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Distributed order-statistic CFAR detection of dependent sensor observations

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Abstract

Distributed signal detection schemes have received significant attention recently, but usually under the assumption of independent and stationary observations at each sensor. Here, ordered statistics constant false alarm rate (OS-CFAR) detection techniques are applied to a distributed detection system with nonstationary and possibly dependent observations from sensor to sensor. Cases with weak narrowband random signals in additive Gaussian noise-plus-clutter of unknown power are considered. Necessary conditions for best weak-signal performance are given for n sensor cases and solutions are obtained for some specific two sensor cases. The best schemes may use either AND or OR fusion rules depending on the specific false alarm probability and the number of reference observations used in the OS-CFAR scheme. Distributed OS-CFAR and CA-CFAR schemes are compared in terms of their capability to maintain false alarm probability in nonhomogeneous backgrounds. At least for the specific cases we have studied, there are OS-CFAR schemes which generally outperform the CA-CFAR schemes in this regard.

I Introduction

Ordered statistics constant false alarm rate (OS-CFAR) detection techniques have been studied in [1] for some distributed detection problems, but only for the case of independent observations from sensor to sensor. We investigate some distributed detection problems for cases with dependent observations. Such problems are generally more difficult than similar problems with independent observations and they generally provide interesting results [2]. The results in [1] assume that the signal-to-noise ratios of the observations are known which may be unrealistic in some cases. This is especially true for cases where a CFAR scheme is to be used to overcome difficulties associated with unknown noise power. Our results apply for some interesting cases with unknown signal-to-noise ratios.

We assume that the in-phase and quadrature components of the received narrowband observations at each of the n sensors consist of a weak random signal in additive Gaussian noise-plus-clutter and that these observations are processed by matched filters. The matched filtered in-phase and quadrature components (Z_{Ij}, Z_{Qj}) at the jth sensor $(j = 1, 2, \ldots, n)$ are given by an additive combination of the received random signal components (S_{Ij}, S_{Qj}) and the received random noise-plus-clutter components (W_{Ij}, W_{Qj}) as

$$\begin{aligned} Z_{Ij} &= \theta S_{Ij} + W_{Ij} \\ Z_{Qj} &= \theta S_{Qj} + W_{Qj} \end{aligned} \tag{1}$$

where the parameter θ, which corresponds to the amplitude of the signal return, indicates the presence of a signal when $\theta \neq 0$. In order to simplify our presentation we consider the case where each sensor receives observations with identical Gaussian signal components $S_{Ij} = S_I$, $S_{Qj} = S_Q$; $j = 1, ..., n$, each with unit variance and zero mean. The signals are also assumed to have symmetric power spectral densities. Since we consider cases with weak signals, specifically locally optimum detection, our results can be shown to be valid for non-Gaussian (S_{Ij}, S_{Qj}) [3]. The in-phase and quadrature noise-plus-clutter returns (W_{Ij}, W_{Qj}) are also assumed to be Gaussian, each with variance σ_j^2 and zero mean. The noise returns at each of the different sensors are assumed to be independent.

We shall restrict our attention to cases where the jth sensor decision will be based only on a single received pulse (at the jth sensor) augmented with N_j reference observations (noise only). We assume non-coherent processing, so our sensor decisions will be based only on the square envelope of the observed returns. The reference observations are also assumed to be square envelope observations which come from the model in (1) when θ =0.

Our OS-CFAR detection scheme, as implemented at the jth sensor, $j = 1, 2, \ldots, n$, is described as follows. The reference observations at the jth sensor are first rank ordered according to increasing magnitude. This new sequence is expressed as $Y_{j(1)} \leq Y_{j(2)} \leq \ldots \leq Y_{j(N_j)}$. The square envelope of the observation under test X_j is compared to an adaptive threshold given by the product $t_j V_j$, where t_j is a fixed part of the threshold and $V_j = Y_{j(K_j)}$ is the K_jth order statistic of the reference observations at the jth sensor. If $X_j \geq t_j V_j$, then we announce signal is present. If $X_j < t_j V_j$, we announce signal is absent. This test was used previously in [1] for cases with independent observations.

In our distributed detection scheme we assume each sensor will transmit a single binary decision to a fusion center. The fusion center will make a final decision based on the complete set of sensor decisions. To determine the best set of sensor thresholds under signal present conditions generally requires a knowledge of the relative magnitude of the noise-plus-clutter power at each sensor. Here we assume the noise-plus-clutter powers are equal, $\sigma_j^2 = \sigma^2$, but our results can be easily extended to cases where each σ_j^2/σ_k^2; $j = 1, 2, ..., n$, $j \neq k$ is some other known ratio. Note that this assumption is not needed to assure that our scheme is CFAR, but it is needed to optimize performance under the signal present alternative. In [4], where CA-CFAR is considered in a distributed detection setting, the authors make a similar assumption.

II Distributed CFAR Detection

Under the assumptions we have outlined it is straight forward to calculate

$$E[Z_{Ii}Z_{Ij}] = E[Z_{Qi}Z_{Qj}] = \begin{cases} \theta^2 + \sigma^2 & \text{if} \quad i = j \\ \theta^2 & \text{if} \quad i \neq j \end{cases}$$
$$\text{for} \quad i, j = 1, 2, ..., n \tag{2}$$

Since we assume our signal and noise observations come from random processes with symmetric power spectral densities we also find

$$E[Z_{Ii}Z_{Qj}] = E[Z_{Qi}Z_{Ij}] = 0; \text{ for } \ i, j = 1, 2, ..., n \tag{3}$$

Using (1) through (3) in the equation for the probability density function (pdf) of a zero-mean Gaussian random vector and transforming components to square envelope and phase variables with $x_i \triangleq Z_{Ii}^2 + Z_{Qi}^2$ and $\phi_i \triangleq \arctan(Z_{Qi}/Z_{Ii})$ for $i = 1, ..., n$, we obtain the expression for the joint pdf of the envelope observations as

$$f_{X_1,X_2,...,X_n}(x_1, x_2, ..., x_n|\theta) = \frac{1}{2^n(2\pi)^n(n\sigma^{2(n-1)}\theta^2 + \sigma^{2n})}$$
$$exp\left(-\frac{(n-1)\theta^2 + \sigma^2}{2\sigma^2(n\theta^2 + \sigma^2)} \sum_{i=1}^{n} x_i\right)$$
$$\int_{\phi_1=0}^{2\pi} \cdots \int_{\phi_n=0}^{2\pi} exp\left(\frac{\theta^2}{\sigma^2(n\theta^2 + \sigma^2)} \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \sqrt{x_i}\sqrt{x_j} cos(\phi_i - \phi_j)\right) d\phi_1 ... d\phi_n \tag{4}$$

The pdf of the variable part of the threshold at the jth sensor is (Appendix A)

$$f_{V_j}(v_j) = \frac{K_j}{2\sigma^2} \binom{N_j}{K_j} exp\left(-\frac{v_j(N_j + 1 - K_j)}{2\sigma^2}\right)$$
$$\left(1 - exp\left(-\frac{v_j}{2\sigma^2}\right)\right)^{K_j - 1} \tag{5}$$

where N_j is the number of reference observations available at the jth sensor.

Denote the probability of detecting a signal of amplitude θ by $P_d(\theta)$. $P_d(\theta)$ can be evaluated for any given fusion rule by using (4) and (5). For example, for the AND fusion rule $P_d(\theta) = Prob[X_1 \geq t_1V_1 \; and \; X_2 \geq t_2V_2 ... \; and \; X_n \geq t_nV_n|\theta]$ and for the OR fusion rule $P_d(\theta) = Prob[X_1 \geq t_1V_1 \; or \; X_2 \geq t_2V_2 ... \; or \; X_n \geq t_nV_n|\theta]$.

A focus on weak signals [3] suggests we maximize $P_d''(0) = \frac{d^2}{d\theta^2}P_d(\theta)|_{\theta=0}$ under the constraint of fixed global false alarm probability, since for our zero-mean random signal model we have $\frac{d}{d\theta}P_d(\theta)|_{\theta=0} = 0$. Define $P(U_0 = 1|U_1 = u_1, \ldots, U_n = u_n)$ as the probability that the fusion center decides for $\theta \neq 0$ for a given set of sensor decisions. Define $P(U_1 = u_1, \ldots, U_n = u_n|\theta)$ as the joint probability of a given set of sensor decisions. Expanding $P_d(\theta)$ [2] and taking the appropriate derivatives gives

$$\begin{aligned} P_d''(0) &= \sum_{u_1=0}^{1} \cdots \sum_{u_n=0}^{1} P(U_0 = 1|U_1 = u_1, \ldots, U_n = u_n) \\ & \quad \frac{d^2}{d\theta^2} P(U_1 = u_1, \ldots, U_n = u_n|\theta)|_{\theta=0} \end{aligned} \tag{6}$$

where

$$\begin{aligned} &\frac{d^2}{d\theta^2} P(U_1 = u_1, \ldots, U_n = u_n|\theta)|_{\theta=0} = \int_{v_1=-\infty}^{\infty} \cdots \int_{v_n=-\infty}^{\infty} \int_{R_1} \cdots \int_{R_n} \\ &\frac{d^2}{d\theta^2} f_{X_1,\ldots,X_n}(x_1, \ldots, x_n|\theta)|_{\theta=0} \prod_{i=1}^{n} f_{V_i}(v_i) dx_1 \cdots dx_n dv_1 \cdots dv_n \end{aligned} \tag{7}$$

and $R_j = \{x_j : x_j \geq v_jt_j\}$ if $u_j = 1$, otherwise it is the complement of this region.

