The analysis of constructional alloy steel bolted plate splices, Ph.D. Dissertation 1967

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Large Bolted Connections

THE ANALYSIS OF CONSTRUCTIONAL ALLOY STEEL BOLTED PLATE SPLICES

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Geoffrey L. Kulak

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>1.1 Behavior of Bearing-Type Connections</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Summary of Theoretical Studies</td>
<td>7</td>
</tr>
<tr>
<td>1.3 Summary of Experimental Studies</td>
<td>9</td>
</tr>
<tr>
<td>1.4 Objectives of the Study</td>
<td>10</td>
</tr>
<tr>
<td>2. THEORETICAL SOLUTION FOR THE LOAD DISTRIBUTION AMONG FASTENERS IN A514 STEEL JOINTS</td>
<td>12</td>
</tr>
<tr>
<td>2.1 Characteristics of Constructional Alloy Steel</td>
<td>12</td>
</tr>
<tr>
<td>2.2 Formulation of the Equilibrium and Compatibility Equations</td>
<td>15</td>
</tr>
<tr>
<td>2.3 Shear - Deformation Relationship for Mechanical Fasteners</td>
<td>21</td>
</tr>
<tr>
<td>2.4 Tensile Stress - Strain Relationship for Plate Material</td>
<td>26</td>
</tr>
<tr>
<td>(1) Introduction</td>
<td>26</td>
</tr>
<tr>
<td>(2) Development of the Mathematical Model</td>
<td>29</td>
</tr>
<tr>
<td>(3) General Stress - Strain Relationship</td>
<td>32</td>
</tr>
<tr>
<td>(4) Comparison of Experimental Results and Theory</td>
<td>38</td>
</tr>
<tr>
<td>2.5 General Solution of the Equilibrium and Compatibility Equations</td>
<td>41</td>
</tr>
<tr>
<td>(1) Assumptions</td>
<td>41</td>
</tr>
<tr>
<td>(2) Load - Deformation Relationships</td>
<td>41</td>
</tr>
<tr>
<td>(3) Solution of the Compatibility and Equilibrium Equations</td>
<td>46</td>
</tr>
</tbody>
</table>
3. EXAMINATION OF IMPORTANT PARAMETERS

3.1 Basis of the Examination
3.2 Location of the Plate Failure - Fastener Failure Boundary
3.3 Effect of Joint Length
3.4 Effect of $A_n/A_s$ Ratio
3.5 Effect of Fastener Pitch
3.6 Effect of Fastener Diameter

4. EXPERIMENTAL VERIFICATION OF THE THEORY

4.1 Material Properties of Test Joints
4.2 Analysis of Test Specimens
   (1) Pilot Tests
   (2) Large Joints

5. PHILOSOPHY OF DESIGN OF CONSTRUCTIONAL ALLOY STEEL TENSION MEMBERS

5.1 Review of Current Design Philosophy
5.2 Characteristics of A514 Steel Members Designed According to Current Practice
5.3 Proposed Design Criteria for Bolted Joints in Constructional Alloy Steel Members
5.4 Notes on the Use of Hybrid Connections

6. SUMMARY AND CONCLUSIONS

7. TABLES AND FIGURES

8. REFERENCES
ABSTRACT

This dissertation extends a general theoretical solution for the distribution of load among the fasteners in bolted butt splices to constructional alloy (A514) steel joints. In addition, it examines the philosophy of design of metallic tension members and evaluates the extent to which mechanically fastened constructional alloy steel members are able to fulfill this philosophy.

The solution of the load distribution problem is shown to require two mathematical models. These establish the relationship between load and deformation for the component parts of a connection. The expressions developed are applicable to both the elastic and inelastic regions of behavior. The general theoretical development has been verified by an extensive testing program. Only the results of this program are reported herein. Excellent agreement between theoretical predictions and actual test results was obtained.

A study of the parameters that might be expected to affect the behavior of bolted A514 splices shows that the ultimate strength of these joints is a function of fastener grade, joint length and relative plate - fastener proportions. It is independent of fastener diameter or pitch, per se.
The performance of A514 steel members containing bolted splices is evaluated. The examination shows that A514 steel joints using high strength bolts do not produce desirable behavior in their members if the elements of the joint are designed according to current stress levels. Some improvement in behavior is obtained if higher allowable bolt stresses, in line with those suggested as a result of studies of mechanically fastened joints of other grades of steel, are used.

It is shown that satisfactory member behavior for a large class of A514 steel joints can only be obtained in one of two ways. One way is to use a different fabrication technique for forming the connections. Some type of upset end, such as an increase in plate width or thickness in the region of the joint, could be used to force yielding of the gross cross-section of the member before the ultimate load of the joint is reached. The other way of accomplishing the same end is to change the specified mechanical properties of A514 steel. In particular, a limitation on the spread between yield and ultimate stresses to about 25 ksi minimum would allow satisfactory member behavior under normal fabrication methods.
1. INTRODUCTION

1.1 Behavior of Bearing-Type Connections

The use and importance of the constructional alloy steels have increased steadily since they were first introduced as proprietary products starting in 1952. These steels, namely United States Steel Corporation's "T-1", Great Lakes Steel Corporation's "N-A-XTRA", and many others, had their first uses mainly in the fabrication of pressure vessels and as components in heavy construction equipment. Their favorable strength to cost ratio has also made these steels attractive for structural applications, and they are now commonly used where large loads must be carried. Their increasing importance is evidenced by the fact that these proprietary products are now covered by an ASTM specification, A514.¹

Although steel meeting ASTM A514 is weldable, in common with other grades of structural steel mechanical fastening by means of rivets or high strength bolts is one of the primary methods of forming connections. An examination of the tensile behavior of such connections forms the basis for this dissertation.
Two distinct phases are involved in the behavior of a mechanically fastened joint. From the condition of no load up to a certain governing value, the load transfer mechanism is principally one of friction between the faying surfaces in the joint. The frictional force is a direct function of the clamping force provided by the fasteners and of the number and condition of the faying surfaces.

Once the frictional resistance in the joint has been overcome, the plates slip into contact against the fasteners and load transfer is then by bearing. The time of change between load transfer by friction and load transfer by bearing is termed "major slip".

The behavior of a bolted joint has been described in detail by Rumpf and Fisher. Only those phases of the load history after major slip, that is when the load transfer is by bearing of the component plates against the fasteners, will be reviewed here.

Plate accelerations during slip are generally so large that once the motion has started it does not stop until the fasteners come into bearing against the sides of the holes. Under theoretically ideal conditions, the end fasteners in the longitudinal line of a joint will come into bearing first.
Under a bearing load a fastener will suffer deformation due to shear, bending, and bearing. In addition, the adjacent plate will deform locally. With increasing load, the deformation of the bolt, holes, and intervening plate between bolts will be such that eventually all bolts in the joint will be in bearing.

If the plate material were perfectly rigid, each fastener would be subject to the same amount of deformation. Under this hypothetical condition, then, the total load in a given joint would be distributed equally to all the fasteners. Designers today generally make this assumption of equal distribution to each fastener.

The situation in reality can be quite different, however. A qualitative assessment of the actual distribution of load can be made by considering Fig. 1. Here, ten fasteners are shown connecting plates A and B. Examining conditions between lines 1 and 2, it can be observed that plate B must be highly stressed since it has just "entered" the joint. On the other hand, plate A will have only a very low stress in this region since it is adjacent to the "free" end of the joint. Plate strains, then, will be correspondingly high in B and low in A. Near the ultimate load of the joint, for example, these strains will probably be in the inelastic range in the former but will be elastic in the latter. Since the bearing connector "1"
receives its load by virtue of the difference in strains between
the plates it connects, the deformation and hence the load in
this fastener can be expected to be high.

A similar line of reasoning applied to a portion of
the joint near the center will show that the differential plate
strains must necessarily be lower there. Qualitatively, then,
the higher loading of end fasteners and gradual decrease in
fastener load as one proceeds toward the center of a joint is
physically acceptable.

The behavior of a bearing joint as described here has
been observed in many tests.\textsuperscript{4,5} Accurate theoretical predic-
tions also are available for certain of these cases.\textsuperscript{2}

There are two possible modes of failure in a bearing-
type connection acting under a tensile load. If the differen-
tial plate strains near the ends of a joint are high as com-
pared to those in the central portion, the shear failure of a
single end fastener can occur. The resulting distribution of
load from the failed connector to those remaining may or may
not cause further failure, depending upon joint proportions.
In either event, single or multiple fastener failure, the entire
joint is considered to have reached its ultimate capacity.
When the tensile capacity of the plate at its net section is less than the shear capacity of the fasteners, failure will occur by fracture of the plate. If it is known that a particular joint is in this plate failure region, its capacity can be readily calculated. This can be done on the basis of plate - with - holes tensile coupons. The results of standard tensile coupons can also be used if a suitable correction factor is applied. Determination of the location of this plate failure - fastener failure boundary still involves the computation of the total shear capacity of the fasteners, however.

Whether a joint is expected to fail by fastener shear or by tensile fracture of the plates, then, the distribution of load among the fasteners must be known. In the former case, this defines the failure load directly. In the latter, it defines the basis upon which the failure load must be computed.

1.2 Summary of Theoretical Studies

Previous theoretical studies into the behavior of mechanically fastened joints have been reviewed extensively in Refs. 6 and 7. A summary will be included here along with an extension into more recent studies.
The early theoretical studies of mechanically fastened joints considered behavior only in the elastic range. These studies showed that the end fasteners carried the greatest proportion of the load and that adding more fasteners to a joint was of decreasing benefit.

The first extension of these studies into the inelastic range was made by Vogt who used a bi-linear relationship to represent the combined load - deformation behavior of the fastener and hole. Francis laid the basis for all subsequent work to date with his study on riveted aluminum joints. He showed that the distribution of load among fasteners was unequal in both the elastic and inelastic ranges of joint behavior. A semi-graphical procedure, based on consideration of equilibrium and compatibility conditions, was presented for predicting this load behavior.

The work of Francis was extended by Rumpf for use in analyzing bearing-type joints of A7 steel fastened by A325 bolts. The theoretical work was confirmed by tests.

Although the method used by Francis and by Rumpf gave excellent results, its semi-graphical technique meant that the amount of work required for the solution of any but the simplest joints was formidable. Fisher overcame this difficulty by the
introduction of mathematical models which established the load-deformation relationships throughout the complete load range of the components in the joint. With these models available, the iterative type of solution became a logical candidate for the digital computer. Fisher's contribution meant that the variation of parameters, such as fastener pitch and plate proportions, could be examined with relative ease. His theoretical studies were also confirmed by tests and excellent agreement between the two was obtained. Although this work covered only the behavior of A325 bolts in A7, A36, and A440 steels, it has since been extended to cover A490 bolts in A440 steel as well.  

1.3 Summary of Experimental Studies

Although a considerable number of tests have been performed on large, bolted plate splices, the only ones reported to date involving constructional alloy steels are those that have been carried out in conjunction with the present study. These tests were done in two series, a pilot study and a program involving large joints.

The pilot study was completed in 1964. It used specimens with two lines each of four bolts. Six joints used 1-1/8 in. diameter A325 bolts, and four used 1 in. diameter A490 bolts. In all cases, a total thickness of 4 in. of A514 plate was gripped.
The second test series, that on large joints, was completed in 1966. Here, A490 bolts of various grips were used to fasten A514 steel. Seven joints used 7/8 in. diameter bolts and one used 1-1/8 in. diameter bolts. All fasteners were arranged in a single line and all joints used a fastener pitch of 3.5 inches. Two joints each of seven, 13, 17, and 25 fasteners in line were tested. Although the latter are among the largest bolted or riveted joints that have ever been tested, joints of this or even greater length are frequently encountered in the construction of large bridges.15

1.4 Objectives of the Study

As will be shown later, the distribution of load among the fasteners in A514 steel joints cannot be obtained by a simple extension of the present theory. The characteristics of constructional alloy steel are significantly different from those of previously investigated grades of steel. The first objective, then, is to obtain a general theoretical solution for the distribution of load among fasteners in mechanically fastened constructional alloy steel tension splices.

The second, and perhaps the major area of the study, is to explore the need for new design criteria when dealing with constructional alloy steel joints. The current procedure used
to design riveted and bolted connections is based on their ultimate strength. The allowable stresses in the fasteners have been established by applying a factor of safety to their ultimate strength in joints. The connected material, that is, the member itself, is designed according to an allowable stress based on yielding.

The design philosophy set forth for metallic tension members considers that yielding should take place through the gross cross-section of the member before failure occurs in the connections. Principally because of the low ultimate to yield stress ratio of A514 steel, tension members of this material may fracture at their net section before the yield stress is reached through the gross section. The resulting lack of deformation which such a member would have is undesirable. The possibility of the occurrence of this condition and subsequent need for new design criteria will be examined.
2. THEORETICAL SOLUTION FOR THE LOAD DISTRIBUTION AMONG FASTENERS IN A514 STEEL JOINTS

2.1 Characteristics of Constructional Alloy Steel

Steels of the grade ASTM A514 were first introduced in 1952 when United States Steel Corporation introduced its proprietary steel "T-1". Other manufacturers subsequently either were licensed to produce this steel or produced their own proprietary steels of similar characteristics. In 1964, most of these steels were covered by ASTM Specification A514. The mechanical properties of A514 steel are summarized in Table 1.

The high yield strength alloy steels were developed to meet the need for a constructional steel, primarily in plate form, which had a yield strength in the order of 90,000 psi, good low temperature toughness and good weldability. The attainment of these qualities is largely a reflection of two factors; a micro-structure of tempered martensite and a low carbon content.

Tempered martensite is characteristically tough and this toughness is most pronounced at low levels of carbon. Furthermore, this micro-structure permits the attainment of the very
high strength desired at this low carbon level. The principal alloying elements employed are manganese, molybdenum, boron, chromium, nickel, and vanadium, titanium, or zirconium. The latter is included to maintain the high yield strength characteristic of the material in the face of tempering at high temperature.

