustain such loads is due to the development of alternate load paths consisting mainly of the bottom lateral bracing system and the cross bracing. The behavior of the structure by mobilizing these load paths is explained conceptually in Art. 5.4.1.

5.4.1 Conceptual Explanation of Observed Structural Behavior

The logical starting point for this discussion is the girder in which the midspan fracture is imposed. Figure 5.28(a) shows the idealized girder that a designer normally deals with when designing a girder for primary bending. The girder is considered to be a line element subjected to transverse in-plane loading.

Figure 5.28(b) shows how most designers might model the presence of a midspan fracture in the idealized girder. In the figure a moment hinge is imposed at midspan since a moment hinge is thought to have the same effect as a full-depth fracture in which the bending capacity is destroyed at that cross section. The beam in Fig. 5.28(b) is now clearly a mechanism and will collapse. Thus, it is concluded, based on this simplistic idealization, that a two-girder bridge is nonredundant. However, this conclusion is contrary to the findings of the lower bound analyses just reported.

A closer look at the assumptions underlying the idealization of the girder reveals a significant problem with this approach. The line
element shown in Figs. 5.28(a) and (b) is actually the neutral axis of the girder. The true meaning of the idealized fractured girder is the model shown in Fig. 5.28(c). The supports are at the level of the neutral axis, and the hinge representing the fracture is also at the neutral axis. However, Fig. 5.28(c) still does not faithfully

Figure 5.28(d) shows the actual fractured girder. The supports are at the bottom flange, and a hinge develops where the fracture is assumed to arrest in the top flange at midspan. The consequence of the downward deflection of the girder at the fracture is the longitudinal movement, $\delta_F$, at the expansion bearing.

Now consider the effect of this longitudinal movement on the bottom lateral bracing system.

The horizontal plane containing the bottom lateral bracing system functions as a truss incorporating the bottom lateral diagonals and the cross bracing horizontals. This truss system can be viewed as a backup bottom flange that becomes activated when the main girder fracture is imposed. It is thus an alternate load path, transferring forces that the fractured girder tension flange would otherwise have sustained, across to the uncracked girder and into the bearings. When the main girder fracture is imposed, however, it is not the usual type of truss since one of its chords (the tension flange of the now-fractured girder) is in effect removed in bay 3.

Figure 5.29(a) shows a plan view of the deflections taking place in the bottom lateral system as a result of the elongation, $\delta_F$, of the
fractured girder. It may be helpful to think of this deflected shape as being displacement-induced rather than load-induced. The applied displacement, $\delta_F$, tries to drag along the rest of the lateral bracing system. There are at least two direct consequences of this.

First, each bay becomes a shear panel to transfer the elongation of the fractured girder over to the unfractured girder. As a result, the fixed bearing of the unfractured girder shears off (at the upper left of the figure), allowing longitudinal movement of the entire unfractured girder. As another result, one lateral is in tension and one is in compression in each bay except the bay containing the fracture. In the bay containing the fracture, both laterals are in tension, in effect taking over the load shed by the bottom flange of the now-fractured girder in that bay.

The signs of the forces in the bottom laterals due to this shear panel action are shown in Fig. 5.29(b). By considering the equilibrium of the joints where the bottom laterals connect to the girders, it is reasonable to expect several cross bracing horizontals to develop high compressive forces to maintain equilibrium. The highest compressive forces occur in the horizontals shown darkened in the figure. These are the same ones that buckle in the finite element analysis discussed in this Chapter. Once the interior cross bracing horizontals buckle, the bottom flange of the unfractured girder takes up the slack in bending horizontally along with the floor beam connection plate at that location, as discussed in the description of Fig. 5.15. This results in the plastic hinge development in the bottom flange of the
unfractured girder and in the connection plates of the floor beam location.

Second, the imposition of the longitudinal displacement $\delta_F$ pulls the unfractured girder towards the fractured girder, resulting in a sideways deflection of the bottom lateral system at midspan, as shown in Fig. 5.29(a). This behavior is analogous to the way a lap joint tries to align itself when subjected to axial load. This sideways deflection at midspan, in addition to the shear lag in the bottom lateral shear panels, means that the longitudinal deflection of the unfractured girder, $\delta_{UF}$, is less than $\delta_F$.

Much of the remaining damage in the steel superstructure can be understood by identifying what components resist this sideways deflection of the bottom lateral system at midspan and identifying by what means they do so.

The deck cannot be expected to deflect laterally to the extent that the bottom lateral system does in Fig. 5.29(a), since it has a much higher in-plane rigidity as can be seen by comparing the relative horizontal deck displacement in Fig. 5.30(a), (b), and (c). But the deck is connected to the bottom lateral system through the cross bracing and transverse stiffeners (ignoring the girder webs, which have little out-of-plane strength).

The cross bracing behavior shown in Fig. 5.30 is the result. Figure 5.30(a) shows how the bottom lateral system at midspan, in attempting to deflect sideways towards the fractured girder, causes plastic moment
hinges to develop in the floor beam connection plates and cross bracing horizontals when the deck does not want to deflect sideways quite so much. The deck does deflect sideways somewhat, however, but because of flexibility of the end cross bracing, not because of significant flexural and shear distortion of the deck itself. The only thing preventing wholesale rigid body sideways deflection of the deck is the fact that it is anchored to the bearings through the end cross bracing.

This anchoring is reflected in the deflected cross bracing shown in Fig. 5.30(c). The direction of shearing in the end cross bracing is thus opposite to that occurring in the second interior cross bracing near the girder fracture. Note that Fig. 5.30(c) does not suggest that lateral reactions at the bearings are present since there are no applied horizontal loads, and only three horizontal boundary restraints need be present. Thus there cannot be any lateral reactions at the bearings.

This discussion thus far has not mentioned the downward (out-of-plane) deflection of the deck. This is because the behavior of most of the components in the bridge can be best understood as consequences of the elongation $\delta_P$ of the fractured girder rather than the differential vertical deflection of the girders. But the flexural behavior of the deck does play a role. Figs. 5.13 and 5.21 suggest the bending and warping behavior that occurs in the deck as it resists the differential deflection of the two girders. However, it is important to observe that redundancy in this two-girder simple span bridge can be understood without relying on the out-of-plane rigidity of the deck.
The understanding of structural behavior revealed by the lower bound analyses has several useful ramifications. First, the limit states indicated in Figs. 5.26 and 5.27 suggest mechanisms to try in the upper bound analyses. The upper bound analysis mechanisms presented in Chapter 6 follow directly from the behavior discussed in this Chapter.

Second, many failures occur because components are not specifically designed for the forces that cause those failures. A design procedure that accounts for these forces and optimizes bracing geometries to resist them can make a two-girder bridge intentionally redundant, not just accidentally so. Just such a design procedure is described in Chapter 7, where the design procedure is developed directly from the understanding of the alternate load paths revealed by the analyses presented in this Chapter as well as those in Chapter 6.

5.4.2 Load Path Identification
Although it has been used in this report and in the technical literature, the phrase "alternae load path(s)" has not been rigorously defined. Phrases used in Ref. 14 such as "structures are said to possess multiple load paths ...." and "so-called redundant load paths" indicate the lack of a clear definition of the term. In essentially one-dimensional members which carry primarily axial stresses, the notion of load path seems intuitively clear. A load path through such a member in a structural system transfers forces from one end to the other. But in the more general case of a 2-dimensional component such as a bridge deck, the meaning of the term "load path" is not clear. It appears that, in general, what is meant by the phrase
"alternate load paths" is actually load redistribution capability, which is the term used for decades to refer to the way a building frame carries load beyond the elastic range of behavior and into the plastic range. (16, 22, 23)

The load redistribution capability of the simple span right bridge model has been described in Art. 5.4.1. It may be summarized as follows. With a full-depth fracture imposed at midspan, the girder sheds the moment that it had previously carried at that cross section. What had been tension in its bottom flange and web is partially redistributed to the bottom lateral bracing system, which transfers forces over to the intact girder. The cross bracing develops significant forces as they connect the sideways-moving bottom lateral system to the deck and as they resist the differential vertical displacement of the girders near the fracture. Thus, the primary "alternate load paths" or load redistribution capability is provided by the bottom lateral bracing system and the cross bracing.

These concepts are developed more fully in Chapter 7. Design procedures and guidelines are proposed in that chapter together with redesign of the simple span right and skew study bridges not only to ensure redundancy but also serviceability in the form of a displacement limitation which is specified by the design engineer.

5.5 Lower Bound Results, Two-Span Right Bridge

Figure 5.31 shows the load-displacement curve generated by the finite element technique for the lower bound analysis of the two-span right bridge. The dashed line indicates, for reference, the load-deflection
curve of the unfractured bridge discussed in Art. 5.2.2. The deflection indicated as a solid line is measured at the midspan of the fractured girder. The curve has a relatively high slope up to load level HS-40, becomes a flat line up to about 21 in. deflection, and continues from there at a much lower slope.

The load deflection curve in Fig. 5.31 is constructed as discussed in Art. 4.7 using 9 stages. Table 5.4 summarizes the load levels and corresponding stages. Points A, B, C, D, and E in Fig. 5.31 and Table 5.4 mark stages for which results are summarized in Articles 5.5.1 through 5.5.5. Discussion in these articles is mainly focused on failures in the steel superstructure and deck damage, whose sequential appearances are similar to those of the simple span bridge, except that they appear at higher load levels. Also comparative studies concerning deflection and force patterns are presented in Art. 5.5.3.

5.5.1 Results at Point A

Point A in Fig. 5.31 is the first departure from linear elastic behavior. The cumulative load applied so far is 0.6 times the dead load, which is approximately twice the elastic limit of the simple span bridge.

Figure 5.32 shows the reactions acting on the girders at load level A. The boundary conditions in the horizontal plane correspond to those shown in Fig. 4.16 with 8 horizontal restraints. First, the lateral reactions at the keeper plates of the fixed bearings reach their limit state indicated in Table 4.2. Consequently it is decided to eliminate
both the lateral restraints. The limit state load described in Art. 4.7.2 is not applied because the failure of keeper plates is a fracture. Secondly, vertical reactions at the center supports increase, compared to the unfractured bridge. This behavior indicates cantilever action of the fractured girder near the center support. Thirdly, longitudinal reactions induced to balance the moments about a vertical axis are 40.3 kips, which is smaller than in the simple span right bridge. This is because in the two-span bridge there is less deflection at midspan.

Figure 5.33 shows the failures imposed in the steel superstructure at load level A. The vertical reactions at the bearings are illustrated as vertical arrows. In addition to the keeper plate failures (not shown), one cross bracing horizontal is considered buckled. The limit state load of the buckled member is not sustained.

Figure 5.34 shows the deck damage imposed at load level A. A transverse crack is imposed above the fractured girder due to longitudinal stresses on the deck's bottom surface there. Compare Fig. 5.34 with Fig. 5.12, both corresponding to the end of elastic behavior.

5.5.2 Results at Point B

At point B in Fig. 5.31, full dead load is applied. The longitudinal reactions at the fixed bearing, as discussed in Art. 5.5.1 and in Fig. 5.32, reach the anchor bolt capacity. Although both anchor bolts are failed simultaneously in the analysis, only one longitudinal restraint at the fractured girder side is eliminated as would occur in reality.
The remaining restraint prevents rigid body motion. The failure is considered a fracture and thus the limit state load is not sustained in the next analysis.

Figure 5.35 shows the failures imposed in the steel superstructure at load level B. This figure is the same as Fig. 5.33, since only anchor bolt failure (not shown) is imposed on the superstructure.

Figure 5.36 shows the cumulative deck damage imposed in the model at load level B (full dead load). In addition to the discrete crack shown in Fig. 5.34, the non-composite deck above the interior support becomes cracked as shown by additional heavy lines in the figure. Compare Figs. 5.36 and 5.20 where both structures carry the full dead load.

The smaller number of failures and damage under full dead load suggests that the two-span bridge is more redundant than the simple span bridge at this stage.

5.5.3 Results at Point C

At point C in Fig. 5.31, full dead load plus HS-20 live and 30% impact load are applied. Figure 5.37 shows the deflected shape of the bridge resulting from the cumulative load level at C. It shows the enlarged girder fracture, and full length transverse deck crack at the center support. Span 1 shows an overall deflected shape similar to the simple span bridge shown in Fig. 5.13.

Figure 5.38 shows the forces and deflections in the bottom lateral bracing system at load level C. The pattern of forces in Fig. 5.38(a) is similar to the simple span right bridge except in bay 5. Tensile
forces in the middle bay become smaller than those in the first bay. The highest bottom lateral forces are compressive forces in bays 2 and 4.

Figure 5.38(b) shows the deflections, amplified 100 times, of the bottom lateral system resulting from the load level C. The unfractured girder span shows small deflections compared to the fractured girder span. This deflection pattern of the fractured girder span is similar to that of the simple span bridge shown in Fig. 5.16(b). This similarity suggests that both bridges have similar redundant load paths in using bottom lateral bracing.

Figure 5.39 shows the amplified deflections in cross bracing at load level C. The figure may be compared with the deflection in cross bracing of the simple span bridge shown in Fig. 5.17. The deflection modes of all the cross bracing, except cross bracing 5-6 at the center support, are similar to those of the simple span right bridge. Cross bracing 5-6 is horizontally deflected without noticeable shearing, because of continuity of girders and stringers. Therefore no failure occurs at cross bracing 5-6.

Figure 5.40 shows the cross bracing resultants in the same 6 cross bracing. Except for cross bracing 5-6, the behavior is similar to that of the simple span right bridge shown in Fig. 5.18. Comparing with the results from the undamaged bridge carrying dead load only, it is observed that the web bending stresses of the floor beam and outriggers do not vary in cross bracing 5-6 even though the girder fracture is imposed and HS-20 live and impact loads are additionally applied.
These comparative results imply again that at the middle support, continuity of girders and stringers resists cross bracing shearing.

Figure 5.41 shows the failures imposed so far in the superstructure at load level C, which corresponds to full dead load plus HS-20 live and impact loads. In addition to the failures shown in Fig. 5.35, two more cross bracing horizontals are buckled, and plastic moment hinges have developed in the girder flange above the fracture and in floor beam connection plates. The additional length of vertical arrows indicates the increased reaction due to live and impact loads.

Figure 5.42 shows the current deck damage imposed in the model at load level C. Compare Figs. 5.42 and 5.23 where both spans are carrying full dead load plus HS-20 truck loading. In addition to the discrete cracks shown in Fig. 5.36, a transverse crack at the center support and a longitudinal crack above the girder fracture have been imposed, as shown by additional heavy lines.

Figures 5.41 and 5.42 illustrating the failures and damage at dead load plus HS-20 live and 30% impact loads occur at a midspan deflection of approximately 2.5 inches. The two-span bridge is obviously more redundant at this stage than the simple span right bridge under dead load plus HS-20 live and 30% impact loads.

