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D. S. Baillie

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FAILURE DOMAINS FOR SINGLE BAY, FLAT ROOFED, 
ONE AND TWO STORY PORTAL FRAMES 

by 

David S. Baillie 
June 1960 

A Report Submitted to Professor 
George C. Driscoll, Jr. in 
Fulfilment of the Requirements 
of CE 406 
Special Problems in Engineering
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LEHIGH UNIVERSITY
Bethlehem, Pennsylvania
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\[ \{ = 1 \quad k_1, k_2 = 1 \quad A = B \quad \frac{a}{D} = 1 \]

Case III --- Two story, flat roofed portal frames

\[ \{ l \quad k, \# k_2 \quad A \# B \quad \frac{a}{D} = C \]

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ACKNOWLEDGEMENTS

The data contained in the following report was compiled and reviewed at Fritz Engineering Laboratory, Lehigh University (Professor William J. Eney - Director). The author wishes to especially express his deepest appreciation to Professor George C. Driscoll, Jr. without whose supervision, encouragement, and critical review, this study would not have been possible.
SYNOPSIS

The Mechanism Method of analysis, as utilized in plastic design of structural steel, is characterized by the selection and investigation of possible hinge patterns required to produce failure. Each of the possible mechanisms will have associated with it a certain critical load required to cause its formation. That mechanism which forms under the lowest load is the mode by which the structure will eventually fail. The design engineer is continually faced with the problem of omitting sometimes obscure critical mechanisms. It is normally necessary to complete an equilibrium check on the proposed critical case. If the plastic moment is not exceeded at any section within the structure, the proposed failure mechanism is assumed correct.

The object of this study is to establish domains or combinations of load-geometry which will define these failure mechanisms and eventually eliminate the need for the time consuming, tedious moment check.
INTRODUCTION

The design of single and two story, flat roofed, portal frames in accordance plastic theory utilizes the mechanism method of analysis due to the difficulty incurred in construction of correct equilibrium moment diagrams. The final design mechanism would be that mechanism which resulted in the lowest possible load (upper bound theorem) and for which the moment at any section of the frame would be less than the plastic moment. From the viewpoint of the designer, the primary obstacle in frame design was the moment check required to insure that the proposed mechanism was the actual failure mechanism.

Professors George C. Driscoll, Jr. and Robert L. Ketter, under the supervision of Professor Lynn S. Beedle at Lehigh University, Bethlehem, Pennsylvania, initiated a program of study to investigate the intricacies and establish the relationships between load and geometry of structural frames. The information contained herein is but a minor contribution toward that overall objective.

This study commenced with a short literature review of the applicable background information including, primarily, a review of the progress of Dr. Driscoll and Dr. Ketter. Failure domains were then established for the single story
flat roofed portal frame subject to uniform vertical loads and concentrated side loads (wind). A logical progression to the two story flat roofed frame followed. Within the frame under initial study, the moment capacity of all members was equal; the side loads were equal and the story heights were equal. Using the data and computations from these limited cases as a basis of study, the final frame was investigated. Loads and geometry were varied and the equations for $M_p$ derived. These equations were then equated and failure domains were established.
ASSUMPTIONS

1) Standard moment curvature relationship for structural steels applicable

2) No structural instability prior to ultimate load

3) First order theory applicable - equilibrium conditions can be formulated on the undeformed structure

4) Connections provide full continuity for transmission of plastic moment

5) Reduction of plastic moment due to application of shearing and normal forces is neglected

6) Proportional loading
GENERAL CONCEPTS

Failure domains were established throughout this paper by the following general approach:

a) All possible mechanisms were established

b) Equilibrium is formulated and the principal of virtual displacement is utilized on each mechanism to compute the plastic moment capacity (the concept of instantaneous center is used to advantage on the more complex structures).

c) The $M_p$ expressions for individual mechanisms are equated to establish the final domains or regions of significance.

NOTE: The simplifying assumption of replacing the uniformly distributed horizontal loads with a concentrated load at the windward joints is utilized. The simplification leads to slightly conservative results.
CASE I  SINGLE STORY, FLAT ROOFED, PORTAL FRAME

\[ \frac{AwL}{2a} \]

\[ M_p = a \text{ constant} \]

<table>
<thead>
<tr>
<th>MECHANISM</th>
<th>( M_p )</th>
<th>( \frac{M_p}{wL^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \frac{wL^2}{16} )</td>
<td>( \frac{1}{16} )</td>
</tr>
<tr>
<td>II</td>
<td>( \frac{AwL^2}{8} )</td>
<td>( \frac{A}{8} )</td>
</tr>
<tr>
<td>III</td>
<td>( \frac{wL^2}{4} \left( \frac{(e-\alpha)(e+\alpha)}{(e-\alpha)} \right) )</td>
<td>( \frac{1}{4} \left( \frac{(e-\alpha)(e+\alpha)}{(e-\alpha)} \right) )</td>
</tr>
</tbody>
</table>

\[ \alpha = 2 \left[ 1 - \frac{1}{2} + \frac{A}{4} \right] \]
CASE I

Single Story Flat Roofed Portal Frame (Constant $M_p$)

All possible failure mechanisms for the frame under study are tabulated on the following pages. The associated $M_p$ values have been calculated by equating $W_{ext}$ and $W_{int}$, as determined by the standard virtual displacement method and are listed beside their corresponding mechanism. Domains are then established by equating pairs of $M_p/w_l^2$ values.

Example of Domain Establishment

Case I - Equating $M_p/w_l^2$ values for Mechanisms I and III.