The fundamental difference in behavior between A514 and other grades of structural steel can be seen by examining Fig. 2. The curves shown are indicative of the minimum stress-strain patterns which may be expected from actual testing of specimens. These curves show that constructional alloy steel is characterized by its very low ultimate vs. yield stress ratio. This can be as low as about 1.07 while the ratio for A440 and A36 steels is about 1.46 and 1.67, respectively. As already noted, this can be a significant feature in the behavior of constructional alloy steel members. In addition, this steel, like most other alloy steels, does not exhibit any well-defined yield point.

Examination of the stress-strain curves also shows that while the total energy absorbed by each of the various grades is not greatly different, the elongation of a structural carbon steel is about twice that of A514. The lesser amount of ductility of A514 and its low ultimate to yield stress ratio
may be of concern. A certain amount of engineering opinion exists that believes there should be as large a spread as possible between yield and ultimate.\(^\text{17}\) If this opinion is well-founded, there is a paradox in the fact that constructional alloy steels have met with wide acceptance and been successfully used for some fifteen years.

Part of the answer to this paradox lies in the applications for which A514 has been used over this period. It is quite clear that the primary use for which A514 was intended was in the construction of pressure vessels.\(^\text{18}\) The important criterion here is the amount of energy absorbed and a large amount of discussion is given to this point in the early literature.\(^\text{17}\)

The problems with respect to structural engineering are somewhat different, however. It will be shown later that member ductility is considered to be of considerable importance in tension members. It will also be shown that for a wide range of practical cases, A514 steel members may not provide sufficient ductility if designed according to current practice. This topic is covered in detail in Chapter 5.
2.2 Formulation of the Equilibrium and Compatibility Equations

The formulation of the equilibrium and compatibility equations as presented herein is not an original part of this dissertation. The theoretical work of Francis,\(^\text{13}\) which formed the basis both for Rumpf’s work\(^\text{6}\) and Fisher’s presentation,\(^\text{7}\) is also used as the basis here. Since mathematical models substantially different from those proposed by Fisher will be presented, however, it is necessary to review this formulation again.

A physical interpretation for a mathematical model is useful in obtaining a clear understanding of the latter. Reference will again be made to the ten-bolt specimen shown in Fig. 1. Although numerous other cases are possible, the determination of the ultimate load of such a joint may be examined for purposes of a qualitative discussion.

Presuming that the proportions are such as to ensure that the joint is in the fastener failure region, the ultimate load will be defined by the failure in shear of one or more fasteners. As was described in Art. 1.1, an end fastener, such as "1" in Fig. 1, will be critical and, accordingly, the load in this fastener can be set at its ultimate value as obtained from a shear test.\(^\text{19}\) A trial value of the ultimate load of the joint is chosen. Now, it is known that the load in plate B
between lines 1 and 2 must be the total assumed load less the load in fastener 1. If a mathematical model which describes the load - deformation behavior of this plate, that is, a plate with holes, is available, the strain in plate B between lines 1 and 2 can be computed from the known load.

The load in plate A between lines 1 and 2 can now be computed from a consideration of equilibrium at this section. Again using the plate - with - holes model, the strain in plate A between lines 1 and 2 can then be calculated.

A consideration of the shear strain in fastener 1 and the strains in plates A and B between lines 1 and 2 gives the shear strain to which fastener 2 must be subjected. If a mathematical model relating fastener deformation to fastener load is available, the load in fastener 2 can be obtained from this computed shearing strain.

This procedure is repeated from fastener to fastener until all have been considered. The sum of all the fastener loads thus computed can then be compared to the assumed value of the ultimate load of the joint. If these are equal or sufficiently close in value, the solution has been obtained. If not, a new trial value of the ultimate load of the joint is picked and the process repeated.
The qualitative discussion above is intended to point out three things:

1. Both equilibrium and compatibility must be satisfied.

2. Two mathematical models are needed, namely;
   a. A model describing the load - deformation behavior of a plate - with - holes.
   b. A model describing the load - deformation behavior of a single fastener acting in shear.

3. Implicit in the discussion is the fact that normally many trials will be necessary before satisfactory convergence is reached between assumed and computed values. Considering also that many intermediate steps are required in each trial, it is obvious that the use of an electronic digital computer is a necessity if joints of any practical size are to be analyzed.

The mathematical formulation of the compatibility and equilibrium equations can now be presented. Although differing slightly, this formulation is also based on the work of Rumpf⁶
and of Fisher. It is included here again for completeness.

It is necessary to consider only a single line of bolts, as shown in Fig. 3, when treating the analysis of double shear, symmetrical butt splices. Previous research has shown that the behavior of multiple, identical lines of bolts is directly proportional to the number of such lines present. The behavior of multiple, staggered lines is not within the scope of this dissertation.

The solution of the determination of the individual fastener forces in such a joint is accomplished by means of meeting the requirements of equilibrium, compatibility, and initial value conditions.

The requirements of equilibrium can be visualized with the aid of Fig. 4. The load in the main plate between fasteners $i$ and $i+1$ is equal to the total load in the plate, $P$, minus the sum of the fastener loads preceding the portion of the joint under consideration. The latter will be designated as $\Sigma R_i$. In algebraic form then,

$$ P_{i,i+1} = P - \sum_{i=1}^{i} R_i $$

(1)
From equilibrium at the section under consideration, the load in the lap plates between fasteners $i$ and $i+1$, $Q_{i,i+1}$, must be

$$Q_{i,i+1} = \sum_{i=1}^{i} R_i \quad (2)$$

The fulfillment of the condition of compatibility between these same two fasteners can now be formulated. As was pointed out in Art. 1.1, the total load in the joint is assumed to be transmitted by bearing of the plates against the fasteners once major slip has occurred. The compatibility conditions, then, will assume that the joint has either already slipped into bearing or was erected in bearing.

The distance between the fasteners $i$ and $i+1$ in a joint with no load is equal to the fastener pitch, $p$. Upon application of load, the main plate will elongate an amount $e_{i,i+1}$ and the lap plate on amount $e'_{i,i+1}$ in this pitch length.

In addition, fastener $i$ will suffer deformation of an amount $\Delta_i$ and fastener $i+1$ deformation of an amount $\Delta_{i+1}$. These quantities include deformations due to shear, bending, and bearing of the fastener and deformations due to bearing of the plate adjacent to the fastener.
The compatibility of deformations between these two fasteners, from Fig. 5, gives

\[ \Delta_i + p + e_{i,i+1} = \Delta_{i+1} + p + e_{i,i+1} \]  

(3)

Or, simplifying,

\[ \Delta_i + e_i = \Delta_{i+1} + e_{i,i+1} \]  

(4)

Each term in Eq. 4 is expressible in terms of a corresponding load, either that in a fastener or that in a component plate between adjacent fasteners. Using \( f(R_i) \) and \( f(R_{i+1}) \) as fastener deformations corresponding to \( \Delta_i \) and \( \Delta_{i+1} \), and \( g(Q_i,i+1) \) and \( g(P_i,i+1) \) as plate deformations corresponding to \( e_{i,i+1} \) and \( e_{i,i+1} \), Eq. 4 can be written

\[ f(R_i) + g(Q_{i,i+1}) = f(R_{i+1}) + g(P_{i,i+1}) \]  

(5)

Substituting the expressions obtained in Eqs. 1 and 2, the compatibility condition can be finally expressed as

\[ f(R_i) + g(\sum_{i=1}^{i} R_i) = f(R_{i+1}) + g(P - \sum_{i=1}^{i} R_i) \]  

(6)
In this form of the equation, each deformation is expressed in terms of a function of the unknown fastener loads. Equation 6 can be written \( n-1 \) times for a joint containing \( n \) connectors. The remaining equation necessary for the solution is the equilibrium equation

\[
P - \sum_{i=1}^{n} R_i = 0
\]  

(7)

As can be seen from Eq. 6 and as was previously developed when the qualitative description of this problem was presented, two mathematical models are required. One describes the load - deformation response of a single fastener acting in shear (the "f"-functions in Eq. 6). The other describes the load - deformation response of a plate - with - holes specimen acting under a tensile load (the "g"-functions in Eq. 6).

### 2.3 Shear - Deformation Relationship for Mechanical Fasteners

The analytical expression developed by Fisher\(^{22}\) relating the load and deformation of a fastener subjected to a tension-induced shear was based on the following assumptions:

1. For small values of deformation, the load - deformation relationship is approximately linear.
2. As the deformation approaches its ultimate value, the bolt load increases at a decreasing rate.

3. The deformation includes those components due to shear, bending, and bearing of the fastener and those due to bearing of the plate adjacent to the fastener.

In addition, the model had to meet the boundary condition that at zero load the deformation also had to be zero.

On the basis of these requirements, the expression chosen was

\[ R = \tau \left[ 1 - e^{-\mu \Delta} \right] \lambda \]  \hspace{1cm} (8)

where \( R \) is the fastener load, \( \Delta \) is the deformation, \( e \) is the base of natural logarithms, and \( \tau, \mu, \) and \( \lambda \) are regression coefficients. The range of Eq. 8 is \( 0 \leq R \leq R_{\text{ult}} \) and \( 0 \leq \Delta \leq \Delta_{\text{ult}} \).

The regression coefficients \( \tau, \mu, \) and \( \lambda \) have been determined for a wide range of cases involving A325 or A490 bolts in A7, A36, or A440 steel plate.\(^{19}\) However, this source reports only one series (three tests) involving A490 bolts in A514 plate.
The variables known to influence the load-deformation relationship are (1) bolt diameter, (2) bolt grade, (3) grade of steel, and (4) thickness of plates. In addition, the type of loading (tension or compression) that is used to induce the shear is known to be important. Although it is the less simple test of the two, the tension-type test best represents the case in a tension splice.\textsuperscript{19} Since the model as expressed by Eq. 8 had already been successfully used, its use again in this study seemed indicated.

The analysis used by Fisher\textsuperscript{22} showed that the coefficient $\tau$ must be equal to $R_{\text{ult}}$. The other coefficients are best found by trial. In this form, Eq. 8 becomes

$$R = R_{\text{ult}} \left[ 1 - e^{-\mu \Delta} \right]^\lambda$$  \hspace{1cm} (8a)

Three different lots of bolts were calibrated in connection with the present study. The basic strength properties of these bolts and of those in the tests previously reported are shown in Table 2, along with the data relating to the shear study of these bolts.

The success of Eq. 8 and the empirical parameters chosen in fitting the test data can be measured by means of Figs. 6 through 9. Bolt lots B, C, and D (Figs. 6, 7, and 8, respectively)
show very little scatter when individual tests are compared within a given lot. The use of \( \mu = 40 \) and \( \lambda = 0.95 \) along with the appropriate value of \( R_{\text{ult}} \) in Eq. 8 gives excellent agreement between the resulting curve and the corresponding test data for lots B and D. These two lots were both 7/8 in. diameter bolts, lot B being bolts of 4 in. grip and lot D, 8 in. grip.

Figure 7 shows that the amount of scatter in the tests of 1-1/8 in. diameter bolts (lot C) is again very small and that the use of the coefficients \( R_{\text{ult}} = 192 \) kips, \( \mu = 40 \), and \( \lambda = 1.50 \) gives a curve in excellent agreement with these data.

It should be noted in Figs. 6 through 9 that the test data show the value of the fastener load reaching an ultimate value and then dropping off slightly. Equation 8 approaches \( R_{\text{ult}} \) asymptotically and is not expected to cover this drop-off portion of the curve. In addition, \( \Lambda_{\text{ult}} \) is taken as the value of the deformation corresponding to \( R_{\text{ult}} \). Although deformations greater than this value are obtained when the calibration is performed in a testing machine, they would not occur if a dead load test were used.

The fact that the coefficient \( \lambda \) is the only variable in fitting Eq. 8 to the test data (\( R_{\text{ult}} \) being set at the appropriate value in each case) would lead to the hope that these
coefficients could perhaps be determined without the need of the calibration test.

The trend observed in the coefficients used for lots B, C, and D does not extend to lot JJ, however, in spite of the fact that all lots had similar ultimate strength properties (see Table 2). In Fig. 9, the test data for lot JJ is shown along with the theoretical curve obtained by using Fisher's suggested values of $\mu = 28$ and $\lambda = 0.35$. Although there is considerable scatter in the test results, this theoretical curve does give a good approximation. A large range in values of $\lambda$ has been tried in combination with a value of $\mu = 40$. None give as good a fit as the curve plotted in Fig. 9.

It would be desirable to be able to predict the load-deformation response of a fastener loaded in shear completely independently of any physical calibration test. The lack of such an approach, however, does not present any serious problem other than the fact that the calibration test must be performed. Neither Fisher, who reported the results of shear calibration tests of 23 lots of bolts and rivets (probably about 70 individual tests), nor Wallaert and Fisher, who investigated the parameters involved in considerable detail, were able to present such a solution. The development of this theoretical approach
is not within the scope of this dissertation and further consideration will not be given to it.

2.4 Tensile Stress - Strain Relationship for Plate Materials

(1) Introduction

The need for a mathematical model describing the tensile behavior of a plate - with - holes coupon has been outlined in Art. 2.2. In order to develop such a model, the stress - strain relationship must first be established experimentally.

A "standard" plate - with - holes coupon is shown diagrammatically in Fig. 10. Although the coupon should be prepared from the same material as used to fabricate the test connections, it will be shown later that, within practical limits, there appears to be no need to duplicate the geometry of the plate in the test joint.