Using (4) and (5) in (7) and scaling the variables of integration yields (Appendix B)

$$\begin{aligned} &\frac{d^2}{d\theta^2} P(U_1 = u_1, \ldots, U_n = u_n|\theta)|_{\theta=0} = \frac{2}{\sigma^2} \Big\{ \int_{v_1=-\infty}^{\infty} \cdots \int_{v_n=-\infty}^{\infty} \int_{R_1} \cdots \int_{R_n} \\ &\sum_{i=1}^{n} (x_i - 1) \prod_{j=1}^{n} \Big[K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} \exp\left(-v_j(N_j + 1 - K_j)\right) \end{aligned}$$

$$(1-\exp(-v_j))^{K_j-1}\exp(-x_j)\Big]dx_1\cdots dx_n dv_1\cdots dv_n\Big\} \quad (8)$$

From (6) and (8) we see that for any fixed fusion rule we can choose the thresholds $t_1,\ldots,t_n$ to maximize $\sigma^2 P_d''(0)$ without knowing σ^2. Choosing the thresholds to maximize $\sigma^2 P_d''(0)$ will insure that these thresholds will maximize $P_d''(0)$ for any $\sigma^2 > 0$. One can use a number of numerical techniques to find thresholds to maximize $\sigma^2 P_d''(0)$ subject to the false alarm constraint $P_f = \alpha_0$. We outline one approach here which uses necessary conditions. Let us define $\mathbf{t} = (t_1,\ldots,t_n)$ where t_j is the threshold at the jth sensor. Restating a theorem by Luenberger [7] (p. 224) we have

Theorem 1

Let $\sigma^2 P_d''(0)$ denote our performance measure, a real-valued continuous function of $\mathbf{t} = (t_1,\ldots,t_n)$. Let P_f denote the overall false alarm probability which is also a real-valued continuous function of $\mathbf{t}$. Let $\mathbf{t}^* = (t_1^*,\ldots,t_n^*)$ be a local extremum of $\sigma^2 P_d''(0)$ under the constraint $P_f = \alpha_0$ and assume that $(\frac{dP_f}{dt_1},\ldots,\frac{dP_f}{dt_n}) \neq (0,\ldots,0)$ at $\mathbf{t}^*$. Then there exists a real-valued ξ such that

$$\begin{aligned}
\sigma^2\frac{dP_d''(0)}{dt_1} - \xi\frac{dP_f}{dt_1} &= 0 \\
&\vdots \\
\sigma^2\frac{dP_d''(0)}{dt_n} - \xi\frac{dP_f}{dt_n} &= 0 \\
P_f &= \alpha_0
\end{aligned} \quad (9)$$

□

Assuming an optimum solution exists which satisfies the conditions of theorem 1, we can find it by finding all solutions to the set of equations (9) and checking each solution for optimality. Using this approach we can search for the best solution which uses any one of a set of possible fusion rules. We illustrate the use of theorem 1 with examples of these calculations for the AND and OR fusion rules.

AND Rule

For the AND rule we find, using (6) and (8) (Appendix C)

$$P_d''(0) = \frac{2}{\sigma^2}\prod_{j=1}^{n}\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\sum_{i=1}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right) \tag{10}$$

An expression for the false alarm probability is (Appendix D)

$$P_f = \prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}} = \prod_{i=1}^{n}\left(\prod_{L=0}^{K_i-1}\frac{N_i-L}{N_i+t_i-L}\right) = \alpha_0 \tag{11}$$

The ℓth necessary condition from (9) for $\ell = 1,\ldots,n$ reduces to (Appendix E)

$$2\prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\left\{\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2} - \sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L} - \right.$$
$$\left.\sum_{i=1,i\neq\ell}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right\} + \xi\prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}$$
$$= 0 \qquad \text{for} \quad \ell = 1,\ldots,n \tag{12}$$

Alternately, we can obtain the relationships

$$\sum_{L=0}^{K_i-1}\frac{1}{N_i+t_i-L}\sum_{L=0}^{K_j-1}\frac{N_j-L}{(N_j+t_j-L)^2} =$$
$$\sum_{L=0}^{K_j-1}\frac{1}{N_j+t_j-L}\sum_{L=0}^{K_i-1}\frac{N_i-L}{(N_i+t_i-L)^2}$$
$$\text{for} \quad i,j = 1,2,\ldots,n, \qquad i \neq j. \tag{13}$$

from (12) (Appendix F) which show the symmetry in the set of equations in (13). It is clear that if $N_i = N_j$ and $K_i = K_j$ then $t_i = t_j$ will be a solution of the $i-j$th equation in (13).

The thresholds t_j ($j = 1, 2, ...n$) can be obtained by solving the set of equations given by (11) and (12) or by solving (11) along with the appropriate set of n-1 equations of the form given in (13). For example, if n=3 we choose the $i = 1, j = 2$ equation and the $i = 2, j = 3$ equation in (13).

Note that for the AND rule $(\frac{dP_f}{dt_1}, \ldots, \frac{dP_f}{dt_n}) = (0, \ldots, 0)$ when any of the t_j ($j = 1, 2, \ldots, n$) are set to ∞. This is an uninteresting case since in this case $P_f = 0$ and $P_d = 0$.

For the case of n=2, the necessary conditions become

$$\sum_{L=0}^{K_2-1} \frac{1}{N_2 + t_2 - L} \sum_{L=0}^{K_1-1} \frac{N_1 - L}{(N_1 + t_1 - L)^2} = \sum_{L=0}^{K_1-1} \frac{1}{N_1 + t_1 - L} \sum_{L=0}^{K_2-1} \frac{N_2 - L}{(N_2 + t_2 - L)^2} \qquad (14)$$

and

$$\prod_{L=0}^{K_1-1} \frac{N_1 - L}{N_1 + t_1 - L} \prod_{L=0}^{K_2-1} \frac{N_2 - L}{N_2 + t_2 - L} = \alpha_0 \qquad (15)$$

OR Rule

For the OR fusion rule we find, using (6) and (8) (Appendix G) yields

$$P_d''(0) = \frac{2}{\sigma^2} \sum_{i=1}^{n} \left\{ \left[\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}} \sum_{L=0}^{K_i-1} \frac{t_i}{N_i + t_i - L} \right] \prod_{j=1, j\neq i}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \right] \right\} \qquad (16)$$

An expression for the false alarm probability is (Appendix D)

$$P_f = 1 - \prod_{i=1}^{n} \left(1 - \frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}} \right) = 1 - \prod_{i=1}^{n} \left(1 - \prod_{L=0}^{K_i-1} \frac{N_i - L}{N_i + t_i - L} \right) = \alpha_0 \qquad (17)$$

The ℓth necessary condition from (9) for $\ell = 1, \ldots, n$ reduces to (Appendix H)

$$2 \sum_{i=1, i\neq \ell}^{n} \left\{ \left[\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}} \sum_{L=0}^{K_i-1} \frac{t_i}{N_i + t_i - L} \right] \left[\prod_{j=1, j\neq i,\ell}^{n} \left(1 - \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \right) \right] \frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}} \right.$$

$$\sum_{L=0}^{K_\ell-1} \frac{1}{N_\ell+t_\ell-L}\Bigg\} + 2\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}}\left(\sum_{L=0}^{K_\ell-1} \frac{N_\ell-L}{(N_\ell+t_\ell-L)^2} - \sum_{L=0}^{K_\ell-1} \frac{t_\ell}{N_\ell+t_\ell-L}\right.$$

$$\left.\sum_{L=0}^{K_\ell-1} \frac{1}{N_\ell+t_\ell-L}\right) \prod_{j=1,j\neq\ell}^{n}\left[1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right] - \xi\left[\prod_{j=1,j\neq\ell}^{n}\left(1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right)\right]$$

$$\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell-1} \frac{1}{N_\ell+t_\ell-L} = 0 \qquad \text{for} \quad \ell = 1,2,\ldots,n \tag{18}$$

Simplifying equation (18), we obtain (Appendix I)

$$\left(\sum_{L=0}^{K_j-1} \frac{N_j-L}{(N_j+t_j-L)^2}\right) \Big/ \left(\sum_{L=0}^{K_j-1} \frac{1}{N_j+t_j-L}\right) -$$
$$\left(\sum_{L=0}^{K_i-1} \frac{N_i-L}{(N_i+t_i-L)^2}\right) \Big/ \left(\sum_{L=0}^{K_i-1} \frac{1}{N_i+t_i-L}\right) +$$
$$\left(\sum_{L=0}^{K_j-1} \frac{t_j}{N_j+t_j-L}\right) \Big/ \left(\prod_{L=0}^{K_j-1} \frac{N_j-L}{N_j+t_j-L} - 1\right) -$$
$$\left(\sum_{L=0}^{K_i-1} \frac{t_i}{N_i+t_i-L}\right) \Big/ \left(\prod_{L=0}^{K_i-1} \frac{N_i-L}{N_i+t_i-L} - 1\right) = 0$$
$$\text{for} \quad i,j = 1,2,\ldots,n, \qquad i \neq j. \tag{19}$$

The OR rule thresholds t_j ($j = 1,2,...,n$) are obtained by solving (17) along with the set of the equations in (18). We could also solve (17) along with the appropriate set of n - 1 equations from (19). The equations in (19) show the symmetry in the necessary conditions. Clearly if $N_i = N_j$ and $K_i = K_j$ then $t_i = t_j$ is a solution of the $i-j$th equation in (19).

Note that for the OR rule $(\frac{dP_f}{dt_1},\ldots,\frac{dP_f}{dt_n}) = (0,\ldots,0)$ when two or more of the t_j are set to 0. This is an uninteresting case where $P_f = 1$ and $P_d = 1$. Note that $(\frac{dP_f}{dt_1},\ldots,\frac{dP_f}{dt_n}) = (0,\ldots,0)$ also when $t_1 = \cdots = t_n = \infty$, which applies for the uninteresting case $P_f = 0$ and $P_d = 0$. It is important to note that there may be more then one solution to the set of equations (9) which may complicate matters.