$\frac{M_p}{w_l^2}$ (Mechanism I) = $\frac{1}{16}$

$\frac{M_p}{w_l^2}$ (Mechanism III) = $\frac{1}{4} \left[ 3 + A - 2 \sqrt{2 + A} \right]$

$\frac{1}{16} = \frac{1}{4} \left[ 3 + A - 2 \sqrt{2 + A} \right] \quad$ or $\quad 16 A^2 + 24 A - 7 = 0$

Solution by the Quadratic Equation yields $A = \frac{3}{2}$

(Domain line between Mechanisms I and III)

Similarly the $M_p/w_l^4$ value for Mechanisms I and II were equated and the finally established domains are shown on the following graph.
FAILURE DOMAINS FOR CASE I

SIDE LOAD FACTOR

A

COLUMN HEIGHT FACTOR
INSTANTANEOUS CENTER CONFIGURATION FOR CASES II & III
CASE II TWO STORY, FLAT ROOFED FRAME

\[ M_p = a \text{ constant} \]

<table>
<thead>
<tr>
<th>MECHANISM</th>
<th>( M_p )</th>
<th>( M_p/\text{WL}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \frac{A_w L^2}{8} )</td>
<td>( A )</td>
</tr>
<tr>
<td>II</td>
<td>( \frac{w L^2}{16} )</td>
<td>( \frac{1}{16} )</td>
</tr>
<tr>
<td>III</td>
<td>( \frac{A_w L^2}{4} )</td>
<td>( A )</td>
</tr>
<tr>
<td>IV</td>
<td>( w )</td>
<td>( n )</td>
</tr>
<tr>
<td>V</td>
<td>( \frac{3}{2} \frac{A + \alpha}{1 - \alpha} ) ( w L^2 )</td>
<td>( \frac{3}{2} \frac{A + \alpha}{6 + 4\alpha} )</td>
</tr>
</tbody>
</table>

\[ \alpha = 3 \pm 3 \sqrt{\frac{2 + A}{3}} \]
CASE III  TWO STORY, FLAT ROOFED FRAME (Variable loads & geometry)

DEPENDENT VARIABLES $M_p, k_1, k_2$

INDEPENDENT VARIABLES $wL, a, b, A, B, I$

**MECHANISM**

<table>
<thead>
<tr>
<th>MECHANISM</th>
<th>$M_p$</th>
<th>$M_p/wL^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\frac{3wL^2}{16k_2}$</td>
<td>$\frac{3}{16k_2}$</td>
</tr>
<tr>
<td>II</td>
<td>$\frac{wL^2}{16k_1}$</td>
<td>$\frac{1}{16k_1}$</td>
</tr>
<tr>
<td>III</td>
<td>$\frac{BwL^2}{8k_2}$</td>
<td>$\frac{B}{8k_2}$</td>
</tr>
<tr>
<td>IV</td>
<td>$\frac{AwL^2}{8} + \frac{BwL^2}{8b}$</td>
<td>$\frac{A}{8} + \frac{Ba}{8b}$</td>
</tr>
<tr>
<td>V</td>
<td>$\frac{M_p}{wL^2} = \frac{A + B(1+\xi) + 2\xi + \omega}{4} \left[ 1 + \frac{k_1}{1-\alpha} + \frac{k_2}{1-\alpha} \right]$</td>
<td>$\omega = \frac{1 + \frac{k_1}{k_2} + \frac{k_2}{k_1} + \frac{1}{L}}{Z}$</td>
</tr>
</tbody>
</table>

$Z = \sqrt{4k_2 + 4k_1w + 4k_2 + k_1 + \frac{1}{k_2} + \frac{1}{k_1}} - \frac{2k_1 + 4k_2}{Z}$
CASES II AND III

Since Case II is nothing more than a special limitation of Case III the ensuing discussion is applicable to both. The possible failure mechanisms listed were analyzed in accordance with the outline listed under "General Concepts." The mechanisms are listed with these corresponding $M_p$ values. The $\frac{M_p}{W}$ expressions were then equated in a manner similar to the example previously discussed. The resulting domains for Case II are shown in the following graph.
FAILURE DOMAINS FOR CASE II

Side load factor

Column Height Factor
CONCLUSION

The original purpose of this report, as previously stated, was to establish failure domains for the one and two story, flat roofed, single bay frames. Although Failure Domain Charts are plotted for Cases I and II the domains for the two story frame (General Terms) are not plotted. The expressions can be equated and the failure domains be established for almost any combination of load and geometry. Due to the presence of ten variables, the plotting of domain charts can only be of use if predetermined practice values are assigned to certain of the variables. As an example the factor $\ell$, which relates the uniformly distributed vertical loads at floor level and roof level can be assigned the value $\frac{1}{2}$ if the roof and floor loading are selected from most standard building codes as 20 and 40 lbs. per square foot respectively. Similar variable elimination can be accomplished and a series of domain charts applicable to specially limited cases can be plotted.

The expansion of the concept of establishing failure domains to include the multi-story, multi-bay case is of questionable value. The additional variables incurred would very likely result in a unduly intricate, tedious and involved mathematical analysis.
NOMENCLATURE

Mp = fully plastic moment value.

a, b = none-dimensional parameter, relating the height of a column to the span length.

f = function value.

k, kL = none-dimensional parameter, relating the fully plastic moment values of members.

w = distributed vertical load per unit length.

A, B = none-dimensional parameter, relating the horizontal force acting on a structure (or the hypothetical "overturning" moment of one part of a structure on the adjacent part) to the vertical loads. It is assumed that A results in positive work being done as the structure fails.

L = length measurement. Can be total span length or fractional part of it.

C = constant.

We xt = external work associated with a virtual displacement of an assumed mechanism.

Wint = internal work associated with a virtual displacement of an assumed mechanism.

α, θ = none-dimensional parameters, defining the distance to the plastic hinge in the rafter of a structure.

θx = virtual rotation.

f = none-dimensional parameter relating vertical loads.
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