5.5.4 Results at Point D

After point C in Fig. 5.31, load is increased until full dead load plus HS-40 live and 30% impact loads are applied. The 6th stage of loading
described in Table 5.4 results in 4.13 inches deflection at the midspan under this load level. This corresponds to the leftmost point on the plateau shown in Fig. 5.31. The imposed failures at this stage are similar to the failures at a load level between points B and C of the simple span bridge shown in Fig. 5.5.

The failures in the steel superstructure are:

1. Cross bracing horizontal plastic hinges and floor beam connection plate plastic hinges at all the cross bracing except cross bracing 5-6.
2. Bottom lateral buckling in bay 2.
3. Stringer plastic hinges at the midspan and at the middle support of the exterior west stringer, and
4. Lateral buckling of the fractured girder bottom flange at the middle support.

All these failures are imposed and corresponding limit state loads are sustained, except those for the buckled bottom lateral and girder bottom flange.

The laterally buckled girder is subsequently modeled not to sustain normal stress but to sustain shear stress only. Consequently the fractured girder span becomes similar to the simple span right bridge especially in carrying eccentric live loads. While maintaining dead load plus HS-40 live and impact loads, substantial failures occur. Three consequent stages of analysis including additional failures and limit state loads are performed in order to reach point D in Fig. 5.31, since many limit states are exceeded along the plateau in Fig. 5.31.
Figure 5.43 shows the failures imposed in the steel superstructure at load level D. This figure shows similar failures at load level D of the simple span bridge which is shown in Fig. 5.24. Additional failures, however, occur in the bottom laterals in bay 5 of the two-span bridge.

Figure 5.44 shows the cumulative deck damage imposed in the model at load level D. Additional damage resulting from the above-mentioned 3 analyses is significant and shown cross-hatched in the figure.

Although the level of damage is significantly more extensive than at load level C, it can be concluded that the two-span right bridge is still redundant under dead load plus HS-40 live and 30% impact loads.

5.5.5 Results at Point E

At point E in Fig. 5.31, full dead load plus HS-50 live and 30% impact loads are applied. The slope of the load-deflection curve is significantly reduced from the initial slope.

Figure 5.45 shows the failures imposed in the steel superstructure at load level E. In addition to the failures shown in Fig. 5.43, the bottom laterals in bays 1 and 6 are buckled, a plastic moment hinge develops at cross bracing 6-7, and the floor beam bottom flange at cross bracing 0-1 has yielded. At this stage very little of the structure in the fractured span remains to carry additional load, and the analyses are terminated. The total deflection of between two and three feet shown in Fig. 5.31 is approaching the serviceability limit for this span also.
Figure 5.46 shows the cumulative deck damage imposed in the model at load level E. Significant regions of the deck have developed cracking. Most of the cracking is similar to that of the simple span right bridge, except the additional damage in the negative moment region.

5.6 Discussion of Lower Bound Analysis Results, 2 Span

Figure 5.47 shows the comparison of the lower bound load-deflection curves for the simple span and the two-span right bridges. The two-span right bridge has larger initial stiffness up to HS-40 level before the fractured girder bottom flange buckles laterally.

In the first stage the fractured two-span right bridge results in 0.47 inch deflection under 0.6 times dead load, while the fractured simple span right bridge results in 0.48 inch deflection under 0.29 times dead load. This comparison implies that redundant load path forces are induced by deflections, because the same deflection causes the same first failure on both bridges. In fact, the failure pattern of the two-span bridge is similar to the simple span bridge under the same deflection, except that it appears later and under higher load.

The lateral buckling failure of the fractured girder at the middle support results in extensive failures of members causing even larger deflection than that of the simple span right bridge under the same dead load plus HS-40 live and impact loads. The reason it results in relatively larger deflection is that the two-span right bridge has thinner girder flanges in the critical region.
The two-span bridge becomes, under more than the HS-40 loading, similar to the simple span bridge in stiffness. Since the remaining unfractured girder and stringers acting as a cantilever are able to carry additional forces, the two-span bridge has slightly higher stiffness than the simple span right bridge.

The behavior of the two-span bridge discussed in this Chapter led directly to the selection of the mechanism presented in Chapter 6.

The redistribution of load shown by the analysis of the two-span bridge and the manner in which this span approaches the ultimate load led directly to the design procedure discussed in Chapter 7 for ensuring redundancy of the two-span study bridge.
6. RESULTS OF UPPER BOUND ANALYSIS

6.1 Initial Concepts

Chapter 3 indicated that in the calculation of the upper bound capacity of a ductile, elastic-plastic structure only two conditions must be satisfied, namely, the equilibrium and mechanism conditions. Any mechanism which transforms the structure, or any part of it, into a linkage is satisfactory. Equilibrium is formulated for the mechanism using the principle of virtual displacements and virtual work as demonstrated in Chapter 3.

The difficult part of the upper bound analysis is the definition of the mechanism condition, especially for complex structures such as the three study bridges. Although any mechanism is suitable, only those achieving the lowest upper bound solutions are of practical interest.

One approach to finding practical mechanisms is to study the behavior of a structure during incremental lower bound load-deflection analyses such as those described in Chapters 4 and 5. As load increases and steadily approaches the lower bound capacity the successive attainment of member limit states throughout the structure gradually transforms the structure into a collapse mechanism. Although the resulting collapse mechanism and collapse load are not true states, since they are based on a lower bound approach, they are reasonably accurate providing the incremental lower bound analysis is accurately performed. A study of the successive attainment of limit states during such an analysis is of valuable assistance in suggesting practical mechanism conditions for the upper bound analyses. This approach, which has a
long history at Lehigh University in steel framed building research, is
used to establish practical least upper bound mechanism conditions for
the three study bridges.

Once the mechanism condition is established, the equilibrium condition
is formulated by equating the total internal virtual work to the total
external work corresponding to an arbitrary virtual displacement of the
mechanism as discussed in Chapter 3. The pattern of internal and
external virtual displacements resulting from a virtual displacement of
the mechanism is referred to as the virtual displacement field.

Although the real collapse mechanism involves real elastic and plastic
distortions of the mechanism, the principle of virtual displacements is
not limited to the use of the real displacements. Any displacement
field, real or imaginary, may be used to define the virtual
displacement field. The resulting upper bound capacity is independent
of the nature of the displacements selected. The virtual displacement
field normally used is one that results in a reduction in numerical
computation. This criterion is satisfied by requiring the entire
structure to be rigid except at the plastic hinges, yield lines etc.
which are needed to define the mechanism.

Relative virtual shear displacement between the concrete deck and the
supporting steel girders is not required for the mechanism condition
for any of the three study bridges. For the simple span bridges the
girders also remain rigid, except at the fracture. Thus the upper
bound capacities of the simple span bridges are the same for both
composite and noncomposite spans. For the two-span bridge, a hinge is assumed in the fractured girder at the inflection point adjacent to the fracture. The limit state at this point is computed neglecting the web and using a reduced bottom flange width. As for the simple span bridges the girders elsewhere remain rigid and virtual shear displacement between deck and girders is not required. Thus the upper bound capacity for the two-span bridge is also the same for both composite and noncomposite spans.

In the following, all components of each bridge are assumed to be rigid and to undergo rigid body displacements defined by the virtual displacement field selected. Any member required to shorten during a virtual displacement is assumed to buckle and does not participate in the formation of equilibrium. It is also assumed that the deck and supporting girders do not separate.

6.2 Limit States

The limit states used for calculating internal virtual work are shown in Table 6-1.

For the concrete deck, positive bending refers to cracking on the bottom surface. Negative bending refers to cracking on the top surface. Transverse refers to development of the transverse reinforcement. Since the longitudinal steel is not symmetric about a horizontal axis, the positive and negative limit states are different.

The limit state of the girder bottom flange refers to bending about a vertical axis lying in the web.
To account for the fact that the 7" x 14.75 channel is not compact, the limit state is reduced to $0.50 M_p$ where $M_p$ is the plastic moment capacity.

To account for local buckling of the floor beam connection plates, and transverse web stiffeners their limit states are reduced to $0.50 M_p$.

The limit state for the two-span girder at the inflection point neglects the web, assumes a bottom flange width of 12.6 in. ($b/t = 8.4$), assumes a plastic centroid in the top flange and computes the plastic moment using a maximum stress of 34 ksi in the bottom flange (AASHTO Art. 10.48.4). (2)

6.3 Simple Span Right Bridge

6.3.1 Description of Mechanism

Figures 6.1, 6.2 and 6.3 describe the assumed rigid-plastic mechanism for the simple span right bridge.

The virtual displacement field for the concrete deck and stringers is shown in Fig. 6.1. The fractured girder is given unit downward (positive) virtual displacements at the two first interior cross bracing locations as shown. Since only rigid-plastic deformation is permitted, yield lines form in the concrete deck to allow the unit displacements to occur. The yield lines are shown by the jagged lines. The deck is assumed to be connected to the girders to prevent separation. All virtual displacements shown in the figure are relative.
to the unit virtual displacements of the fractured girder. The undisplaced position of the deck is shown dashed.

The virtual displacement fields for the two girders and the bottom lateral system are shown in Fig. 6.2. All virtual displacements shown in the figure are relative to the unit virtual displacements of the fractured girder shown in Fig. 6.1. Yield lines in the girder webs are shown by the jagged lines. Plastic hinges occurring in the bottom flange plates are shown by black circles. These hinges develop about a vertical axis lying in the web. The fracture is shown by the gap at midspan in Fig. 6.2 (b). Local yield lines in the top flanges at the floor beam locations, which are required by the mechanism, are ignored since the internal virtual work at these locations is negligible compared to the total work for the structure. The original undisplaced positions of both girders are shown by the dashed lines in (a) and (b).

The undisplaced position of the bottom lateral system is shown by the dashed lines in Fig. 6.2 (c). The displaced position shown in the figure is defined by two conditions. First, the lower bound analysis indicated that the tension diagonal members will not likely yield. So they are assumed rigid in the upper bound analysis. Second, referring to Fig. 6.2(c), the lower bound analysis showed that the longitudinal and transverse displacements at joint B' are relatively small. Thus, in the upper bound analysis no displacements are allowed at joint B' and at the corresponding joint near the other end of the girder. The resulting virtual displacement field shown in (c) is compatible with the displaced positions along the bottom flanges of the two girders.
shown in (a) and (b). This displacement field requires that diagonal C-B' buckles as shown by the jagged line. Also part of the horizontal cross bracing member at joint C' buckles.

The undisplaced positions of the cross bracing systems are shown dashed in Figs. 6.3 (a), (b), and (c). The virtual displacement fields shown in these figures are compatible with the displacements of the deck, girders and bottom lateral system shown in the previous two figures. The plastic hinges in the floor beam connection plates and in the 7" x 14.75 horizontal channel member of the cross bracing are shown as black circles. The buckled member near joint C' is shown in (c). All virtual displacements shown in the figures are relative to the unit virtual displacements of the fractured girder shown in Fig. 6.1.

6.3.2 Equilibrium Condition and Upper Bound Capacity

The computed external and internal virtual work quantities corresponding to the virtual displacement fields shown in Figs. 6.1, 6.2, and 6.3 and to the limit states shown in Table 6.1 are shown in the first column of Table 6.2 for the simple span right bridge.

In computing the external virtual work a unit load factor for dead load is used, as is the case in the lower bound analysis. The external virtual work for the live load is computed in terms of an upper bound factor U times two lanes of HS-20 truck loading with 30% impact. Figure 6.3 (d) shows the location of the two lanes of HS-20 wheel loads on the deck. This pattern maximizes the external virtual work due to
truck loading, thus minimizing the upper bound factor \( U \). Impact is 30% as in the lower bound analysis.

With reference to Table 6.2 the upper bound factor, \( U \), is determined from the following equilibrium equation:

\[
408 + 292 \ U = 1,459 \quad (6.1)
\]

or

\[
U = 3.6
\]

giving an upper bound live load capacity of the simple span right bridge of HS-72. This capacity is shown on Fig's. 5.5 and 5.47.

6.4 Simple Span Skew Bridge

6.4.1 Description of Mechanism

Figures 6.4 through 6.7 describe the assumed rigid-plastic mechanism for the simple span skew bridge.

The virtual displacement field for the concrete deck and stringers is shown in Fig. 6.4. The fractured girder is given unit downward (positive) virtual displacements at the two locations shown in the figure. Yield lines in the deck are shown by the jagged lines. Separation between the deck and steel girders is not permitted. The undisplaced position of the deck is shown dashed. All virtual displacements are relative to the unit virtual displacements of the fractured girder.

The virtual displacement fields for the two girders and the bottom lateral system are shown in Fig. 6.5. All virtual displacements are relative to the unit virtual displacements of the fractured girder shown in Fig. 6.4. Yield lines in the girder webs are shown by the
jagged lines. Plastic hinges occurring in the bottom flange plates are shown by black circles. The original undisplaced positions of both girders are shown by dashed lines.

The undisplaced position of the bottom lateral system is shown by dashed lines in Fig. 6.5 (c). The virtual displacement field in (c) is defined assuming that tension diagonals remain rigid and that points B' and E' do not displace. Buckled members are shown by jagged lines in (c).

The undisplaced positions of the cross bracing systems are shown dashed in Fig. 6.6 and Fig. 6.7 (a) and (b). The virtual displacement fields shown in these figures are compatible with the displacements of the deck, girders and bottom lateral system shown in the previous figures. Plastic hinges are shown by black circles. All virtual displacements shown in the figures are relative to the unit virtual displacements of the fractured girder shown in Fig. 6.4.

6.4.2 Equilibrium Condition and Upper Bound Capacity

The computed external and internal virtual work quantities corresponding to the virtual displacement fields shown in Figs. 6.4 through 6.7 and to the limit states shown in Table 6.1 are shown in the second column of Table 6.2 for the simple span skew bridge.

A unit load factor is used for dead load. Fig. 6.7 (d) shows the location of the two lanes of HS-20 wheel loads on the deck which minimize the upper bound load factor, U. Impact is taken as 30%.
With reference to Table 6.2 the upper bound load factor, \( U \), is determined from the following equilibrium equation:

\[
402 + 293 \ U = 1,472 \\
\text{or} \\
U = 3.65
\]  

(6.2)

giving an upper bound live load capacity of the simple span skew bridge of HS-73. This is nearly identical to the upper bound live load capacity of the simple span right bridge of HS-72.

6.5 Two-Span Right Bridge

6.5.1 Description of Mechanism

Figures 6.8 through 6.11 describe the assumed rigid-plastic mechanism for the two-span right bridge.

The virtual displacement field for the concrete deck and stringers is shown in Fig. 6.8. The fractured girder is given a unit downward (positive) virtual displacement at the first interior cross bracing location nearest the simply supported end of the structure as shown in the figure. The fractured girder is assumed to develop a plastic hinge at the first interior cross bracing location (point of inflection) nearest the middle support as shown. To recognize the fact that the plate girder in this study bridge cannot develop a full plastic hinge at this location, the moment capacity is substantially reduced as described in Art. 6.2. Yield lines are shown by the jagged lines. Separation between the deck and steel girders is not permitted. The undisplaced position of the deck is shown dashed. All virtual
displacements are relative to the unit virtual displacement of the fractured girder.