For the case of n=2, the necessary conditions for the OR rule become

$$\left(\sum_{L=0}^{K_2-1} \frac{N_2 - L}{(N_2 + t_2 - L)^2}\right) \Bigg/ \left(\sum_{L=0}^{K_2-1} \frac{1}{N_2 + t_2 - L}\right) -$$
$$\left(\sum_{L=0}^{K_1-1} \frac{N_1 - L}{(N_1 + t_1 - L)^2}\right) \Bigg/ \left(\sum_{L=0}^{K_1-1} \frac{1}{N_1 + t_1 - L}\right) +$$
$$\left(\sum_{L=0}^{K_2-1} \frac{t_2}{N_2 + t_2 - L}\right) \Bigg/ \left(\prod_{L=0}^{K_2-1} \frac{N_2 - L}{N_2 + t_2 - L} - 1\right) -$$
$$\left(\sum_{L=0}^{K_1-1} \frac{t_1}{N_1 + t_1 - L}\right) \Bigg/ \left(\prod_{L=0}^{K_1-1} \frac{N_1 - L}{N_1 + t_1 - L} - 1\right) = 0 \qquad (20)$$

and

$$\prod_{L=0}^{K_1-1} \frac{N_1 - L}{N_1 + t_1 - L} + \prod_{L=0}^{K_2-1} \frac{N_2 - L}{N_2 + t_2 - L} -$$
$$\prod_{L=0}^{K_1-1} \frac{N_1 - L}{N_1 + t_1 - L} \prod_{L=0}^{K_2-1} \frac{N_2 - L}{N_2 + t_2 - L} = \alpha_0 \qquad (21)$$

III Numerical Results for Two Sensors

We solved (14) and (15) for some cases with specific values for α_0, N_1, N_2, K_1, and K_2 to find the best thresholds t_1 and t_2 when an AND fusion rule is used. We also solved (20) and (21) for the same cases to find the best thresholds when an OR fusion rule is used. For each of the AND rule cases we found only one solution to the necessary conditions which we verified was optimum by examining $P_d''(0)$ as a function of t_1 and t_2 under the false alarm probability constraint. For the OR rule cases we sometimes found more than one solution but we only give the best solution here. For the OR rule cases we also verified our solutions were optimum. In each of these cases, regardless of whether an AND fusion rule or an OR fusion rule was used, we found the best thresholds to be equal if $N_1 = N_2$ and $K_1 = K_2$, but we found the best thresholds were generally not equal if $N_1 \neq N_2$ or $K_1 \neq K_2$.

Initially, we considered cases with an equal number of reference samples at each sensor. We provide results here for a representative case with $N_1 = N_2 = 32$ with $K_1 = K_2 = 24$. Table 1 gives the best thresholds and the normalized performance $\sigma^2 P_d''(0)$ for the AND fusion rule case. Table 2 gives these quantities for the OR fusion rule case. These results show that the best AND rule solution achieves better performance for the smaller false alarm probabilities that we investigated (with $\alpha_0 \leq 10^{-4}$) while the best OR rule solution gives better performance for larger false alarm probabilities $\alpha_0 \geq 10^{-3}$. We also studied the best choice of K_1 and K_2 to maximize $P_d''(0)$. For cases with $N_1 = N_2 =$ N we constrained our search to cases with $K_1 = K_2 =$ K and we found the best $P_d''(0)$ was generally obtained for a K near 0.75N. The best K varied slightly around 0.75N for different false alarm probabilities, but this variation was quite small. Choosing K to be 0:75N was suggested in previous work [6] for single sensor cases.

Next we studied some cases with $N_1 = N_2 = 32$ and with $K_1 \neq K_2$. We provide results here the representative case of $K_1 = 27$ and $K_2 = 24$. The best thresholds and the normalized performance for this case are given in Table 3 and Table 4. In this case we find the OR rule is better for all the false alarm probabilities we investigated with $\alpha_0 > 10^{-4}$. Comparing the performance of this case with the performance for the $K_1 = K_2 = 24$ case in Table 1 and Table 2 shows that there is only a slight advantage to using $K_1 = 24$ and $K_2 = 27$.

We also studied some case with $N_1 \neq N_2$ and $K_1 = K_2 =$ K. We provide results here for the representative case of $N_1 = 16$, $N_2 = 20$ and K = 15. Table 5 and Table 6 show the best thresholds and the normalized performance for this case. Here the OR rule is better for all false alarm probabilities we investigated with $\alpha_0 > 10^{-3}$. These results are similar to those for the $N_1 = N_2$ and $K_1 \neq K_2$ case.

The last case we considered used $N_1 \neq N_2$ and $K_1 \neq K_2$. We provide results here for the representative case of $N_1 = 16$, $N_2 = 24$, $K_1 = 12$ and $K_2 = 18$, and the best thresholds and the normalized performance for this case are given in Table 7 and Table 8. The OR rule provides better performance for all false alarm probabilities we investigated with $\alpha_0 > 10^{-3}$.

α_0	$t_1 = t_2$	$\sigma^2 P_d''(0)$
10^{-6}	6.086	2.368E-5
10^{-5}	4.933	2.022E-4
10^{-4}	3.838	1.658E-3
10^{-3}	2.800	1.275E-2
10^{-2}	1.816	8.726E-2
0.05	1.158	0.2891
0.1	0.883	0.4481
0.3	0.456	0.7120

Table 1: Best AND fusion rule parameters found for cases with $N_1 = N_2 = 32$, $K_1 = K_2 = 24$.

α_0	$t_1 = t_2$	$\sigma^2 P_d''(0)$
10^{-6}	15.38	2.140E-5
10^{-5}	12.23	1.881E-4
10^{-4}	9.382	1.597E-3
10^{-3}	6.810	1.284E-2
10^{-2}	4.488	9.368E-2
0.05	2.996	0.3334
0.1	2.380	0.5397
0.3	1.412	0.9490

Table 2: Best OR fusion rule parameters found for cases with $N_1 = N_2 = 32$, $K_1 = K_2 = 24$.

α_0	t_1	t_2	$\sigma^2 P''_d(0)$
10^{-6}	4.743	5.863	$2.376E-5$
10^{-5}	3.830	4.776	$2.027E-4$
10^{-4}	2.969	3.735	$1.661E-3$
10^{-3}	2.157	2.739	$1.277E-2$
10^{-2}	1.392	1.786	$8.732E-2$
0.05	0.885	1.144	0.2892
0.1	0.674	0.873	0.4482
0.3	0.347	0.452	0.7121

Table 3: Best AND fusion rule parameters found for cases with $N_1 = N_2 = 32$, $K_1 = 27$, $K_2 = 24$.

α	t_1	t_2	$\sigma^2 P''_d(0)$
10^{-6}	11.29	15.61	2.161E-5
10^{-5}	9.043	12.36	1.895E-4
10^{-4}	6.983	9.443	1.606E-3
10^{-3}	5.097	6.834	1.289E-2
10^{-2}	3.374	4.495	9.389E-2
0.05	2.257	2.998	0.3338
0.1	1.794	2.380	0.5402
0.3	1.066	1.412	0.9495

Table 4: Best OR fusion rule parameters found for cases with $N_1 = N_2 = 32$, $K_1 = 27$, $K_2 = 24$.

α_0	t_1	t_2	$\sigma^2 P_d''(0)$
10^{-6}	3.810	7.215	2.145E-5
10^{-5}	2.976	5.817	1.856E-4
10^{-4}	2.226	4.509	1.543E-3
10^{-3}	1.557	3.281	1.205E-2
10^{-2}	0.964	2.125	8.384E-2
0.05	0.595	1.355	0.2814
0.1	0.447	1.033	0.4386
0.3	0.225	0.533	0.7039

Table 5: Best AND fusion rule parameters found for cases with $N_1 = 16$, $N_2 = 20$, $K_1 = K_2 = 15$.

α	t_1	t_2	$\sigma^2 P_d''(0)$
10^{-6}	12.27	18.89	1.810E-5
10^{-5}	9.304	14.51	1.622E-4
10^{-4}	6.795	10.77	1.405E-3
10^{-3}	4.683	7.583	1.155E-2
10^{-2}	2.921	4.857	8.645E-2
0.05	1.871	3.183	0.3139
0.1	1.457	2.509	0.5131
0.3	0.836	1.472	0.9181

Table 6: Best OR fusion rule parameters found for cases with $N_1 = 16$, $N_2 = 20$, $K_1 = K_2 = 15$.

α_0	t_1	t_2	$\sigma^2 P''_d(0)$
10^{-6}	5.560	8.151	2.179E-5
10^{-5}	4.434	6.494	1.883E-4
10^{-4}	3.394	4.968	1.565E-3
10^{-3}	2.437	3.563	1.220E-2
10^{-2}	1.555	2.272	8.463E-2
0.05	0.981	1.433	0.2833
0.1	0.745	1.086	0.4411
0.3	0.381	0.556	0.7061

Table 7: Best AND fusion rule parameters found for cases with $N_1 = 16$,$N_2 = 24$, $K_1 = 12$, $K_2 = 18$.

α_0	t_1	t_2	$\sigma^2 P''_d(0)$
10^{-6}	34.50	16.32	1.897E-5
10^{-5}	21.79	12.70	1.677E-4
10^{-4}	14.19	9.623	1.436E-3
10^{-3}	9.121	6.958	1.172E-2
10^{-2}	5.465	4.588	8.735E-2
0.05	3.445	3.064	0.3167
0.1	2.672	2.434	0.5172
0.3	1.527	1.444	0.9239

Table 8: Best OR fusion rule parameters found for cases with $N_1 = 16$, $N_2 = 24$, $K_1 = 12$, $K_2 = 18$.

