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Implementation of Prestress Loss Estimation Procedures

A NEW PROCEDURE FOR ESTIMATION OF PRESTRESS LOSSES

by

Ti Huang

Report No. 470.1
Implementation of Prestress Loss Estimation Procedures

A NEW PROCEDURE FOR ESTIMATION OF

PRESTRESS LOSSES

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Ti Huang

Prepared in cooperation with the Pennsylvania Department of Transportation and the U.S. Department of Transportation, Federal Highway Administration. The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Pennsylvania Department of Transportation or the U.S. Department of Transportation, Federal Highway Administration. This report does not constitute a standard, specification or regulation.

LEHIGH UNIVERSITY
Office of Research
Bethlehem, Pennsylvania

Fritz Engineering Laboratory Report No. 470.1

May 1982
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ABSTRACT

A new procedure for the estimation of prestress losses is proposed for implementation by the AASHTO Subcommittee on Bridges and Structures. The procedure is a simplified version of a more general procedure based on rational consideration of the equilibrium, compatibility and material characteristic relationships. The new procedure is superior to the current method in its wide range of applicability and flexibility. All types of prestressed concrete beam members can be analyzed by the procedure, including those subjected to multistage post-tensioning. Prestress loss at any specified time is calculated directly, without the need of summing over several time intervals. Parts II and III are written in a format suitable for direct incorporation into the AASHTO Specifications and the Commentaries. A brief description of the research work leading to this proposal is given in Part V. Part IV contains a complete listing of technical reports emanating from this research effort.
I. IMPLEMENTATION STATEMENT

A new procedure is proposed for the estimation of prestress losses in highway bridge members. The procedure is based on a rational consideration of the stresses and deformations in the materials, and is applicable to prestressed concrete members of all types, including those subjected to multi-stage post-tensioning and segmental construction.

The current AASHTO method for prestress loss estimation deals only with the total loss at an unspecified time "after losses have occurred."(7) Losses at other times are not considered. Such a procedure may be sufficient for the simpler pretensioned and post-tensioned members, but becomes severely inadequate for newer structural systems involving prestressing at more than one time. Introduction of new steel or concrete materials also presents difficulties, since the empirically developed coefficients in the current procedure cannot be adjusted properly without extensive experimental data on the new combination of materials. The proposed procedure overcomes both of these shortcomings.

The new procedure provides significantly more flexibility and versatility, in comparison with the current AASHTO procedure. Specifically, the advantages of the new procedure include the following:

1. It enables the determination of prestress loss at any time throughout the service life of the structural member.

2. It enables the estimation of prestress loss in individual prestressing elements.

3. It is directly applicable to structural systems involving multi-stage prestressing, including pre-post-tensioning and segmental construction.
4. It gives reasonable allowance to the effect of permanent loads applied at different times.

5. It recognizes the variability of the shrinkage and creep characteristics of concrete.

6. It allows for a variation of the initial tensioning stress of the prestressing elements.

7. It permits easy expansion to accommodate new steel and concrete materials.

8. It enables the establishment of the total history of prestress variation (and stress variation) throughout the service life of the structural member.

9. It enables the load rating of an existing beam (e.g. for the purpose of overload permit control) according to its age. In general, a higher overload will be accepted for the earlier ages of a bridge structure.

10. It enables a realistic evaluation of stresses in a prestressed concrete beam at any given time. In particular, it points out the potential risk of overstressing as the age of the structure increases.

It is proposed that Article 1.6.7 of the AASHTO Standard Specification for Highway Bridges(7) be replaced by a new procedure with related changes in Article 1.6.2, as presented in part II. The new procedure is substantially the same as given in an earlier report (FL report 402.4, Appendix C). Editorial changes have been made for clarification and conformity with AASHTO format.
II. PROPOSED SPECIFICATIONS

1.6.2 Notations and Definitions

(A) Notations

Listed below are new notations to be added. In conjunction, the following existing notations should be deleted: \( \text{CR}_c, \text{CR}_s, \text{CR}_{\text{sp}}, f_{\text{cd}}, f_{\text{cr}}, \Delta f_s, L, \text{SH}, T_0, T_x, E_{\text{ci}}, f_{\text{cir}}, f_{\text{cds}} \) and FR. Unless specifically indicated otherwise, all quantities are expressed in consistent kip-inch-day units. All stresses are positive in tension.

\( a_{si} \) = Area of the prestressing element tensioned at the i-th stage

\( A \) = Area of member cross section

\( \text{ACF} \) = Prestress loss due to friction and anchorage seating

\( C_s \) = Coefficient for estimation of shrinkage correction, see Eq. 7

\( \text{CR} \) = Prestress loss due to creep of concrete

\( \text{CRA} \) = Correction to prestress loss for multistage post-tensioning

\( e \) = Eccentricity of prestress

\( E_s \) = Modulus of elasticity of prestressing steel

\( *f_c \) = Fiber stress in concrete

\( f_{c_{gi}} \) = Concrete fiber stress, at level of steel, caused by member's own weight activated at the time of i-th stage \( t_{si} \)

\( f_{c_{gi}} \) = Concrete fiber stress, at level of steel, caused by permanent loads (including members weight), activated at the i-th stage

\( *f_{cp} \) = Prestress in concrete

\( *f_p \) = Prestress in steel

*Notations preceded by an asterisk are used in Commentary only.
\( f_{pj} \) = Initial tensioning stress in steel

\( f_s' \) = Specified ultimate tensile strength of prestressing steel

\( *f_s \) = Stress in prestressing steel

\( f_{sdi} \) = Increment of concrete stress, at level of steel, due to the prestressing of the i-th stage

GL = Component of prestress loss, used for post-tensioned steel only, for the effect of member's own weight

IL = Initial prestress loss, immediately after introduction of concrete prestress

\( k \) = Combined friction and curvature coefficient = \( K + \mu \phi \), in ft\(^{-1}\)