The virtual displacement fields for the two girders and bottom lateral system are shown in Fig. 6.9. All virtual displacements are relative to the unit virtual displacement of the fractured girder shown in Fig. 6.8. Yield lines in the girder are shown by jagged lines. Plastic hinges in the bottom flange plates are shown by black circles. The original undisplaced positions of both girders are shown dashed.

The undisplaced position of the bottom lateral system is shown dashed in Fig. 6.9 (c). The virtual displacement field in (c) is defined assuming that tension diagonals remain rigid and that points B' and E' do not displace, both conditions indicated by the results of the lower bound analysis. Buckled members are shown by jagged lines.

The undisplaced positions of the cross bracing systems are shown dashed in Fig's. 6.10 and 6.11 (a). The virtual displacement fields shown in these figures are compatible with the displacements of the deck, girders and bottom lateral system shown in the previous figures. Plastic hinges are shown by black circles. All virtual displacements shown in the figures are relative to the unit virtual displacement of the fractured girder shown in Fig. 6.8.

6.5.2 Equilibrium Condition and Upper Bound Capacity

The computed external and internal virtual work quantities corresponding to the virtual displacement fields shown in Figs. 6.8
through 6.11 and to the limit states shown in Table 6.1 are shown in
the third column of Table 6.2 for the two-span right bridge.

A unit load factor is used for dead load which is the case in the lower
bound analysis. Figure 6.11 (b) shows the location of the two lanes of
HS-20 wheel loads on the deck which minimize the upper bound load
factor, U. Impact is taken as 30%.

With reference to Table 6.2 the upper bound load factor, U, is
determined from the following equilibrium equation:

\[ 351 + 226 U = 1,485 \]
\[ \text{or} \quad U = 5.0 \]  

(6.3)

giving an upper bound live load capacity of the two-span right bridge
of HS-100. This capacity is shown on Fig's. 5.31 and 5.47.

6.6 Discussion

6.6.1 Comparison of Simple Span Bridge Results

The upper bound analysis of the simple span right bridge was performed
when the lower bound load-displacement analysis was nearing completion.
At the same time the finite element model of the simple span skew
bridge was prepared. However, before proceeding with the finite
element analysis of the skew bridge the upper bound analysis of that
bridge was completed. With nearly identical upper bound results for
the two simple span bridges it was concluded that the lower bound
analyses would likely be similar. For this reason the finite element
analysis of the skew bridge was not performed. Instead, more detailed
upper and lower bound analyses of the two-span bridge than originally anticipated, were performed.

6.6.2 Significance of Upper and Lower Bound Analyses of the Simple Span Right Bridge

Excellent agreement is achieved between the upper and lower bound analyses as shown in Fig. 5.5. Although the finite element analysis is not carried out to the stability limit load it appears from the figure that this load level is likely to be quite near the upper bound load level. Of course, the upper bound mechanism is made to resemble the damage experienced during the finite element analysis, so this agreement is expected.

The significance in performing the two analyses lies not in attempting to achieve such good agreement between the stability limit loads, although this is quite significant in itself, but in identifying the members and components involved in the progressive collapse of the structure and those involved in the final collapse mechanism. This information is the foundation upon which the design for redundancy concepts in Chapter 7 is based. These analyses firmly identified the roles of the deck, composite action, cross bracing and bottom lateral systems in providing redundancy. They also indicated the bracing changes needed in order to enable these systems to more efficiently provide the required redundancy.

6.6.3 Comparison of Single Span and Two-Span Bridges

At an early stage of this investigation it was tentatively felt that a major weakness of the two-span study bridge with respect to redundancy
lay in the reduced cross section at the inflection point (or inflection region, considering the live load envelope). However, this feeling is dramatically confirmed by the upper and lower bound analyses of the two-span bridge as shown in Fig's 5.31 and 5.47. The two-span bridge, in effect, is transformed into a simple span bridge with the flexural failure of the fractured girder near the inflection point. The fractured girder behaves as though there are two hinges in the span, one at the inflection point and one at the midspan fracture. This girder span becomes a mechanism or linkage, supported by the cross bracing and bottom lateral systems and the deck (diaphragm action) as in the simple span bridge. An obvious way to provide for redundancy of the two-span bridge is to redesign the girder between the fracture and the middle support to carry the redistributed bending moments. This is dealt with in Chapter 7.

6.6.4 Upper Bound Result for Two-Span Bridge

At first glance it appears that the upper bound capacity of the two-span bridge is unnecessarily higher than the lower bound result. This may or may not be the case for the following reasons.

The upper bound mechanism was established at an early stage of the lower bound finite element analysis. This is because time did not permit waiting for the lower bound results before selecting the mechanism. Thus, the best mechanism may not be achieved. The result is an increase in the upper bound capacity.

On the other hand, time also did not permit carrying out the finite element analysis further than that shown in Fig. 5.31. However, at
stage E shown in the figure the essential stages of the analysis are
developed and incorporated into the upper bound mechanism. Although
the behavior of the two-span and simple span bridges are similar after
failure of the two-span girder at the inflection point, as shown in
Fig. 5.47, the load-displacement behavior of the two-span bridge is not
carried out far enough to provide an accurate indication of the
stability limit load. It is quite possible that the stability limit
load approaches the upper bound capacity, which would indicate that the
upper bound mechanism is quite good. However, for purposes of the
design for redundancy presented in Chapter 7, it is not necessary to
further refine the upper and lower bound analysis.
7. DESIGN FOR REDUNDANCY

7.1 Redundancy of Study Bridges

Chapters 5 and 6 present and discuss the results of the upper and lower bound analyses of the study bridges. These results demonstrate that both the simple span and two-span bridges are redundant since the live load capacities of both bridges are relatively high. The lower bound results show the progression of member yielding and buckling which occurs as dead and live loads are distributed. The main significance of the upper and lower bound analyses is the identification of the role each component of the bridge plays in achieving redundancy.

For the simple span bridges, the cross bracing and lateral bracing members are primarily responsible for redundancy. For the two-span bridge, redundancy is also dependent upon the continuity of the fractured girder over the interior support.

The fractured and unfractured girders participate mainly in redistributing internal forces (stress resultants) to the cross bracing and lateral bracing members. Flexural strength of the deck is not significant. The deck participates mainly by providing a high level of in-plane (membrane) stiffness in order to maintain the alignment of the top flanges of the girders. This is accomplished in the study bridges by providing a shear connection between the deck and the girders. If this shear connection is not provided, girder alignment can also be maintained with a top lateral bracing system having an in-plane stiffness comparable to the deck. However, this is not likely to be an economical solution.
In the usual composite steel-concrete design the shear connectors are provided to develop the flexural strength of the composite girders. For the simple span study bridges a reasonable level of redundancy is achieved, independent of the flexural strength of the girders, whether composite or noncomposite. Therefore, the shear connectors serve two functions: (1) to develop the flexural strength of the composite girders in normal design, and (2) to maintain girder alignment in design for redundancy.

In order for the flexural strength of the deck to significantly contribute to redundancy both the transverse and longitudinal strength must be significant and the deflections (resulting curvatures) must be large. In normal deck design, the transverse strength is not very large and the longitudinal strength is somewhat lower. Also, in a practical design for redundancy deflections should not be large. Although an increase in deflection is desirable, following the fracture of a girder, so that the possibility of early detection of fracture is enhanced, the bridge should remain serviceable at normal highway speeds. Excessive deflection could lead to vehicle damage, collisions and possible injury and death to vehicle occupants.

The results of Chapter 5 and 6 indicate that the study bridges, even though significantly redundant, are not well designed to achieve a practical level of redundancy. Although the lateral bracing and cross bracing systems function reasonably well, their designs can be improved by making connections to the bottom flanges of the girders, rather than to the floor beam connection plate at some distance above...
the flange. This enables the girders to transmit forces to the bottom lateral system more efficiently as well as reducing the potential for displacement induced fatigue cracking during normal service.\(^6\) The design of the cross bracing system from the point of view of redundancy is poor. The main function of the cross bracing system is to resist the lateral bracing member forces after fracture and to maintain the cross sectional shape of the span. This can best be achieved with full depth interior and end cross bracing having X or K framing consisting of stable triangles.

Because of continuity of the fractured girder over the interior support, many bridge engineers assume that a two-span bridge is automatically more redundant than the simple span bridge. The results presented in Chapters 5 and 6 show that this is not the case. In fact the simple span and two-span bridges achieved about the same level of redundancy. The reason for this, of course, is the reduced cross section capacity of the continuous girders near the dead load point of inflection. After fracture, the cantilever portion of the fractured girder is unable to carry the higher negative bending moment resulting from the redistribution of positive moment towards the interior support. Design of the two-span study bridge for redundancy can be achieved simply by maintaining the cross section necessary to resist the redistributed moments. Making the improvements to the bottom lateral and cross bracing systems discussed above, will also help.

This chapter presents economical design modifications for the simple span and two-span study bridges which ensure both a reasonable level of
redundancy and deflection control following nearly full depth midspan fracture of a girder. The design procedures discussed in this Chapter suggest application to the redundant design of other two-girder and multi-girder bridges with different configurations and with the same and different fracture conditions.

7.2 Selection of Load Factors

The AASHTO truck and lane loading criteria and combinations of loads are intended for use in the design of new bridges and retrofitting and rehabilitation of existing bridges. Among other things they provide for "an increase in traffic volume" and "extreme conditions of long continued loading".\(^{(29)}\)

Following the midspan fracture of a girder it is highly unlikely that the fracture would go undetected long enough that the bridge would experience a significant increase in traffic volume or extreme conditions of long continued loading. It is more likely that the fracture would be detected within a reasonably short period of time either as a result of excessive deflections or other noticeable distress or by bridge inspection. Thus an argument can be made for reducing design load factors or increasing allowable stresses. Table 3.22.1A of the AASHTO specifications could be modified at the appropriate time to include load factors and allowable stresses for design for redundancy. The values selected should reflect the practical consideration that at the time of fracture some expected deterioration of the bridge has already occurred. Load factors or allowable stresses should also be selected considering the sudden
energy release at fracture and the resulting dynamic loading of the structure.

Research into the appropriate load factors and allowable stresses to select for design for redundancy is not considered to be within the scope of this investigation. At the same time, in order to redesign the study bridges to ensure redundancy and deflection control following midspan fracture of a girder, it is necessary to assume some reasonable values.

For the study bridges designed for redundancy in this chapter, a load factor approach is used. The following loading conditions and load factors are assumed:

1. HS-20 loading of two design traffic lanes (same as the original design).
2. 30% impact (to account for the effect of increased deck deflections with traffic maintaining normal highway speeds).
3. Dead load factor of 1.1.
4. Live load factor of 1.3.

7.3 Simple Span Right Bridge

In the following the simple span right study bridge is redesigned for redundancy. The approach taken is fairly simple and is similar to the normal design procedure for two-girder bridges. In normal design, the two girders are identified as the load paths for all dead and live loading. This is a lower bound (safe) approach in that the resistance of all other bridge members to the dead and live loads is ignored. The resulting design for static dead load and live loads is safe. (Such a
procedure for design against fatigue due to repetitive live loads is not necessarily safe however).

In the redesign for redundancy an extension of this lower bound load path approach is followed. It is necessary only to recognize that the bottom lateral bracing is an alternate load path which functions together with the cross bracing and then to design the members of these bracing systems to carry the redistributed dead and live loads.

7.3.1 Design Assumptions

1. The longitudinal and transverse alignment of the top flanges of both girders are maintained, either through adequate composite connection to the concrete deck or by the addition of a top lateral bracing system with an in-plane stiffness comparable to the in-plane or membrane stiffness of the deck.

2. Full depth identical interior and end X or K-type cross bracing are provided with sufficient strength and stiffness to maintain the shape of the cross section, and to resist forces from the bottom lateral system.

3. An X-type bottom lateral system is provided at the level of the bottom flanges of both girders, with identical members in each bay.

4. Connections of cross bracing and lateral bracing members are to develop the full strength of the members.

5. Fatigue design of all components of the structure is to be in accordance with the redundant load path allowable stress range provisions of the AASHTO specifications, Art. 10.3.1. (2)

6. A midspan fracture of one girder of the simple span bridge extends through the bottom (tension) flange and through the full web depth. The top (compression) flange is intact and capable (along with the deck) of resisting the relatively small midspan shear.

7. The flexural strength, and shear strength and stiffness of the deck, the stringers and the floor beams are
8. Linear behavior of all members comprising the redundant load path systems is assumed.

9. The design of the lateral bracing system will conservatively ignore any effect of the cross bracing system in reducing the forces in the lateral bracing members.

10. As was done in the PADOT design calculations for the Betzwood bridge the design span length is rounded off from 89'-2" to 89'-0" c/c brgs.

7.3.2 Bottom Lateral and Cross Bracing Systems as Redundant Load Paths

In the normal design of a simple span two-girder steel bridge the two girders are considered to be the design load paths for all dead plus live loading. The live load is positioned for maximum effect in one girder and that girder designed for the resulting dead and live load moments and shears. The other girder is designed the same way. When a midspan fracture occurs in one of the two girders, the midspan bending moment is redistributed to other members of the as-built three dimensional structure.

In the following it is demonstrated that this bending moment can be redistributed to produce forces in the cross bracing and bottom lateral systems. Therefore, a safe lower bound design for redundancy can be based on the lateral bracing and cross bracing systems acting together with the girders to provide the redundant load paths. The resistance of other elements such as the flexural strength of the deck are ignored.
Figure 7.1 shows three schematic views of the simple span bridge. In (a) the bridge is shown with a midspan fracture of the near girder. The opposite girder is not fractured. The span is subjected to 1.1D plus two lanes of 1.3(L+I) where the truck loading is HS-20 with 30% impact. The two lanes of trucks are positioned over the fractured girder for maximum midspan bending moment. Stability of the span is maintained by forces developed in the cross bracing and lateral bracing systems.

The bridge shown in (b) is identical to the one in (a) except that both girders are unfractured. This represents the normal design condition for the bridge where it is assumed that no forces are generated in the cross bracing and bottom lateral bracing under the dead and live loading. Static equilibrium requires that the vertical reactions at the four girder bearings in (a) be identical to those in (b).

The bridge shown in (c) is identical to the one in (a) except that it is not subjected to dead and live loading. However, a moment, M, is applied at the midspan fracture location, where M is equal and opposite to the internal bending moment at midspan of the near girder in (b).