IV Effects of Clutter Power Variations

Here we investigate false alarm probability variations due to step changes in clutter power across the group of reference samples of either one or both sensors. Thus each reference sample is characterized as either being in increased clutter, with higher noise-plus-clutter power, or in the clear, with a lower noise-plus-clutter power. At sensor j, $N_j/2$ of the reference samples are assumed to come, for example, from neighboring "range cells" on one side of the observation under test and the other $N_j/2$ of them come from "range cells" on the other side [6]. Thus, after the step change in clutter power passes over $N_j/2$ of the reference observations, the observation under test also experiences a change in clutter power. Let r_j of the reference samples at sensor j be subjected to a $C_j \times 100$ percent increase in noise-plus-clutter power. Thus the total noise-plus-clutter power at each of these r_j samples is $\sigma^2(1+C_j)$. The other $N_j - r_j$ reference samples at sensor j, which we say are in the clear, are subjected to a total noise-plus-clutter power of σ^2. The false alarm probability α_j of the jth sensor decision is given in [6] as

$$P_{f_j} = t_j \sum_{i=K_j}^{N_j} \sum_{L=max(0,i-r_j)}^{min(i,N_j-r_j)} \binom{N_j-r_j}{L} \binom{r_j}{i-L}$$
$$\sum_{j_1=0}^{L} \sum_{j_2=0}^{i-L} \frac{\binom{L}{j_1}\binom{i-L}{j_2}(-1)^{j_1+j_2}}{N_j - r_j - L + t_j + j_1 + \frac{(j_2+r_j-i+L)}{(1+C_j)}} \tag{22}$$

provided the observation under test X_j is in the clear ($r_j < N_j/2$). When the observation under test is also in the increased noise-plus-clutter ($r_j > N_j/2$) the false alarm probability is

$$P_{f_j} = t_j \sum_{i=K_j}^{N_j} \sum_{L=max(0,i-r_j)}^{min(i,N_j-r_j)} \binom{N_j-r_j}{L} \binom{r_j}{i-L}$$

$$\sum_{j_1=0}^{L}\sum_{j_2=0}^{i-L}\frac{\binom{L}{j_1}\binom{i-L}{j_2}(-1)^{j_1+j_2}}{(1+C_j)(N_j-r_j-L+j_1)+j_2+r_j-i+L+t_j} \tag{23}$$

Since our observations are independent under the null hypothesis, the overall false alarm probability for distributed detection systems using either the AND or the OR fusion rules are easily found from using $P_{f,AND} = \alpha_1\alpha_2$ and $P_{f,OR} = \alpha_1+\alpha_2-\alpha_1\alpha_2$ where α_1 and α_2 are the false alarm probabilities at each of the two sensors. Using (22) and (23) we can find the variation in overall false alarm probability as a step change in clutter, sweeps across the groups of reference samples of one or both of the sensors.

In our example the increase in noise-plus-clutter power occurs at r_1 reference samples at sensor 1 and the reference samples at sensor 2 all have the same noise-plus-clutter power σ^2. We consider the specific case where the percent increase in noise-plus-clutter power at sensor 1 is C_1 = 10dB. We compare CA-CFAR and OS-CFAR schemes which use AND and OR fusion rules by investigating the variations in false alarm probability as a function of r_1. The CA and OS schemes we studied were those which maximize $\sigma^2 P_d''(0)$ for the specific case of $N_1 = N_2 = 24$ and $\alpha_0 = 10^{-6}$. We studied OS schemes for several values of $K_1 = K_2 =$ K.

In general we find that if $r_1 \leq N_1/2$ then the false alarm probability will be below the required value which leads to a loss in detection performance due to corresponding loss in false alarm probability [6]. If $r_1 > N_1/2$ then the false alarm probability will be above the required value which is usually considered a more serious violation.

Figure 1 shows the variation in overall false alarm probability for CA and OS schemes using an AND fusion rule. If the number of observations in increased clutter,

r_1, is less than $N_1/2$ then the OS schemes are better for each value of K shown and the variation in P_f is less for the smaller K. If $r_1 > N_1/2$, then the performance is a little less sensitive to whether an OS or CA scheme is used and also the specific value of K used in the OS scheme. The OS schemes with K $\geq$ 20 (best for $P_d''(0)$) are clearly better than the CA scheme.

Figure 2 shows the variation in false alarm probability for CA and OS schemes using an OR fusion rule. Now if $r_1 \leq N_1/2$, then again the OS schemes are better than the CA scheme for all values of K. If $r_1 > N_1/2$, again the OS schemes with larger K provide smaller variations in false alarm probability and are better than the CA scheme if $K \geq 21$.

Figure 3 compares the OS-CFAR schemes using AND and OR fusion rules. These results indicate that we can minimize the increase in P_f when $r_1 > N_1/2$ by using an OS scheme with an AND rule with the largest possible K but this will lead to a larger decrease in P_f if $r_1 \leq N_1/2$ and thus a loss in detection performance. Thus there is a trade-off in the choice of K. The OR fusion rule gives larger increases in P_f if $r_1 > N_1/2$ but smaller decreases if $r_1 \leq N_1/2$ so the choice of fusion rule is also a trade-off. In any case the best OS schemes do better than the best CA schemes.

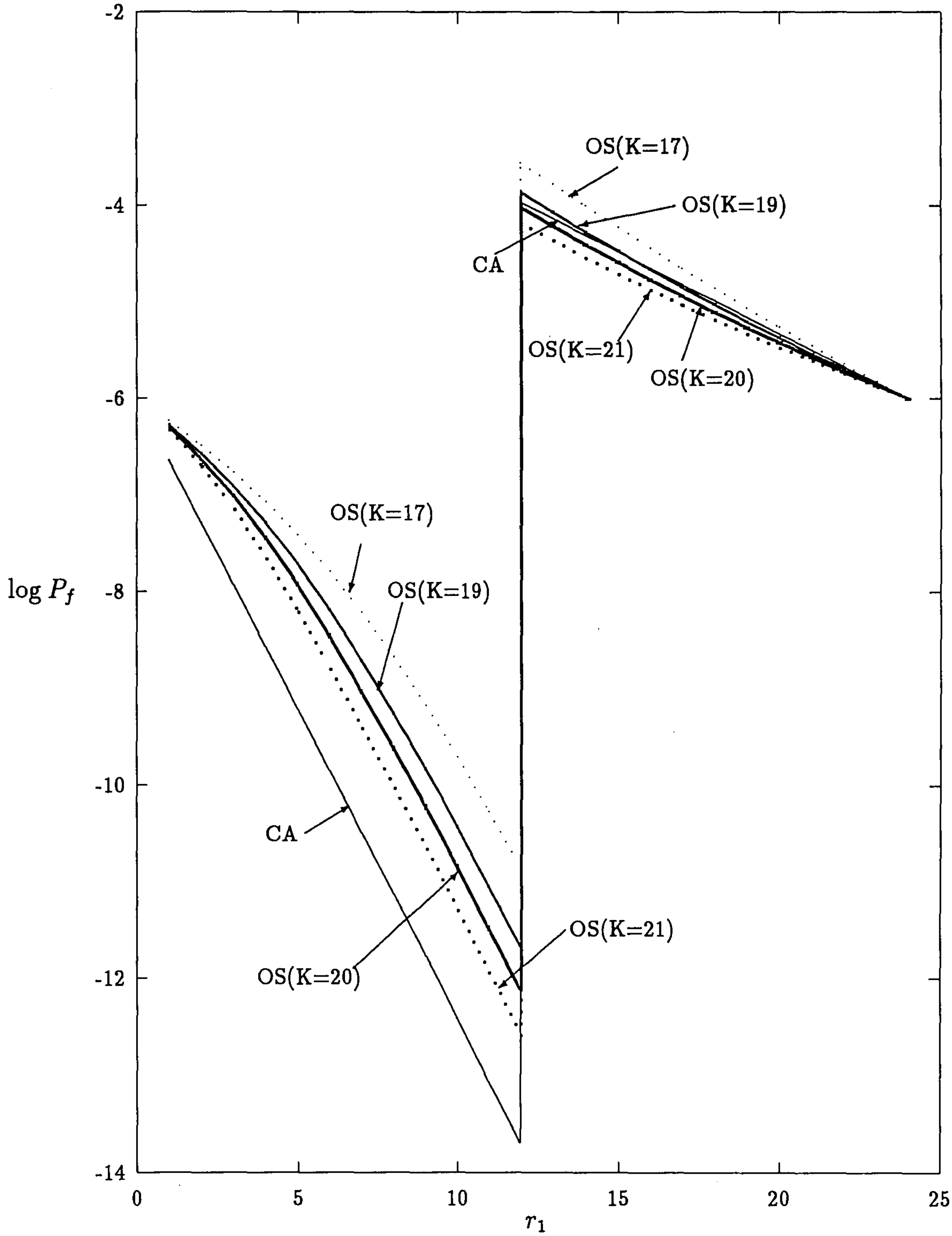

Figure 1: False alarm probability variation versus number of reference cells in increased clutter r_1 for CA-CFAR and OS-CFAR schemes using an AND rule.