\( l_a \) = Anchorage length, in feet

LD = Effect of applied permanent load on final prestress loss

\( *M \) = Bending moment on section caused by applied load, see Fig. C.1

\( M_{gi} \) = Moment caused by member's own weight, activated at the i-th stage

\( M_i \) = Bending moment activated at the i-th stage (\( M_i \) includes \( M_{gi} \))

\( n \) = Modular ratio of steel to concrete

\( *P \) = Axial compressive force on section caused by applied load, see Fig. C.1

PL = Prestress loss at concrete age of \( t \)

REL\( _1 \) = Relaxation loss in pretensioned strands occurring before transfer

S = Correction to prestress loss, accounting for the shrinkage occurring before post-tensioning

SRL = One part of the final prestress loss, independent of concrete stress

\( t \) = Age of concrete, starting from end of curing

\( t_{si} \) = Age of concrete when the i-th stage event takes place (for pre-tensioned steel, \( t_{si} \) is negative for Fig. 1 but taken as zero elsewhere)
TL = Final prestress loss at end of service life
x = Distance of a given section from the jacking end, in ft
x a = Distance of a given section from the end of the anchorage length, in ft.
*y = Distance of elementary area from the centroid axis of cross section, see Fig. C.1
A a = Anchorage seating distance, in feet
*ϕ = Curvature of tendon profile, in radians per foot.

The last subscript in a, f, c, f, f, M, M, and t stands for the specific individual stage or element, and may take on any numerical or symbolic value. E.g., a sk refers to the area of k-th element, t sm refers to the time of m-th stage, M 2 refers to the moment at stage 2, etc.

(B) Definitions

(1) Prestressing Element: A prestressing element designates a group of prestressing steel which are tensioned, and induce prestress in the concrete, at a common time. An element may refer to one or more post-tensioned tendons, or the entire collection of pretensioned strands.

(2) Stage: A stage is a specific time in the life history of a prestressed concrete member, when a permanent change of loading or prestressing takes place. The event occurring may be the introduction of additional prestress, the activation of additional permanent load, or both.

(3) Prestress: Prestress refers to the material stress which is not directly caused by the external loads. In practice, it is evaluated as the difference between the actual material stress
and the direct elastic stress caused of the external loads, which include the weight of the members itself.

(4) Initial Prestress: The initial prestress of a prestressing element refers to its stress at the jacking end immediately before the anchoring and releasing of the tensioning device.

(5) Loss of Prestress: Loss of prestress refers to steel only, and is measured with reference to the initial prestress. At any given time, the prestress loss in a prestressing element is the difference between its current prestress and its initial prestress.

1.6.7 Loss of Prestress

(A) General

Loss of prestress is calculated for each element and each time interval separately. The loss in the $k^{th}$ element $a_{sk}$ at time $t$, within the time interval between the $m^{th}$ and the $(m+1)^{th}$ stages, $(m \geq k)$, is calculated by the following equation.

$$ PL = IL + 0.22(TL-IL) \log(t-t_{sk}) $$

(1)

where $t_{sk}$ is taken as zero for pretensioned tendons. However, $PL$ shall not be taken as less than the value calculated for time $t_{sm}$ from the preceding time interval. The quantities $IL$ and $TL$ are calculated according to (B), (C) and (D) of this article.

(B) Initial Loss $IL$, Corresponding to the Initial Time $t_{sk}$

$$ IL = REL + ES + ACF + GL $$

(2)
where REL₁ = initial relaxation loss in pretensioned strands occurring before transfer, from Fig. 1. This term is omitted for post-tensioned tendons.

ES = Elastic shortening loss

\[ \sum_{i=k}^{m} f_{sdi} \] for post-tensioned elements

\[ \sum_{i=k}^{m} f_{sdi} \] for pre-tensioned elements

\[ f_{sdi} = -a_{si} f_{pj} \left( \frac{1}{A} + \frac{e_{i}e_{k}}{I} \right) \]

ACF = Loss due to friction and anchorage seating, see Article 1.6.7(C)

GL = Loss in post-tensioned elements due to member weight

\[ \sum_{i=1}^{k} f_{cgi} \] This term is omitted for pretensioned elements.

\[ f_{cgi} = M_{gi} \frac{e_{k}}{I} \]

(C) **Friction and Anchorage Seating Loss, ACF**

The loss of prestress due to friction and anchorage seating shall be calculated by the basic equation

\[ ACF = f_{pj} \left[ 1 - e^{-(Kx+\mu\alpha)} \right] \]  

(3)

where \( K,\mu = \) Wobble and curvature friction coefficient, respectively

\( x = \) Distance from the jacking end, but see below

\( \alpha = \) Total angle change in distance \( x, \) including those in vertical as well as horizontal planes.
The anchorage seating loss is nonuniformly distributed within a length $l_a$. To include this loss component in Eq. 3, the distance $x$ shall be taken as $l_a + x_a$, where $x_a$ is the distance from the end of the anchorage length $l_a$ to the section in question. The anchorage length $l_a$ is controlled by the seating slippage distance $\Delta_a$. For the case where the tendon profile has uniform curvature within the distance $l_a$, this length is determined as follows:

$$l_a = -\frac{1}{k} \ln \left[ 1 - \sqrt{\frac{E \Delta}{s} \frac{k}{f_p}} \right]$$

(4)

where $k = K + \mu \frac{A}{x}$

If the friction coefficients are small, the tendon profile is flat and the seating slippage distance is small ($kx \leq 0.30$), simpler equations may be used for ACF and $l_a$

$$ACF = f_p (Kx + \mu \alpha)$$

(5)

$$l_a = \sqrt{E \Delta / k} \frac{f_p}{f}$$

(6)

$K$ and $\mu$ values shall be determined experimentally for the materials used. When experimental data are not available, the values in the following table may be used.