The principle of superposition of linear structures requires that the stress resultants for the bridge in Fig. 7.1(a) be equal to those in (b) plus those in (c). Since the cross bracing and bottom lateral bracing systems develop no forces in the bridge shown in Fig. 7.1(b), then the forces generated in the cross bracing and bottom lateral systems of (a) are a function only of the moment, M, released at the fracture location and distributed to those systems. A change in
flexural stresses also occurs in the two girders. It will be shown later that these stresses will not exceed the stresses obtained under normal design conditions.

The design of the lateral bracing system can therefore be based on a relatively simple modification of the usual procedure for designing one of the girders of a two-girder bridge. It is assumed that the moment capacity of the top flange of the fractured girder above the fracture is negligible and that the design can be based on a zero moment capacity of the fractured girder at midspan.

7.3.2.1 Design of the Bottom Lateral System

Figure 7.2 shows the analytical model used to calculate the bottom lateral bracing forces. A free body of the fractured girder is shown in (a). The girder is subjected to a uniformly distributed dead load of 3.537 k/ft, which is 1.1 times the dead load used to design the Betzwood bridge. With two lanes of HS-20 truck loading positioned for maximum midspan bending moment the girder is subjected to 1.65 lanes of live load, the same as that used to design the Betzwood bridge. With 30% impact and a 1.30 load factor the girder is loaded with two concentrated forces of 89.23k and one of 22.31k, as shown in (a). The bottom flange is also subjected to horizontal forces F1, F2 and F3 which are imposed by the lateral bracing system after the fracture occurs. The vertical reactions are also shown in (a). Although the cross bracing systems will also apply supporting forces to the girder shown in (a) these forces are ignored in the design of the bottom lateral system, which is consistent with the lower bound approach.
The total force $F = F_1 + F_2 + F_3$ acting at the level of the bottom flange on half the span can be calculated on the condition of zero bending moment at midspan as discussed in relation to Fig. 7.1. Alternatively, since the girder behaves as a 3 hinged tied arch, $F$ can be calculated using the influence line for $F$ shown in Fig. 7.2 (b) as follows:

$$F = \frac{1}{2} (89)(3.537)(2.871) + 89.23(2.871+1.968) + 22.31(1.968)$$

or

$$F = 927.57 \text{ kips} \quad (7.1)$$

The arrangement of the bottom lateral system is shown in Fig. 7.2(c). The spacing center-to-center of the girder webs is 18.5ft. The forces $F_1$ and $F_2$ are each developed by two diagonal members framing in to the girder flange. The force $F_3$ is developed by only one diagonal member. It is assumed that all the diagonal members of the bottom lateral system are identical, having equal cross section areas, $A_b$, and properties.

Figure 7.3 shows the displacements of the fractured girder and the bottom lateral system after fracture. In (a) the fractured girder is shown in its deflected position. The horizontal displacement of the bottom flange at the fracture is $d$, as shown. The vertical displacement at the fracture therefore is $44.5d/7.75 = 5.742d$. It is shown later that the horizontal displacement $d$, and thus the vertical displacement $5.742d$ can be selected by the design engineer thus controlling maximum stress level in the bottom lateral system (Eq. 7.26).
In Fig. 7.3(b) the after-fracture displacements of the bottom lateral system are shown and result from unloading of the fractured girder. Both of the diagonals in Bay 3 are in tension. In Bays 1 and 2 one diagonal is in tension, the other in compression as shown. The flange of the fractured girder in the region A-B-C is subjected to compression. The flange of the unfractured girder in the region D-E-F-G is subjected to tension. The dashed lines show the original position of the bottom lateral and cross bracing members. The solid lines show the positions after fracture.

The horizontal displacement of joint A is \( d_1 = d \). Since no girder shortening occurs between Joint A and the fracture, \( s_1 = 0 \). The displacement, \( d \), is entirely controlled by the level of stress, selected by the design engineer, in the tension diagonals in Bay 3. The horizontal displacement \( d_2 \) of joint B is less than \( d \) by the amount of girder shortening, \( s_2 \), between A and B. Similarly the horizontal displacement of joint C is less than \( d \) by the amount of girder shortening, \( s_3 \), between A and C. The horizontal displacements of joints D, E, F and G on the unfractured girder due to girder elongation are \( e_1 \), \( e_1 \), \( e_2 \) and \( e_3 \), respectively. It is assumed in the design of the lateral bracing system that no relative displacement occurs between the girders. That is, the cross bracing horizontal members connecting the bottom flanges of the girders are assumed to be axially rigid.

The displacements \( s \) and \( e \) shown in Fig. 7.3(b) cannot be calculated until the forces in the bottom lateral system are known. In turn these forces are dependent upon the displacements \( s \) and \( e \). However, as a
result of these displacements the forces in the diagonal members are smallest in Bay 1 and largest in Bay 3. A trial and error approach can be used but is not necessarily needed to obtain reasonably accurate diagonal member forces for this study bridge. In this approach, displacements $s$ and $e$ are estimated from an assumed distribution of forces in the diagonal members. Based on these estimates the member forces are computed and used to revise the estimates of $s$ and $e$. The process is repeated, if necessary, until the desired accuracy is obtained.

It is shown later that good estimates of the forces in the diagonal members can be obtained from a first trial by assuming a distribution of forces that increases from Bay 1 to Bay 3. Such a distribution is shown in Fig. 7.3(c). This distribution is used herein to derive expressions for the required area of the diagonal members, $A_b$, the forces in the diagonal members, and the vertical deflection of the fractured girder, all as functions of the level of stress selected by the design engineer in the diagonals in Bay 1. These results will be compared with computer generated values in Fig. 7.4, using a modified version of the finite element model described in Chapter 4.

In Fig. 7.3(c), the forces in Bay 2 are assumed to be double those in Bay 1. In Bay 3 the forces are three times those in Bay 1. The resulting forces acting on both girders are also shown in (c) where $F = 927.57$ kips as derived in Eq. 7.1.

Consider, for example, the segment of the fractured noncomposite girder from A to B in Fig. 7.3(c). If $N$ is the sum of the forces applied at
joints B and C, then the displacement, $u$, of joint B relative to joint A is,

$$u = \frac{NL}{AgE} + \frac{NL}{EIg} \frac{h^2}{4}$$  \hspace{1cm} (7.2)$$

where $E = 29,000$ ksi (Young's Modulus)

$L = 17.8$ ft. \hspace{1cm} (Bay length)

$h = \text{overall girder depth (95 in. in Bay 1; 96 in. in Bays 2 and 3)}$

$Ag = \text{noncomposite area of girder (85.5 in.}^2 \text{ in Bay 1; 102.5 in.}^2 \text{ in Bays 2 and 3)}$

$Ig = \text{noncomposite moment of inertia of girder (135,803 in.}^4 \text{ in Bay 1; 174,546 in.}^4 \text{ in Bays 2 and 3)}$

Equation 7.2 can be used to calculate the relative displacement between any two joints on the fractured and unfractured girders except, of course, in Bay 3 of the fractured girder.

Equation 7.2 is used to calculate the following values of the displacements $s$ and $e$ shown in Fig. 7.3 (b).

$$s_1 = 0 \hspace{1cm} e_1 = +0.0261 \text{ in.}\n$$

$$s_2 = -0.0697 \text{ in.} \hspace{1cm} e_2 = +0.0958 \text{ in.}\n$$

$$s_3 = -0.0911 \text{ in.} \hspace{1cm} e_3 = +0.1173 \text{ in.}\n$$

If $k$ is the axial stiffness of a diagonal member, then

$$k = \frac{29,000A_b}{25.67 \times 12} = 94.144 A_b \text{ (kips/in.)}\n$$

where $E = 29,000$ ksi; $A_b = \text{area of the diagonal member and the length of the member is 25.67 ft.}$
Consider the tension diagonal in Bay 2. As shown in Fig. 7.3(b) this member is subjected to a displacement of $d - s_2$ at joint $B$ and $e_1$ at Joint $E$. The resulting tension force, $P$, in the diagonal member is

\[ P = 94.144 \ A_b \frac{17.8}{25.67} (d - s_2 - e_1) \]  \hspace{1cm} (7.5)

The tension or compression force in any diagonal member is therefore given by

\[ P = 65.281 \ A_b \ (d - s \pm e) \]  \hspace{1cm} (7.6)

where the values of $s$ and $e$ at the ends of the diagonal are provided in Eq's. (7.3).

The component, $P_h$, of $P$ in the direction of the girder is

\[ P_h = 45.27 \ A_b \ (d \pm e) \]  \hspace{1cm} (7.7)

The forces $F_1$, $F_2$, and $F_3$ acting on the fractured girder as shown in Fig. 7.2(a) can now be calculated in terms of $A_b$ and $d$. For example, at joint $A$, $F_1 = F_{11} + F_{12}$ where $F_{11}$ refers to Bay 3, $F_{12}$ to Bay 2, and

\[ F_{11} = 45.27 \ A_b \ (d + 0.0261) \]  \hspace{1cm} (7.8)
\[ F_{12} = 45.27 \ A_b \ (d - 0.0958) \]  \hspace{1cm} (7.9)

and

\[ F_1 = 45.27 \ A_b \ (2d - 0.0697) \]  \hspace{1cm} (7.10)

Similarly at joint $B$, $F_2 = F_{21} + F_{22}$, where

\[ F_{21} = 45.27 \ A_b \ (d - 0.0697 - 0.0261) \]  \hspace{1cm} (7.11)
\[ F_{22} = 45.27 A_b \left( d - 0.0697 - 0.1173 \right) \]  \hspace{1cm} (7.12)

and \[ F_2 = 45.27 A_b \left( 2d - 0.2828 \right) \]  \hspace{1cm} (7.13)

At joint C, \[ F_{31} = 45.27 A_b \left( d - 0.0911 - 0.0958 \right) \]  \hspace{1cm} (7.14)

or \[ F_3 = 45.27 A_b \left( d - 0.1870 \right) \]  \hspace{1cm} (7.15)

Let \( P_t \) be the tension force in the diagonal in Bay 3 in kips, and \( S_t \) be the tension stress in that diagonal in ksi. Then by Eq. 7.6,

\[ P_t = S_t A_b = 65.281 A_b \left( d + 0.0261 \right) \]  \hspace{1cm} (7.16)

or \[ d = \frac{S_t - 1.704}{65.281} \text{ (in)} \]  \hspace{1cm} (7.17)

and the vertical displacement, \( v \), at midspan of the fractured girder is, from Fig. 7.3 (a),

\[ v = 5.742 d = \frac{S_t - 1.704}{11.369} \text{ (in)} \]  \hspace{1cm} (7.18)

Equation 7.18 indicates that the midspan deflection of the fractured girder, a measure of the serviceability of the bridge after fracture, is a function of the tension stress, \( S_t \), in the diagonal members in Bay 3.

The required area for all the diagonal members, \( A_b \), is found from Eq. 7.1 by adding Eq's. 7.10, 7.13 and 7.15, and substituting for \( d \) from Eq. 7.17.

\[ \text{Required } A_b = \frac{267.518}{S_t - 8.748} \text{ (in.}^2) \]  \hspace{1cm} (7.19)
The stress, \( S_t \), in the diagonal tension members in Bay 3 is limited by the yield strength, \( F_y \). The limit state for the compression diagonal in Bay 2 is the AASHTO critical stress, 0.85 \( F_{cr} \). The component of force in this member in the direction of the girder is given by Eq. 7.9. Since \( A_b \) is required to be constant for all diagonal members, then the stress in the compression diagonal in Bay 2 is

\[
0.85 F_{cr} = 65.281 (d - 0.0958) \quad (7.20)
\]

or \( F_{cr} = 76.801 d - 7.358 \quad (7.21) \)

In terms of the midspan deflection, \( v \), of the fractured girder, given by Eq. 7.18

\[
F_{cr} = 13.375 v - 7.358 \quad (7.22)
\]

Substituting \( d \) from Eq. 7.17 into Eq. 7.21, the maximum stress, \( S_t \), in the tension diagonals in Bay 3 is given by

\[
S_t = 0.850 F_{cr} + 7.958 \quad (7.23)
\]

Article 10.54.1.1 of the AASHTO Bridge Specifications defines \( F_{cr} \) in the elastic and inelastic ranges of buckling. (2) Substitution of the AASHTO provisions into Eqs. 7.22 and 7.23 produces the following design equations, for the simple span right study bridge, based on the distribution shown in Fig. 7.3 (c).

The required \( (KL)^2/r \) for the compression diagonal in Bay 2 is

\[
(KL)^2/r = \frac{4\pi^2 E}{F_y^2} (F_y - 13.375 v + 7.358) \quad (7.24)
\]
when \( \frac{(KL)^2}{r} \leq \frac{2\frac{v^2E}{F_y}}{\text{Eq. 7.25}} \)

and \( \frac{(KL)^2}{r} = \frac{2\frac{v^2E}{F_y}}{13.375v + 7.358} \) (7.25)

when \( \frac{(KL)}{r} \geq \frac{2\frac{v^2E}{F_y}}{\text{Eq. 7.26}} \)

The maximum stress, \( S_t \), in the tension diagonals in Bay 3 is (Eq. 7.18)

\[ S_t = 11.369v + 1.704 \] (7.26)

It is now possible to design the diagonal members of the bottom lateral system for a specified limiting value of midspan deflection, \( v \), of the fractured girder as follows:

Let \( v \) be limited to the span length over 300, say. Then

\[ v = \frac{89 \times 12}{300} = 3.56 \text{ in.} \] (7.27)

For \( F_y = 50 \text{ ksi} \)

\[ \frac{(KL)^2}{r} = \frac{2\frac{v^2E}{F_y}}{50} = 11,449 \] (7.28)

or \( \frac{KL}{r} = 107 \) (7.29)

Assuming inelastic buckling, Eq. 7.24 gives

\[ \text{Required} \quad \frac{(KL)^2}{r} = 4,462 \]

or Required \( \frac{KL}{r} = 66.8 < 107 \) (7.30)

From Eq. 7.26, \( S_t = 42.2, \text{ ksi} \). From Eq. 7.19 \( A_b = 8.00 \text{ in}^2 \). and from Eq. 7.17, \( d = 0.62 \text{ in.} \)
The member forces $P_{11}$ in Bay 3, $P_{12}$ and $P_{21}$ in Bay 2 and $P_{22}$ and $P_{31}$ in Bay 1 are now calculated from Eqs. 7.8, 7.9, 7.11, 7.12 and 7.14, respectively, as follows:

\[
\begin{align*}
P_{11} &= +338 \text{ kips} \\
P_{12} &= -274 \text{ kips} \\
P_{21} &= +274 \text{ kips} \\
P_{22} &= -226 \text{ kips} \\
P_{31} &= +226 \text{ kips} \\
\end{align*}
\]

(7.31)

These forces are shown in Fig. 7.4 (a). The assumed distribution of forces (Fig. 7.3(c)) is also shown in parentheses in Fig. 7.4(a). The sum of all these forces over a half span must be constant and equal to

\[
25.67 \times 927.57 = 1,338 \text{ kips}
\]

The distribution of member forces based on the calculated values shown in Fig. 7.4 (a) is somewhat different from the assumed distribution. Rather than the assumed distribution of 3:2:1 in Bays 3, 2 and 1, respectively, the resulting distribution is 1.52 : 1.22 : 1.0.