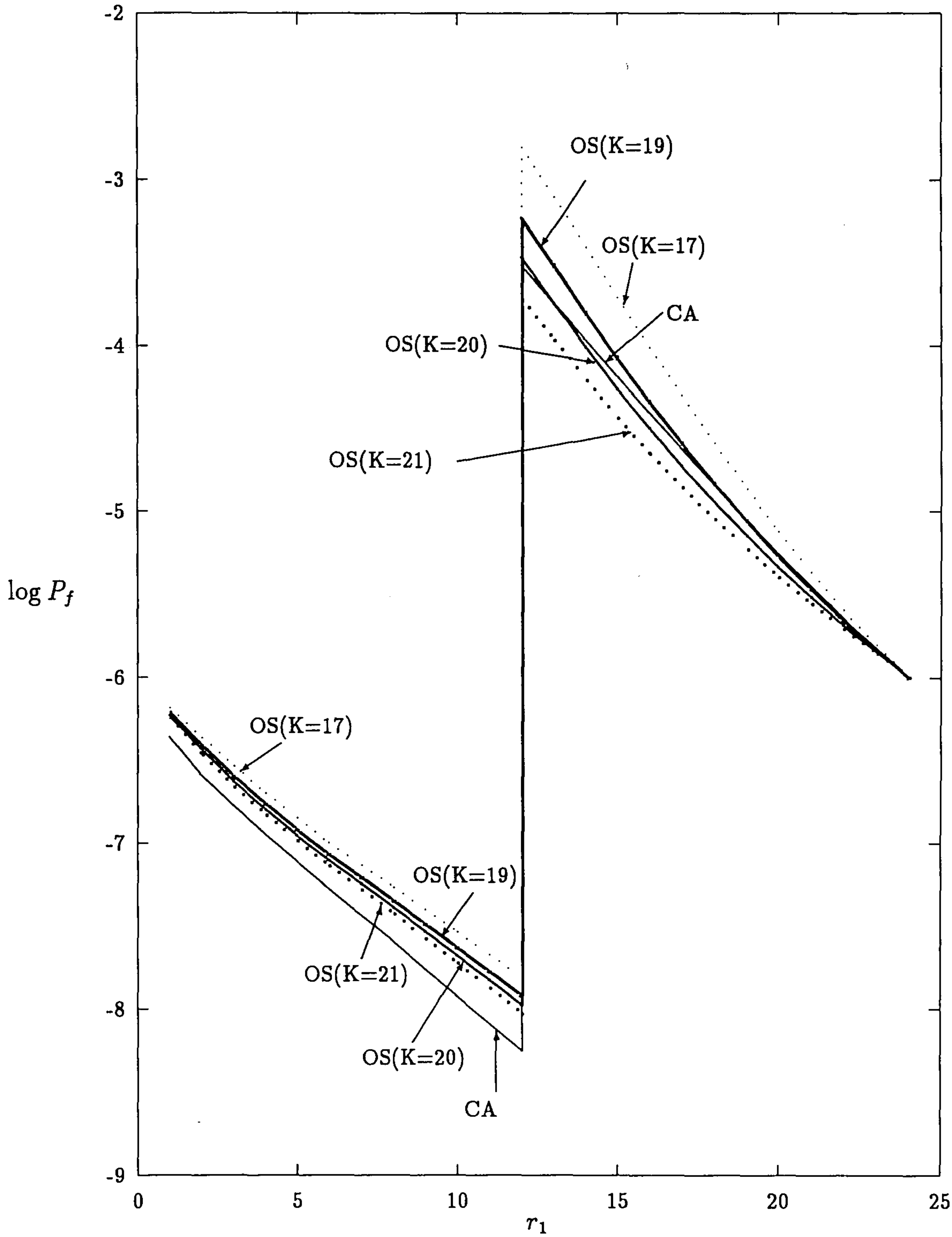

Figure 2: False alarm probability variation versus number of reference cells in increased clutter r_1 for CA-CFAR and OS-CFAR schemes using an OR rule.

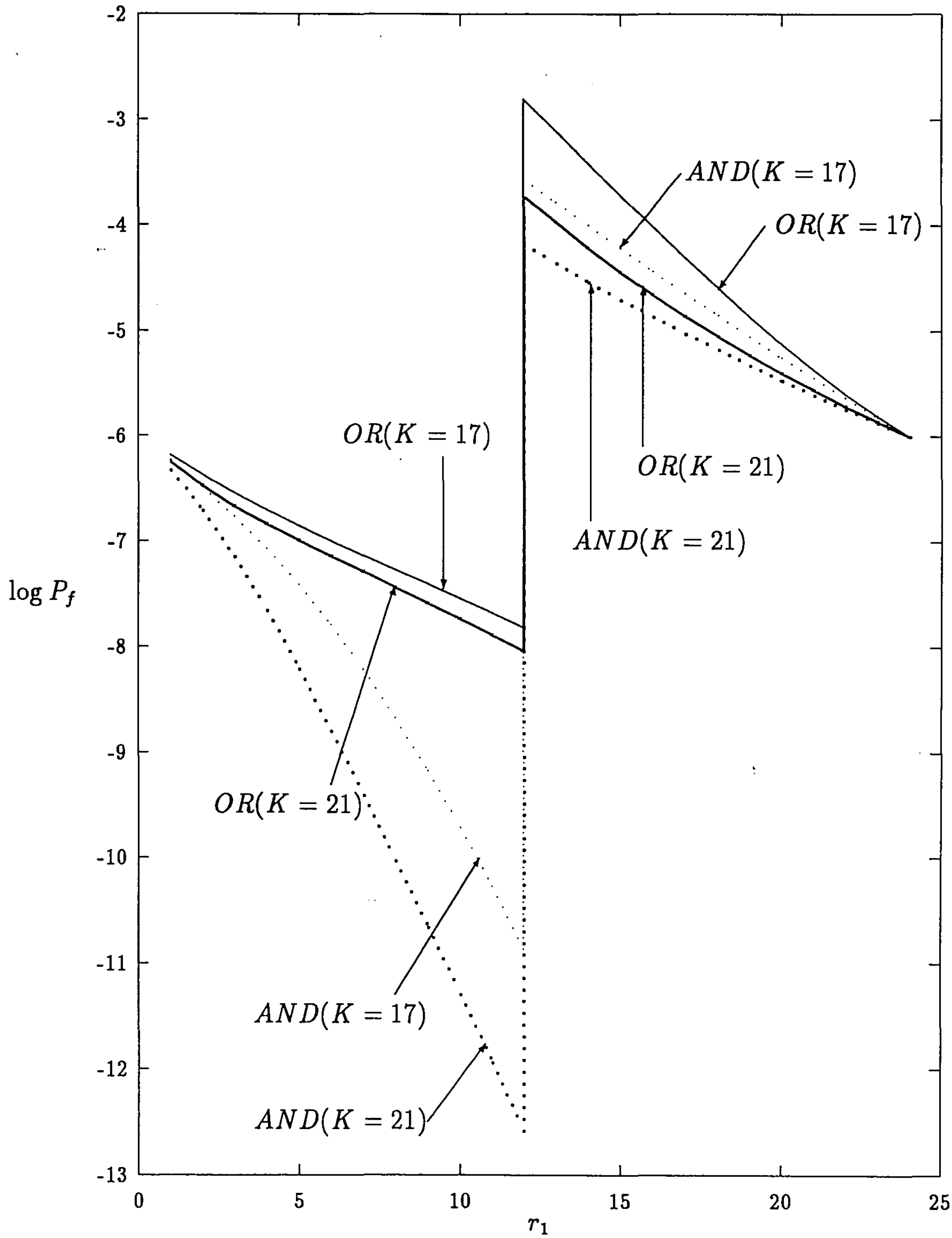

Figure 3: False alarm probability variation versus number of reference cells in increased clutter r_1 for the OS-CFAR schemes using AND and OR rules.

V Conclusions

We have applied OS-CFAR detection techniques to a distributed detection system with dependent observations from sensor to sensor under the assumption of weak signals. The best thresholds were given for schemes employing either an AND or OR fusion rule for specific cases. In all of the cases we investigated the OR fusion rule provided better performance than the AND fusion rule for false alarm probabilities larger than some critical value. This critical value depends on the fusion rule used and on the N_1, N_2, K_1 and K_2 used.

We have also investigated the ability of some CFAR distributed detection schemes to maintain constant false alarm probability in the presence of noise-plus-clutter power variations in the reference observations. Our results indicate that some distributed OS-CFAR schemes can provide much better overall performance than CA-CFAR schemes under similar conditions, assuming the order statistic chosen to form the adaptive part of the threshold and the fusion rule are used correctly.

An interesting discussion of a fast implementation of an OS-CFAR detection scheme was recently given in [8]. A similar implementation could be useful in the distributed scheme we discuss here.

