Experimental and Numerical Development of the core-Drilling Method for the Nondestructive Evaluation of In-situ Stresses in Concrete Structures

Michael McGinnis
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Experimental and Numerical Development of the Core-Drilling Method for the Nondestructive Evaluation of In-situ Stresses in Concrete Structures

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ATLSS Report No. 05-05

October 2006
ACKNOWLEDGEMENTS

This research was funded by the Pennsylvania Infrastructure Technology Alliance, the Precast/Prestressed Concrete Institute and the Center for Advanced Technology for Large Structural Systems at Lehigh University. Additional financial support was provided by Trilion Quality Systems, Dywidag Systems International, Accurex Measurement Systems, the Gibson Family, and Lehigh University. The financial support noted above is gratefully acknowledged.

A special thanks are due to Timothy Schmidt for the generous donation of his time to perform the digital image correlation measurements that were part of this work. Without his generosity this work would never have been completed.
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ABSTRACT

The core-drilling method (CDM) is a technique to assess the in-situ stresses in concrete structures. In the method a circular core hole is cut into the concrete in a structure and the surface displacements that occur as a result are measured. These displacements are used to estimate the in-situ stresses through elasticity theory.

The current research investigates the effects on the CDM of three issues that were previously unexplored: (1) core-drilling water induced swelling displacements; (2) differential concrete shrinkage; and (3) steel reinforcement proximate to a cored hole in the concrete. These issues were probed analytically and numerically with the finite element method and other techniques, and experimentally in core-drilling tests of concrete plates. Displacements in the experiments were measured with digital image correlation.

The relationships relating in-situ stresses to relieved displacements proposed by previous researchers accurately describe the behavior that occurs in the CDM, and digital image correlation is an acceptable measurement technique for this application. The effects of swelling displacements, shrinkage, and reinforcement must be considered in the calculation of in-situ stresses to obtain acceptable accuracy. The average error in the experiments dropped from 28.4% to 9.5% if these factors were addressed. Any of the three factors may influence calculated in-situ stresses, depending on the condition and history of the interrogated concrete structure. Parameters that determine which of these factors are important are the age, sorptivity and thickness of the interrogated concrete element, relative humidity, and the size and proximity of reinforcement. Absorption of drilling water by the concrete around a core hole causes swelling of this concrete and swelling displacements. These displacements introduce fictitious apparent stresses that appear primarily as hydrostatic tension. An approach to correct for these apparent stresses was developed. The apparent stresses from differential shrinkage also appear as hydrostatic tension. Differential shrinkage does not significantly affect the CDM except in certain circumstances. Proximate reinforcement causes a significant under-prediction in stress if the reinforcement is neglected, however the effect reduces significantly with increasing concrete cover or increasing clear spacing to the nearest bar.
CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Reliable information about the in-situ state of stress in the concrete in an existing structure is often needed as part of the evaluation of the structure. The evaluation may be performed as part of the determination of the load rating for the structure, or to support a decision about the repair or replacement of the structure. As just one example, information about the in-situ state of stress in a prestressed concrete bridge girder can be used to estimate the effective prestress remaining in the girder. This information is useful in predicting the service load behavior and ultimate strength of the girder.

The current research involves a technique that can be used to assess the in-situ stresses in a concrete structure called the core-drilling method. In the core-drilling method a circular core hole is cut into the concrete in a structure, and the surface displacements that occur in the concrete as the hole is cut are measured. These measured displacements are then related to the in-situ state of stress in the structure. Figure 1.1 illustrates the method. In the figure, three points, \(i\), \(j\), and \(k\) are shown on the surface of a test object. As the core hole is drilled, each point undergoes a relieved displacement, \((u, v)\) relative to the center of the hole, where \(u\) and \(v\) are the radial and tangential components of the overall displacement respectively (additional information shown in Figure 1.1 is discussed in the Section 2.2.5.1). Measured displacements are the relative displacement between any two of these three points, and are denoted with a capital \(U\). These measured displacements are then related to the in-situ stresses in the structure prior to drilling the hole. In practice the location and number of measurement points is somewhat arbitrary. However, at least as many measurements as unknown stresses must be captured, and the measurement points should be located fairly close to the core hole to increase the magnitude of the observed displacements, and thus the accuracy of the technique.