<table>
<thead>
<tr>
<th>Type of Steel</th>
<th>Type of Duct</th>
<th>$(K/ft)$</th>
<th>$\mu$</th>
<th>$(K/m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire or ungalvanized</td>
<td>Bright Metal Sheathing</td>
<td>0.0020</td>
<td>0.30</td>
<td>0.0066</td>
</tr>
<tr>
<td>strand</td>
<td>Galvanized Metal Sheathing</td>
<td>0.0015</td>
<td>0.25</td>
<td>0.0049</td>
</tr>
<tr>
<td></td>
<td>Greased or asphalt-coated and wrapped</td>
<td>0.0020</td>
<td>0.30</td>
<td>0.0066</td>
</tr>
<tr>
<td></td>
<td>Galvanized rigid</td>
<td>0.0002</td>
<td>0.25</td>
<td>0.0007</td>
</tr>
<tr>
<td>High-strength bars</td>
<td>Bright Metal Sheathing</td>
<td>0.0003</td>
<td>0.20</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td>Galvanized Metal Sheathing</td>
<td>0.0002</td>
<td>0.15</td>
<td>0.0007</td>
</tr>
</tbody>
</table>
(D) **Final Total Loss, TL, for End of Service Life at** \( t = 36500 \) **Days**

\[
TL = IL + SRL - REL - S + CR - CRA - LD
\]

(7)

where SRL = Component of loss independent of concrete stress, from Fig. 2.

\( S = \) Correction for shrinkage occurring prior to \( t_{sk} \), for post-tensioned tendons only.

\( S = C_s \log t_{sk} \)

\( C_s = 4.0 \) and 2.2, respectively, for upper bound and lower bound loss estimates.

\( CR = \) Loss due to creep = \(-1.2 \sum_{i=1}^{m} f_{sdi} \)

\( CRA = \) Correction for creep strain occurring prior to \( t_{sk} \)

\( LD = \) Long term effect of applied loads

\( f_{cli} = \) Concrete stress caused by permanent load, including member weight, activated at stage \( i \)

\( f_{cli} = M_i \frac{e_k}{I} \)
Fig. 1 Initial Relaxation Loss in Pretensioned Strands

Stress Relieved Strands

\[ fp_j = 0.8 f_s \]

Low Relaxation Strands

\[ fp_j = 0.8 f_s \]
Fig. 2 SRL Part of Final Prestress Loss
III. COMMENTARY

The proposed procedure is a simplified version of a general procedure which is more suitable for computer operations. This simpler version is designed to facilitate manual applications.

The general procedure makes use of stress-strain-time characteristic relationships of the concrete and steel materials. Linking these relationships with the compatibility and equilibrium conditions enables a complete analysis of the prestressed concrete member at any time during its service life. The procedure is completely rational, the determination of stresses is direct, and the interaction of the long-term behavior of the two materials is automatically taken care of. On account of the rational approach, extension for new material or new construction procedure can be done easily. Only the material characteristic relationship needs to be determined experimentally. Details of the general procedures are given in several published reports (Fritz Laboratory Report 339.9, 382.5, 402.4). A brief summary is contained in part V of this document.

The simpler procedure proposed here for specification implementation was developed based on a parametric study using the general procedure. Like the general procedure, it is applicable to prestressed concrete members of all types, including those with multi-stage prestressing.

The following specific commentary is keyed to the sections and subsections of the proposed specifications, part II.

1.6.2(B)
(2) Stage: The individual stages are identified by the age of concrete when the event takes place. Pretensioned strands are stretched before the casting of concrete, but do not induce concrete prestress until
transfer at the end of curing \((t = 0)\). Therefore, the associated
time \(t_{sl}\) is negative for the evaluation of \(\text{REL}_l\) in Fig. 1, but taken
as zero for all other purposes.

In this document, quantities related to a certain stage are designated
by a subscript signifying the sequential order of the stage. Thus,
the \(i\)-th stage takes place at time \(t_{sl}\), when prestress is induced by
element \(a_{sl}\), and simultaneously an increment of permanent load is
activated, causing bending moment \(M_i\) (which may include a part \(M_{gi}\)
due to member weight). Either \(a_{sl}\) or \(M_i\) may be zero.

(3) Prestress: The definition of prestress recognizes its load-independent
and self-equilibrated characteristics. It is directly derivable from
the familiar equations for fiber stresses in a prestressed member.
These equations show the material stress as the sum of the prestress
and the direct effect of the applied loads.

\[
f_s = f_p + n\left(-\frac{P}{A} + \frac{M}{I}\right)
\]

\[
f_c = f_{cp} + \left(-\frac{P}{A} + \frac{M}{I}\right)
\]

where \(A, I, e\) and \(y\) refer to the transformed cross section of the member,
in which the steel is replaced by \(n\) times its area in concrete. The
sign convention for the load effects and distances are given in Fig.
C.1. These equations imply that the prestresses are a fixed character-
istic of the section in question, and are not influenced by the present
loading condition. On the other hand, prestresses are time dependent,
and the creep and relaxation behaviors are both controlled by the total
stress history. Consequently, any sustaining (permanently applied)
load would have an indirect effect on the prestresses. The weight of the member is a load exerted by an external agent (i.e., the earth), therefore, it has no direct effect on the prestress, and must be included in the load effects to be removed in the evaluation of prestress.

Under the zero load condition, only prestresses are present in a prestressed member, and they must satisfy the static equilibrium. Therefore, corresponding to the i-th stage prestressing:

\[ f_{cp} = -\frac{a_{si} f_p}{A} + \frac{a_{si} f_p e_i y}{I} \]

where \( A, I, e \) and \( y \) refer to the cross section effective in resisting the tensioning of prestressing element \( a_{si} \). For pre-tensioned members, this would be the transformed section at the transfer time, although ordinarily the gross section properties can be used without inducing serious errors. (Exceptional cases would be where the section is heavily prestressed and the modular ratio is high, a detailed discussion is given in project report 339.9.) For post-tensioned members, the effective section includes the concrete and all previously anchored steel, and changes from stage to stage. Any empty space occupied by the ducts should be deducted.