To investigate whether the design of the diagonal members will change the member forces are recalculated using a distribution of 2.26 : 1.61 : 1.0 which is an average of the above two distributions. These values are shown in parentheses in Fig. 7.4 (b). The revised member forces corresponding to this distribution are also shown in Fig. 7.4 (b), together with the computed values of $A_b$, $S_t$ and $KL/r$. The resulting distribution is 1.82 : 1.23 : 1.0. Further refinement is not necessary.
A suitable steel shape can now be selected for the diagonal members of the bottom lateral system based on the conditions shown in Fig. 7.4(b). The diagonal members are laterally supported in both directions at the ends and the compression member is assumed to be braced only in the horizontal plane at mid-length, so that \( K = 0.75 \), \( L_x = 308 \) in. and \( L_y = 154 \) in. With \( F_y = 50 \) ksi,

\[
\frac{2F_y^2E}{F_y} = (107)^2
\]

Try WT 10.5 x 31

\[
A = 9.13 \text{ in}^2 \quad r_x = 3.21 \text{ in.} \quad r_y = 1.77 \text{ in.}
\]

\[
\frac{Kl}{r_x} = 72.0 \quad \frac{Kl}{r_y} = 65.3
\]

\[
S_t = 42.7 \text{ ksi} \quad d = 0.59 \text{ in.} \quad v = 3.39 \text{ in.}
\]

The WT 10.5 x 31 meets all the design requirements. The member forces corresponding to the area provided by this member are shown without parentheses in Fig. 7.4 (c).

For the design forces shown without parentheses in Fig. 7.4 (c) the lateral member between joints A and E (bottom horizontal member of the cross bracing at that location) is subjected to compression. The other lateral members are unloaded. Joints A and E will displace towards each other. The effect is to reduce forces \( P_{11} \), \( P_{12} \) and \( P_{21} \) in Bays 3 and 2. Since the sum of forces in the diagonal members must be constant then \( P_{22} \) and \( P_{31} \) must increase. However, design of the diagonal members assuming no shortening between joints A and E is seen to be conservative and safe.
A modified version of the finite element model of the simple span right bridge described in Chapter 4 was used to generate forces in the diagonal members. The resulting member forces are shown in parentheses in Fig. 7.4 (c). The tension forces in Bay 3 are in excellent agreement. In Bays 2 and 3 the compression forces are lower and the tension forces larger than calculated. However, none of the computer generated forces exceed the design conditions used to select the WT10.5 x 31. In Bays 2 and 3 the average forces obtained in the finite element model are in close agreement with the calculated forces.

The midspan displacement of the fractured girder obtained in the finite element model is 3.46 in. This is to be compared with \( v = 3.39 \) in. calculated using the WT 10.5 x 31 diagonal member.

The finite element model described in Chapter 4 was modified to conform as closely as possible with the assumptions used in developing the design equations presented in this article. The following specific modifications were made:

1. The concrete deck and stringer elements were removed.

2. A top lateral bracing system was added at the level of the top flanges of the girders, consisting of truss elements over both girders and over all floor beams, plus beam elements forming X bracing similar to that used in the bottom lateral system. The cross section areas of all top lateral bracing members was set equal to 10,000 in\(^2\) in order to create a very stiff horizontal membrane as is assumed in the design calculations.

3. The sloping members of all interior and end cross bracing were removed. The bottom horizontal members were replaced with truss elements having areas of 10,000 in\(^2\). to eliminate relative displacement between girders and moved down to the level of the bottom flanges as assumed in the design calculations.
4. The flexural and shear stiffness of the floor beams was reduced to near zero.

5. Both girders were constrained to remain vertical and to deflect at interior nodes in a vertical plane.

6. The bottom lateral X bracing members of the study bridge were removed and new WT 10.5 x 31 X bracing added at the level of the bottom flanges of the girders as assumed in the design calculations.

The dead, live and impact loading of the model is identical to that used in the design.

7.3.2.2 Design of Cross Bracing System

In design for redundancy the cross bracing system acting with the floor beams serves four basic functions:

1. Provide sufficient strength to resist the forces imposed by the bottom lateral diagonal members.

2. Transfer these forces into the deck where they are resisted by the lateral (horizontal) bending and shear strength of the deck.

3. Provide sufficient stiffness to allow only minimal distortion of the cross section.

4. Provide sufficient stiffness to minimize the relative displacement of the bottom flanges of the two girders.

For the study bridge all six end and interior cross bracing designs are assumed to be identical. The design is based on the maximum force condition which occurs for the cross bracing between joints A and E. Although other cross bracing could be designed for smaller forces in this case of a midspan fracture, larger cross bracing would be needed if the fracture occurs in other bays. In addition, the larger cross
bracing provides needed increased stiffness especially at the ends of the span.

The design forces at joints A and E are shown in Fig. 7.5 (a). These forces are the components of the forces in the diagonal members of the lateral bracing system at the two joints. The configuration of K bracing is also shown in (a). The member forces for the K bracing are shown without parentheses in (b), together with the sections selected for the members based on $F_y = 50$ ksi. The WT $10.5 \times 41.5$ spans between joints A and E and is assumed to be braced in a horizontal direction at midlength with L/r bracing extending from the intersections of the bottom lateral diagonal members in Bays 2 and 3. The WT $6 \times 20$ is used for both sloping members.

Member forces and sections for X bracing are shown in Fig. 7.5 (c). The WT $10.5 \times 31$ is also assumed to be braced in a horizontal direction at midlength. The WT $10.5 \times 46.5$ is used for both sloping members.

A comparison of Figs. 7.5 (b) and (c) indicates that for the study bridge the K bracing is more efficient.

The modified finite element model of the simple span bridge was again used to generate the forces shown in parentheses in Fig. 7.5 (b). The model is the same as that described in Art. 7.3.2.1 except that the K-type cross bracing members shown in (b) were added at each of the six end and interior locations. The six truss elements with $A = 10,000$ in$^2$ joining the bottom flanges of the girder were removed.
7.3.3 Girder Bending Moments and Stresses

In Fig. 7.6(a) and (b) the girders are shown together with their loading. The loading conditions and load factors are described in Art. 7.2. The unfractured girder is subjected to 0.35 lanes of truck loading. The fractured girder is subjected to 1.65 lanes of truck loading as is assumed in the design for redundancy described in Art. 7.3.2.1. The forces along the bottom flanges are computed from the member forces without parentheses in Fig. 7.4 (c). The 85 kip vertical force at the inner two cross bracing locations is computed from Fig. 7.5(b). The 20 kip vertical forces are computed in a similar manner.

Forces from the cross bracing are not shown in Fig. 7.6(b). The bottom flange forces are computed assuming no restraining forces from the cross bracing as discussed in Art. 7.3.2.1. Since they are the reverse of those shown in (a) it is conservative to exclude them when computing moments and stresses in the fractured girder.

The bending moments are shown in Fig. 7.6(c) for both girders. All moments are well within the capacity of the girders which is also shown in the figure.

The maximum stresses in the unfractured girder are 33 ksi tension and 24 ksi compression and occur at the cross bracing locations nearest midspan. These stresses are computed considering both axial and flexural effects.

Except at midspan, the maximum stresses in the fractured girder are 12 ksi tension and 21 ksi compression and also occur at the cross bracing
locations nearest midspan. At midspan, the top flange is assumed continuous over the fracture. In the design for redundancy, the moment capacity of the flange at midspan is ignored. In reality the flange will likely develop the reduced plastic capacity, $M_{pc}$, where

$$M_{pc} = \left(1 - \frac{P}{P_y}\right)^2 M_p$$  \hspace{1cm} (7.32)

for a rectangular section. For a 2" x 17" flange, and with $P_y = 36$ ksi

$$M_p = \frac{1}{4} \times 17 \times (2)^2 \times 36 = 612 \text{ k-in}$$

$$P_y = 17 \times 2 \times 36 = 1,224 \text{ k}$$

$$P = 928 \text{ k}$$

and

$$M_{pc} = 260 \text{ k-in} = 22 \text{ k-ft.}$$

This small moment capacity at midspan can be ignored. However, to develop $M_{pc}$ the girder reaches the yield stress, 36 ksi at midspan.

Except at midspan of the fractured girder all stresses are well within the 36 ksi design stress in tension and compression, where the compression flange is assumed to be laterally supported by the concrete deck.

7.3.4 **Influence of Concrete Deck, Stringers and Floor Beams**

A modified version of the finite element model of the simple span right bridge described in Chapter 4 was used to generate forces in the cross bracing and bottom lateral systems. The modified model is identical to that described except as follows:

1. The bottom lateral system is moved to the level of the bottom flanges of the girders. The diagonal members are WT 10.5 x 31 as shown in Fig. 7.4(c).
2. The K-type cross bracing system shown in Fig. 7.5(b) is used. The members shown in the figure are used at all six cross bracing locations.

Figure 7.7 shows the design of cross bracing and lateral bracing systems and the effect of the deck, stringers and floor beams on the forces in the bracing members. The forces calculated using the design for redundancy procedures described in Art. 7.3.2 are shown without parentheses. The forces generated using the modified finite element model are shown in parentheses. All stress resultants in all elements of the finite element model are below their respective limit states, except for a very local overstress in the deck directly above the fracture. The influence of this local overstress on the member forces shown in Fig. 7.7 is insignificant.

The effect of the deck, stringers and floor beams is to reduce virtually all the forces. Since the design is based on the largest calculated forces then Fig. 7.7 confirms that the design for redundancy presented in Art. 7.3.2 is conservative and safe for the midspan fracture case assumed.

The midspan deflection of the fractured girder obtained from the finite element model is 2.8 in. This is to be compared with the deflection \( v = 3.39 \) in. calculated in Art. 7.3.2.1 using the WT 10.5 x 31 diagonal members for the bottom lateral system. Part of this difference is attributed to the influence of the deck, stringers, and floorbeams, and part is due to the restraining effect of the cross bracing system which is ignored in the design of the lateral bracing diagonals for redundancy.
7.4 Simple Span Skew Bridge

The design for redundancy of the simple span skew bridge follows the same procedures that were developed in Art. 7.3 for the simple span right bridge.

Figure 7.8(a) shows the free body of the fractured girder. The dead and live loading condition is the same as that for the right bridge. The bottom flange is also subjected to horizontal forces imposed by the lateral bracing members after the fracture occurs. As before, the sum of these forces to each side of the fracture must be 927.57 kips as given by Eq. 7.1. However, as shown in Fig. 7.8(b) this force is developed by six diagonals to the left of the fracture but only four to the right. The forces in the diagonals to the right will be larger and the design therefore is based on the tension stress, $S_t$, in member A-D, shown in (c) and on the critical compression stress in member A-F of that figure. In Fig. 7.8(c) the member forces shown are based on the assumed distribution in parentheses. The member selected for all the diagonal members to the left and right of the fracture is a WT 10.5 x 34 with $F_y = 50$ ksi.

Although the cross bracing design is not included in this report, the design follows the same procedures discussed in Art. 7.3.2.2.

7.5 Two-Span Right Bridge

Chapter 5 clearly shows that the redundancy of the two-span study bridge is significantly reduced by the strength of the girder in the
negative moment region. Failure of the girder in this region essentially transforms the fractured girder into two simple spans.

Although design for redundancy of the two span study bridge can be based on the design of the cross bracing and bottom lateral systems, as for the simple span study bridge, a more cost effective design is based simply on a redesign of the girder.

Figure 7.9 shows the redesign for redundancy of the two-span girder. In (b) the end of the girder is at A. The fracture (an assumed girder hinge) is at H. The middle support is at O.

The D+L moment envelope with 23.3% impact used in the original design of the 2-span continuous girders is shown by curve A-B-C-D-E-F-G. The existing girder design is shown in Fig. 7.9(a). The capacity of the existing girder is shown in (b).

The girder is redesigned for redundancy assuming a moment hinge at H. The resulting D+L moment envelope is shown as curves A-H-J-K and L-M-N. Loading conditions, load factors and impact are as given in Art. 7.2. The resulting increased moments in the negative moment region exceed the girder capacity over about a 50 ft. length.

Figure 7.9(c) shows the redesign of the girder to carry the increased moments corresponding to a midspan fracture at H. The design stress in the compression flange near the middle support is calculated using the unbraced provisions of AASHTO Art. 10.48.4. (2)
7.6 Discussion

7.6.1 Simple Span Two-Girder Bridges

7.6.1.1 Role of the Bottom Lateral System

When fracture occurs two actions of the span occur which counter each other. The bottom lateral system provides the countering action.

First, vertical deflection of the fractured girder produces a torsional displacement or rotation of the span about its longitudinal axis which displaces the bottom flange of the unfractured girder laterally outwards with respect to that axis.

Second, vertical deflection of the fractured girder produces rotation of the ends of the girder about a horizontal axis. This rotation causes the ends of the bottom flange to displace outwards in the direction of the girder.

The bottom lateral diagonal members, being connected to both girders, resist these motions. One set of diagonals is subjected to tension, the other to compression. Both sets of diagonals are needed. The motions of the girders described above tend to elongate the one set of diagonals. This elongation is resisted by the compression diagonals which are anchored at the ends of the unfractured girder.

For this reason a design procedure for redundancy based on tension diagonals alone, and assuming that compression diagonals buckle is not possible. In this case, tension forces cannot develop in the diagonals except in the bay containing the fracture. Computer models simulating this condition show that the entire force $F$ given by Eq. 7.1 is
developed by these two tension diagonals and that large horizontal displacements of the bottom lateral system take place where the fractured girder displaces outward and the unfractured girder inward with respect to the longitudinal axis of the bridge.

7.6.1.2 Role of the Cross Bracing System

The cross bracing system maintains the spacing of the bottom flanges of the two girders, resists distortion of the cross section, is required to develop forces in the bottom lateral diagonal members and transmits these forces to the deck.

This investigation did not study how the forces in the cross bracing diagonal members are anchored by the deck. These members are connected to the floor beams in the study bridges. This would also be the case for essentially all other practical bridges. Thus the floor beams transmit the forces to the deck. The horizontal component of these forces act transverse to the direction of the deck. An efficient way to transmit the forces from the floor beams to the deck is through shear connectors. Composite floor beams therefore would be designed, not for flexural behavior, although they could be, but simply to carry the shear in the direction of the floor beam into the deck much in the same way as a drag strut or member functions in seismic design.

7.6.1.3 Role of the Deck

The deck is subjected to transverse forces at each interior floor beam location. These forces are carried by flexure and shear to the ends of the deck where they are transmitted to the end cross bracing members, again by providing composite floor beams at the ends of the bridge.
This investigation did not investigate the flexural and shear requirements of the deck to transmit the forces from the composite floor beams.