The basic objection to any of the previously developed models describing the stress - strain relationship of a plain plate coupon for use with the plate - with - holes specimen is simply that none fit the test data. The introduction of holes into a plate affects the tensile behavior of such a specimen in two principal ways. For practical values of specimen geometry, the absence of any distinct yield plateau is noted in the plate - with - holes coupon, whether or not a standard tensile coupon of
the same material shows this behavior. Second, in a fashion similar to the introduction of a circumferential groove around a bar,\textsuperscript{24} the holes restrict necking in the section and thus give the coupon a higher ultimate stress than would be obtained in the same plate without holes.

Fisher developed a model which he found to be generally applicable for the grades of steel that he investigated.\textsuperscript{22} He used the simple relationship utilizing Hooke's Law in the elastic range, that is,

\[ \sigma = E \varepsilon \]  

whereas for values of strain greater than the yield strain he used

\[ \sigma = \sigma_y + (\sigma_u - \sigma_y) \left[ \frac{1 - e^{-\frac{\sigma_u - \sigma_y}{q} \frac{d}{p}}} {1 - e^{-\frac{\sigma_u - \sigma_y}{q} \frac{d}{p}}} \right]^{3/2} \]  

where
- \( E \) = modulus of elasticity of the plate material
- \( \varepsilon \) = unit strain
- \( p \) = pitch (distance center-to-center of holes)
- \( e \) = total deformation in pitch
- \( \sigma_u \) = ultimate stress of the plate - with - holes coupon
- \( \sigma_y \) = yield stress of standard tensile coupon
- \( e \) = base of natural logarithms
\[ d = \text{hole diameter} \]
\[ g = \text{width of specimen} \]

This relationship was based upon dynamic stress-strain measurements made on the plate-with-holes coupon. In addition, since the yield point of such a coupon is not a well-defined point, it was found convenient, as a reasonable approximation, to use the static yield point of a standard tensile coupon of the same material.

Fisher's model (Eq. 10) was found to be not suitable for use with A514 steel specimens. Figure 11 shows an A440 steel specimen containing holes being tested. It can be seen that the yield bands extend outward from the holes at approximately 45° to the direction of the applied load. It is apparent that a large portion of the volume of the plate between the holes is in the inelastic range. It is also apparent that the extent to which the yield bands intersect will be reflected by the plate geometry.

Figure 12 shows a similar test being performed on an A514 steel specimen. Because of the high yield to ultimate ratio, the gross cross-section of the specimen will not reach yield for the practical range of plate geometry. As the test shows, the yield bands extend from the edges of the holes to the edges of
the plate at approximately right angles to the line of the load. This behavior is observed up to the ultimate load of the plate. Thus, even when the response of the specimen as a whole indicates inelastic behavior, there are essentially two different regions within the pitch length -- an inelastic region in the vicinity of the holes and an elastic region between the holes. Any relationship suitable for use with A514 steel must account for these physically observable differences.

(2) Development of the Mathematical Model

A total of twelve plate calibration coupons were prepared for use in developing the new model. All came from the same rolling of steel but covered a wide range of geometry. Details of these specimens are shown in Table 3.

The major variables influencing the behavior of the plate - with - holes coupon are:

1. Hole diameter
2. Yield strength of the coupon
3. Ultimate strength of the coupon
4. Speed of testing of the coupon

It should be noted that within the practical range of geometry, the spacing of the holes and the specimen width are no longer
considered to be significant. From Table 3 it can be seen that a sufficiently broad range of specimen geometry was included in order that this assumption might be verified.

As was pointed out in Art. 2.4.1, Ref. 22 related the yield strength of the plate - with - holes coupon to the static yield of the material as obtained from a standard tensile coupon. This was convenient since the plate - with - holes specimen does not exhibit a well-defined yield point whereas, for the grades of steel investigated in Ref. 22, the standard tensile coupon did. This approach offers no advantage when dealing with A514 steel, however, as neither the plate - with - holes nor the standard tensile coupon shows a definite yield point. Accordingly, the proportional limit of the plate - with - holes coupon will be defined as the load at first departure from linearity on its load - elongation diagram.

The speed of testing of the coupon is known to be important. Primarily, the question is whether to use static load and elongation readings or to use those values obtained while running the crosshead of the testing machine at a slow rate of travel. Since the dynamic test corresponds more closely to the behavior of a joint under test, this is the approach that was used here. Further discussion of this point is available in Ref. 22.
In line with the foregoing discussion, the following assumptions were made in the development of the analytical relationship desired:

1. The proportional limit is defined by the point where the stress - strain curve of the plate - with - holes coupon becomes non-linear.

2. When strain is less than the proportional limit strain as defined above, stress is proportional to strain.

3. In the inelastic range, dynamic stress and strain values are used to describe the behavior of the specimen.

4. Stress increases at a decreasing rate with increasing values of strain in the inelastic range of the specimen.

5. Computed stresses are based upon the initial net area of the specimen.

6. In the inelastic range of the coupon, as defined in (1), the material is assumed to remain elastic between the
holes and to be inelastic in a region equal to the diameter of the holes.

(3) General Stress - Strain Relationship

In the elastic range, the relationship between stress and strain can be expressed as

$$\sigma = E \varepsilon \quad (11)$$

For the plate - with - holes coupon, it is necessary to consider this expression in the light of the cross-sectional area involved, that is, either the net area or the gross area. If Eq. 11 is solved for the strain and thence the total elongation in the pitch length $p$ which contains a hole of diameter $d$, the following expression results:

$$e = \frac{P}{A_n E} (p-d) + \frac{P}{A_n E} (d) \quad (12)$$

Of course, the stress distribution across the width of a plate containing a hole is not uniform, as implied by Eq. 12. If this expression is to be used in determining the elongations of the plate - with - holes coupon within the elastic range, its performance in meeting the boundary conditions must be evaluated. These boundary conditions are that $e = 0$ when $P = 0$ and that $e = e_y$ when $P = P_y$. The values of $P_y$ and $e_y$ are those values of
load and elongation at the departure of the load - elongation curve from linearity.

By inspection, Eq. 12 fulfills the first boundary condition. The degree to which this expression meets the second requirement is examined in Art. 3.4. It will be shown that there is satisfactory agreement between the observed values of $e_y$ and corresponding values computed using Eq. 12. This equation, then, will be used to express the load - deformation response of the plate - with - holes coupon over the range $0 \leq P \leq P_y$.

The expression chosen for the inelastic portion of the curve also had to fulfill certain boundary conditions. These are that $\sigma = \sigma_y$ when $\varepsilon = \varepsilon_y$ and that $\sigma = \sigma_u$ when $\varepsilon = \varepsilon_u$. In addition, the physical behavior of the plate - with - holes coupon as hypothesized in Art. 2.4.1 means that in this region the total elongation in a pitch length is composed of two distinct portions. One is the plastic deformation, assumed to be confined to a region equal to the depth of the hole, and the other is the elastic deformation still taking place in the region between the holes. Although for the usual hole spacings the latter could be expected to be only a very small proportion of the total inelastic elongation of the coupon, it will be included here for completeness. No restrictions need then be placed on the model with regard to
any maximum governing hole spacing. Accordingly, the total elongation in a pitch length at any point in the inelastic range of the plate - with - holes coupon is assumed to be given by

\[ e = \varepsilon_e (p - d) + \varepsilon_y d + \varepsilon_p d \]  \hspace{1cm} (13)

where \( \varepsilon_y \) is the limiting elastic strain achieved through the net section, given by

\[ \varepsilon_y = \frac{P}{A_y E} \]  \hspace{1cm} (14)

and, as just described, \( \varepsilon_e \) is the elastic strain in the gross section, given by

\[ \varepsilon_e = \frac{P}{A_y E} \]  \hspace{1cm} (15)

and \( \varepsilon_p \) is the plastic portion of the strain. Again, the plastic strain is assumed to be concentrated in a region equal to the depth of the hole. Its formulation is described below.

Breaking down the total elongation into its components, as was done in Eq. 13, will be convenient later when formulating the necessary equilibrium equations. However, in setting up an expression relating the plastic strain to the applied load, it
was found more convenient to work first with stresses. A number of expressions were examined. The one chosen was

\[
\sigma = E \varepsilon_y + K \left[ 1 - \varepsilon \left( \frac{\epsilon^p}{M + N \epsilon^y_p} \right)^\alpha \right] \tag{16}
\]

where \( \sigma \) is the stress on the net section and \( K, M, N, \alpha, \beta, \) and \( \gamma \) are empirical parameters. The other terms in Eq. 16 have previously been defined.

For small values of \( \epsilon_p \), the term

\[
-\alpha \left( \frac{\epsilon^p}{M + N \epsilon^y_p} \right)^\alpha
\]

goes to unity and Eq. 16 reduces to \( \sigma = E \varepsilon_y = \sigma_y \). Thus, the requirements of the first boundary condition are met. Further, the parameters \( \alpha, \beta, \gamma, M, \) and \( N \) were chosen so that as \( \epsilon_p \) goes to \( \epsilon_u \), its ultimate value, the term

\[
-\alpha \left( \frac{\epsilon^p}{M + N \epsilon^y_p} \right)^\alpha
\]
goes to zero. In this case, Eq. 16 becomes

\[ \sigma = E \varepsilon_y + K = \sigma_u \]  \hspace{1cm} (17)

Solving for the unknown parameter, \( K = \sigma_u - E \varepsilon_y \), that is,

\[ K = \sigma_u - \sigma_y \]  \hspace{1cm} (18)

The value of \( K \) expressed by Eq. 18 automatically ensures that Eq. 16 will satisfy the second boundary condition.

The remaining parameters were obtained by trial on the basis of the twelve test specimens previously described. The values finally chosen were \( \alpha = \sigma_u - \sigma_y \), \( \beta = 0.4 \), \( \gamma = 2.15 \), \( M = 5.50 \), and \( N = 160 \). The dimensions of \( K \) are ksi, while \( \alpha \), \( \beta \), \( \gamma \), \( M \), and \( N \) are all dimensionless. The final form of the expression, using the substitution \( E \varepsilon_y = \sigma_y \), becomes

\[ \sigma = \sigma_y + (\sigma_u - \sigma_y) \left[ 1 - e^{-\frac{0.4 \left( \varepsilon_p - 2.15 \right)}{5.50 - 160 \varepsilon_p}} \right] \]  \hspace{1cm} (19)

It should be noted again that all the stress terms in Eq. 19 are based on the net area of the coupon. The evaluation
of the parameters in the expression was made when these stresses were expressed in units of ksi. Equation 19 is valid for the stress range $\sigma_y \leq \sigma \leq \sigma_u$.

This section has formulated the stress-strain relationship for a plate-with-holes coupon for both the elastic range (Eq. 11) and the inelastic range (Eq. 19). The expressions for the total elongation in a pitch length of such a coupon have also been developed. Equation 12 is applicable to the elastic range and Eq. 13 is valid thereafter.

Within the assumptions that have been made, the development is general. It is subject to one limitation, however. Implicit in the development of Eq. 13 is the restriction that the stress in the gross cross-section within the joint length does not exceed the yield stress. It will be shown in Chapter 5 that the goal sought in tension member behavior is that the gross section of the member should yield at or before joint failure. It would take an unusual combination of joint geometry and fastener spacing to produce yielding in the gross cross-section within the joint before joint failure, however. While the elongation formulated in Eq. 13 does not account for the case where $P > P_y$ (gross section), it provides a lower bound and practical limit to the solution of the problem.
(4) Comparison of Experimental Results and Theory

The suitability of the use of Eqs. 11 and 19 in describing the stress-strain behavior of A514 steel plates with holes can be evaluated on the basis of Fig. 13. Here, the computed relationships are shown for ten of the twelve specimens described in Table 3. Shown also on each plot are the values recorded during the test of that specimen.

In the elastic portion of each of the plots, the agreement between the experimental values and the curve obtained by use of Eq. 11 is excellent. The experimental values of load and elongation at the proportional limit were obtained from enlarged scale plots of the load-elongation curves for the individual specimens. The smoothness of the theoretical curves in relation to the test data shown in Fig. 13 indicates that this upper limit of Eq. 11 (or, its more specific form, Eq. 12) is satisfactorily obtained.

The use of Eq. 19 to describe the inelastic behavior of the plate-with-holes specimen also provides a curve which is in close agreement with the experimental results. The specimens tested provided a reasonable range of geometric properties. On the basis of the ratio of net area to gross area ($A_n/A_g$), they provided a spread of from 0.86 to 0.71. The upper limit represents, for example, a 1 in. diameter hole in a plate 7 in. wide, while the lower limit represents a 1 in. diameter hole in
a plate 3.46 in. wide. As can be seen by comparing Figs. 13(b) and 13(f), the model worked equally well at both extremes.

Two different pitches were used, 3.50 in. and 4.00 inches. In agreement with the assumed elastic - inelastic division described in Art. 2.4.1, the difference in pitches was found to have no influence upon specimen behavior. For example, specimens JS-7 and JS-8 had nearly identical $A/A_g$ values but different pitches. Figures 13(g) and 13(h) show that there was no substantial difference in the degree of fit of these two curves.

Specimens JS-1 through JS-8 were made of 1 in. thick plate. The remaining four coupons were $1/2$ in. thick, milled from the same plate as the first eight. Again, the use of Eq. 19 to describe the inelastic behavior of these specimens gave good results. The only effect of the thinner plate was to give higher ultimate and yield stresses. The average ultimate stress of specimens JS-1 through JS-8 was 118.2 ksi while that of JS-9 through JS-12 was 122.7 ksi. The mean ultimate stress of all twelve coupons was 119.8 ksi. Significantly, the value of $(\sigma_u - \sigma_y)$ was reasonably constant for all specimens, irrespective of plate geometry, hole pitch, or plate thickness. The mean value of $(\sigma_u - \sigma_y)$ for all twelve specimens was 24.8 ksi with a standard deviation of 0.7 ksi.