On account of the varying effects of external loads along the length of a member, the prestress in a given element also varies along the length. Therefore, to be strictly meaningful, prestress values must be identified not only with the time, but also with the location.
(4) Initial prestress: The initial prestress is the last amount subjected to direct control. After anchorage and release, stresses and pre-stresses change as dictated by the material and loading characteristics but are no longer directly controllable.

For pre-tensioned strands, the friction and anchorage seating losses are very small, and the initial prestress may be taken as the stress in the strands after anchoring to the prestressing bed.

(5) Prestress loss: The principal sources of prestress loss include friction and anchorage seating, elastic deformation, creep, shrinkage, and relaxation. While the first two components can be accurately predicted on the basis of rational theories, the understanding of the last three components is not complete. Furthermore, these three time-dependent effects are strongly interdependent such that it is not appropriate to separately estimate each effect and then sum them together. The proposed procedure takes full account of the inter-relation among the several components.

1.6.7(A) General

Prestress loss differs for each prestressing element, and for different locations along the length of the member. For the purpose of design, evaluation of losses is usually needed only at the section of maximum service load moment. For a simply supported member, this section may be taken at the midspan.

From a computerized parametric study, it was found that without introducing additional prestress or permanent load, the growth of prestress
loss with time can be closely approximated by the linear semi-logarithmic relationship represented by Eq. 1. This equation shows that the prestress loss is IL at the initial time $t_{sk}$ and TL at the end of service life, taken to be 100 years after the termination of curing ($t = 36500$ days).

It is emphasized that the linear relationship is valid only within each time interval between two consecutive stages. At the end of an interval, the addition of prestress (tensioning of new tendons), or permanent load (caused by members weight or other additional dead load), or both, causes increment of stresses in concrete. These stress increments, $f_{sd1}$, $f_{cg1}$ and $f_{cgl}$, change IL and TL for all previously anchored steel elements, resulting in changed line segments for the new time interval, as shown in Fig. C.2.

If the stage event involves only the addition of a permanent load (e.g. casting of deck slab), there will be no change in IL, but a decrease in TL, resulting in an abrupt decrease of prestress loss at the stage time. This is caused by certain approximations used in this simplified procedure. The real behavior, as obtained by the basic general procedure, is more gradual, but distinctly nonlinear (Fritz Laboratory Report 382.5). For the sake of simplicity, such sudden decrease is ignored, and the prestress loss is taken as remaining constant for a part of the next time interval. Fig. C.2 shows the typical variation of prestress loss over several time intervals.
1.6.7(B) Initial Loss

IL is the loss of prestress calculated for the time of tensioning \((t = t_{sk})\), and represents the difference between the jacking stress \((f_{pj})\) and the prestress at the desired location immediately after anchorage. For pretensioned elements, it is calculated for \(t_{sl} = 0\).

REL\_1 applies to pretensioned strands only, it accounts for the relaxation loss in the strands before transfer. For its determination from Fig. 1, the actual negative values for \(t_{sl}\) prestressing is used.

ES represents the effect of elastic deformation caused by successive prestressing. The lower limit of summation is different for pre- and post-tensioned elements. In pretensioned strands, the shortening of concrete upon transfer causes a corresponding loss in the strands. In contrast, the post-tensioned tendons are tensioned against the concrete members, the jacking stress being measured after concrete has already shortened. Consequently, post-tensioned tendons do not cause an EL loss in themselves.

GL represents a nominal loss of prestress in post-tensioned tendons, on account of the dead load at the time of tensioning. The cambering of the member upon post-tensioning causes the \(f_{cgl}\) stress to develop at the time when \(f_{pj}\) is being measured. In line with the definition of prestress given in section 2.3, this effect must be removed to arrive at the prestress. This term is not used for pretensioned strands, since the tensioning of pretensioned strands does not coincide with the cambering of the member.

For a simple pretensioned member, IL is evaluated at \(t = 0\), ACF and GL are omitted, and
IL = REL₁ + ES

$$ES = n A_{ps} f_p j (\frac{1}{A} + \frac{e^2}{I})$$

where

$$A_{ps} = \text{Total area of pretensioning strands.}$$

1.6.7(C) Anchorage and Friction Losses

These losses are caused by the friction between the prestressing tendons (or strands) and the conduit along which its slides during the tensioning or releasing process. The combined amount of these losses is controlled by the surface roughness, the curvature of the tendon profile, and the seating slippage distance, and is dependent on the distance from the jacking end. The basic equation (3) is derived from the principles of statics.

It is important to note that $\alpha$ represents the sum of all changes in directions, within the distance $x$, in all longitudinal planes (vertical, horizontal as well as inclined). Resolving an angle change in an inclined plane into its horizontal and vertical components for summation into $\alpha$ results in a conservative but reasonable approximation, slightly over-estimating the loss.

For pretensioned strands, friction exists only at the bulkheads and deflecting devices and can be relieved before placing concrete. Also, the seating distance is insignificant in comparison with the length of the prestressing bed. Consequently, the component ACF can be, and usually is, neglected.
Because of friction, the anchorage seating loss in a post-tensioned tendon is severely nonuniform in its distribution, being concentrated near the jacking end. As distance from the jacking end increases, the anchorage seating loss decreases rapidly, while the frictional loss increases at a slower rate, resulting in a gradual decrease of ACF. Beyond a certain distance $l_a$, the anchorage seating loss disappears, and the ACF reflects only the gradually increasing friction effect. When using Equation 3 to calculate ACF at a location within the anchorage length, the distance $x$ should be taken as $(l_a + x_a)$, where $x_a$ is the distance from the end of the anchorage length $l_a$, as shown in Fig. C.3.