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Appendix A

Under the null hypothesis, the reference observations have an exponential probability density function with [6]

$$p(v) = \begin{cases} \frac{1}{2\sigma^2} exp(-\frac{v}{2\sigma^2}) & \text{for} \quad v \geq 0 \\ 0 & \text{for} \quad v < 0 \end{cases} \tag{24}$$

The corresponding cumulative distribution function is

$$P(v) = \int_{-\infty}^{\infty} p(v)dv = \begin{cases} 1 - exp(-\frac{v}{2\sigma^2}) & \text{for} \quad v \geq 0 \\ 0 & \text{for} \quad v < 0 \end{cases} \tag{25}$$

The pdf of the K_j^{th} ordered statistics of the reference observation at sensor j is

$$f_{V_j}(v_j) = K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} (1 - P(v_j))^{N_j - K_j} (P(v_j))^{K_j - 1} p(v_j)$$

$$= K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} \left(1 - 1 + exp(-\frac{v_j}{2\sigma^2})\right)^{N_j - K_j} \left(1 - exp(-\frac{v_j}{2\sigma^2})\right)^{K_j - 1}$$

$$\frac{1}{2\sigma^2} exp(-\frac{v_j}{2\sigma^2})$$

$$= \frac{K_j}{2\sigma^2} \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-\frac{v_j(N_j + 1 - K_j)}{2\sigma^2}\right) \left(1 - exp\left(-\frac{v_j}{2\sigma^2}\right)\right)^{K_j - 1}$$

$$\text{for} \quad v_j \geq 0 \tag{26}$$

Appendix B

In equation (7),we have [2]

$$\frac{d^2}{d\theta^2} f_{X_1,\ldots,X_n}(x_1, \ldots, x_n|\theta)|_{\theta=0} = \frac{1}{\sigma^2} \sum_{i=1}^{n} (\frac{x_i}{\sigma^2} - 2)$$

$$\prod_{j=1}^{n} \left(\frac{1}{2\sigma^2} exp(-\frac{x_j}{2\sigma^2})\right) \tag{27}$$

Putting (27) and (5) into (7), we obtain

$$\frac{d^2}{d\theta^2}P(U_1 = u_1, \ldots, U_n = u_n|\theta)|_{\theta=0} = \frac{2}{\sigma^2}\Big\{\int_{v_1=-\infty}^{\infty} \cdots \int_{v_n=-\infty}^{\infty} \int_{R_1} \cdots \int_{R_n}$$
$$\sum_{i=1}^{n}\left(\frac{x_i}{2\sigma^2} - 1\right)\prod_{j=1}^{n}\Big[\frac{K_j}{4\sigma^4}\begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-\frac{v_j(N_j + 1 - K_j)}{2\sigma^2}\right)$$
$$\left(1 - exp\left(-\frac{v_j}{2\sigma^2}\right)\right)^{K_j - 1} \exp(-\frac{x_j}{2\sigma^2})\Big] dx_1 \cdots dx_n dv_1 \cdots dv_n\Big\} \tag{28}$$

Changing variables to $v'_j = \frac{v_j}{2\sigma^2}$ and $x'_j = \frac{x_j}{2\sigma^2}; j = 1, 2, \ldots n$, yields

$$\frac{d^2}{d\theta^2}P(U_1 = u_1, \ldots, U_n = u_n|\theta)|_{\theta=0} = \frac{2}{\sigma^2}\Big\{\int_{v'_1=-\infty}^{\infty} \cdots \int_{v'_n=-\infty}^{\infty} \int_{R_1} \cdots \int_{R_n}$$
$$\sum_{i=1}^{n}(x'_i - 1)\prod_{j=1}^{n}\Big[K_j\begin{pmatrix} N_j \\ K_j \end{pmatrix} \exp\left(-v'_j(N_j + 1 - K_j)\right)$$
$$\left(1 - \exp(-v'_j)\right)^{K_j - 1} \exp(-x'_j)\Big] dx'_1 \cdots dx'_n dv'_1 \cdots dv'_n\Big\} \tag{29}$$

Replacing v'_j by v_j and x'_j by x_j in (29) yields equation (8).

Appendix C

Using AND rule, equation (6) becomes

$$P''_d(0) = \int_{v_1=-\infty}^{\infty} \cdots \int_{v_n=-\infty}^{\infty} \prod_{i=1}^{n} f_{V_i}(v_i)\Big[\int_{x_1=v_1t_1}^{\infty} \cdots \int_{x_n=v_nt_n}^{\infty}$$
$$\frac{d^2}{d\theta^2}f_{X_1,\ldots,X_n}(x_1, \ldots, x_n|\theta)|_{\theta=0}dx_1 \cdots dx_n\Big] dv_1 \cdots dv_n \tag{30}$$

Let Q denote the part of (30) in the square brackets. Using equation (27), we have

$$Q = \int_{x_1=v_1t_1}^{\infty} \cdots \int_{x_n=v_nt_n}^{\infty} \frac{1}{\sigma^2}\sum_{i=1}^{n}(\frac{x_i}{\sigma^2} - 2)\prod_{j=1}^{n}\frac{1}{2\sigma^2}exp(-\frac{x_j}{2\sigma^2})dx_1 \cdots dx_n$$
$$= \frac{2}{\sigma^2}\int_{x'_1=\frac{v_1t_1}{2\sigma^2}}^{\infty} \cdots \int_{x'_n=\frac{v_nt_n}{2\sigma^2}}^{\infty} \sum_{i=1}^{n}(x'_i - 1)\prod_{j=1}^{n} exp(-x'_j)dx'_1 \cdots dx'_n$$
$$= \frac{2}{\sigma^2}\sum_{i=1}^{n}\frac{v_it_i}{2\sigma^2}\prod_{j=1}^{n} exp(-\frac{v_jt_j}{2\sigma^2}) \tag{31}$$

where we used the change of variable $x'_j = \frac{x_j}{2\sigma^2}; \quad j = 1, 2, ..., n$. Putting (31) and (26) into (30), we have

$$P''_d(0) = \int_{v_1=0}^{\infty} \cdots \int_{v_n=0}^{\infty} \prod_{j=1}^{n} \frac{K_j}{2\sigma^2} \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-\frac{v_j(N_j+1-K_j)}{2\sigma^2}\right)$$

$$\left(1 - exp\left(-\frac{v_j}{2\sigma^2}\right)\right)^{K_j-1} \frac{2}{\sigma^2} \sum_{i=1}^{n} \frac{v_i t_i}{2\sigma^2} \prod_{\ell=1}^{n} exp(-\frac{v_\ell t_\ell}{2\sigma^2}) dv_1 \cdots dv_n$$

$$= \frac{2}{\sigma^2} \int_{v'_1=0}^{\infty} \cdots \int_{v'_n=0}^{\infty} \prod_{j=1}^{n} \left\{ K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v'_j(N_j+1-K_j)\right)\right.$$

$$\left.(1 - exp(-v'_j))^{K_j-1} exp(-v'_j t_j)\right\} \sum_{i=1}^{n} v'_i t_i dv'_1 \cdots dv'_n \tag{32}$$

where we changed the variable with $v'_j = \frac{v_j}{2\sigma^2}; \quad j = 1, 2, ...n$. Interchanging the summation and integrations, we get

$$P''_d(0) = \frac{2}{\sigma^2} \sum_{i=1}^{n} \left(\int_{v'_1=0}^{\infty} \cdots \int_{v'_n=0}^{\infty} \prod_{j=1}^{n} \left\{ K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v'_j(N_j+1-K_j)\right)\right.\right.$$

$$\left.\left.(1 - exp(-v'_j))^{K_j-1} exp(-v'_j t_j)\right\} (v'_i t_i) dv'_1 \cdots dv'_n \right)$$

$$= \frac{2}{\sigma^2} \sum_{i=1}^{n} \left\{ \left[\int_{v'_i=0}^{\infty} \left(K_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} exp\left(-v'_i(N_i+1-K_i)\right)\right.\right.\right.$$

$$\left.\left.(1 - exp(-v'_i))^{K_i-1} exp(-v'_i t_i)\right) (v'_i t_i) dv'_i \right]$$

$$\left[\prod_{j=1, j\neq i}^{n} \int_{v'_j=0}^{\infty} K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v'_j(N_j+1-K_j)\right)\right.$$

$$\left.\left.(1 - exp(-v'_j))^{K_j-1} exp(-v'_j t_j) dv'_j \right] \right\}$$

$$= \frac{2}{\sigma^2} \sum_{i=1}^{n} \{[A]\,[B]\} \tag{33}$$

where A denotes the integral over v'_i and B denotes the product of the other integrals in (33). Evaluating the integral in A gives

$$A = \int_{v_i'=0}^{\infty} \Bigg(K_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} \exp\left(-v_i'(N_i + 1 - K_i)\right) \left(1 - exp(-v_i')\right)^{K_i - 1}$$

$$exp(-v_i't_i)\Bigg) (v_i't_i) dv_i' \tag{34}$$

A can be further simplified by using $(v_i't_i)exp(-v_i't_i) = -t_i\frac{d}{dt_i}exp(-v_i't_i)$. We can interchange the derivative with respect to t_i and the integration to obatin

$$A = -t_i \frac{d}{dt_i} \int_{v_i'=0}^{\infty} \Bigg(K_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} \exp\left(-v_i'(N_i + 1 - K_i + t_i)\right)$$

$$(1 - exp(-v_i'))^{K_i - 1} \Bigg) dv_i'$$

$$= -t_i \frac{d}{dt_i} \frac{\binom{N_i}{K_i}}{\binom{N_i + t_i}{K_i}} = -t_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} \frac{d}{dt_i} \frac{1}{\binom{N_i + t_i}{K_i}}$$

$$= -t_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} K_i! \frac{d}{dt_i} \prod_{L=0}^{K_i - 1} \frac{1}{N_i + t_i - L} = -t_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} K_i! \frac{d}{dt_i} \mu \tag{35}$$

where

$$\mu = \prod_{L=0}^{K_i - 1} \frac{1}{N_i + t_i - L} \tag{36}$$

Taking natural logrithm and then derivative with respect to t_i on both sides of (36), we have

$$\frac{d}{dt_i}(ln\mu) = \frac{\frac{d}{dt_i}\mu}{\mu}$$

$$= \frac{d}{dt_i} ln\left(\prod_{L=0}^{K_i - 1} \frac{1}{N_i + t_i - L}\right) = \frac{d}{dt_i}\left(\sum_{L=0}^{K_i - 1} ln \frac{1}{N_i + t_i - L}\right)$$

$$= -\frac{d}{dt_i}\left(\sum_{L=0}^{K_i - 1} ln(N_i + t_i - L)\right) = -\sum_{L=0}^{K_i - 1} \frac{1}{N_i + t_i - L} \tag{37}$$

Rewriting (37), we have

$$\frac{d}{dt_i}\mu = -\mu \sum_{L=0}^{K_i-1} \frac{1}{N_i+t_i-L} = -\prod_{L=0}^{K_i-1} \frac{1}{N_i+t_i-L} \left(\sum_{L=0}^{K_i-1} \frac{1}{N_i+t_i-L} \right) \tag{38}$$