The method is similar to the ASTM hole-drilling drain-gauge method (ASTM 837-01e1 2001) that consists of measuring strains at the surface of a specimen as a hole is drilled. The ASTM hole-drilling strain gauge method is often used to determine residual stresses in homogeneous materials such as metals. However the hole-drilling strain gauge method is not applicable to concrete structures because of the heterogeneous nature of concrete.
Hole drilling methods for assessing stresses in structures have been pursued for
decades (Mathar 1934, Soete and Vacrombrugge 1950), and many researchers have
contributed to the extremely wide body of literature describing these methods. Turker
(2003) summarizes aspects of hole-drilling research for use with the core-drilling
method applied to concrete structures, formalizing the problem in terms of measured
displacements at specific location in configurations around a core hole. In keeping
with the historical hole-drilling approach, it is assumed that a small hole is drilled
completely through a thin infinite plate that is stressed only in the plane of the plate
and comprised of a linear elastic, isotropic, homogenous material. The relationships
between the displacements that occur as a result of drilling a core hole and the in-situ
stresses in the concrete are determined through idealized elasticity relationships.
Turker (2003) also documents studies that account for differences in the behavior due
to coring objects with finite widths and depths, coring objects whose stresses vary
through the depth, and coring blind holes rather than through holes. Turker (2003)
was primarily an analytical and numerical study of the application of the core-drilling
method to concrete; no experiments were performed.

There are several aspects that have not previously been addressed that complicate
determination of in-situ stresses with the core-drilling method in real structures.
Water used for lubricating the coring drill may be absorbed by the concrete
surrounding the hole. The absorption can cause the concrete around the hole to swell
slightly, resulting in swelling displacements difficult to differentiate from those due to
stress relief. Differential shrinkage of concrete can result in differential shrinkage
stresses that perturb the method. Steel reinforcement typically present in structural
concrete is significantly stiffer than the surrounding concrete and violates the
assumption of isotropic behavior noted above. The following paragraphs describe
each of these aspects in more detail.

The typical current practice for cutting a core hole in concrete involves flushing the
core hole with water to cool and lubricate the coring drill bit, flush coring debris from
the core hole, and minimize air-borne dust. Hardened concrete typically swells when
exposed to water (Neville 1981). However this swelling is not accounted for in the
core-drilling method equations for relating relieved displacements to in-situ stresses.
This swelling induces displacements around a core hole that are difficult to
differentiate from displacements due to stress relief, resulting in errors in in-situ stress
predictions when applying the method.

It is well known that hardened concrete will shrink upon drying. The interior of a
concrete member will remain at a higher moisture content than the exterior for some
time. The resulting moisture content gradient causes differential shrinkage, with the
outer surface of the concrete shrinking at a faster rate than the inner portions. This
interior region provides restraint to the free shrinkage that would take place if the
entirety of the concrete member was at the same moisture content. This restraint in
turn causes a stress gradient to develop through the thickness of the concrete member.
Thus any displacements measured around a core hole will reflect not only relief of the
stresses due to known applied loads, but also relief of these unknown differential shrinkage stresses.

As mentioned, one of the primary assumptions made in the development of the core-drilling method is that concrete is an isotropic, heterogeneous material. This simplification is clearly not exact for the case of steel reinforced concrete, as the steel and concrete have different material properties. Because steel is much stiffer than concrete, steel reinforcement can constrain the displacements that take place during drilling, resulting in an under-prediction in stress when reinforcement is in the vicinity of the core hole.

1.1.1 Statement of Objectives

The objective of the current research was to investigate the effects of core-drilling water induced swelling, differential shrinkage and proximate steel reinforcement on the core-drilling method. A further objective was to experimentally investigate the core-drilling method, probing the simplified behavior summarized in Turker (2003) as well as these more complex issues. The results of the current research should allow for the practical application of the core-drilling method to concrete structures.

1.2 ORGANIZATION OF THE DISSERTATION

The remainder of this dissertation is organized into 7 chapters (Chapters 2-8) each discussing an important aspect of the research.