The method for evaluating the anchorage length $l_a$, and the derivation of equations for several typical conditions have been presented in several reports and publications (Report 402.3 and publication 1). Equations 4 and 6 are for the cases where the tendon profile has uniform curvature over the length $l_a$ (e.g. a flat parabolic profile). In this case, the angle change $\alpha$ is directly proportional to the distance $x$, and a combined friction coefficient can be formulated

$$k = K + \mu \phi$$

where $\phi = \text{Curvature of the tendon profile, in radians per foot}$. Equation 4 is derived by equating the shortening of the tendon within the anchorage length to the seating slipping distance $\Delta_a$. At $x = l_a$ (or $x_a = 0$), the ACF is smallest,

$$(\text{ACF})_{\min} = \sqrt{E\Delta \frac{k_f}{s_a \rho_j}}$$
If the friction effect is small \((k x < 0.30)\), the exponential function in Eq. 3 can be approximated by a linear expansion, resulting in Eq. 5. Equation 4 is similarly simplified into Eq. 6. The equation for \((ACF)_{\text{min}}\) remains unchanged for this case.

1.6.7(D) Final Loss

The final prestress loss is calculated for the end of the assumed service life of the member. It includes the long-term effects of shrinkage creep and relaxation.

SRL is a quantity independent of the stresses in concrete. It is rather dependent upon the shrinkage and relaxation characteristics of the materials. Its evaluation is empirically obtained from Fig. 2.

\(S\) is a correction term for SRL, reflecting the shrinkage taking place prior to time \(t_{sk}\). That portion of the shrinkage strain will have no effect on the losses in the element \(a_{sk}\).

CR represents essentially the prestress loss due to creep. The negative sign in the defining formula is needed because \(f_{sdi}\) stresses are generally negative (compression). CRA is a correction term for the creep strain taking place prior to the tensioning of the \(k\)-th element. Similar to the \(S\) correction, these creep strains also have no effect on the loss in the element \(a_{sk}\).

LD represents the long-term effect of the sustaining external loads. Transient loads are not included in the calculation. These loads appear in short periods of time only, and their effects on prestress losses can be safely ignored.
For a simple pretensioned member, S and CRA are zero, and Equation 6 is simplified

\[ TL = SRL + EL + CR - LD \]

Once again, it is emphasized that IL and TL change for each time interval. However, most of the terms in Equations 2 and 6 remain unchanged. It is only necessary to calculate the changes in EL, CR and LD which are controlled by the two stress increments \( f_{sd} \) and \( f_{cl} \) (or \( f_{cg} \)).
Fig. Cl  Sign Convention for Applied Loads and Distances
Fig. C2 Typical Variation of Prestress Loss with Time
Fig. C3 Friction and Anchorage Seating Losses
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V. BACKGROUND

The proposed new procedure is the result of a fourteen-year research effort at Lehigh University. Initially, the investigation was limited to pretensioned concrete members then used on Pennsylvania highways. An earlier version of the procedure, applicable to pretensioned members only, was presented to the regional meetings of the AASHTO Subcommittee on Bridges and Structures in 1973\(^{(4)}\) and the Transportation Research Board in 1975.\(^{(6)}\)

The original procedure was later modified in line with a two-year field study on an experimental bridge, and expanded to include other prestressed concrete bridge members. The present version is directly applicable to pretensioned members, post-tensioned members, pre-post-tensioned members, as well as segmentally-constructed structures. Details of the research work and the evolution of the estimating procedures are contained in numerous project reports. The bibliography in Part IV of this document includes a list of these project reports, as well as related publications and references.

The fundamental concept used in the development of this procedure involves the representation of the material characteristics in the form of stress-strain-time relationships. These relationships reflect not only the instantaneous (elastic) material behavior, but also the long-term effects, namely, shrinkage and creep of concrete, and relaxation of steel. These relationships are then combined with the compatibility conditions of time and deformation, and the equilibrium of stresses. Such a rational consideration of all necessary conditions enables a direct solution of the stresses in the member at any specified time within its service life. Also, as the long-term rheological effects are included in the material characteristic equations, their inter-interference is automatically taken into account.
The experimental study of material characteristics involved long-term stress and strain measurements of specially designed specimens over a period of approximately six years. Initially, the scope of experiment included only the materials for pretensioned bridge beams used in the state of Pennsylvania. At that time, the state standard required a concrete compressive strength of 5000 psi at transfer and 5500 psi at 28 days. Prestressing steel was predominantly 7-wire stress-relieved strands of the 270 K grade. Other materials, including the low-relaxation strands, were also tested at later stages.

The parameters considered in the study of concrete characteristics included the following:

1. The material ingredients and composition, as represented by the several fabricating plants of pretensioned bridge beams for Pennsylvania Department of Transportation (seven plants)
2. Amount of embedded prestressing steel (six variations)
3. The average initial compressive stress (six stress levels, including zero stress)
4. The lateral stress gradient (five stress gradients)
5. The storing environment (laboratory or outdoors)

The volume-to-surface ratio of the specimen, commonly recognized to be an important factor affecting shrinkage and drying creep, was not varied in this study. It was found that all standard beam sections of PennDOT as well as AASHTO have the volume-to-surface ration values within a narrow range. All concrete specimens were made 12 in. by 24 in. in cross section. The volume to surface ratio of 4 in. was approximately the mean value for all the standard I and box shapes. Complete descriptions of these specimens are contained in project report 339.3
Longitudinal strain measurements were taken at pre-selected time intervals over the entire study period. Extensive analysis was made on the elastic, shrinkage and creep strain data in order to determine the significance of the several controlled parameters. The concretes from different fabrication plants were found to behave quite differently, although they all conform to the same strength requirements. In recognition of this variation, two concrete characteristic equations were developed, representing the upper and lower bounds of concrete deformability. On the other hand, the lateral stress gradient had very little effect on the strain behavior, and hence was removed from the formulation.