Inserting the expression for $\frac{d}{dt_i}\mu$ from (38), we obtain

$$\begin{aligned} A &= -t_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} \left(-K_i! \prod_{L=0}^{K_i-1} \frac{1}{N_i+t_i-L} \right) \sum_{L=0}^{K_i-1} \frac{1}{N_i+t_i-L} \\ &= \frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}} \sum_{L=0}^{K_i-1} \frac{t_i}{N_i+t_i-L} \end{aligned} \tag{39}$$

Evaluating integral in B gives

$$\begin{aligned} B &= \prod_{j=1,j\neq i}^{n} \int_{v_j'=0}^{\infty} K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} \exp\left(-v_j'(N_j+1-K_j)\right) \\ &(1-exp(-v_j'))^{K_j-1} exp(-v_j' t_j) dv_j' \\ &= \prod_{j=1,j\neq i}^{n} \int_{v_j'=0}^{\infty} K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} \exp\left(-v_j'(N_j+1-K_j+t_j)\right) \\ &(1-exp(-v_j'))^{K_j-1} dv_j' \\ &= \prod_{j=1,j\neq i}^{n} \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \end{aligned} \tag{40}$$

Putting A and B into (33), we have

$$P_d''(0) = \frac{2}{\sigma^2} \prod_{j=1}^{n} \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \sum_{i=1}^{n} \left(\sum_{L=0}^{K_i-1} \frac{t_i}{N_i+t_i-L} \right) \tag{41}$$

Appendix D

For a single sensor (take the first sensor for example), the false alarm probability is

$$\begin{aligned} P_{f1} &= Prob[X_1 \geq tV_1 | \theta = 0] \\ &= \int_{v=0}^{\infty} \int_{x=tv}^{\infty} f_{X_1}(x|\theta=0) f_{V_1}(v) dx dv \end{aligned}$$

$$= \int_{v=0}^{\infty} \left(\int_{x=tv}^{\infty} \frac{1}{2\sigma^2} exp(-\frac{x}{2\sigma^2}) dx \right) \frac{K}{2\sigma^2} \left(\begin{array}{c} N \\ K \end{array} \right) exp\left(-\frac{v(N+1-K)}{2\sigma^2} \right)$$

$$\left(1 - exp\left(-\frac{v}{2\sigma^2} \right) \right)^{K-1} dv$$

$$= \int_{v=0}^{\infty} exp(-\frac{vt}{2\sigma^2}) \frac{K}{2\sigma^2} \left(\begin{array}{c} N \\ K \end{array} \right) exp\left(-\frac{v(N+1-K)}{2\sigma^2} \right)$$

$$\left(1 - exp\left(-\frac{v}{2\sigma^2} \right) \right)^{K-1} dv$$

$$= \int_{v'=0}^{\infty} K \left(\begin{array}{c} N \\ K \end{array} \right) exp\left(-v'(N+1-K+t) \right) \left(1 - exp(-v') \right)^{K-1} dv'$$

$$= \frac{\binom{N}{K}}{\binom{N+t}{K}} \tag{42}$$

where $f_{X_1}(x|\theta = 0)$ comes from (4) and $f_{V_1}(v)$ comes from (5), and we changed variable with $v' = \frac{v}{2\sigma^2}$. In a similar way, the false alarm probability in jth sensor will be

$$P_{fj} = \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \tag{43}$$

Using AND rule, the overall false alarm probability is

$$P_f = \prod_{j=1}^{n} P_{fj} = \prod_{j=1}^{n} \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \tag{44}$$

Using OR rule, the overall false alarm probability is

$$P_f = 1 - \prod_{j=1}^{n} (1 - P_{fj}) = 1 - \prod_{j=1}^{n} \left(1 - \frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}} \right) \tag{45}$$

Appendix E

We first rewrite (10) as

$$P_d''(0) = \frac{2}{\sigma^2} \frac{\prod_{i=1}^{n} \binom{N_i}{K_i}}{\prod_{i=1, i\neq \ell}^{n} \binom{N_i+t_i}{K_i}} \left\{ \left[\frac{1}{\binom{N_\ell+t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell - 1} \frac{t_\ell}{N_\ell + t_\ell - L} \right] \right.$$

$$+\sum_{i=1,i\neq\ell}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)\frac{1}{\binom{N_\ell+t_\ell}{K_\ell}}\Bigg\} \tag{46}$$

Taking a derivative of $\sigma^2 P_d''(0)$ with respect to t_ℓ (see (34) - (39)) gives

$$\sigma^2\frac{dP_d''(0)}{dt_\ell}=\sigma^2\frac{2}{\sigma^2}\frac{\prod_{i=1}^{n}\binom{N_i}{K_i}}{\prod_{i=1,i\neq\ell}^{n}\binom{N_i+t_i}{K_i}}\Bigg\{\frac{d}{dt_\ell}\left[\frac{1}{\binom{N_\ell+t_\ell}{K_\ell}}\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}\right]$$

$$+\sum_{i=1,i\neq\ell}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)\frac{d}{dt_\ell}\frac{1}{\binom{N_\ell+t_\ell}{K_\ell}}\Bigg\}$$

$$=2\frac{\prod_{i=1}^{n}\binom{N_i}{K_i}}{\prod_{i=1,i\neq\ell}^{n}\binom{N_i+t_i}{K_i}}\Bigg\{\frac{1}{\binom{N_\ell+t_\ell}{K_\ell}}\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}+\left(\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}\right)\frac{-1}{\binom{N_\ell+t_\ell}{K_\ell}}$$

$$\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}+\sum_{i=1,i\neq\ell}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)\frac{-1}{\binom{N_\ell+t_\ell}{K_\ell}}\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\Bigg\}$$

$$=2\prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\Bigg\{\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}-\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}$$

$$-\sum_{i=1,i\neq\ell}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\Bigg\} \tag{47}$$

Taking a derivative of ξP_f with respect to t_ℓ by using equation (11) (see (34) - (39)) gives

$$\xi\frac{dP_f}{dt_\ell}=\xi\frac{d}{d_\ell}\prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}=-\xi\prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L} \tag{48}$$

Combining (47) and (48) yields (12)

Appendix F

Starting with the necessary condition for the AND rule given in (12), omitting the common factor $\prod_{i=1}^{n}\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}$ and dividing every term by $2\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right)$, we get (note that we have added and subtracted the last term).

$$-\frac{\xi}{2} = \left(\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}\right)\Bigg/\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right)-\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}$$

$$-\sum_{i=1}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)+\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L} \tag{49}$$

The second term and fourth term on the right side of equation (49) are equal but of opposite sign. The third term is moved to left side to obtain

$$\sum_{i=1}^{n}\left(\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\right)-\frac{\xi}{2}=\frac{\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}}{\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}} \tag{50}$$

Because the left side of equation (50) does not change with ℓ, we are able to obtain an algebraic relationship between t_i and t_j as equation (13)

Appendix G

For an OR rule, (6) becomes

$$P_d''(0)=-\int_{v_1=-\infty}^{\infty}\cdots\int_{v_n=-\infty}^{\infty}\prod_{i=1}^{n}f_{V_i}(v_i)\Big[\int_{x_1=0}^{v_1t_1}\cdots\int_{x_n=0}^{v_nt_n}\frac{d^2}{d\theta^2}f_{X_1,\dots,X_n}(x_1,\dots,x_n|\theta)|_{\theta=0}dx_1\cdots dx_n\Big]dv_1\cdots dv_n \tag{51}$$

Let Z denote the part of (51) in the square brackets. Using (27), we have

$$\begin{aligned}
Z &= \int_{x_1=0}^{v_1t_1}\cdots\int_{x_n=0}^{v_nt_n}\sum_{i=1}^{n}\frac{1}{\sigma^2}\sum_{i=1}^{n}(\frac{x_i}{\sigma^2}-2)\prod_{j=1}^{n}\left(\frac{1}{2\sigma^2}exp(-\frac{x_j}{2\sigma^2})\right)dx_1\cdots dx_n\\
&= \int_{x_1'=0}^{\frac{v_1t_1}{2\sigma^2}}\cdots\int_{x_n'=0}^{\frac{v_nt_n}{2\sigma}}\sum_{i=1}^{n}\frac{2}{\sigma^2}(x_i'-1)\prod_{j=1}^{n}exp(-x_j')dx_1'\cdots dx_n'\\
&= \frac{2}{\sigma^2}\sum_{i=1}^{n}\left\{\int_{x_i'=0}^{\frac{v_it_i}{2\sigma^2}}(x_i'-1)exp(-x_i')dx_i'\prod_{j=1,j\neq i}^{n}\int_{x_j'=0}^{\frac{v_jt_j}{2\sigma^2}}exp(-x_j')dx_j'\right\}\\
&= \frac{2}{\sigma^2}\sum_{i=1}^{n}\left\{-\frac{v_it_i}{2\sigma^2}exp(-\frac{v_it_i}{2\sigma^2})\prod_{j=1,j\neq i}^{n}\left(1-exp(-\frac{v_jt_j}{2\sigma^2})\right)\right\}
\end{aligned} \tag{52}$$

where we used the change of variable $x_j'=\frac{x_j}{2\sigma^2};\quad j=1,2,...,n$. Putting (52) and (26) into (51), we obtain

$$P_d''(0)=\frac{2}{\sigma^2}\int_{v_1=0}^{\infty}\cdots\int_{v_n=0}^{\infty}\prod_{j=1}^{n}\Bigg\{\frac{K_j}{2\sigma^2}\begin{pmatrix}N_j\\K_j\end{pmatrix}exp\left(-\frac{v_j(N_j+1-K_j)}{2\sigma^2}\right)$$