7.6.2 Development of Shear at Midspan Fracture

It is assumed in this investigation that the midspan fracture propagates from a fatigue crack at a midspan welded detail. Based on observations of the fractures which occurred in the girders of the I-79 and Lafayette St. bridges, it is further assumed that they will not propagate into the compression flange. In fact, the fracture will not likely reach the compression flange due to compression in the upper portion of the web. This compression is developed by the bottom lateral system. Although development of shear is not studied in this investigation it is assumed that at midspan the remaining web, top flange and deck in a practical situation will develop the small shear at midspan.

7.6.3 Other Fracture Scenarios

Other assumed fracture scenarios need to be studied in order to complete the design approach for redundancy developed in this investigation.

Cross bracing can exist at midspan. A midspan fracture might render this cross bracing ineffective, or change the way this cross bracing develops the forces in the bottom lateral members in the bays on either side.
Girder fractures can occur at other locations along the girder. Forces in the bottom lateral diagonals will be larger on the side with fewer bays, as is seen in the design of the skew bridge for redundancy.

As the fracture location moves away from midspan, the investigation of shear at the fracture becomes more important.

7.6.4 Simple Span Multigirder Bridges

The design for redundancy procedures developed in this investigation for simple span two-girder bridges are equally valid for multigirder bridges, provided that the fractured girder has a bottom lateral system on at least one side of the girder. In fact, the bottom lateral system is more effective in multigirder bridges since the remaining girders, cross bracing and deck allow the forces in the bottom lateral system to be developed more efficiently. The span does not tend to rotate about the longitudinal axis as much as for the two-girder bridge.

An alternate load path for multi-girder bridges exists in the form of the cross-bracing system acting as a continuous truss on three or more supports (girders). Providing this continuity, however, may be more difficult than it first appears. Tie plates are required to connect the top and bottom members across the girder flanges. Unless these tie plates are carefully designed they can fail by displacement induced fatigue cracking, rendering them useless. In addition, the cross-bracing "truss" nearest the fracture will likely carry most of the load, resulting in uneconomical designs.
7.6.5 **Two-Span Bridges**

As demonstrated in this investigation the most likely cost effective design for redundancy consists of redesigning the girders in the regions subjected to negative moment after fracture. The bottom lateral system is also a valid load path but less cost effective.

7.6.6 **Multiple Redundant Load Paths**

Redundant designs which proportion the forces between two or more load path systems such as the bottom lateral system and the redesigned continuous girders is likely to be very difficult and is not recommended at this time. Such designs require involved redistribution and may also require elastic-plastic design procedures.

7.6.7 **Computer Generated Design of Simple Spans For Redundancy**

Although hand calculations are developed and demonstrated for the design of the simple spans for redundancy, a computer can be used just as well. However, the following points should be observed:

1. The hand calculated design of the bottom lateral and cross bracing systems would serve as preliminary design for input to the computer.

2. The computer model must be based on a three-dimensional modeling of the two girders, the floorbeams, cross bracing, and bottom lateral system.

3. The two girders cannot be modeled as beam elements, otherwise a grid type model results which is unsatisfactory. (Refer to discussion in Art's. 5.4.1 and 5.4.2.)

4. The two girders should be modeled in a manner similar to that described in Chapter 4.

5. The model can include the deck. However, modeling of the deck must be done with care.
6. Alternatively, the deck can be ignored. In this case the in-plane (horizontal) stiffness or rigidity of the deck must be modeled in a manner similar to that discussed in this chapter.

7. The four supports must be carefully modeled. Include only the four vertical displacement restraints plus only three horizontal displacement restraints positioned to prevent free body displacement in the horizontal plane.
8. SUMMARY AND CONCLUSIONS

8.1 Summary

Design of welded steel two-girder bridges against fatigue by use of the allowable stress range provisions of the AASHTO bridge specifications does not ensure that fracture of the steel structure cannot occur. To minimize the consequences of fracture, should it occur, AASHTO requires the use of reduced allowable fatigue stress ranges for nonredundant load path structures. As a guide to design engineers, AASHTO classifies, by example, redundant and nonredundant load path structures. Multi-beam bridges are classified as redundant and two-girder bridges are classified as nonredundant. The purpose of this investigation is to study the behavior of three real two-girder steel bridge spans, determine whether or not redundant load paths exist, and if so, suggest design procedures and guidelines for ensuring redundancy of these two-girder bridges.

The three bridge spans selected for investigation are:

1. Simple-span right with 90-ft. span.
2. Simple-span skew with 90-ft. span and 45° skew
3. Two-span right with two 90-ft. spans

All three spans are from the Betzwood Bridge carrying LR 1046-1 over the Schuylkill River and Reading Railroad in Montgomery County, Pennsylvania, and were designed to HS20 truck loading.

Very little research has been done to quantify the degree of redundancy needed and available not only for two-girder bridges but for steel bridges in general. Those few relevant studies found that the cross
bracing and bottom lateral bracing systems effectively create redundancy in two-girder bridges but stop short of providing design guidelines which make use of these systems.

In recent years computer techniques have been developed to make the evaluation of redundancy through research more quantifiable than previously possible. These techniques now allow the development of guidelines suitable for routine design office use to ensure redundancy.

The computer-integrated engineering system owned and operated by the Department of Civil Engineering, Lehigh University, enables realistic, three-dimensional bridge models to be constructed and analyzed. This makes possible an examination of bridge behavior to a more detailed level of accuracy than possible only a few years ago. The use of this system to generate the lower bound elastic-plastic load-displacement curves is directly responsible for the insight which enabled the research team to understand the redistribution of load and the alternate load paths developed by the structures. This understanding led to the development of guidelines for the redundant designs of the study bridges. Modifications of the same finite element models used to develop the lower bound analyses were then used to verify the design guidelines. The use of computer graphics coupled with the finite element discretizations was quite helpful in suggesting practical mechanisms for use in the upper bound analyses.

Elastic-plastic analyses of the two-girder study bridges were conducted. Lower bound elastic-plastic finite element incremental dead, live and impact loading of the simple span right and two-span
bridges were conducted. The lower bound load-displacement curves of these two bridges were determined up to near the stability limit load. Upper bound rigid-plastic analyses were also conducted for all three bridges using discontinuous virtual displacement fields to satisfy the mechanism condition. Both types of analysis are of direct value in understanding the alternate load paths available after midspan fracture of a girder. This understanding led to the analytical approach taken to suggest guidelines for the economical design of the study bridges, and similar bridges, for redundancy following a midspan girder fracture.

Economical framing and design modifications are presented for the two single span study bridges based on guidelines developed in the investigation. These guidelines suggest the use of the bottom lateral and cross bracing system to ensure both a reasonable level of redundancy (strength) plus deflection control (serviceability) following a midspan girder fracture. An economical design modification is also presented for the two-span bridge to ensure a reasonable level of redundancy and deflection control.

8.2 Conclusions

In this investigation both upper and lower bound analyses are performed on actual, three dimensional, welded steel two-girder bridges. Upper bound analyses of all three bridges provided estimates of the stability limit loads. Lower bound analyses of a simple span and a two-span bridge provided elastic-plastic load-deflection curves up to near the
stability limit loads. Excellent agreement is achieved between the upper bound and estimated lower bound stability limit loads.

These analyses led to an understanding of load redistribution in the three bridges, to the identification of the alternate load paths that develop and to the formulation of design guidelines to ensure both redundancy (strength) and deflection control (serviceability) of the study bridges and similar bridges.

The following conclusions are based on the results of this investigation:

8.2.1 Simple Span Two-Girder Steel Bridges

1. Studies of redundancy and after fracture serviceability of two-girder bridges requires the use of three-dimensional analytical models in order to simulate the role each bridge member and component plays during load redistribution.

2. The Lehigh University, Department of Civil Engineering, computer-integrated engineering system employed in this investigation, making use of computer graphics coupled with finite element discretization of the real three-dimensional bridge is extremely valuable in the understanding of the redistribution of loads, identifying alternate load paths and the development of design guidelines and procedures for the study bridges and similar bridges.

3. For the two-girder study bridges the bottom lateral system is the primary alternate load path following midspan girder fracture.

4. The cross bracing systems of the simple span study bridges together with the deck are required to develop the forces which develop in the bottom lateral system.

5. The cross bracing systems of the simple span study bridges are also required to provide sufficient stiffness to prevent significant distortion of the cross section.
6. The bottom lateral system can be economically designed to provide both redundancy (strength) and deflection control (serviceability) following midspan fracture.

7. Design procedures and guidelines are developed for the design of the bottom lateral and cross bracing systems.

8. Reframing and redesign of the simple span study bridges is performed to demonstrate the validity of the proposed design procedures and guidelines for redundancy and serviceability.

9. The redundant designs of the simple span study bridges are verified by finite element modeling and analyses of the redesigned three-dimensional bridges.

10. It is suggested that the redundant design procedures developed for the simple span study bridges are applicable to similar bridges as well as to simple span multi-girder bridges.

8.2.2 Two-Span Two-Girder Steel Bridges

1. The two-span study bridge developed a reduced level of redundancy similar to the simple span study bridges and, as expected, is not automatically more redundant than a simple span bridge.

2. The major weakness of the two-span study bridge, from a redundancy point of view, is the reduced cross section at the point (region) of inflection, which is a normal situation in traditionally designed continuous girders.

3. The two-span study bridge can be redesigned for redundancy using the bottom lateral system as the redundant load path, similar to the simple span study bridges, although a more economical redundant design was chosen in this investigation.

4. In this investigation the two-span study bridge is designed for redundancy and serviceability after-midspan-fracture by redesigning the continuous girder over the negative moment region.

8.2.3 General Conclusions

1. Girder redesign for continuous steel girder bridges is a relatively simple and economical procedure to ensure redundancy and serviceability after midspan girder fracture.
2. The design procedures and guidelines developed in this investigation for the study bridges suggest themselves to the redundant design of other two-girder and multi-girder steel bridges with different configurations and with either the same or different girder fracture conditions.

3. The design procedures and guidelines developed in this investigation were demonstrated by means of hand calculations to show that they are relatively simple and easy to apply.

4. The design procedures and guidelines developed in this investigation produce lower bound, safe, redundant designs since the strength of other components such as the flexural strength of the deck are ignored.

5. These procedures may be performed by computer, but require a three-dimensional discretization of simple span two-girder bridges in order to develop accurate forces in the bottom lateral and cross bracing systems.

6. Hand calculations can be performed following the procedures developed in this investigation to provide a preliminary design of the bottom lateral and cross bracing systems for input to a finite element model for final analysis and design.
9. RECOMMENDATIONS

9.1 Guidelines for Design for Redundancy

The intent of research into redundancy is to provide guidelines for the design for redundancy of steel bridges in the form of design procedures and proposed bridge specification provisions. The investigation of the three study bridges is a major step in this direction. It has resulted in the identification of alternate load paths for simple and two-span, two-girder bridges following midspan girder fracture, developed economical design procedures to ensure redundancy and after fracture serviceability of the study bridges, and demonstrated by means of relatively simple hand calculations the application of these procedures to the three study bridges.

Although more research is needed to extend these procedures to other simple and continuous two-girder bridges, and to propose specification provisions which are applicable to two-girder steel bridges, the following guidelines, based on this investigation, suggest the direction such provisions may eventually take:

9.1.1 Simple Span Two-Girder Steel Bridges

1. Design the bottom lateral and cross bracing systems to provide redundancy and after fracture serviceability of simple span two-girder steel bridges.

2. Require the bottom lateral system to be framed to the girder bottom flanges with full strength connections. (30)

3. Alternatively, if the bottom lateral system must be framed above the bottom flanges of the girder, it is required that the forces from the bottom lateral system be transmitted directly to the girder bottom flanges so that minimum in-plane or out-of-plane distortions of the girder web occur.
4. Bottom lateral system to be designed with tension and compression members and to be continuous over the entire span.

5. Cross bracing system to be provided as currently specified but all interior cross bracing to extend and connect to the bottom lateral system.

6. Cross bracing at all interior and end locations to be designed for the forces transmitted by the bottom lateral system.

7. End cross bracing to extend full girder depth and to connect to the bottom lateral system so that all lateral forces from the bottom lateral system are transmitted directly into the end cross bracing.

8. Framing of cross bracing to consist of X or K type stable triangles with a horizontal bottom member.

9. Cross bracing forces to be developed by adequate connections to the concrete deck.

10. Concrete deck to be designed for the in-plane (horizontal) bending moments and shears transmitted by the cross bracing.

9.1.2 Two-Span Two-Girder Steel Bridges

1. Redesign the two continuous girders to provide for redundancy by developing the after fracture dead and live load moments and shears.

2. Alternatively, provide for redundancy with bottom lateral and cross bracing systems designed as for simple span two-girder steel bridges.

3. Suggest that redundancy not be provided for by sharing the after fracture design loads between redesigned girders as in 1 above, and bracing systems as in 2 above, until further research establishes the basis on which such sharing can or cannot be performed.

9.1.3 General Guidelines

1. Reduced dead load factor, such as 1.1, be used for load factor redundant design.

2. Reduced live load factor, such as 1.3, be used for load factor redundant design.
3. Higher allowable stresses be used for allowable stress redundant design.

4. Number of design traffic lanes loaded and reduction of live load intensity as currently specified.

5. Impact be increased to 30%, say, for design for redundancy.

6. After fracture serviceability be specified through a dead plus live load deflection limitation such as span over 300 ft.

7. Design for fatigue of two-girder steel bridges including design for redundancy use the AASHTO Bridge Specification provisions for redundant load path structures.

8. Design for fatigue of the redundant load path system to ensure that its full strength is available following girder fracture.

9.2 Further Research Needs

9.2.1 Two-Girder Bridges

1. Extension of research to consider other realistic girder fracture scenarios.

2. Perform computer simulation studies for each significant realistic fracture scenario to confirm known redundant load paths and to investigate new redundant load paths.

3. Justify the redundant load paths as the theoretical basis of the formulation of guidelines and proposed AASHTO specification provisions to ensure redundancy of all two-girder bridges.

4. Develop recommended guidelines and proposed AASHTO Bridge Specification provisions for the redundant design of two-girder bridges, for presentation at appropriate AASHTO Regional Meetings.

9.2.2 Multi-Girder Bridges

1. Extension of research to consider the possibility of guidelines developed for two-girder bridges being extended to multi-girder bridges.

2. Investigation of cross bracing systems to develop redundancy through continuous "truss" action.
3. Investigation of fracture scenarios peculiar to multi-girder bridges.

4. Computer simulation studies to investigate and confirm new load paths.

5. Development of design guidelines and procedures for redundant design of multi-girder bridges.

6. Investigation of load sharing between two or more alternate load paths.

7. Justification of proposed design procedures & specifications.


9.2.3 Other Bridge Configurations

1. Extension of research to consider two-girder and multi-girder steel bridges which are horizontally curved, straight and curved articulated and straight, curved and continuous skewed bridges.