Considerable effort was devoted to the selection of time functions for the long-term strains. Stability of long-term prediction was used as the primary criterion for selection. For this purpose, numerous regression analyses were made using strain data from different blocks of time, and variations of the predicted strain at the end of a long service life were compared. A modified logarithmic time function was finally selected on account of the relatively stable long-term predictions. For these comparisons, the length of the service life of a prestressed concrete bridge member was taken to be 100 years (project report 339.7).

The concrete stress-strain-time relationship initially was developed in the following form, intended for pretensioned members only.

\[ S_c = -Cf_c + D_1 + D_2 \log(t_c+1) \]

\[ + E_1 + E_2 \log(t_c+1) - \left[ E_3 + E_4 \log(t_c+1) \right] f_c \]  

(B1)

Notations in Part V follow the definitions given in Part II. The few notations not included in Part II are defined where they first appear.
where \( S_c \) = Concrete strain, shortening positive, in \( 10^{-2} \)

\( f_c \) = Concrete stress, tension positive, in ksi

\( t_c \) = Age of concrete, in days.

In the above equation, the first term represents the elastic strain, the next two, with D coefficients, represent the shrinkage strain, and the last terms, with E coefficients, represent the creep strain. The empirical values for the coefficients are listed in Table B.1.

In later phases of the research program, the concrete characteristic equation was modified to accommodate post-tensioning as well as incremental loading at various times. The more complete stress-strain-time relationship, used in the general procedure, is as follows (project report 4Q2.3).

\[
S_c = -Cf_c + D_1 + D_2 \log(t_c + 1) \\
+ E_1 + E_2 \log(t_c - t_{s1} + 1) - E_3 f_c \\
- E_4 (f_c - \sum f_{ci}) \log(t_c - t_{s1} + 1) \\
- E_4 \sum f_{ci} \log(t_c - t_{si} + 1)
\]

(B2)

where \( t_{s1} \) = Age of concrete when concrete is first prestressed. (If pre-tensioned, initial concrete stress occurs at transfer, and \( t_{s1} = 0 \)).

\( t_{si} \) = Age of concrete when the \( i \)-th increment of concrete stress is introduced.

\( f_{ci} \) = Increment of concrete stress introduced at the \( i \)-th stage.

The steel characteristic equations were developed from a similar regression analysis of experimental data. Strand specimens were tested for the instantaneous as well as long-term stress-strain behavior. Controlled parameters varied in the study included the following:
1. Manufacturing procedures (three manufacturers)
2. Size of strands (two sizes)
3. Initial stress (or strain) levels (three levels)
4. Controlled variation of stress or strain (three loading programs)

Initially, the study was restricted to stress-relieved strands of the 270 K grade. A total of 40 strand specimens were tested over an effective length of 10 ft. Most were tested in structural steel load frames, with one series encased but unbonded to slender concrete members. Similar tests for low-relaxation strands were conducted at a later stage.

Analysis of the stress and strain data over a nearly-six-year period revealed that neither the manufacturing source nor the strand diameter affects the stress-strain behavior significantly. Consequently, a single expression is recommended for all stress-relieved strands. Following a comparison procedure similar to that used for the concrete characteristic equations, a nonlinear stress-strain-time relationship for prestressing steel was selected in the following form:

\[
\frac{f_s}{f_s'} = \left( A_1 + A_2 S_s + A_3 S_s^2 \right) \left\{ \frac{B_1 + B_2 \log(t_s + 1)}{t_s} - \frac{B_3 + B_4 \log(t_s + 1)}{t_s} \right\} S_s^2 \]

(B3)

when \( f_s \) = Steel stress, in ksi
\( f_s' \) = Guaranteed ultimate tensile strength of steel, in ksi.
\( S_s \) = Steel strain, in \( 10^{-2} \)
\( t_s \) = Time after tensioning, in days.

In the above equation, the first terms, with the A coefficients, represent the instantaneous stress relationship, and the last terms, with B coefficients, represent the loss due to relaxation. The empirical values for the coefficients are listed in Table B.2.
Detailed descriptions of the experimental study on strand specimens are contained in project reports 339.4, 339.5 and 339.6.

For the analysis of a prestressed concrete member, the material characteristic equations are connected by the following conditions.

(1) Competibility of time, for the \( i \)-th steel element, tensioned at time \( t_{si} \)

\[
\frac{t_s}{t_c} = t_{si} \tag{B4}
\]

For a post-tensioned element, tensioning of steel and stressing of concrete occur simultaneously, hence \( t_s = 0 \) when \( t_c = t_{si} \). For a pretensioned element, tensioning of steel precedes the hardening of concrete, hence \( t_{si} \) is negative. However, stress is not induced in concrete until transfer. Therefore, the corresponding \( t_{si} \) in equation (B2) is taken as zero.

(2) Compatibility of strains for all times after anchoring, for the \( i \)-th steel element

\[
S_c + S_a = k_{4i} \tag{B5}
\]

where \( k_{4i} \) is a constant determined by the condition immediately after tensioning of steel element at time \( t_{si} \).

(3) Equilibrium of stresses and loads

\[
\int f_c \, da + \sum (f_{si} \, a_{si}) = -P \tag{B6}
\]

\[
\int f_c \, y \, da + \sum (f_{si} \, a_{si} \, e_i) = M \tag{B7}
\]

where \( f_{si} \), \( a_{si} \) and \( e_i \) are stress, area and eccentricity,
respectively, of the steel element tensioned at time $t_{si}$.

For sign conventions, see Fig. C.1.

(4) In addition to the compatibility and equilibrium requirements, it is assumed that the concrete stress exhibits a linear distribution over the height of the member section. Hence

$$f_c = g_1 + g_2 y$$

(B8)

Where $g_1$ and $g_2$ are parameters to be determined.