As has been pointed out, plate - with - holes coupons may be expected to reach a higher net section ultimate stress
than standard tensile coupons of the same material. The mean tensile strength of five standard bar coupons tested was 111.9 ksi. The mean tensile strength of the plate - with - holes coupons was, therefore, 107% of the mean tensile strength of the standard bar coupons. It can also be noted that the mean proportional limit stress of the plate - with - holes coupons was 93% of the mean yield stress of the standard bar coupons.

In order that the curves shown in Figs. 13(a) through 13(j) could be compared to one another, they were plotted on stress vs. strain axes. The strain values were obtained by dividing the elongation as computed by Eqs. 12 or 13 by the appropriate pitch length. Stress was computed using the net cross-sectional area of the coupon.

There is no substantial difference in the degree of fit of these curves when Eqs. 11, 12, 13, and 19 are used directly. Figure 13(k) shows actual and theoretical values for specimen JS-4 on total load vs. total elongation axes. Comparison with Fig. 13(d) shows that substantially the same degree of fit is obtained on either basis.

Discrepancies between observed values and the curve computed using Eqs. 11 and 19 are completely random. The degree of any such discrepancy is small and these equations in all respects provide a satisfactory means of expressing the relationship between stress and strain for plate - with - holes coupons made of A514 steel.
2.5 General Solution of the Equilibrium and Compatibility Equations

(1) Assumptions

Using the load - deformation relationships developed in Art. 2.3 and 2.4, the solution of the equilibrium and compatibility equations can now be formulated. In this development, the following assumptions have been made:

1. Load transfer is by bearing of the component plates against the fasteners, that is, frictional forces are neglected.

2. The behavior of the elements of the joint can be described by the mathematical models developed in Art. 2.3 and 2.4.

3. All fasteners in the connection are of the same diameter and all plates are of constant thickness.

(2) Load - Deformation Relationships

Equation 6 was an expression of compatibility in which each deformation was expressed in terms of a function of the unknown fastener loads. From the mathematical models developed in Art. 2.3 and 2.4, these functions can now be expressed explicitly.
The first term in Eq. 6

$$\Delta_i = f \left[ \frac{R_i}{R_{\text{ult}}} \right]$$  \hspace{1cm} (20)

becomes, from Eq. 8a,

$$\Delta_i = -\frac{1}{\mu} \ln \left[ 1 - \left( \frac{R_i}{R_{\text{ult}}} \right)^{\frac{1}{\lambda}} \right]$$ \hspace{1cm} (21)

for $0 \leq R_i \leq R_{\text{ult}}$.

Similarly, the term

$$\Delta_{i+1} = f \left[ \frac{R_{i+1}}{R_{\text{ult}}} \right]$$

in Eq. 6 becomes

$$\Delta_{i+1} = -\frac{1}{\mu} \ln \left[ 1 - \left( \frac{R_{i+1}}{R_{\text{ult}}} \right)^{\frac{1}{\lambda}} \right]$$  \hspace{1cm} (22)

for $0 \leq R_{i+1} \leq R_{\text{ult}}$.

The plate deformation terms can be similarly evaluated by using Eq. 12 or 13. Using the notation of Eq. 5, the elongation in the main plate
becomes, in the elastic range of the plate,

\[ e_{i, i+1} = g(P_{i, i+1}) \]  \hspace{1cm} (23)

\[ e_{i, i+1} = \frac{P_{i, i+1}}{A_{g}E} (p-d) + \frac{P_{i, i+1}}{A_{n}E} (d) \]  \hspace{1cm} (24)

Equation 24 is valid for the range \( 0 \leq P_{i, i+1} \leq \sigma_{y} A_{n} \).

For values of \( P_{i, i+1} \geq \sigma_{y} A_{n} \) and \( P_{i, i+1} \leq \sigma_{y} A_{g} \), Eq. 23 becomes

\[ e_{i, i+1} = \frac{P_{y}}{A_{n}E} \cdot d + \frac{P}{A_{g}E} (p-d) + e_{p} \cdot d \]  \hspace{1cm} (25)

where, from Eq. 19, \( e_{p} \) is given by

\[ \frac{e_{p}}{5.50-160} = \frac{2.15}{\varepsilon_{p}} = \frac{0.4}{\varepsilon_{p}} = \frac{1 - \frac{(P_{i, i+1} - \sigma_{y} A_{n})}{(\sigma_{u} - \sigma_{y} A_{n})}}{\ln \left( 1 - \frac{(P_{i, i+1} - \sigma_{y} A_{n})}{(\sigma_{u} - \sigma_{y} A_{n})} \right)} \]  \hspace{1cm} (26)

Equation 26 must be solved by iteration. Both Eq. 25 and Eq. 26 are valid for the range \( \sigma_{y} A_{n} \leq P_{i, i+1} \leq \sigma_{u} A_{n} \).
In a similar fashion, the elongation in the lap plates,

\[ e_i', i+1 = g(Q_i', i+1) \],

will be

\[ e_i', i+1 = \frac{Q_i', i+1}{A_g E} (p-d) + \frac{Q_i', i+1}{A_n E} (d) \]  

(27)

in the elastic range, \( 0 < Q_i, i+1 < \sigma_{y}A_i' \). In the inelastic range it will be given by

\[ e_i', i+1 = \frac{P_y}{A_n E} \cdot d + \frac{P}{A_E} (p-d) + \epsilon_p' \cdot d \]  

(28)

where \( \sigma_{y}A_i' < Q_i, i+1 < \sigma_n' \) and \( Q_i, i+1 < \sigma_{y}A_i' \).

Again, \( \epsilon_p' \) must be solved by iteration from the expression

\[ \frac{\epsilon_p'0.4}{5.50-160 \epsilon_p'2.15} = \frac{1}{\left( \sigma_n' - \sigma_{y} \right)} \ln \left[ 1 - \frac{(Q_i, i+1 - \sigma_{y}A_i')}{(\sigma_n' - \sigma_{y}) A_n'} \right] \]  

(29)

By means of the relationships expressed in Eqs. 1 and 2, Eqs. 24, 25, 27, and 28 can be expressed in terms of the load on the joint and the bolt forces. The elongations in the main plate are then
\[ e_{i',i+1} = \frac{(P - \Sigma R_i)}{A_E} (p-d) + \frac{(P - \Sigma R_i)}{A_n} (d) \] (30)

or

\[ e_{i',i+1} = \frac{P_y}{A_n} (d) + \frac{(P - \Sigma R_i)}{A_n} (d) + \epsilon_p \cdot d \] (31)

where

\[ \frac{\epsilon_p^{0.4}}{5.50-160 \epsilon_p^{2.15}} = -\frac{1}{(\sigma_u - \sigma_y)} \ln \left[ 1 - \frac{(P - \Sigma R_i - P_y)}{(\sigma_u - \sigma_y) A_n} \right] \] (32)

Similarly, in the lap plates,

\[ e_{i',i+1} = \frac{\Sigma R_i}{A_E} (p-d) + \frac{\Sigma R_i}{A_n} (d) \] (33)

or

\[ e_{i',i+1} = \frac{P_y}{A_n} + \frac{\Sigma R_i}{A_n} (d) + \epsilon_p \cdot d \] (34)
where

\[
\frac{e_p^{0.4}}{5.50-160 e_p^{2.15}} = \frac{1}{(\sigma_T - \sigma_y)} \ln \left[ 1 - \frac{\left( \Sigma R_i - P'_i \right)}{\left( \sigma_T - \sigma_y \right) A_n} \right]
\] (35)

When the expressions obtained in Eqs. 21, 22, 30 or 31, and 33 or 34 are substituted into Eq. 4, the form of Eq. 6 desired for the solution of the problem is obtained. It is applicable for any plate thickness, plate width, fastener pitch, fastener diameter, or fastener type. It is restricted to plates of ASTM grade A514, however.

(3) Solution of the Compatibility and Equilibrium Equations

If \( i \) is set equal to 1, then three of the four terms in Eq. 6 are functions of \( R_1 \). For an assumed value of \( R_1 \) and a trial value of \( P \), the remaining term \( R_2 \) can be obtained. With \( R_2 \) known, \( i \) is set equal to 2 and \( R_3 \) can be evaluated. In like fashion, each value of \( R_{i+1} \) can be obtained, all based on the assumed value of \( R_1 \) and trial value of \( P \). For any such assumed \( R_1 \), a correct solution will be obtained only if Eq. 7, the equilibrium equation, is also satisfied. If this expression is not satisfied, a new trial value of \( P \) is chosen and the process repeated.
The choice of the assumed value of $R_1$, the load in the end fastener, is straightforward. Generally, it is the ultimate load of the joint that is desired. Since this is defined as the load at which an end fastener fails, $R_1$ is set at the ultimate value of a fastener as obtained in the fastener calibration test. If the load distribution among the fasteners corresponding to some load less than ultimate is desired, $R_1$ is set at some value less than its ultimate load and a trial-and-error procedure again used.

The iterative type of solution indicated here, along with the fact that Eq. 32 and Eq. 35 also require an iterative type of solution, means that the use of an electronic digital computer is practically mandatory. The details of the programming of this type of problem are given in Ref. 7.
3. Examination of Important Parameters

3.1 Basis of the Examination

In examining the application of the theory developed in this dissertation to hypothetical joints, it is appropriate to base the approach on the use of plate and fasteners of minimum strength. A lower bound to the behavior of constructional alloy steel joints will thus result.

Studies on the shear strength of high strength bolts\textsuperscript{19} have shown that the minimum shear strength can be approximated by

\[ \tau_{\text{min}} = \frac{\sigma_{\text{min}}}{\sigma_{\text{ult}}} \tau_{\text{ult}} \]  

(36)

where \( \tau_{\text{ult}} \) is the double shear strength of a single fastener tested in plates subjected to a tensile load, \( \sigma_{\text{min}} \) is the minimum specified tensile strength of the bolt material and \( \sigma_{\text{ult}} \) its actual tensile strength. This same study also showed that the use of Eq. 36 does not depend upon the type of steel used in the jig.
The minimum ultimate shear strength obtained by these investigators for A325 bolts was 75.1 ksi, based on tests of 7/8 in. diameter bolts. Shear calibration tests of 1-1/8 in. diameter A325 fasteners were reported in connection with the pilot tests referred to in Art. 1.3. These gave a minimum ultimate shear strength of 65.0 ksi. The tensile strength requirements of A325 bolts vary with fastener diameter and this must be taken into account if a comparison is to be made. Reducing the results obtained in the pilot study by the ratio of required tensile strengths for the diameters involved, that minimum shear stress becomes 68.6 ksi. Extrapolation of the results to fastener diameters not specifically tested thus seems reasonable if account is taken of the differences in required tensile strengths. In this study, the principal diameter investigated is 1-1/8 inches. Bolts of this size represent perhaps a minimum practical diameter of A325 bolt when this type of fastener is used in A514 steel.

The minimum ultimate shear strength of A490 bolts obtained by the same investigators was 91.9 ksi. The three lots of A490 bolts used in the experimental portion of the present study had an ultimate shear strength of 91.5 ksi. Although only 7/8 in. and 1 in. diameter bolts were tested, all A490 bolts of diameters 1/2 through 2-1/2 in. have the same specified strength.
properties. The value chosen for use in this study was 91.5 ksi.

Table 4 summarizes the bolt parameters used in this article. They are based on the above discussion and on average values of μ, λ, and Δult as given in Art. 2.2 and 2.3.

The plate elements making up the joint were all assumed to be 1 in. plies. The parameters needed here for the analysis of hypothetical joints are the ultimate and yield stresses of a plate - with - holes coupon made of constructional alloy steel of minimum strength. The studies reported in Art. 2.4 showed that these parameters were, within a practical range, independent of geometry. Accordingly, the minimum values were obtained by multiplying the actual values by the ratio of observed to minimum specified tensile strength of the plate material. These have been computed as $\sigma_u = 121.3$ ksi and $\sigma_y = 97.5$ ksi.

3.2 Location of the Plate Failure - Fastener Failure Boundary

As was pointed out in Art. 1.1, the first step in any examination of the ultimate strength of a bolted joint must be to determine the mode of failure. The establishing of the plate failure - fastener failure boundary line is most conveniently done by a converging process. At a given joint length, the ultimate strengths of joints with decreasing values of $A_n/A_s$ are
computed. The process must start in the fastener failure region, that is, at a value of $A_n/A_s$ high enough to ensure this failure mode. With each calculation, the ultimate load so computed is compared to the ultimate load of the plates, as represented by $A_n \times \sigma_{ult}$ at that step. If the two values of load are equal, or within an acceptable limit, a point on the boundary has been obtained. This process is repeated for other joint lengths until the complete curve has been obtained for the desired range.

This searching process was used to obtain the plate failure - fastener failure boundary shown in Fig. 14. Plotted here as the average shear stress in the fasteners vs. joint length, the computations have been based on the use of 7/8 in. diameter A490 bolts of minimum strength connecting minimum strength A514 plate. The fastener pitch was taken as 3.5 inches. (The effects of different pitches and bolt diameters are examined later in this chapter.) Calculations show that differences in grip length have only a very small influence on the location of the boundary or on ultimate strength.

The dashed horizontal line extending across at a shear stress level of 91.5 ksi represents the "ideal" joint, that is, one in which all fasteners carry an equal load. This occurs, of course, only at $A_n/A_s = \infty$. Shown between this limiting line and
the other limit, the plate failure boundary, are joint strength curves for $A_n/A_s$ values of 1.00, 0.80, 0.70, and 0.60. The same boundary is plotted in Fig. 15 as a function of $A_n/A_s$ vs. joint length.

If A514 steel joints fastened by A490 bolts are designed according to current practice, it is unlikely that proportions will be such that failure will occur in the fasteners. For example, using an allowable stress value of 60 ksi for A514 plate in combination with the current allowable stress value for A490 bolts used in buildings,$^{28}$ an $A_n/A_s$ ratio of 0.53 results. As seen in Fig. 15, this $A_n/A_s$ value intersects the plate failure - fastener failure boundary at a joint length of about 85 inches. In other words, at these stress levels, joints would have to be longer than 85 in. before the fasteners would be the critical element.