$$\left(1 - exp\left(-\frac{v_j}{2\sigma^2}\right)\right)^{K_j-1}\Bigg\} \sum_{i=1}^{n} \left\{ \frac{v_i t_i}{2\sigma^2} exp(-\frac{v_i t_i}{2\sigma^2}) \prod_{\ell=1,\ell\neq i}^{n} \left(1 - exp(-\frac{v_\ell t_\ell}{2\sigma^2})\right) \right\}$$

$$dv_1 \cdots dv_n$$

$$= \frac{2}{\sigma^2} \int_{v_1'=0}^{\infty} \cdots \int_{v_n'=0}^{\infty} \prod_{j=1}^{n} K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v_j'(N_j + 1 - K_j)\right)$$

$$(1 - exp(-v_j'))^{K_j-1} \sum_{i=1}^{n} \Big\{ v_i' t_i exp(-v_i' t_i)$$

$$\prod_{\ell=1,\ell\neq i}^{n} (1 - exp(-v_\ell' t_\ell)) \Big\} dv_1' \cdots dv_n' \qquad (53)$$

where we changed variables to $v_j' = \frac{v_j}{2\sigma^2}$; $j = 1, 2, ..., n$. Interchanging the summation and the integration, we get

$$P_d''(0) = \frac{2}{\sigma^2} \sum_{i=1}^{n} \Big(\int_{v_1'=0}^{\infty} \cdots \int_{v_n'=0}^{\infty} \prod_{j=1}^{n} \Big\{ K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v_j'(N_j + 1 - K_j)\right)$$

$$(1 - exp(-v_j'))^{K_j-1} (v_i' t_i) exp(-v_i' t_i) \Big\} \prod_{\ell=1,\ell\neq i}^{n} (1 - exp(-v_\ell' t_\ell))\, dv_1' \cdots dv_n' \Big)$$

$$= \frac{2}{\sigma^2} \sum_{i=1}^{n} \Big\{ \Big[\int_{v_i'=0}^{\infty} \Big(K_i \begin{pmatrix} N_i \\ K_i \end{pmatrix} exp\left(-v_i'(N_i + 1 - K_i)\right) (1 - exp(-v_i'))^{K_i-1} \Big)$$

$$(v_i' t_i) exp(-v_i' t_i) dv_i' \Big] \Big[\prod_{j=1,j\neq i}^{n} \int_{v_j'=0}^{\infty} \left(1 - exp(-v_j' t_j)\right) K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix}$$

$$exp\left(-v_j'(N_j + 1 - K_j)\right) (1 - exp(-v_j'))^{K_j-1} dv_j' \Big] \Big\}$$

$$= \frac{2}{\sigma^2} \sum_{i=1}^{n} \{[C][D]\} \qquad (54)$$

where C denotes the integral with respect to v_i' and D denotes the other integrals in (54). Note that C is equal to A in (39), therefore

$$C = \frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}} \sum_{L=0}^{K_i-1} \frac{t_i}{N_i + t_i - L} \qquad (55)$$

Taking the integral in D gives

$$D = \prod_{j=1, j\neq i}^{n} \int_{v'_j=0}^{\infty} \left(1 - exp(-v'_j t_j)\right) K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v'_j(N_j + 1 - K_j)\right)$$

$$(1 - exp(-v'_j))^{K_j - 1} dv'_j$$

$$= \prod_{j=1, j\neq i}^{n} \left\{1 - \int_{v'_j=0}^{\infty} K_j \begin{pmatrix} N_j \\ K_j \end{pmatrix} exp\left(-v'_j(N_j + 1 - K_j + t_j)\right)\right.$$

$$\left.(1 - exp(-v'_j))^{K_j - 1} dv'_j\right\}$$

$$= \prod_{j=1, j\neq i}^{n} \left\{1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}}\right\} \tag{56}$$

Putting equation C and D into equation (54), we obtain

$$P''_d(0) = \frac{2}{\sigma^2} \sum_{i=1}^{n} \left\{ \left[\frac{\binom{N_i}{K_i}}{\binom{N_i + t_i}{K_i}} \sum_{L=0}^{K_i - 1} \frac{t_i}{N_i + t_i - L} \right] \prod_{j=1, j\neq i}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}} \right] \right\} \tag{57}$$

Appendix H

$P''_d(0)$ in (16) can be rewritten as

$$P''_d(0) = \frac{2}{\sigma^2} \left\{ \sum_{i=1, i\neq \ell}^{n} \left(\left[\frac{\binom{N_i}{K_i}}{\binom{N_i + t_i}{K_i}} \sum_{L=0}^{K_i - 1} \frac{t_i}{N_i + t_i - L} \right] \left(\prod_{j=1, j\neq i, \ell}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}} \right] \right) \right.\right.$$

$$\left.\left. \left[1 - \frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell + t_\ell}{K_\ell}} \right] \right) + \left[\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell + t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell - 1} \frac{t_\ell}{N_\ell + t_\ell - L} \right] \prod_{j=1, j\neq \ell}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}} \right] \right\} \tag{58}$$

Taking derivative of $\sigma^2 P''_d(0)$ with respect to t_ℓ (see (34) - (39)) gives

$$\sigma^2 \frac{dP''_d(0)}{dt_\ell} = \sigma^2 \frac{2}{\sigma^2} \left\{ \sum_{i=1, i\neq \ell}^{n} \left(\left[\frac{\binom{N_i}{K_i}}{\binom{N_i + t_i}{K_i}} \sum_{L=0}^{K_i - 1} \frac{t_i}{N_i + t_i - L} \right] \prod_{j=1, j\neq i, \ell}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}} \right] \right.\right.$$

$$\left.\left. \frac{d}{dt_\ell} \left[1 - \frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell + t_\ell}{K_\ell}} \right] \right) + \frac{d}{dt_\ell} \left[\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell + t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell - 1} \frac{t_\ell}{N_\ell + t_\ell - L} \right] \prod_{j=1, j\neq \ell}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}} \right] \right\}$$

$$= 2 \sum_{i=1, i\neq \ell}^{n} \left\{ \left[\frac{\binom{N_i}{K_i}}{\binom{N_i + t_i}{K_i}} \sum_{L=0}^{K_i - 1} \frac{t_i}{N_i + t_i - L} \right] \left(\prod_{j=1, j\neq i, \ell}^{n} \left[1 - \frac{\binom{N_j}{K_j}}{\binom{N_j + t_j}{K_j}} \right] \right) \frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell + t_\ell}{K_\ell}} \right.$$

$$\sum_{L=0}^{K_\ell-1} \frac{1}{N_\ell+t_\ell-L}\Bigg\} + 2\Bigg\{-\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell-1} \frac{1}{N_\ell+t_\ell-L} \sum_{L=0}^{K_\ell-1} \frac{t_\ell}{N_\ell+t_\ell-L}$$

$$+\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell-1} \frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}\Bigg\} \prod_{j=1,j\neq\ell}^{n} \left[1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right] \tag{59}$$

Taking a derivative of ξP_f in (17) with respect to t_ℓ (see (34) - (39)) gives

$$\xi\frac{dP_f}{dt_\ell} = \xi\frac{d}{dt_\ell}\left(1-\prod_{j=1}^{n}\left(1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right)\right)$$

$$= \xi\left[\prod_{j=1,j\neq\ell}^{n}\left(1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right)\right]\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}} \sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L} \tag{60}$$

Combining (59) and (60) yields (18)

Appendix I

Starting with the necessary condition for the OR rule in (18), omitting the common factor $\prod_{j=1,j\neq\ell}^{n}\left(1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right)\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}$ and dividing every term by $2\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right)$, we get

$$\frac{\xi}{2} = \sum_{i=1,i\neq\ell}^{n}\left\{\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\Bigg/\left(1-\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\right)\right\} +$$

$$\left(\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}\right)\Bigg/\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right) - \sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L} \tag{61}$$

Adding and subtracting the missing term in the first summation of (61) yields

$$\frac{\xi}{2} - \sum_{i=1}^{n}\left\{\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\Bigg/\left(1-\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\right)\right\} =$$

$$-\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}}\Bigg/\left(1-\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}}\right)\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L} -$$

$$\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L} + \left(\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}\right)\Bigg/\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right)$$

$$-\left\{\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}}\Big/\left(1-\frac{\binom{N_\ell}{K_\ell}}{\binom{N_\ell+t_\ell}{K_\ell}}\right)+1\right\}\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}$$

$$+\left(\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}\right)\Big/\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right) \qquad (62)$$

After simplifying and rearranging (62), we have

$$\frac{\xi}{2}-\sum_{i=1}^{n}\left\{\frac{\binom{N_i}{K_i}}{\binom{N_i+t_i}{K_i}}\sum_{L=0}^{K_i-1}\frac{t_i}{N_i+t_i-L}\Big/\left(1-\frac{\binom{N_j}{K_j}}{\binom{N_j+t_j}{K_j}}\right)\right\}$$

$$=\left(\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{(N_\ell+t_\ell-L)^2}\right)\Big/\left(\sum_{L=0}^{K_\ell-1}\frac{1}{N_\ell+t_\ell-L}\right)+$$

$$\left(\sum_{L=0}^{K_\ell-1}\frac{t_\ell}{N_\ell+t_\ell-L}\right)\Big/\left(\sum_{L=0}^{K_\ell-1}\frac{N_\ell-L}{N_\ell+t_\ell-L}-1\right) \qquad (63)$$

Because the left side of equal sign in equation (63) does not depend on ℓ, we are able to obtain the algebraic relationship between t_i and t_j as given in equation (19).

Vita

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END

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