2. Extension of research to consider steel through and deck type truss bridges, simple span and continuous.

3. Extension of research to single cell, multicell and multi box steel girder bridges.

4. Research into redundancy of steel bridge components such as tension hangers at the ends of suspension spans (Mianus River Bridge for example) and eye-bar bridges.

9.2.4 Loading Conditions, Load Factors, Allowable Stresses and Impact

Table 3.22.1A of the AASHTO Bridge Specifications (2) provides for combinations of loads for use in service load and load factor design. These are likely inappropriate in design for redundancy. Research needs therefore include the following items:

1. Appropriate loading conditions, such as number of lanes of truck or lane loading to consider.
2. The appropriate load factors for dead and live loading.

3. The appropriate load combinations.

4. The appropriate allowable stresses to use for dead and live loading.

5. The appropriate level of impact.

6. Definition of after fracture serviceability of the bridge.

9.2.5 Rating of Bridges

The AASHTO Manual for Maintenance Inspection of Bridges (26) provides for the rating of steel bridges by allowable stress and load factor methods. These provisions are applicable to bridges in which members and components have suffered loss of strength due to corrosion and damage, the extent of which has been evaluated through bridge inspection. They also apply to older bridges which were designed to earlier specifications and must be evaluated in terms of today's loading.

However, these provisions, as they currently stand are inappropriate for application to steel bridges which have suffered the kind of catastrophic damage envisioned in design for redundancy.

Research is needed to extend the design for redundancy concepts presented in this report and new concepts coming from future research not only to design specification provisions but also to provisions for the rating of bridges.
10. TABLES AND FIGURES
Table 3.1 - Illustration of Comparison of Conditions for Correct Elastic and Plastic Analyses

<table>
<thead>
<tr>
<th>Elastic Analysis</th>
<th>Plastic Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Continuity* Mechanism" /></td>
<td><img src="image" alt="Mechanism" /></td>
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<tr>
<td><img src="image" alt="Equilibrium" /></td>
<td><img src="image" alt="Equilibrium" /></td>
</tr>
<tr>
<td><img src="image" alt="First Yield" /></td>
<td><img src="image" alt="Plastic Condition" /></td>
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</table>

*Or compatibility
Table 4.1 Summary of Finite Elements Used

<table>
<thead>
<tr>
<th>Type</th>
<th>GTSTRUDL Designation</th>
<th>Shape</th>
<th>Degrees of Freedom</th>
<th>Bridge Components Modelled</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D truss</td>
<td>SPACE TRUSS</td>
<td></td>
<td>3 translations at each node</td>
<td>Cross frame diagonal</td>
</tr>
<tr>
<td>3-D beam</td>
<td>SPACE FRAME</td>
<td></td>
<td>3 translations and 3 rotations at each node</td>
<td>girder flanges girder stiffeners stringer flanges floor beam flanges floor beam stiffeners outrigger flanges outrigger stiffeners bottom laterals cross frame horizontals</td>
</tr>
<tr>
<td>Plane Stress</td>
<td>CSTG (1)*</td>
<td></td>
<td>2 translations at element centroid</td>
<td>outrigger web</td>
</tr>
<tr>
<td>Flat Shell</td>
<td>PSHQ (2)*</td>
<td></td>
<td>2 translations at each node</td>
<td>girder web stringer web floorbeam web outrigger web</td>
</tr>
<tr>
<td>Flat Shell</td>
<td>SBHQ6 (3)*</td>
<td></td>
<td>3 translations and 3 rotations at each node</td>
<td>deck</td>
</tr>
</tbody>
</table>

* Parentheses refer to notes in Art. 4.2
### Table 4.2 Limit State Criteria Employed for Simple Span Right Bridge

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter of Interest</th>
<th>Limit Criterion</th>
<th>Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansion bearing keeper plate</td>
<td>lateral load capacity</td>
<td>plastic capacity (yield line analysis)</td>
<td>11.9 k (1)</td>
</tr>
<tr>
<td>Fixed bearing anchor bolts</td>
<td>anchor bolt capacity</td>
<td>bolt shear capacity</td>
<td>42 k (2)</td>
</tr>
<tr>
<td>Bottom lateral</td>
<td>compressive capacity</td>
<td>buckling load</td>
<td>188 k (3)</td>
</tr>
<tr>
<td></td>
<td>tensile capacity</td>
<td>ultimate tensile strength</td>
<td>281 k (4)</td>
</tr>
<tr>
<td>Cross frame diagonal</td>
<td>compressive capacity</td>
<td>inelastic column buckling strength</td>
<td>58.1 k (5)</td>
</tr>
<tr>
<td>Cross frame horizontal</td>
<td>beam-column capacity</td>
<td>stability limit x 1/2 varies</td>
<td>varies (6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>strength limit varies</td>
<td>varies (7)</td>
</tr>
<tr>
<td>RC deck</td>
<td>cracking stress</td>
<td>tensile cracking</td>
<td>440 psi (8)</td>
</tr>
</tbody>
</table>

**Notes:**

1. Assume rigid to limit state
2. Assumes A307 steel
3. AASHTO Formula (10–151)
4. AASHTO Art. 10.46
5. AASHTO Formula (10–151)
6. AASHTO Formula (10–155)
7. AASHTO Formula (10–156)
8. AASHTO Art. 8.15.2.1.1 (See Art. 4.6.2 for further discussion)
<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter of Interest</th>
<th>Limit Criterion</th>
<th>Limit State</th>
<th>Notes: (cont’d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor beam connection plate</td>
<td>strong axis bending capacity</td>
<td>strength limit</td>
<td>Varies (9)</td>
<td>(9) AASHTO Formula (10-156) substituting M for M</td>
</tr>
<tr>
<td>Transverse stiffener</td>
<td>strong axis bending capacity</td>
<td>1/2 MP</td>
<td>243k-in</td>
<td>(10) AASHTO Formula (10-156)</td>
</tr>
<tr>
<td>Bearing stiffener</td>
<td>strong axis beam column capacity</td>
<td>strength limit</td>
<td>varies (10)</td>
<td>(11) AASHTO Art. 10.46</td>
</tr>
<tr>
<td>Stringer</td>
<td>bending capacity</td>
<td>plastic moment, as indicated by axial force in flange</td>
<td>134.6 k</td>
<td>(12) AASHTO Formula (10-156)</td>
</tr>
<tr>
<td>Floor beam</td>
<td>bending capacity</td>
<td>plastic moment, as indicated by axial force in flange</td>
<td>250 k</td>
<td>(13) AASHTO Formula (10-104)</td>
</tr>
<tr>
<td>Girder bottom flange (17&quot; x 1 3/4&quot;)</td>
<td>strong axis beam-column capacity</td>
<td>strength limit</td>
<td>varies (12)</td>
<td></td>
</tr>
<tr>
<td>Girder bottom flange (17&quot; x 1 3/4&quot;)</td>
<td>lateral buckling</td>
<td>inelastic lateral torsional buckling</td>
<td>1,011 k</td>
<td></td>
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Table 5.1 Summary of Base-Line Results  
(Undamaged Simple Span Right Bridge)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Finite Element Model</th>
<th>Hand Calculation (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Deflection</td>
<td>at midspan</td>
<td>0.61&quot;</td>
<td>0.49&quot;</td>
</tr>
<tr>
<td>Tensile stress in bottom flange</td>
<td>at midspan</td>
<td>9.4 ksi</td>
<td>9.3 ksi</td>
</tr>
<tr>
<td>of girder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal displacement of</td>
<td>at expansion</td>
<td>0.24&quot;</td>
<td>0.12&quot;</td>
</tr>
<tr>
<td>bottom flange</td>
<td>bearing (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal in-plane stress in</td>
<td>at midspan</td>
<td>0.24 ksi compression</td>
<td>0.21 ksi compression</td>
</tr>
<tr>
<td>deck</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum axial force in cross</td>
<td>in cross</td>
<td>18.3 k compression</td>
<td>---</td>
</tr>
<tr>
<td>frame horizontal</td>
<td>frame 2-3 (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum axial force in bottom</td>
<td>in bay 3 (2)</td>
<td>12.8 k tension</td>
<td>---</td>
</tr>
<tr>
<td>lateral</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:

(1) Based on treating the entire bridge section (including the deck) as a simple span composite beam, subjected to uniform dead load.

(2) Refer to Fig. 4.1
Table 5.2 Summary of Base-Line Results  
(Undamaged Two-Span Right-Bridge)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Finite Element Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical deflection</td>
<td>at midspan</td>
<td>0.39&quot;</td>
</tr>
<tr>
<td>Longitudinal displacement</td>
<td>at expansion bearing</td>
<td>0.09&quot;</td>
</tr>
<tr>
<td>Tensile stress in bottom flange of girder</td>
<td>at midspan</td>
<td>7.69 ksi</td>
</tr>
<tr>
<td>Compressive stress in bottom flange of girder</td>
<td>at fixed bearing</td>
<td>10.35 ksi</td>
</tr>
<tr>
<td>Longitudinal in-plane stress in deck</td>
<td>at midspan</td>
<td>0.19 ksi compression</td>
</tr>
<tr>
<td>Maximum axial force in cross frame horizontal</td>
<td>in cross frames 2-3 and 7-8</td>
<td>15 kips compression</td>
</tr>
<tr>
<td>Maximum axial tensile force in bottom lateral</td>
<td>in bays 2 and 9</td>
<td>10.7 kips</td>
</tr>
<tr>
<td>Maximum axial compressive force in bottom lateral</td>
<td>in bays 5 and 6</td>
<td>17.7 kips</td>
</tr>
</tbody>
</table>
Table 5.3 Load Increments, Simple Span Right Bridge

<table>
<thead>
<tr>
<th>Load Increment</th>
<th>Points Indicated in Fig. 5.5</th>
<th>Incremental Load Applied</th>
<th>Cumulative Load Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>0.29 D</td>
<td>0.29 D</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.28 D</td>
<td>0.57 D</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>0.43 D</td>
<td>1.0 D</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.42 (L+I)</td>
<td>D+0.42 (L+I)</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.21 (L+I)</td>
<td>D+0.63 (L+I)</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>0.40 (L+I)</td>
<td>D+1.03 (L+I)</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.50 (L+I)</td>
<td>D+1.53 (L+I)</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0.30 (L+I)</td>
<td>D+1.83 (L+I)</td>
</tr>
<tr>
<td>9</td>
<td>D</td>
<td>0.85 (L+I)</td>
<td>D+2.68 (L+I)</td>
</tr>
<tr>
<td>10 (1)</td>
<td></td>
<td>0.11 (L+I)</td>
<td>D+2.79 (L+I)</td>
</tr>
</tbody>
</table>

Note:

(1) The applied live load at this point is equivalent to that of HS56 truck loading (2.79 x 20 = 56).
Table 5.4  Load Increment - Two Span Bridge

<table>
<thead>
<tr>
<th>Stage</th>
<th>Points Indicated in Fig. 5.31</th>
<th>Load Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>0.6 D</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>1.0 D</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>D+0.15 (L+I)</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>D+1.0 (L+I)</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>D+1.2 (L+I)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>D+2.0 (L+I)</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>D+2.0 (L+I)</td>
</tr>
<tr>
<td>8</td>
<td>D</td>
<td>D+2.0 (L+I)</td>
</tr>
<tr>
<td>9</td>
<td>E</td>
<td>D+2.5 (L+I)</td>
</tr>
</tbody>
</table>
Table 6.1 Limit States

<table>
<thead>
<tr>
<th>Component</th>
<th>Comment</th>
<th>Limit States</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Deck</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transverse</td>
<td>Pos. &amp; Neg. Bending</td>
<td>11.00 kft/ft</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>Pos. Bending</td>
<td>6.26 kft/ft</td>
</tr>
<tr>
<td></td>
<td>Neg. Bending</td>
<td>4.11 kft/ft</td>
</tr>
<tr>
<td>Plate Girder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Web</td>
<td>3/8&quot; thickness</td>
<td>1.27 kft/ft</td>
</tr>
<tr>
<td>Bottom Flange</td>
<td>17&quot; x 1 1/2&quot; plate</td>
<td>325.13 kft</td>
</tr>
<tr>
<td></td>
<td>17&quot; x 2&quot; plate</td>
<td>433.50 kft</td>
</tr>
<tr>
<td>Cross Section</td>
<td>2-span at Inflection Point</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross Bracing</td>
<td>7&quot; x 14.75 Channel</td>
<td>11.67 kft</td>
</tr>
<tr>
<td>Stringers</td>
<td>W18 x 45</td>
<td>269.00 kft</td>
</tr>
<tr>
<td>Bearing</td>
<td>2 - 1&quot; x 7 1/2&quot; plates</td>
<td>168.80 kft</td>
</tr>
<tr>
<td>Stiffeners</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connection</td>
<td>2 - 1/2&quot; x 7 1/2&quot; plates</td>
<td>42.20 kft</td>
</tr>
<tr>
<td>Plate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transverse Web</td>
<td>2 - 3/8&quot; x 6&quot; plates</td>
<td>27.00 kft</td>
</tr>
<tr>
<td>Stiffeners</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: All structural steel is A36. Reinforcing steel $F_y = 40$ ksi. For concrete deck $f'_c = 3,500$ psi.
Table 6.2 Internal and External Virtual Work

<table>
<thead>
<tr>
<th></th>
<th>Simple Span Right (kft)</th>
<th>Simple Span Skew (kft)</th>
<th>Two-Span (kft)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Internal Virtual Work</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete Deck</td>
<td>260</td>
<td>316</td>
<td>203</td>
</tr>
<tr>
<td>Steel Girder</td>
<td>120</td>
<td>120</td>
<td>355</td>
</tr>
<tr>
<td>Steel Stringers</td>
<td>835</td>
<td>761</td>
<td>753</td>
</tr>
<tr>
<td>Cross Bracing</td>
<td>76</td>
<td>81</td>
<td>54</td>
</tr>
<tr>
<td>Connection Plates and Stiffeners</td>
<td>168</td>
<td>194</td>
<td>120</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,459</td>
<td>1,472</td>
<td>1,485</td>
</tr>
</tbody>
</table>

| **External Virtual Work** |                         |                        |               |
| **Dead Load**            |                         |                        |               |
| Concrete Deck and W.S.   | 263                     | 260                    | 241           |
| Curb and Parapet         | 68                      | 70                     | 52            |
| Steel Stringers          | 9                       | 9                      | 9             |
| Steel Girders            | 49                      | 45                     | 35            |
| Steel Floor Beams        | 13                      | 13                     | 10            |
| Cross Bracing            | 2                       | 1                      | 1             |
| Bottom Lateral System    | 4                       | 4                      | 3             |
| **Total**                | 408                     | 402                    | 351           |

| **Live Load**            |                         |                        |               |
| HS-20 Truck Loading      | 292U                    | 293U                   | 226U          |
Fig. 2.1 Overall Plan and Elevation of the Betzwood Bridge
(a) Plan View of Superstructure Below Deck

(b) Cross Section A-A
Elevation of Stiffened Girder

Fig. 2.2 Plan and Elevation Views of Simple Span Right Bridge
Fig. 2.4 Plan View of Simple Span Skew Bridge
Fig. 2.6(a) Bottom Laterals
See Fig. 2.6(a)

Girders
Stringers
Floor Beam and Cross Bracing

(a) Plan View of Superstructure

Notes
1. Web, stiffeners same as shown in Fig. 2.2(b)
2. Lateral connection plates not shown

Symmetrical Around Except Field Splice

(b) Cross Section A-A Elevation of Stiffened Girder

Flange (17" x 3/4")
Top and Bottom
Flange (17" x 1 1/8")
Top and Bottom
Field Splice (See Fig. 2.7)

87' - 2"
cc. Brgs.