Recently, an allowable stress of 40 ksi has been suggested for A490 bolts used in bearing-type connections.$^{29}$ Again using an allowable stress of 60 ksi in the plate material, the $A_n/A_s$ ratio for this case is 0.67. Examining Fig. 15, it can be seen that plate failure would now control up to a joint length of 60 inches.
The failure mode boundary has been obtained in a similar fashion for minimum strength A325 bolts of 1-1/8 in. diameter contained in minimum strength A514 plate. The pitch chosen was again 3.5 inches. Figure 16 shows the boundary plotted on average shear stress vs. joint length axes and Fig. 17 shows it on the basis of $\frac{A_n}{A_s}$ vs. joint length. Also shown in Fig. 16 are joint strength curves for selected values of $\frac{A_n}{A_s}$.

The behavior of A325 bolts in A514 plate is very similar to that of the A490 bolts as just described. The $\frac{A_n}{A_s}$ ratio for A514 joints using A325 bolts will be 0.37, based again on an allowable stress of 60 ksi in the plate material and the current allowable shear stress for A325 bolts used in buildings. It can be seen in Fig. 17 that at this $\frac{A_n}{A_s}$ value, plate failure is the governing failure mode throughout the range of joint length investigated.

A higher allowable shear stress has also been suggested for A325 bolts. Using this value, 30 ksi, the $\frac{A_n}{A_s}$ ratio is now 0.50. At these stress levels, plate failure will control up to a joint length of about 65 inches.
3.3 Effect of Joint Length

Part of the effect of joint length upon joint behavior is implicit in the discussion of the location of the plate failure - fastener failure boundary. The type of failure mode at a given $A_n/A_s$ ratio is a function of joint length. It was shown in Art. 3.2 that the plate failure mode is of considerable importance in the examination of the behavior of constructional alloy steel joints. The behavior of this type of joint when failing in the fasteners is also of importance, however. This will be particularly true if the higher bolt stresses mentioned in Art. 3.2 are adopted. Fastener type failure would then be the governing failure mode for joints longer than about 60 inches. In practice, many joints can be expected to exceed this value.

The importance of joint length in the determination of ultimate joint strength has been shown by previous investigators. These studies showed that the end fasteners carried the highest load and in many of the experimental studies the amount of this inequality of fastener loads was enough that the test could be stopped when an end fastener failed. The same phenomenon was expected to occur in A514 steel joints to a lesser degree. Because of the much higher yield strength of this material as compared to those grades previously investigated, a more uniform distribution of fastener load should occur.
For a given number of fasteners, joint length is a function of fastener spacing (pitch). In this article, a constant pitch taken as 3.5 in. is used. This represents what would often be used in practice for fasteners in the range of 3/4 in. to 1-1/8 in. diameter. The separate effect of fastener pitch upon joint strength is examined in Art. 3.5.

The strength curves plotted for selected $\frac{A_n}{A_s}$ values in Fig. 14 (A490 bolts) and Fig. 16 (A325 bolts) show that the effect of joint length upon strength is not pronounced, as it had been for milder steels. The curves in Fig. 14 show that for a given $\frac{A_n}{A_s}$ ratio, the average shear stress in A490 fasteners undergoes a gradual, almost linear, decrease with joint length. As seen in Fig. 16, this decrease is much less for A325 bolts in A514 steel. For example, the decrease in average shear stress in A490 bolts over the joint length range from 70 in. to 84 in. is only 1.1 ksi for $\frac{A_n}{A_s} = 0.60$. Over the same range and for the same joint proportions, A325 bolts have a 0.2 ksi decrease in average shear stress. Considering a greater length range, an A490 - A514 joint with $\frac{A_n}{A_s}$ equal to 0.70 has an average shear strength of 84.2 ksi at a joint length of 56 inches. If the joint length is increased to 84 in., the same $\frac{A_n}{A_s}$ proportion gives an average shear stress of 81.4 ksi in the bolts.
The effect of joint length upon the behavior of individual fasteners within a joint can be seen in Fig. 18. Here, the shear stresses in the fasteners of a 25-bolt joint are shown. The fasteners are A490 bolts and the $A_n/A_s$ ratio is 0.60. This joint represents an extreme case — the joint length is long (84 in.) and the $A_n/A_s$ ratio chosen puts the specimen only slightly above the failure mode boundary. In such a joint, the degree of load inequality among the fasteners could be expected to be relatively large. This is borne out by the values shown in Fig. 18. The shear stress in the end fasteners of this joint is 91.5 ksi while that in the centerline bolt is only 59.8 ksi.

A similarly extreme case (long joint length and a low $A_n/A_s$ value) is shown in Fig. 19 for A325 bolts. The joint length is again 84 in. and the $A_n/A_s$ ratio has been taken as 0.45. The shear stress in the end fasteners of this joint is 64.8 ksi while that in the centerline bolts is 49.0 ksi. Because of their greater ductility, the A325 bolt allows a more favorable distribution of load among fasteners than does the A490 bolt.

3.4 Effect of $A_n/A_s$ Ratio

The $A_n/A_s$ ratio of a mechanically fastened joint can be thought of as a "modulus of rigidity". At a given joint length, an increasing $A_n/A_s$ ratio means an increasingly more uniform
distribution of load among the fasteners. As has already been pointed out, the ideal case of equal load distribution among fasteners occurs only at the value of $A_n/A_s = \infty$. This represents, then, a perfectly rigid joint. For any lesser value of $A_n/A_s$, the fasteners carry unequal loads.

The strength curves shown in Fig. 14 for A490 bolts and in Fig. 16 for A325 bolts form the basis for this examination of the effect of the $A_n/A_s$ ratio upon joint strength. It should be remembered that this examination is appropriate only to those joints with proportions such that they are in the fastener failure range.

It is obvious from Figs. 14 and 16 that the spacing of the $A_n/A_s$ curves is significant. For example, in Fig. 14 the limiting $A_n/A_s$ values of 0.62 and $\infty$ at a joint length of 70 in. cover a range of shear stress values of only 75.8 ksi to 91.5 ksi. This means that the load carried by a joint with $A_n/A_s = 0.62$ will not be greatly less than a joint of the same length with, say, $A_n/A_s = 1.00$. For this illustration, the theoretical values of joint load for one line of fasteners are 1915 kips and 2258 kips, respectively. In other words, although the plate area was increased 61%, the load that could be carried increased only 18%. The effect of adding more plate area in order to make the fasteners work at a higher stress level and a more uniform stress level is one of decidedly decreasing benefit.
This same observation can be made with regard to the use of A325 bolts in A514 plates. At a joint length of 91 in. the ultimate load of a joint with $A_n/A_s = 0.45$ is 2915 kips. As seen in Fig. 16, this represents a very "flexible" specimen since it is only slightly above the failure mode boundary line. If the plate area of this joint is increased 17% ($A_n/A_s = 0.525$), the increase in load is only 2.9%. Increasing the plate area 100% at this same length gives an increase in load of 20%.

It should be kept in mind, however, that an increase in the $A_n/A_s$ ratio at a given joint length can be obtained in several ways. One way is to increase the available plate area (by decreasing the allowable plate stress) while maintaining a given allowable shear stress in the fasteners. This results in an increased cost and, as already shown, the resulting load increase is markedly disproportionate.

A second way of producing an increase in $A_n/A_s$ is to make the fasteners work at a higher stress level. This produces no increase in material cost and so the benefits in increased load carrying capability, however small, can be accepted without question of economics. What must be examined now is the resulting factor of safety at any suggested higher fastener shear stress. Naturally, the amount of any such increase in fastener shear stress must satisfy a desired minimum factor of safety.
Another possibility would be to use a combination of these two approaches. In addition, since the increased plate area is needed only in the vicinity of the joint and not throughout the length of the member, it may be feasible in some cases to provide this by means of an upset end. These approaches have been examined in detail in Chapter 5.

3.5 Effect of Fastener Pitch

Previous studies of bolted and riveted joints have shown that pitch, or distance center-to-center of fasteners measured parallel to the line of principal stress, is not an important variable in A7, A36, and A440 steels. These studies showed that the important variable was, rather, joint length as determined by pitch. It is of interest to check the validity of this conclusion when dealing with A514 steel joints.

Since the location of the failure mode boundary has been found to be an important variable in the investigation, it was decided first to compute its location for various pitches. This has been done for 7/8 in. diameter A490 fasteners at pitches of 2.625, 3.50, 4.375, and 5.25 in. and for 1-1/8 in. diameter A325 fasteners at pitches of 3.00, 3.50, 4.50, and 6.75 inches. In the case of the A490 bolts, the pitches chosen correspond to 3, 4, 5, and 6 times the fastener diameter. The pitches
investigated for the A325 bolts are 2-2/3, 3-1/8, 4, and 6 fastener diameters.

The effect of the various pitches chosen upon the location of the failure mode boundary for the case of A514 joints using A490 bolts is shown in Fig. 20. There is virtually no effect upon its location for joints whose length is less than about 40 inches. For joints greater than this length, pitch apparently has an effect upon the location of the failure mode boundary and this effect is greater with increasing joint length.

The magnitude of the effect should be viewed in light of the comments made in Art. 3.4, namely, that the loads carried by joints of the same length but with appreciably different \( A_n/A_s \) values do not differ greatly. This was shown to be particularly true in the region near the failure mode boundary. Since the plate failure - fastener failure boundary line is nothing more than a "strength curve" with continuously varying \( A_n/A_s \), the same conclusion should hold.

To investigate this further, the ultimate strengths of two joints with differing pitches were investigated. The two extremes of those pitches investigated were chosen — 2.625 in. and 5.25 in. — and a joint length of 63 in. was used. This means that there would be 25 fasteners in the joint using the
2.625 in. pitch and 13 fasteners in the other. The same $A_n/A_s$ ratio, 0.70, was used in each case. The results of this study are shown in Fig. 21 where the shear stresses in the fasteners of these hypothetical joints are shown. Also shown is the average shear stress in the fasteners in each case. Based on the joint with the smaller fastener pitch, it is seen that a large increase in pitch (from 2.625 in. to 5.25 in.) produced only a minor increase in average fastener shear stress (from 82.4 ksi to 85.2 ksi).

A similar investigation was made of the behavior of A514 joints using A325 bolts. The joints chosen for comparison had an $A_n/A_s$ ratio of 0.50. One used 11 fasteners spaced at 6.75 in. for a total joint length of 67.5 inches. The other used 23 fasteners spaced at 3.00 in. giving a joint length of 66 inches. The average shear stress in the fasteners of the 11-bolt joint is 60.8 ksi and in the fasteners of the 23-bolt joint, it is 59.9 ksi.

Thus, it may be concluded for both A490 - A514 and A325 - A514 combinations that fastener pitch has only a minor effect upon ultimate strength behavior.
3.6 Effect of Fastener Diameter

The effect of different fastener diameters upon the behavior of A514 joints has been investigated both when A490 bolts are used and when A325 bolts are the fasteners. In the former case, the location of the failure mode boundary was determined for three different bolt diameters. This check, in effect, establishes one point on each of an infinite number of joint strength curves. In addition, the average shear stress was computed in hypothetical joints differing only in the fastener diameter used. Joints of A514 steel using A325 bolts were examined for only two fastener diameters. Based on the results found in the A490 - A514 case, it was felt unnecessary to examine the location of the failure mode boundary. The examination was limited to the determination of average shear stress in the fasteners of two hypothetical joints using A325 bolts.

The bolt diameters investigated in the case of A490 bolts gripping A514 plate were 3/4, 7/8, and 1 inch. The failure mode boundary was computed for each fastener diameter with pitch being held constant at 3.5 inches. Within the limits of the accuracy of the solution, the boundaries computed for these three cases were identical. To serve as a further illustration, the average shear stress in the fasteners of a 21-bolt joint with $A_n/A_s = 0.70$ was computed for the same three diameters. For 3/4,
7/8, and 1 in. diameter bolts, the average shear stresses were 82.5, 82.7, and 82.6 ksi, respectively.

Since changes in fastener diameter produced no appreciable change in the ultimate strength behavior of A490 bolts in A514 steel, the A325 - A514 combination was examined only briefly. The average shear stress in 25-bolt joints with $A_n/A_s = 0.50$ was computed for fastener diameters of 1-1/8 and 1-1/2 inches. Again, a pitch of 3.5 in. was maintained. The values obtained here were 59.3 ksi in the 1-1/8 in. bolts and 60.1 ksi in the 1-1/2 in. bolts.

On the basis of these theoretical studies, it is concluded that the ultimate strength behavior of constructional alloy steel joints whose mode of failure is by fastener shear is independent of fastener diameter.
4. EXPERIMENTAL VERIFICATION OF THE THEORY

The test series described in Art. 1.3 was set up to evaluate the validity of the theory presented in this dissertation. As has been pointed out, the tests carried out in conjunction with the present study are the only experimental work on constructional alloy steel joints reported to date. This test series was of sufficient scope, however, to afford an adequate evaluation of the theoretical work.

4.1 Material Properties of Test Joints

The 1 in. plate used for each of the pilot and large joint test specimens came from the same rollings. In each case, it was requested that the material be manufactured to minimum strength properties. Standard tensile coupons from each rolling were tested. For the plate used in the pilot tests, the average values of yield strength (defined by 0.2% offset strain) and tensile strength were 114.0 ksi and 121.4 ksi, respectively. The plate used for the large joints had corresponding values of 101.6 ksi and 111.9 ksi. Five coupons were tested in each case. Plate - with - holes coupons, used in connection with the development of the mathematical model described in Art. 2.4, were also
tested. These allow a more accurate determination of the ultimate load of plate failure type joints. The plate used in the pilot joints had an ultimate strength of 125.6 ksi when such coupons were tested. The plate used for the large joints gave a corresponding value of 118.2 ksi.