Combination of (B2), (B3), (B4) and (B5) leads to the observation that steel stress is a second degree function of the concurrent concrete stress at the same location.

$$f_{si} = R_{1i} + R_{2i} f_c + R_{3i} f_c^2$$

(B9)

where the $R$'s are coefficients dependent upon the material characteristic coefficients, the compatibility constants $t_{si}$ and $k_{4i}$, and the age of concrete $t_c$. It is emphasized that these $R$ coefficients may be different for each steel element, hence the subscript $i$.

Substitution of (B8) and (B9) into the equilibrium equations (B6) and (B7) results in two quadratic equations in $g_1$ and $g_2$, the stress distribution parameters. With all other parameters known, these two quadratic equations can be solved numerically. Substitution into Equations (B8), (B9), (B2) and (B5) then yields the stresses and strains in both concrete and steel materials. Thus, for any specified time $t_c$, a direct and complete analysis of the stress and strain distribution in the cross section is obtained.
In order to enable a rational discussion of the prestress losses, it is most important that prestress be clearly and precisely defined for all times. The basic concept of prestressing is the introduction of stresses "before" the application of service loads. A logical implication is, therefore, that prestress is not directly affected by applied loads. In addition, the natural reference point for the evaluation of prestress losses is the jacking stress immediately before anchoring and releasing of the tensioning device, since that is the last stress value subjected to direct human control. This "initial" prestress value is also obviously unaffected by any would-be applied loads. Based on the above reasoning, prestress is defined here to exclude all direct effects of applied loads. As gravity load (weight of member) is a part of the loads externally exerted, its direct effect is also excluded. Following this definition, prestress at any time is evaluated as the difference between the actual stress (which includes load effects) and the direct (elastic) effects of all applied loads. It should be pointed out that sustaining loads on a member affect prestress indirectly through the influence on creep and relaxation, only their direct effects are excluded from the prestress.

When the procedure was first developed for pretensioned concrete members only, considerable simplification from the method described above was possible. All pretensioned strands are stretched at essentially the same time, and also released at essentially the same time (at transfer of prestress following casting and curing). All strands are usually tensioned to the same initial prestress, which is directly controlled. Moreover, the scattering of the strands in the member section is usually not large, and steel stresses do not differ significantly among different strands.
Consequently, it is not necessary to distinguish among the strands, and all prestressing steel can be treated as forming a single element located at the c.g.s. Only one prestressing stage is involved, and the simpler form of the concrete characteristic equation, \((B1)\), is adequate. With these simplifications, the two quadratic equations in \(g_1\) and \(g_2\) can be conveniently combined into one, in terms of the concrete fiber stress at c.g.s. The details of this early procedure are presented in project report 339.9. A brief description has been published in a Transportation Research Record.\(^{(6)}\)

The newer procedure, applicable to all prestressed concrete structural members, contains many modifications and extensions from the first version. Concrete stress-strain-time relationship is expanded to include creep effects of stress increments applied at various times. Steel characteristic equation is assigned to each element separately to allow for the use of different steels. Wide separation of steel in the cross section is also permitted. Another complication involves the strain compatibility parameter \(k_{41}\). In post-tensioning, concrete is compressed at the same time as the steel element is being stretched, hence \(k_{41}\) is not merely the initial tensioning strain in steel, but its sum with the concrete compression strain. An analysis is needed, treating the newly applied prestressing force as a load on the cross section, including all previous anchored steel but not the element being tensioned. The concrete strain obtained from this analysis is added to the tensioning strain in steel to determine \(k_{41}\). After the anchoring of post-tensioned steel, it becomes part of the cross section resisting any subsequently applied prestress and loading. Therefore, the relevant cross section of a multi-stage post-tensioned member changes with each additional post-tensioning, which further complicates the calculations.
It has been mentioned earlier that for pretensioned members, the stresses at any specified time are directly determined once the initial conditions are known. For members involving multi-stage post-tensioning, analyses at each stage time are needed for the determination of $k_{4i}$ and the concrete stress increments at these times. For any specified time, all information regarding all preceding post-tensioning stages must be known in addition to the initial conditions. Undoubtedly, the work involved is much more complex than the pretensioned problem. Nevertheless, once the required information is known, the analysis of the section yields directly the total stresses and strains in the materials, and hence the prestress losses in each element. This is in contrast with some loss-estimation procedures currently available in literature, whereby loss increments estimated for several time intervals must be added together. (9)

The procedure described above is completely general, being applicable to prestressed concrete structural members of all types, including those involving multi-stage post-tensioning and segmental construction. However, it requires the usage of the large number of material characteristics coefficients, and the development and numerical solution of simultaneous quadratic equations at the time of each post-tensioning or loading stages, as well as any other specified time. These calculations are tedious to perform, and are justifiable only for computer operations. Simpler procedures were developed for application without computers, based on extensive parametric study using the computerized general procedure. The manual procedure for pretensioned members was initially developed in 1973, and has been presented to the regional meetings of the AASHTO Subcommittee.
on Bridges and Structures during that year.\(^{(4)}\) The procedure described in part II of the document is an expansion of the earlier one, and corresponds directly to the expanded general procedure. The basic feature of this manual procedure is the approximately linear growth of prestress losses with respect to the logarithm of time (Eq. 1). For pretensioned members, such a linear relationship prevails for the entire service life of the member, from transfer at \(t_c = 0\) and its assumed end at \(t_c = 36500\). For cases involving multistage post-tensioning, the linear relationship applies only between consecutive stages, necessitating recalculation of IL and TL for each time interval. As additional prestress is introduced, an instantaneous prestress loss occurs in each of the preceding prestressing elements, on account of the elastic deformation of concrete. On the other hand, application of permanent load causes a temporary reversal and retardation of the growth of prestress losses. For the sake of simplification, this reversal is ignored, and prestress loss is taken as remaining unchanged for a short period into the new time interval. Details of the evolution and development of the proposed procedure are presented in project reports 382.5, 402.3 and 402.4