87' - 2"

Fig. 2.5 Plan and Elevation Views of 2-Span Right Bridge
Fig. 2.6 Bottom Lateral Connections

(a) Connection Detail Where Bottom Laterals Cross

(b) Bottom Lateral Connection Plate Detail
Fig. 2.7 Girder Field Splice in Two Span Right Bridge
Fig. 2.8 Bearings
Fig. 2.9 Longitudinal Cross Section of the Deck

# 5 Bars @ 6" c.c.
Top and Bottom

# 4 Bars
@ 9" or 9½"

# 5 Bars
@ 9" or 9½"
(a) Example Structure and Elastic-Perfectly Plastic Stress-Strain Curve

(b) Mechanism Method - Assumed Mechanism

(c) Statical Method - Assumed Equilibrium Bending Moment Diagram

Fig. 3.1 Example of Mechanism and Statical Methods
Fig. 4.1 Simple Span Bridge Viewed from Underneath
Fig. 4.2 Finite Element Model of Simple Span Bridge
Fig. 4.3 Finite Element Discretization of Stringer and Girder in a Typical Bay
Fig. 4.4 Finite Element Discretization of Fractured Girder in Bay 3
(See Fig. 4.1 for Bay Numbering)
Fig. 4.5 Finite Element Discretization of a Cross Section at a Floor Beam Location
Fig. 4.6 Finite Element Discretization of Lateral Bracing System
Fig. 4.7 Finite Element Discretization of Bridge Deck
Fig. 4.8 Support Modeling Alternatives Considered
Expansion Bearing End

209.3k
30.4k
126.2k

0.42"
18.5k
173.3k

0.55"
24.9k
134.1k

(a) 6 Horizontal Restraints (see fig. 4.8 (a))

Fig. 4.9 Results of Support Modeling Study
(b) 3 Horizontal Restraints
(see fig. 4.8(b))

Fig. 4.9 (Continued)
(c) 4 Horizontal Restraints
(see fig. 4.8(c))

Fig. 4.9 (continued)
Fig. 4.10 Finite Element Model of Two-Span Bridge
Fig. 4.11 Finite Element Discretization of Stringers and Girders in Bay 4 of Two-Span Bridges
Fig. 4.12 Finite Element Discretization of a Cross Section at a Floor Beam Location
Fig. 4.13 Finite Element Discretization of Bottom Lateral Bracing System - 2 Span Bridge
Fig. 4.14 Finite Element Discretization of Deck - 2 Span Bridge
Fig. 4.15 Finite Element Discretization of Non-Composite Deck in Bays 5 and 6
Location of Girder Fracture

8 Horizontal Restraints

Fig. 4.16 Boundary Conditions Assumed
Fig. 4.17 Application of Wheel Loads to Simple Span Bridge

Wheel Loads
4 @ $P_1$
4 @ $P_2$

$P_1 = 5.2k$
$P_2 = 20.8k$

(HS 20 Trucks, 30% Impact)
Wheel Loads

$P_1 = 5.2 \text{k}$

$P_2 = 20.8 \text{k}$

( HS-20 Trucks, 30% Impact)

Fig. 4.18 Application of Wheel Loads to 2-Span Bridge
Element of Slab

Unit Length

Ultimate Strain of Concrete Reached

First Yielding of Tension Steel

First Cracking of Concrete

Fig. 4.19 Moment-Curvature Relationship of Reinforced Concrete Element (after Park & Gamble) - Ref. 31
Fig. 4.20 Constructing the Load-Deflection Curve of a Beam
Fig. 4.21 Constructing the Load-Deflection Curve of a Beam by Non-incremental Approach
Fig. 4.22 Constructing the Load-Deflection Curve Considering Buckling
Fig. 5.1 Forces and Deflections in Bottom Laterals due to Dead Load, Undamaged Bridge
Fig. 5.2 Cross Bracing Stress Resultants in Unfractured Simple Span Right Bridge

Contour Interval = 1 ksi
Moment Units = k - in
Applied Load = 1.0 DL

(a) Cross Bracing 2-3

(b) Cross Bracing 1-2

(c) Cross Bracing 0-1 (Expansion Bearing End)
Fig. 5.3 Forces and Deflections in Bottom Laterals due to Dead Load, Undamaged 2-Span Bridge
Fig. 5.4 Cross Bracing Stress Resultants in Unfractured Two-Span Right Bridge
Contour Interval = 1 ksi
Moment Units = k - in
Applied Load = 1.0 DL

Fig. 5.4 Cross Bracing Stress Resultants in Unfractured Two-Span Right Bridge (Cont'd.)
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Fig. 5.5 Upper and Lower Bound Analyses of Simple Span Right Bridge
Fig. 5.6 Reactions at Load Level A (see Fig. 5.5)
Fig. 5.7 Deflected Shape (x100) of Simple Span Right Bridge at Load Level A
(see Fig. 5.5)
Fig. 5.8 Forces and Deflections in Bottom Laterals at Load Level A in Simple Span Right Bridge (see Fig. 5.5)
Fig. 5.9 Deflections (x50) in Cross Bracing at Load Level A in Simple Span Right Bridge (see Fig. 5.5)
Fig. 5.10 Cross Bracing Stress Resultants at Load Level A in Simple Span Right Bridge (see Fig. 5.5)
Applied Load = 0.29 x DL

Fig. 5.11 Failures in Superstructure at Load Level A in Simple Span Right Bridge
(see Fig. 5.5)
Fig. 5.12  Deck Damage at Load Level A in Simple Span Right Bridge (see Fig. 5.5)
Fig. 5.13 Deflected Shape (x50) of Simple Span Right Bridge during the Load Increment Preceding Load Level B (see Fig. 5.5)
Applied Load = 0.43 DL
This Increment Contour Internal = 1 ksi

(a) Longitudinal Stresses

(b) Deflected Shape (x 50)

(c) Bending Moment in Bottom Flange, in the Horizontal Plane.

Fig. 5.14 Fractured Girder Stress Resultants and Deflections in the Simple Span Right Bridge during the Load Increment Preceding Load Level B (see Fig. 5.5)
Fig. 5.15 Unfractured Girder Stress Resultants and Deflections in the Simple Span Right Bridge during the Load Increment Preceding Load Level B (See Fig. 5.5)
Fig. 5.16 Forces and Deflections in Bottom Laterals of the Simple Span Right Bridge due to the Load Increment Preceding Point B (see Fig. 5.5)
Fig. 5.17  Deflection (x50) in Cross Bracing of the Simple Span Right Bridge during the Load Increment Preceding Load Level B (see Fig. 5.5)
Fig. 518 Cross Bracing Stress Resultants in the Simple Span Right Bridge during the Load Increment Preceding Load Level B (see Fig. 5.5)
Fig. 5.19 Failures in Superstructure at Load Level B in Simple Span Right Bridge
(see Fig. 5.5)
Fig. 5.20 Deck Damage at Load Level B in Simple Span Right Bridge (see Fig. 5.5)
Applied Load = 0.40(\text{LL} + I)
This Increment

Fig. 5.21 Deflected Shape (x50) of Simple Span Right Bridge during the Load Increment Preceding Load Level C (see Fig. 5.5)
Total Applied Load = 1.0 x DL
+ 1.03 x (LL + I)

Fig. 5.22 Failures in Superstructure of Simple Span Right Bridge at Load Level C (see Fig. 5.5)
Total Applied Load = 1.0 \times DL + 1.03 \times (LL+I)

Fig. 5.23 Deck Damage in Simple Span Right Bridge at Load Level C (see Fig. 5.5)
Total Applied Load = 1.0 x DL
+ 2.11 x (LL + I)

Fig. 5.24 Failures in Superstructure of Simple Span Right Bridge at Load Level D (see Fig. 5.5)
Total Applied Load = 1.0 x DL + 2.11 x (LL + I)

Fig. 5.25 Deck Damage in Simple Span Right Bridge at Load Level D (see Fig. 5.5)
Total Applied Load = 1.0 x DL
+ 2.79 x (LL + I)

Fig. 5.26 Failures in Superstructure of Simple Span Right Bridge at Load Level E
(see Fig. 5.5)
Total Applied Load = 1.0 × DL + 2.79 × (LL + I)

Fig. 5.27 Deck Damage in Simple Span Right Bridge at Load Level E (see Fig. 5.5)
(a) Idealized Loaded Girder

(b) Idealized Fractured Girder

(c) True Meaning of Idealized Fractured Girder

(d) Actual Fractured Girder

Fig. 5.28 Idealized and Actual Fractured Girder Behavior
Fig. 5.29 Effect of Fractured Girder Elongation on the Bottom Lateral Bracing System
Fig. 5.30 Behavior of Cross Bracing after Girder Fracture is Introduced
Fig. 5.31 Upper and Lower Bound Analyses of Two-Span Bridge
Applied Load = 0.6 x DL

Fig. 5.32 Reactions at Load Level A (see Fig. 5.31)
Fig. 5.33 Failures in Superstructure at Load Level A (see Fig. 5.31)

Applied Load = 0.6 x DL
Fig. 5.34  Deck Damage at Load Level A (see Fig. 5.31)
Applied Load = 1.0 x DL

Fig. 5.36 Deck Damage at Load Level B (see Fig. 5.31)
Applied Load = 1.0 x DL + 1.0 x (LL + I)

Fig. 5.37 Deflected Shape (x30) of the Two-Span Right Bridge at Load Level C
(see Fig. 5.31)
Fig. 5.38  Forces and Deflections in Bottom Laterals of the Two-Span Right Bridge due to the Load Increment Preceding Point C (see Fig. 5.31)
Fig. 5.39 Deflections (x20) in Cross Bracing of the Two-Span Right Bridge during the Load Increment Preceding Load Level C (see Fig. 5.31)
Fig. 5.39 Deflections (x20) in Cross Bracing of the Two-Span Right Bridge during the Load Increment Preceding Load Level C (see Fig. 5.31)
Fig. 5.40 Cross Frame Stress Resultants in the Two-Span Right Bridge during the Load Increment Preceding Load Level C (see Fig. 5.31)
Fig. 5.40 Cross Frame Stress Resultants in the Two-Span Right Bridge during the Load Increment Preceding Load Level C (see Fig. 5.31) (Cont'd)
Fig. 5.41 Failures in Superstructure at Load Level C
(see Fig. 5.31)

Applied Load = 1.0 x DL + 1.0 x (LL + I)

Fixed Brs.
Applied Load = 1.0 × DL + 1.0 × (LL + I)

Fig. 5.42 Deck Damage at Load Level C (see Fig. 5.31)
Fig. 5.43 Failures in Superstructure at Load Level D
(see Fig. 5.31)
Fig. 5.44 Deck Damage at Load Level D (see Fig. 5.31)
Fig. 5.45 Failures in Superstructure at Load Level E
(see Fig. 5.31)
Applied Load = 1.0 x DL
+ 2.5 x (LL + I)

Fig. 5.46 Deck Damage at Load Level E (see Fig. 5.31)
Fig. 5.47 Comparison of Analytical Results for Simple Span Right and Two-Span Bridge
Fig. 6.1 Simple Span Right Bridge - Virtual Displacement Field for the Concrete Deck
Fig. 6.2 Simple Span Right Bridge - Virtual Displacement Fields for the Two Girders and Bottom Lateral System
Fig. 6.3 Simple Span Right Bridge - Virtual Displacement Field for the Cross Bracing/Wheel Load Location
Fig. 6.4 Simple Span Skew Bridge. Virtual Displacement Field for the Concrete Deck

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Fig. 6.5 Simple Span Skew Bridge. Virtual Displacement Fields for the Two Girders and Bottom Lateral System
Fig. 6.6 Simple Span Skew Bridge. Virtual Displacement Fields for the Cross Bracing System.
Fig. 6.7 Simple Span Skew Bridge. Virtual Displacement Fields for the Cross Bracing and Location of Wheel Loads on the Deck
Fig. 6.8 Two-Span Right Bridge. Virtual Displacement Field for the Concrete Deck.
Fig. 6.9 Two-Span Right Bridge. Virtual Displacement Fields for the Two Girders and Bottom Lateral System
Fig. 6.10 Two-Span Right Bridge. Virtual Displacement Fields for the Cross Bracing System
Fig. 6.11 Two-Span Right Bridge. Virtual Displacement Fields for the Cross Bracing and Location of Wheel Loads on the Deck.
Fig. 7.1 Schematic Views of Simple Span Right Bridge
Fig. 7.2 Analytical Model for Calculating Bottom Lateral Forces

(a) Free Body of Fractured Girder

(b) Influence Line for $F = F_1 + F_2 + F_3$

(c) Bottom Lateral System
Fig. 7.3 Displacements of Girder and Bottom Lateral System After Fracture
Fig. 7.4 Design of Bottom Lateral System

(a) $A_b = 8.00 \text{ in.}^2$, $S_f = 42.2 \text{ ksi}$, $KL/r = 66.8$

(b) $A_b = 8.57 \text{ in.}^2$, $S_f = 44.7 \text{ ksi}$, $KL/r = 77.8$

(c) WT 10.5 x 31, $A_b = 9.13 \text{ in.}^2$, $S_f = 42.7 \text{ ksi}$, $KL/r = 72.0$

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Fractured Girder

Fig. 7.5 Design of Cross Bracing
Fig. 7.6 Bending Moments in Girders Compared with Girder Capacity
Fig. 7.7 Effect of Deck, Stringers and Floor Beams on Forces in the Cross Bracing and Bottom Lateral Systems
Fig. 7.8 Simple Span Skew Bridge - Design of Bottom Lateral Diagonal Members

(a) Free Body of Fractured Girder

(b) Bottom Lateral System

(c) WT10.5 x 34  \( A_b = 10.0 \text{ in}^2 \)  \( S_t = 43.5 \text{ ksi} \)  \( KL/r = 72.2 \)
Fig. 7.9 Two-Span Right Bridge. Redundant Design of Girder
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