The high strength bolts used in this program came from several lots, depending upon type, diameter and grip length. All lots were ordered to the minimum strength requirements of the applicable ASTM specification.

The bolts were subjected to a number of calibration tests. These were direct tension, torqued tension and a determination of the load - deformation characteristics of single fasteners contained in A514 steel jigs which subject the bolt to a tension-induced shearing force. The direct tension test forms an acceptance test for the given bolt lot and also establishes the strength of the lot as compared to the minimum specified value. The torqued tension test establishes the load - deformation response of bolts installed by torquing. Having obtained a mean load - deformation curve for a given lot, the clamping force provided by a bolt from that lot can be established by measuring the elongation of the installed bolt. The load - deformation behavior of the single bolts installed in A514 steel jigs is necessary for the theoretical prediction of total joint load. This
was described in Art. 2.3. The details of testing involved in each of these calibrations have been previously described. The pertinent values obtained for the bolts used in the test program also have been reported.

4.2 Analysis of Test Specimens

The analysis of the test specimens was made on the basis of actual material properties as described above. Complete details of the testing procedure and test results have been presented elsewhere. Only those aspects of joint behavior relating to ultimate strength are considered here.

(1) Pilot Tests

Two of the ten pilot joints were expected to fail by fracture of the plates. The predicted ultimate loads for these two joints, designated as F42a and F42b, were 4.1% and 7.5%, respectively, less than the test values. The predicted loads were based on the results of plate-with-holes coupons. However, the double line of holes in these test joints meant that there was even more restriction to necking than that represented by the single hole in the coupons. Hence, the predicted loads could be expected to be less than the actual values obtained in the tests. As a consequence of this higher strength, Specimen F42b, which was very close to the fastener failure – plate failure
boundary, did not fail in the mode predicted.

The theoretical studies showed that the fasteners would carry nearly equal loads in joints of the short length involved here. Hence, predicted ultimate loads for the specimens expected to fail by shearing of the fasteners were taken simply as multiples of the individual fastener strengths. The maximum error on predictions computed in this way was only 2.3%. With one exception, the predicted loads were less than the test values. All predicted and actual ultimate loads are tabulated in Table 5.

(2) Large Joints

The first six joints of this series, those of seven, 13, and 17 fasteners in line were paired. One of each pair was designed to fail by tearing of the plates, the other by shearing of the fasteners. These specimens, then, bracket the plate failure - fastener failure boundary. The location of these specimens in relation to the failure mode boundary is shown in Fig. 22. The remaining two specimens, those of 25 bolts in line, were both designed to fail by fastener shear. As the analytical studies have pointed out, it is not until constructional alloy steel joints reach about this length that any substantial amount of load inequality occurs among the fasteners. It was felt that
these tests were particularly necessary if the mathematical models developed were to be properly evaluated. These two joints are also shown in Fig. 22. (It should be noted that the failure mode boundary in Fig. 22 was computed on the basis of actual, not minimum, strength properties.)

The use of the plate - with - holes coupons to predict the ultimate load of the plate failure specimens gave values virtually identical to the test values. All joints failed in the mode predicted and failure was always through an end bolt hole, either in the main plates or the lap plates. All predicted and actual ultimate loads are shown in Table 6.

The analytical method developed in this dissertation to predict the ultimate load of constructional alloy steel joints failing by fastener shear gave excellent results. As shown in Table 6, the maximum error in predictions of the five large joints tested is 5.6%. The comparison between predicted and actual results is also shown graphically in Fig. 23.

Specimens J072, J132, J172, and J252 all failed by an apparent simultaneous shearing of all of the fasteners. Although the end fasteners should, and probably did, fail first, the high level of load in the remaining bolts meant that they were not able to carry the additional load transferred from the first failed
fastener. That the failure was as hypothesized can be seen in Fig. 24. Here, the sheared bolts from Specimen J132 have been reassembled. Although failure was by apparent simultaneous shear of all fasteners, it is obvious from the deformations that an end bolt did fail first.

The only joint in which the test could be stopped once an end fastener failed was Specimen J251. This joint had been proportioned such that it was very close to the failure mode boundary. A sawed section taken through the end four fasteners, including the failed one, is shown in Fig. 25. The large amount of bolt bending, as seen in this section, is not present in the shear calibration test made on an individual fastener. This means that the calibration test produces a shear failure in the bolt at approximately 90° to its axis while a bolt in a test joint is being sheared on a plane providing more area than this minimum value. For this reason, the actual loads in joints of this type may be expected to be slightly larger than those predicted. The difference is not great, however, and no purpose seems served in trying to account for this increased shear area.

Further verification of the analytical method was obtained by comparing theoretical and actual loads in the plates. The latter were computed from the strain gage readings taken continuously during each test. The gages were located on the
lap plates in all cases. Two joints were chosen for the comparison, one a plate-failure type (Jl31) and the other expected to fail by fastener shear (Jl72). Two locations in each joint were examined, the first pitch from the "loaded" end and a pitch near the centerline of the joint, and only loads above the slip load were considered. Table 7 summarizes the comparison and shows that the theoretical computations gave results in good agreement with the actual values.
5. PHILOSOPHY OF DESIGN OF CONSTRUCTIONAL ALLOY STEEL TENSION MEMBERS

5.1 Review of Current Design Philosophy

The design of structural steel tension members and the design of their connections must be carried out in accordance with an inter-related philosophy. The basis of design and the desired behavior of the member itself has been clearly set forth.\textsuperscript{29,30} The philosophy that has been established considers that the limit of usefulness of a tension member is given by the load at which contained plastic flow commences in the gross section of the member. Beyond this point, "significant and relatively uncontrolled" elongation occurs.\textsuperscript{30}

Up to the present time (1967) none of the commonly used building or bridge specifications have established allowable stresses for tension members of constructional alloy steel.\textsuperscript{31,32,33,34} Suggested allowable stresses have been set forth, however.\textsuperscript{35,36} These have been established by applying the same factor of safety against yield that has been used in conjunction with carbon and high strength structural steels. This factor of safety is set at 1.67 for buildings and at 1.83 for bridges. It is worth noting that the allowable tension stress
in aluminum alloy is set at a much larger factor of safety (2.41) based on yield.\textsuperscript{37} This was considered necessary because of the smaller spread between yield and ultimate of this material as compared to structural alloy steel.\textsuperscript{3} The low ultimate to yield stress ratio of constructional alloy steel may necessitate a similar increase in the factor of safety against yield. Whatever is chosen, it seems clear that the same philosophy will control, that is, the design of constructional alloy steel members will be performed according to an elastic theory. Thus, the factor of safety in the member will be against contained plastic flow.

The philosophy of design of connections in tension members has recently been examined.\textsuperscript{29} In evaluating the behavior of any structure, it is considered desirable that the system have capacity for distortion or geometrical adjustment before failure by fracture. In an axially loaded structure, this means that the connections should be proportioned so that yielding takes place in the gross cross-section of the member before the joint fails. (This joint failure can be either by tearing of the plates through the net section or by fastener shear.) Thus, although the individual member will be beyond its defined limit of usefulness, the criterion for satisfactory behavior of the structure or assemblage of which the member is a part demands
further deformation capacity. The criteria proposed in Ref. 29 ensure this action in mechanically fastened axially loaded structures of carbon or high strength steels.

The same point of view has been expressed in more general terms by Frankland. In discussing the role of ductility in structures in general, the joint in a tension member is referred to as a "stress concentration". It is emphasized that the entire cross-section of the member should be brought to the yield stress level so that the effect of this stress concentration is eliminated.

This approach can be compared to the behavior of a column in a framed structure. The design of the column is done on an ultimate strength basis and is such as to ensure that considerable deformation will have taken place in the connecting flexural members before the column fails. The joint in a tension member is analogous to the column. Its proportions must ensure that there is sufficient ductility in the tension member as the assemblage proceeds toward its ultimate load.
5.2 Characteristics of A514 Steel Members Designed According to Current Practice

It was shown in Art. 3.2 that constructional alloy steel joints proportioned according to suggested allowable stresses in the plate material and current allowable stresses in the fasteners will be governed by the plate failure mode over a large range of joint length. In other words, the factor of safety of the joint as applied to its ultimate strength will be against plate failure at the net section rather than fastener failure. For A514 joints using A490 bolts, it was shown that failure by tearing of the plates governs for joint lengths up to about 85 inches. In A514 joints using A325 bolts, this mode governed throughout the range investigated (up to 91 inches). Since current practice will thus generally result in joints whose potential failure is by fracture of the plates, an examination of the behavior of such joints is pertinent.

As outlined in Art. 5.1, it is considered desirable that tension members yield across their gross cross-section before they fail, either through the net cross-section or in the fasteners. For plate failure type joints, and using minimum specified yield and ultimate stresses for A514 steel, this means that the net area of the member \( (A_n) \) should be equal to or greater than 87% of the gross area \( (A_g) \).
No explicit expression can be set up so that this requirement will be generally satisfied. Involved in the computation of these gross and net area values are complications such as hole patterns, method of hole formation, shape of cross-section, etc. However, if the examination is restricted to consideration of members composed of plates of constant thickness containing non-staggered lines of fasteners, some indication of the effect of the \( \frac{A_n}{A_g} \) requirement can be obtained. Most structural plate will be greater than 1/2 in. thick and since thicknesses larger than this value will contain drilled rather than punched holes, the value of 1/16 in. will be added to any assumed fastener diameter when calculating \( A_n \). For this simplified type of member, it has been shown that the minimum allowable spacing of fasteners, measured perpendicular to the line of the load (gage), is about seven times the bolt diameter. If this requirement is met, the gross cross-section will yield at or before failure occurs at the net section.

It is doubtful whether such a large minimum spacing as seven fastener diameters can be accepted in structural practice, however. Its use would also force the use of longer joints since fewer parallel lines of fasteners than customary for other steels could be used in a given situation.
Whether or not an A514 member can, in fact, attain a joint efficiency \( \left( \frac{A_n}{A_g} \right) \) of 0.87 or greater is also open to question. Previous work which investigated the behavior of riveted joints of A7 steel showed that the joint efficiency reached a limiting value of about 0.87.\textsuperscript{39} As a result of this and similar studies, building specifications typically place an upper limit of 0.85 on joint efficiency.\textsuperscript{31} It is known, however, that the efficiency is influenced by the biaxial stress effect or "reinforcement" of closely spaced holes. An extensive study of plate - with - holes coupons of A7 steel showed that these specimens had a tensile strength about 3% higher than the tensile strength of a standard bar coupon of the same material.\textsuperscript{22} The same study showed that the increase was about 6% in A440 steel. The A514 plate - with - holes coupons tested in conjunction with the present study showed an increase of 7% when compared to standard tensile coupons. (It is probable that these increases are related to the proportions of elastic to inelastic material in the specimens as the ultimate load is approached.)

The much higher increase obtained in the A514 specimens as compared to the A7 specimens indicates that joint efficiencies higher than the present limit of 0.85 are a possibility in constructional alloy steel. Although the large test joints reported in the present study had only a single line of holes, five of the
eight specimens had theoretical efficiencies of 0.85 or greater. As shown in Table 6, the highest were values of 0.90 and 0.91. All of these joints reached or exceeded their theoretical ultimate loads. If the 0.87 requirement is to be met in members fabricated according to current practice, more tests, particularly those involving staggered hole patterns and shapes other than plates, would be needed. An alternate, and more positive approach to this problem is discussed in Art. 5.3, however.

That class of joints which, according to current design practice, falls into the category of those having a factor of safety against shear failure in the fasteners is very small. As has been pointed out, the only joints that could be expected to fail by fastener shear are A514 joints fastened by A490 bolts and whose length is greater than 85 inches. Examination of the ultimate strength of these joints shows that plate stresses in the gross section of the member will be above yield at the time the ultimate load of the joint is reached. (It should be noted that whether or not yield is reached is also dependent upon the disposition of the plate material used. For a given net area, the ratio of net to gross area will be higher for higher values of the width to thickness ratio of the section. In this dissertation, the practical lower limit of the width to thickness ratio as applied to either the main plate or the combined lap plates
The factor of safety against shear failure in the bolts is 2.01 at a joint length of 85 inches. This is slightly above the value in the plate failure type joints designed according to current practice (i.e. 1.92).

5.3 Proposed Design Criteria for Bolted Joints in Constructional Alloy Steel Members

It has been shown that current design practice applied to joints in constructional alloy steel members results in proportions such that the factor of safety in the member generally will be against plate failure. This was shown to be the case for joints up to 85 inches in length when A490 bolts are used and for all joint lengths investigated when A325 bolts are used. It was further shown that if the joint is fabricated by removing material from the main member in the form of holes, it is doubtful whether the requirement that yield should be reached in the gross cross-section before the plates tear through the net section can be met. This article will discuss the effects of higher allowable fastener shear stresses and/or reduced allowable plate stresses upon joint and member behavior.

In attempting to improve the behavior of bolted constructional alloy steel tension members, it seems advisable to
tie in any new proposals to the work that has been done in bolted joints of other steels. For example, there would have to be some marked increase in economy or behavior to justify the use of allowable bolt stresses in A514 steel that are different from those suitable for use in other grades. (In this context, it should be pointed out that previous research has established that the ultimate shear strength of a single high strength bolt is independent of the type of material which it connects.\textsuperscript{22} The deformation capacity of a bolt is affected by the type of connected material, however, and this is of prime importance in the behavior of a line of bolts.) The allowable shear stress suggested for A325 bolts was 30 ksi and that for A490 bolts was 40 ksi.\textsuperscript{29} These recommendations covered the use of these fasteners in A36 and A440 steels.