A three year field study was conducted after the initial development of the procedures for pretensioned members, for the purpose of verification and refinement. The structure studied was a two-span, two-lane experimental bridge on a test road, containing twelve pretensioned concrete I-beams of the PennDOT standard 20/33 cross section. Six of the twelve beams were instrumented for long-term strain measurements. Supplementary specimens of the same cross section were used for control measurements of shrinkage.
and prestress strains. Measured strains were compared with those calculated by the general procedure, and the comparison was found to be reasonably good. The long term trend of continued growth of concrete strains was clearly observed over the entire period when logarithmic scale was used for time. After the placing of the deck structure on the bridge, the growth of concrete strains decreased for a short period. This was recognized as the effect of the concrete stress increments applied at that time. In line with this, the concrete stress-strain-time relationship was modified slightly, adding a correction term for such late stress increment. This refinement was later expanded to allow for several such increments, as shown by Eq. (B2). Details of the field study, and description of the refining of the prestress loss procedure, are given in project reports 382.2 and 382.5.

The effect of elevated temperature during the curing period on prestress losses in pretensioned strands was also studied. There was concern that as temperature decreases after curing, the steel stress loss due to thermal expansion may not be fully recovered. During the fabrication of the experimental bridge beams for the field study, measurements were made on the strand tension until transfer and on concrete compressive strains at transfer. The results indicated virtually full recovery of the thermal loss of strand stress. To further verify this finding, small specimens were fabricated and tested to decompression immediately after transfer. These tests also indicated nearly complete recovery of the thermal loss of strand stress. The details of these two sets of tests are contained in project reports 382.2, 402.2T and 402.4.
In summary, the fourteen-year study of prestress losses at Lehigh University led to the developments, findings and recommendations listed below.

1. A general procedure has been developed, based on the interaction of the stress-strain-time relationships of the concrete and steel materials. The method enables a direct and complete analysis of the stresses and strains in a prestressed concrete member.

2. A simpler procedure has been developed for estimation of prestress losses. This procedure is suitable for manual computation, and yields results comparable to those of the more general procedure.

3. These procedures are applicable to all types of prestressed concrete members, including those subjected to multistage post-tensioning and segmental construction.

4. The long-term characteristics of concrete may vary significantly even when certain strength requirements are satisfied.

5. Without introducing new prestress and/or new load, the growth of long-term prestress losses is nearly linear with respect to the logarithm of time from tensioning (or transfer in case of pretensioned strands). A separate linear relationship applies for different time intervals between consecutive stages.

6. The size and make of the prestressing steel material have only a small effect on the prestress losses, and therefore can be disregarded.

7. Members prestressed with low-relaxation steel elements suffer significantly smaller prestress loss than those with stress-
relieved strands.

8. The loss of stress in pretensioned strands during the curing period is nearly completely recovered upon cooling to normal temperature. There is no need to consider any long-term loss due to this excursion into high temperature.

9. The loss of prestress is a phenomenon of very long duration. The trend of gradual change was clearly observable at the end of the three-to-six-year testing periods for the several phases of the research program.
<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Upper Bound</th>
<th>Lower Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elastic Strain</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C^* )</td>
<td>0.02500</td>
<td>0.02105</td>
</tr>
<tr>
<td><strong>Shrinkage</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_1 )</td>
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<td>-0.00066</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>0.02454</td>
<td>0.01500</td>
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<tr>
<td><strong>Creep</strong></td>
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<tr>
<td>( E_1 )</td>
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<td>-0.00664</td>
</tr>
<tr>
<td>( E_2 )</td>
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</tr>
<tr>
<td>( E_3 )</td>
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<td>-0.00371</td>
</tr>
<tr>
<td>( E_4 )</td>
<td>0.01609</td>
<td>0.01409</td>
</tr>
</tbody>
</table>

*Note: \( C = \frac{100}{E_c} \) where \( E_c \) is modulus of elasticity for concrete, in ksi*
### TABLE B.2 COEFFICIENTS FOR STEEL CHARACTERISTIC EQUATIONS

(Equation B3)

<table>
<thead>
<tr>
<th>Instantaneous Stress-Strain Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1 = -0.04229$</td>
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<tr>
<td>$A_2 = 1.21952$</td>
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<td>$A_3 = -0.17827$</td>
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<table>
<thead>
<tr>
<th>Relaxation Coefficients</th>
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</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Stress-Relieved</td>
</tr>
<tr>
<td>Strands</td>
</tr>
<tr>
<td>Low-Relaxation</td>
</tr>
<tr>
<td>Strands</td>
</tr>
<tr>
<td>$B_1$</td>
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<tr>
<td>-0.01403</td>
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<tr>
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<td>$B_4$</td>
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<tr>
<td>0.01395</td>
</tr>
</tbody>
</table>
VI. ACKNOWLEDGMENTS

This report was prepared under an Implementation agreement with the Pennsylvania Department of Transportation in conjunction with the United States Federal Highway Administration. These same agencies, together with the Reinforced Concrete Research Council, also sponsored the previous research effort. The interest and support of these organizations are gratefully acknowledged.

The research work leading to the proposed procedures described in this report was conducted at the Fritz Engineering Laboratory of Lehigh University. Dr. Lynn S. Beedle is the director of the laboratory.

Many individuals made valuable contributions in the course of the extended research effort. The author wishes to recognize particularly his former assistants, E. G. Schultchen, A. Rokhsar, D. C. Frederickson, R. J. Batal, H. T. Ying, J. Tansu, C. S. Hsieh, P. Rimbos and B. Hoffman.

The manuscript of this report was typed by Ms. S. Matlock and Mrs. R. A. Grimes. The graphs were prepared by Mr. J. Gera and Ms. S. Balogh.