If an allowable shear stress of 40 ksi is used for A490 bolts, it was pointed out in Art. 3.2 that plate failure is the governing failure mode up to joint lengths of about 60 inches. Examination of the strength of joints of this proportion ($A_n/A_s = 0.67$) and lengths greater than 60 in. shows that the factor of safety varies from 2.02 at 60 in. to 1.98 at a joint length of 84 inches. The latter is only fractionally less than the value of 2.0 suggested as a minimum desirable level.\textsuperscript{29} (Factor of safety for fastener failure type joints is taken as the average
shear stress in the bolts at joint failure divided by the allowable bolt shear stress.) Joints of this proportion do produce the desired yielding of the gross cross-section before they reach their ultimate load.

It was also shown in Art. 3.2 that the suggested allowable shear stress of 30 ksi for A325 bolts gives joint proportions which mean that plate failure is the governing failure mode for joints less than about 65 inches. The \( \frac{A_n}{A_S} \) ratio here is 0.50. The factor of safety against fastener failure is 2.02 at a joint length of 65 in. and at 84 in., it is also 1.98. Again, the joint proportions produce satisfactory member behavior, that is, gross section yielding in the fastener failure region.

These suggested allowable bolt stresses used in conjunction with an allowable plate stress of 60 ksi, result in satisfactory joint and member behavior for joints in which the factor of safety is against fastener shear. They provide a high enough factor of safety, a factor of safety reasonably constant with length, and satisfactory behavior in the member itself. On the basis of the tests described in Ref. 5, it appears that the higher bearing stresses which will result from increased allowable fastener shear stresses should not be a problem. In addition, these increased shear stress values are at the same levels as those suggested for the same fasteners when used in joints of
carbon and high strength steels. The only undesirable feature is that the most practical range of joints, those under about five feet in length, still remain as plate failure type joints and failures will occur before yield is reached on the gross section.

The difficulties involved in constructional alloy steel joints in which the factor of safety is against plate failure were discussed in Art. 5.2. Examination of Fig. 15 shows that it would take an increase in the $\frac{A_n}{A_s}$ ratio to about 0.80 to ensure fastener failure for all lengths of A514 steel joints fastened by A490 bolts. If the allowable shear stress in the fasteners is maintained at 40 ksi, this can be accomplished by a decrease in the suggested allowable tensile stress of the plate material from 60 ksi down to 50 ksi. The factor of safety at the maximum joint length considered (84 in.) is 2.14. However, member behavior resulting from these proportions is satisfactory only for the longer joints. At the shorter joint lengths, the requirement that yield be reached in the gross section prior to joint failure is not achieved. For example, at the time of failure of a joint with seven bolts in line (length = 21 in. for a 3.5 in. fastener pitch) the stress in the gross cross-section is only 90 ksi. It is not until the joint length reaches about 40 in. that the yield requirement is met. If the $\frac{A_n}{A_s}$ ratio
were reduced to 0.75, the minimum value to ensure fastener failure for all joint lengths, the stress in the plates of the seven-bolt joint illustrated above rises to only 92 ksi.

The decrease in the allowable plate stress to 50 ksi has a similar effect upon the behavior of A514 steel joints in which A325 bolts are used. With an allowable fastener shear stress of 30 ksi, the $A_n/A_s$ ratio now is 0.60. This gives a satisfactory margin above the theoretical failure mode boundary value of 0.54 at zero joint length. Joints proportioned to this value will have a factor of safety that is against fastener failure for all joint lengths. This factor of safety is 2.11 at a joint length of 84 inches. However, as was the case with joints using A490 bolts, these proportions do not ensure the desired member behavior. Only for joints in excess of about 60 in. does yielding occur in the gross cross-section at or before the ultimate load of the joint is reached.

It is apparent that if the design philosophy established is to be fulfilled for constructional alloy steel tension members, some alternative to the approaches discussed above will have to be used. This examination has shown that, under normal fabrication techniques, there is no reasonable combination of allowable stresses in the plate and fasteners that will satisfactorily meet the criteria established for joint and member behavior for all
joint lengths. Two alternatives are suggested, namely:

(1) The use of some type of upset end, such as an increase in plate width or thickness in the region of the joint.

(2) A change in the specified mechanical properties of constructional alloy steel.

The first alternative, the use of upset ends, means simply that more material would be provided in the region of the connection. This would overcome the difficulties described in meeting the net to gross member area requirement. The net section of the upset end would be established at 87% or more of the gross section of the main member. The allowable stresses could be kept at 60 ksi in the plate (main member) and at 40 ksi in A490 bolts and 30 ksi in A325 bolts. The use of upset ends would then be required for joints up to 60 in. long when A490 bolts were used and up to 65 in. long when A325 bolts were used. If desired, joints of greater lengths could be fabricated in the usual manner, that is, holes for the fasteners would be removed from the main member cross-section. For convenience, the figure of five feet might be established as the "cut-off" point for both fastener types.
If this approach were used, the suggested stress levels appear to be optimum. Although either an increase in allowable shear stress in the fasteners or a decrease in the allowable tension stress in the plate would reduce the length over which such joint reinforcement would be required, the benefits are small. The factor of safety against shear failure in the fasteners is probably at a minimum desirable value at the 40 ksi (A490 bolts) and 30 ksi (A325 bolts) levels. A decrease in allowable stress in the gross section of the member from 60 ksi to 50 ksi would mean that 16% more plate material would be required over the length of the member. The reduction in member stress does reduce the length required for joints in which upset ends are needed to 40 in. when A490 bolts are used and to 60 in. when the fasteners are A325 bolts. Although extra fabrication cost must be included, the material saved by this reduction in allowable member stress occurs only over a relatively short length, that of the joint.

The additional cost involved in providing some type of upset end is difficult to evaluate. In certain cases, however, it may be fairly inexpensive. Since constructional alloy steel is weldable, the situation may often arise in which shop fabrication will be done by welding and field connections made using high strength bolts. If the member were composed of one or more plies of plate, the upset end could be provided by welding plates
of the same thickness but of greater width to the ends of the main member. When rolled shapes rather than plates are used, the provision of upset ends will be more complicated. They can be provided in essentially the same manner, however.

The other approach to the problem of providing satisfactory member behavior is to change the specified mechanical properties of constructional alloy steel. The principal reason for the difficulties arising in plate failure type joints and their members is the low spread between the yield and tensile strengths of A514 steel. The situation would be considerably improved if the minimum specified tensile strength were increased from the present level of 115 ksi to, say, 125 ksi while the minimum specified yield strength is kept at 100 ksi.

This change would reduce the $A_n/A_g$ requirement from 0.87 to 0.80. Assuming that the allowable stresses remain at 60 ksi in the plates and at 40 ksi (A490) or 30 ksi (A325) in the bolts, the proportion of plate failure type joints could be expected to remain about the same. However, fabrication of these joints (those under about five feet in length) could probably be done in the normal manner. The joint could be prepared by drilling holes in the section provided by the main member as long as the area provided in the net section is kept at a minimum of 80% of that of the gross section. The minimum allowable fastener
spacing would now be about 4-1/4 times the fastener diameter. The minimum allowable spacing presently suggested in building codes is three fastener diameters. It is important to note that any change in specified mechanical properties would have to include a limitation on the minimum allowable spread between the yield and tensile strengths. For example, if the values of yield and tensile strengths were set at 100 ksi and 125 ksi but the actual levels in delivered material were 110 ksi and 126 ksi, no advantage would have been gained. The $A_n/A_g$ ratio is again 0.87. Indeed, the figure of 0.87 based on the currently specified minimum values may itself be unconservative. Reference 37 quotes typical mechanical properties of "T-1" steel at 118.4 ksi yield and 127.0 ksi tensile strength. These strengths require a minimum $A_n/A_g$ ratio of 0.93.

A change in specified material properties, as discussed here, would provide a satisfactory solution to the problem from a structural point of view. Metallurgical considerations would, of course, play a leading role in deciding whether such changes could be considered by the steel industry. Although these considerations will not be dealt with in this dissertation, it would be remarked that metallurgists consider that it is already a difficult problem to provide a quenched and tempered steel with a yield - tensile strength ratio less than about 0.90.
5.4 Notes on the Use of Hybrid Connections

Although this dissertation is concerned primarily with the behavior of constructional alloy steel tension splices, it is pertinent to include some discussion of joints in which this may not be the only type of steel used. On the basis of past experience with constructional alloy steel, it seems probable that the use of this material will be limited to the most highly-stressed portions of a structure. This means that at some point constructional alloy material will have to be connected to steel of a lower grade — a hybrid joint.

An extensive theoretical investigation into the behavior of bolted hybrid connections has recently been completed. (Those portions of the work dealing with A514 steel are based on the theoretical developments presented in this dissertation.) Only aspects of Ref. 40 which deal with A514 steel will be reviewed here.

The following points made as a result of the investigation are of interest:

1. The length over which the plate failure mode governs in a hybrid joint which includes A514 steel is intermediate between those lengths which would be obtained for homogeneous joints of the same materials.
2. In a hybrid joint failing in the plates, the higher strength material will govern the value of the ultimate load if the design of the members has been based on a yield criterion.

3. In A514-A36 and A514-A440 hybrid joints using A325 bolts, the governing failure mode is plate failure for practically all joint lengths investigated if the fastener shear stress is set at the current allowable level of 22 ksi. 28

4. In A514-A440 hybrid joints using A490 bolts proportioned to the current allowable shear stress of 32 ksi, 28 plate failure is the governing mode for all lengths investigated.

5. The use of higher allowable bolt stresses, namely 30 ksi for A325 bolts and 40 ksi for A490 bolts, has the effect of reducing the lengths over which the plate failure mode governs. This length is still substantial for most cases, however. For example, the
A514-A440 combination using A325 bolts proportioned at a shear stress of 30 ksi fail by fastener shear only for joints longer than about 85 inches.

Although the theoretical work on hybrid connections still has to be verified by experiments, some observations about the behavior of hybrid joints which use A514 steel can be made.

1. The use of allowable fastener shear stresses of 30 ksi (A325 bolts) and 40 ksi (A490 bolts) is appropriate.

2. Most combinations of plate material and fasteners will result in a joint whose factor of safety is against plate failure.

3. Since most hybrid joints will be governed by plate failure and since the A514 member will govern the ultimate load of the joint, the design criteria outlined in Art. 5.3 for homogeneous A514 joints will also be suitable here.
This dissertation consists of two main portions. One portion is the extension of a general theoretical solution for the distribution of load among the fasteners in bolted butt splices to joints of constructional alloy (A514) steel. The other portion examines the philosophy of design of metallic tension members and evaluates the extent to which mechanically fastened constructional alloy steel members are able to fulfill this desired philosophy.

The solution of the load distribution problem is shown to require two mathematical models. The first model, the load-deformation behavior of a single fastener contained in A514 plates and subjected to a tension-induced shear, was obtained by an extension of existing theory. The second model required is one describing the load-deformation behavior of a plate-with-holes coupon loaded in tension. Because of the significantly different behavior of A514 as compared to that of previously investigated grades of steel, a completely new model was required. The relationship proposed in this dissertation was developed on the basis of a substantial number of tests. Both of the expressions developed are applicable to the elastic and inelastic regions of behavior.
The theoretical work developed in this dissertation has been verified by an extensive testing program. Only the results of this program are reported herein. The comparison shows that theoretical predictions are reliable, both for the ultimate load of a joint and for obtaining the distribution of load among fasteners at loads less than ultimate.

A study of the parameters that might be expected to affect the behavior of bolted splices has been based on this theoretical development. These parameters included fastener type, diameter, and pitch, relative proportions of plate and fasteners, and joint length.

Since experimental verification of the theory was available, an examination of the performance of mechanically fastened A514 steel members could be made with some confidence. This examination showed that the application of current design procedure results generally in joints whose potential failure mode is by fracture of the plates. Under usual fabrication techniques, this will lead to unsatisfactory member behavior in that failure occurs through the net section before yield is reached on the gross section. A number of possible ways of overcoming this situation were explored.
The most important conclusions reached as a result of this study can be itemized as follows:

1. An accurate theoretical solution is available for predicting the ultimate load of bolted, butt splices of constructional alloy steel. The same theoretical development can be used to provide plate or individual fastener loads at levels less than ultimate.

2. The ultimate strength of these joints is a function of fastener grade, joint length, and relative plate - fastener proportions. It is independent of fastener diameter or pitch, per se.

3. Constructional alloy steel joints using high strength bolts do not produce desirable behavior in the members in which they are contained if the elements of the joint are designed according to current stress levels.
4. The use of higher allowable bolt stresses, in line with those suggested as a result of studies of mechanically fastened joints of other grades of steel, is suitable for use in A514 steel.

5. Satisfactory member behavior for a large and important class of joints can only be obtained in one of two ways. These are

a. The use of some type of upset end providing enough plate area through the net cross-section of the joint so that yielding of the gross cross-section of the member will take place before joint failure.

b. A change in the specified mechanical properties of A514 steel sufficient to allow satisfactory member behavior to be attained under standard fabrication methods.
7. TABLES AND FIGURES
### TABLE 1  MECHANICAL PROPERTIES OF A514 STEEL

<table>
<thead>
<tr>
<th></th>
<th>Plates</th>
<th>Structural Shapes</th>
</tr>
</thead>
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<tr>
<td><strong>Yield Strength,</strong> Ext. under load, min, psi</td>
<td>100,000</td>
<td>100,000</td>
</tr>
<tr>
<td><strong>Tensile Strength,</strong> psi</td>
<td>115,000/135,000</td>
<td>115,000/140,000</td>
</tr>
<tr>
<td><strong>Elongation in 2 in. min,</strong> %</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td><strong>Reduction of Area,</strong> min, %</td>
<td>3/4 in. and under-40 over 3/4 in.- 50</td>
<td>3/4 in. and under-45 over 3/4 in.- 55</td>
</tr>
</tbody>
</table>

### TABLE 2  BASIC STRENGTH PROPERTIES OF TEST BOLTS

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<th>LOT B</th>
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<th>LOT D</th>
<th>LOT JJ*</th>
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<td>A490</td>
<td>A490</td>
<td>A490</td>
</tr>
<tr>
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<td>1-1/8</td>
<td>7/8</td>
<td>1</td>
</tr>
<tr>
<td><strong>Grip, in.</strong></td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Connecting Material</strong></td>
<td>A514</td>
<td>A514</td>
<td>A514</td>
<td>A514</td>
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<tr>
<td><strong>Specified Min. Tensile Str., kips</strong></td>
<td>69.3</td>
<td>114.5</td>
<td>69.3</td>
<td>90.9</td>
</tr>
<tr>
<td><strong>Actual Tensile Str., kips</strong></td>
<td>72.2</td>
<td>119.5</td>
<td>75.4</td>
<td>99.1</td>
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<tr>
<td><strong>Shear Test:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{ult}$, kips</td>
<td>116.6</td>
<td>191.8</td>
<td>119.8</td>
<td>151.7</td>
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<tr>
<td>$A_{ult}$, in.</td>
<td>0.127</td>
<td>0.165</td>
<td>0.131</td>
<td>0.155</td>
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<tr>
<td>$\mu$</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>28</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.95</td>
<td>1.50</td>
<td>0.95</td>
<td>0.35</td>
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* From Ref. 21
** Corresponds to a tensile strength of 150 ksi
### TABLE 3 PLATE - WITH - HOLES SPECIMENS

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<tr>
<th>Specimen No.</th>
<th>Width (in.)</th>
<th>Thickness (in.)</th>
<th>Hole Dia. (in.)</th>
<th>Hole Pitch (in.)</th>
<th>( \frac{A_N}{A_g} )</th>
<th>( \sigma_u ) (ksi)</th>
<th>( \sigma_y ) (ksi)</th>
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</thead>
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<tr>
<td>JS-1</td>
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<td>1.017</td>
<td>1.000</td>
<td>4.00</td>
<td>0.86</td>
<td>117.6</td>
<td>93.4</td>
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<tr>
<td>-2</td>
<td>3.004</td>
<td>1.009</td>
<td>0.876</td>
<td>3.50</td>
<td>0.71</td>
<td>119.7</td>
<td>96.0</td>
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<tr>
<td>-3</td>
<td>4.005</td>
<td>1.015</td>
<td>1.010</td>
<td>3.50</td>
<td>0.75</td>
<td>116.3</td>
<td>92.1</td>
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<tr>
<td>-4</td>
<td>4.002</td>
<td>1.013</td>
<td>0.890</td>
<td>3.50</td>
<td>0.78</td>
<td>117.9</td>
<td>93.6</td>
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<tr>
<td>-5</td>
<td>3.504</td>
<td>1.012</td>
<td>0.883</td>
<td>3.50</td>
<td>0.75</td>
<td>119.0</td>
<td>95.5</td>
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<td>1.020</td>
<td>1.012</td>
<td>4.00</td>
<td>0.86</td>
<td>118.9</td>
<td>95.0</td>
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<td>1.015</td>
<td>0.757</td>
<td>3.50</td>
<td>0.81</td>
<td>117.2</td>
<td>93.4</td>
</tr>
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<td>-8</td>
<td>6.003</td>
<td>1.022</td>
<td>1.014</td>
<td>4.00</td>
<td>0.83</td>
<td>119.0</td>
<td>96.0</td>
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<tr>
<td>-9</td>
<td>3.001</td>
<td>0.501</td>
<td>0.755</td>
<td>3.50</td>
<td>0.75</td>
<td>121.8</td>
<td>97.8</td>
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<td>2.999</td>
<td>0.504</td>
<td>0.757</td>
<td>3.50</td>
<td>0.75</td>
<td>123.2</td>
<td>97.3</td>
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<td>0.894</td>
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<td>99.0</td>
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<td>-12</td>
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<td>0.886</td>
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<td>0.71</td>
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*Tensile strength of standard bar coupons was 111.9 ksi

### TABLE 4 HYPOTHETICAL BOLT PROPERTIES

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<th>Diameter</th>
<th>( R_{ult} ) kips</th>
<th>( \Delta_{ult} ) in.</th>
<th>( \mu )</th>
<th>( \lambda )</th>
<th>( R_{ult} ) kips</th>
<th>( \Delta_{ult} ) in.</th>
<th>( \mu )</th>
<th>( \lambda )</th>
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<td>7/8 in.</td>
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<td>--</td>
<td>--</td>
<td>--</td>
<td>110.0</td>
<td>0.125</td>
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<td>0.95</td>
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<tr>
<td>1</td>
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<td>--</td>
<td>--</td>
<td>144.0</td>
<td>0.142</td>
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<tr>
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<td>Item</td>
<td>Units</td>
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<td>F42b</td>
<td>F42c</td>
<td>F42d</td>
<td>F42e</td>
<td>F42g</td>
<td>J42a</td>
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<td>-------</td>
<td>-------</td>
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<tr>
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<td></td>
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<td>A325</td>
<td>A325</td>
<td>A325</td>
<td>A325</td>
<td>A490</td>
<td>A490</td>
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<tr>
<td><strong>Diameter</strong></td>
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<td>1-1/8</td>
<td>1-1/8</td>
<td>1-1/8</td>
<td>1-1/8</td>
<td>1-1/8</td>
<td>1</td>
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<td><strong>No. in Line (n)</strong></td>
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<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<tr>
<td><strong>Shear Area ((A_s))</strong></td>
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<td>15.90</td>
<td>15.90</td>
<td>15.90</td>
<td>15.90</td>
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<td>12.57</td>
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<td>10.5</td>
<td>10.5</td>
<td>10.5</td>
<td>10.5</td>
<td>10.5</td>
<td>10.5</td>
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<td><strong>Plate Width</strong></td>
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<td>6.76</td>
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<td>7.56</td>
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<td><strong>Thickness (t)</strong></td>
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<td>2.04</td>
<td>2.04</td>
<td>2.04</td>
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<td>2.04</td>
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<td><strong>Gross Area ((A_g))</strong></td>
<td>in.(^2)</td>
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<td>12.98</td>
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<td>14.51</td>
<td>15.41</td>
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<td>13.93</td>
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<td>0.61</td>
<td>0.66</td>
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<tr>
<td>(A_n/A_g)</td>
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<td>0.57</td>
<td>0.62</td>
<td>0.65</td>
<td>0.67</td>
<td>0.68</td>
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<td>Predicted</td>
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<td>1050</td>
<td>1050</td>
<td>1050</td>
<td>1210</td>
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<td>Actual</td>
<td>kips</td>
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<td>1052</td>
<td>1064</td>
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<td>1062</td>
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<td>Bolts</td>
<td>Bolts</td>
<td>Bolts</td>
<td>Bolts</td>
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* Pitch = 3.5 in. for all joints
## TABLE 6

**JOINT DIMENSIONS AND TEST RESULTS - LARGE JOINTS**

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<tr>
<th>Item</th>
<th>Units</th>
<th>J071</th>
<th>J072</th>
<th>J131</th>
<th>J132</th>
<th>J171</th>
<th>J172</th>
<th>J251</th>
<th>J252</th>
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<tr>
<td><strong>Bolts Type</strong></td>
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<td>A490</td>
<td>A490</td>
<td>A490</td>
<td>A490</td>
<td>A490</td>
<td>A490</td>
<td>A490</td>
<td>A490</td>
</tr>
<tr>
<td><strong>Diameter</strong></td>
<td>in.</td>
<td>7/8</td>
<td>7/8</td>
<td>7/8</td>
<td>1-1/8</td>
<td>7/8</td>
<td>7/8</td>
<td>7/8</td>
<td>7/8</td>
</tr>
<tr>
<td><em><em>No. in Line</em> (n)</em>*</td>
<td></td>
<td>7</td>
<td>7</td>
<td>13</td>
<td>13</td>
<td>17</td>
<td>17</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td><strong>Shear Area (As)</strong></td>
<td>in.²</td>
<td>8.41</td>
<td>8.41</td>
<td>15.63</td>
<td>25.84</td>
<td>20.44</td>
<td>20.44</td>
<td>30.07</td>
<td>30.07</td>
</tr>
<tr>
<td><strong>Joint Length</strong></td>
<td>in.</td>
<td>21</td>
<td>21</td>
<td>42</td>
<td>42</td>
<td>56</td>
<td>56</td>
<td>84</td>
<td>84</td>
</tr>
<tr>
<td><strong>Plate Width</strong></td>
<td>in.</td>
<td>3.86</td>
<td>4.71</td>
<td>6.38</td>
<td>7.00</td>
<td>8.07</td>
<td>10.09</td>
<td>6.97</td>
<td>9.22</td>
</tr>
<tr>
<td><strong>Thickness (t)</strong></td>
<td>in.</td>
<td>2.03</td>
<td>2.04</td>
<td>2.04</td>
<td>4.08</td>
<td>2.02</td>
<td>4.08</td>
<td>4.08</td>
<td>4.08</td>
</tr>
<tr>
<td><strong>Gross Area (A_g)</strong></td>
<td>in.²</td>
<td>7.82</td>
<td>9.58</td>
<td>12.99</td>
<td>28.55</td>
<td>16.48</td>
<td>20.40</td>
<td>28.35</td>
<td>37.55</td>
</tr>
<tr>
<td><strong>Net Area (A_n)</strong></td>
<td>in.²</td>
<td>5.92</td>
<td>7.66</td>
<td>11.08</td>
<td>23.70</td>
<td>14.55</td>
<td>18.52</td>
<td>24.55</td>
<td>33.73</td>
</tr>
<tr>
<td><strong>A_n/A_s</strong></td>
<td></td>
<td>0.70</td>
<td>0.91</td>
<td>0.71</td>
<td>0.91</td>
<td>0.71</td>
<td>0.90</td>
<td>0.82</td>
<td>1.12</td>
</tr>
<tr>
<td><strong>A_n/A_g</strong></td>
<td></td>
<td>0.76</td>
<td>0.80</td>
<td>0.85</td>
<td>0.83</td>
<td>0.88</td>
<td>0.91</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td><strong>Ultimate Load</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted</td>
<td>kips</td>
<td>700</td>
<td>810</td>
<td>1309</td>
<td>2485</td>
<td>1720</td>
<td>1950</td>
<td>2740</td>
<td>2935</td>
</tr>
<tr>
<td>Actual</td>
<td>kips</td>
<td>710</td>
<td>850</td>
<td>1308</td>
<td>2615</td>
<td>1718</td>
<td>2015</td>
<td>2735</td>
<td>3100</td>
</tr>
<tr>
<td>Failure Mode</td>
<td></td>
<td>Plate</td>
<td>Bolts</td>
<td>Plate</td>
<td>Bolts</td>
<td>Plate</td>
<td>Bolts</td>
<td>Bolts</td>
<td>Bolts</td>
</tr>
</tbody>
</table>

* Pitch = 3.5 in. for all joints
### TABLE 7

**COMPARISON OF THEORETICAL VS. MEASURED PLATE LOADS**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Location (bolt lines)</th>
<th>Total Joint load kips</th>
<th>Theoretical Load in Plates kips</th>
<th>Measured Load in Plates kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>J172</td>
<td>17-16</td>
<td>800</td>
<td>730</td>
<td>694</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1350</td>
<td>1250</td>
<td>1218</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850</td>
<td>1736</td>
<td>1780</td>
</tr>
<tr>
<td>(Ult. load $= 2015^k$)</td>
<td>11-10</td>
<td>800</td>
<td>448</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1350</td>
<td>770</td>
<td>722</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850</td>
<td>1080</td>
<td>1046</td>
</tr>
<tr>
<td>J131</td>
<td>13-12</td>
<td>700</td>
<td>624</td>
<td>616</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1050</td>
<td>950</td>
<td>996</td>
</tr>
<tr>
<td>(Ult. load $= 1308^k$)</td>
<td>7-6</td>
<td>700</td>
<td>316</td>
<td>312</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1050</td>
<td>482</td>
<td>456</td>
</tr>
</tbody>
</table>
Fig. 1 Bolted Plate Splice - Pictorial

Fig. 2 Typical Stress - Strain Curves of Various Steels
Fig. 3 Joint Geometry

Fig. 4 Load Transfer in Bolted Joint
Fig. 5 Bolt and Plate Deformations

Fig. 6 Tension Shear Jig - Lot B Bolts
Fig. 7 Tension Shear Jig - Lot C Bolts

Fig. 8 Tension Shear Jig - Lot D Bolts
Fig. 9  Tension Shear Jig - Lot JJ Bolts

Fig. 10  Schematic of Plate - With - Holes Coupon
Fig. 11  A440 Steel Plate - With - Holes Coupon Under Load

Fig. 12  A514 Steel Plate - With - Holes Coupon Under Load
Fig. 13  Stress – Strain Curves of Plate – With – Holes Coupons
Fig. 13 (cont'd.)
Fig. 14 Failure Mode Boundary ($\tau$ vs. $L$) - A490 Bolts

Fig. 15 Failure Mode Boundary ($A_n/A_s$ vs. $L$) - A490 Bolts
Fig. 16  Failure Mode Boundary ($\tau$ vs. $L$) - A325 Bolts

Fig. 17  Failure Mode Boundary ($A_n/A_s$ vs. $L$) - A325 Bolts
Fig. 18  Load Distribution to Fasteners of 25-Bolt Joint - A490 Bolts

Fig. 19  Load Distribution to Fasteners of 25-Bolt Joint - A325 Bolts
Fig. 20  Effect of Pitch on Failure Mode Boundary - A490 Bolts

Fig. 21  Load Distribution in Joints of Same Length But of Different Pitch - A490 Bolts
Fig. 22  Failure Mode Boundary and Large Test Joints

Fig. 23  Predicted and Actual Results - Large Test Joints
Fig. 24  Sheared Bolts from Specimen J132

Fig. 25  Sawed Section from Specimen J251
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