Chapter 2 provides the technical background necessary for an understanding of the remaining chapters. It provides a review of other methods for measuring in-situ stresses in concrete and other materials, an overview of the theoretical basis of the core drilling method, and a summary of experiments performed as part of the current work to evaluate the applicability of digital image correlation to the core-drilling method. In addition, Chapter 2 provides a review of several concepts involving concrete behavior that are important in relation to the core-drilling method, such as moisture movement in concrete materials and concrete stiffness properties.

Chapter 3 discusses the effect of concrete swelling due to core-drilling water on the core-drilling method. Absorption of the water used as a lubricant during the drilling process by the concrete immediately surrounding the drilled hole can cause this concrete to swell, resulting in displacements that can be difficult to differentiate from those due to stress relief and that serve to distort calculated in-situ stresses. A finite element approach was used to quantify the extent and affect of this swelling on the method. The suitability of this approach was demonstrated by using the analytical findings to adjust the results of a previous hole drilling investigation in concrete plates performed by Buchner (1989).
Chapter 4 discusses the effects that differential shrinkage has on the core-drilling method. As a concrete element dries out and reaches moisture equilibrium with the surrounding environment, a moisture gradient develops through the thickness of the element due to the relative slowness of moisture movement within the concrete. Aging concrete experiences shrinkage that is directly tied to this loss of moisture and is hence differential in nature due to the gradient in moisture content that develops. Differential shrinkage stresses develop as the free shrinkage of the concrete on the exterior of an element (at a relatively low moisture content) is contrained by concrete on the interior that remains at a higher moisture content for some time. A hole drilled into a concrete member thus relieves stresses due to applied physical loads as well as any stresses that develop due to differential shrinkage. A finite element investigation is presented that predicts the development of differential stresses based on input parameters such as the relative humidity of the surrounding environment and the ultimate shrinkage strain of the concrete. Then the effects of drilling through the resulting differential stress field on the core-drilling method are quantified and discussed.

Chapter 5 discusses the ways that steel reinforcement within concrete structural elements affects determination of in-situ stresses with the core-drilling method. Using the finite element method it is shown that steel reinforcement that is in close proximity to a drilled core hole can cause inaccuracies in calculated in-situ stresses due to the stiffness mismatch between the reinforcement and the surrounding concrete. Calculated in-situ stresses using the core-drilling method may be adjusted for the effects of proximate steel reinforcement using the results presented in this chapter.

Chapter 6 presents the results of experiments performed to validate the core-drilling method. Concrete plate specimens were constructed and then the stresses within these specimens while under load were determined using the core-drilling method. Displacement measurements were performed using digital image correlation. Saturated and drying specimens were tested in order to isolate the affects of core-drilling water and moisture movement induced differential shrinkage. In-situ stresses from specimens with and without steel reinforcement were determined. Calculated in-situ stresses were within 9.5% of the applied stresses.

Chapter 7 discusses a sensitivity study that was performed to further probe the experimental and analytical results of the preceding chapters. In particular the relative importance of some of the phenomena covered in detail in the previous chapters is evaluated. Recommendations are provided in regards to future testing procedures.

Chapter 8 provides the conclusions derived from the current work. Avenues of necessary future work are also discussed.
1.3 SUMMARY OF SIGNIFICANT FINDINGS

The analytical formulations relating in-situ stresses to relieved displacements proposed by previous researchers accurately describe the behavior that occurs in the core-drilling method. Digital image correlation is an acceptable measurement technique for application of the core-drilling method to concrete. The effects of core-drilling water induced swelling, differential shrinkage and proximate reinforcements must be considered in the calculation of in-situ stresses using the core-drilling method to obtain acceptable accuracy. The average error in the experiments was 9.5% if these issues were addressed but only 28.4% if they were neglected. Any of the three influencing factors noted above may be the most influential on calculated in-situ stress results, depending on the condition and history of the concrete structure being investigated. The most important parameters that control which of these aspects has the greatest influence are the age of the concrete at test, the relative humidity of the structural environment, the thickness of the structural element being considered, the sorptivity of the concrete and the proximity and diameter of the nearest reinforcing bar.

Absorption of water by the concrete around a core hole causes swelling of this concrete and swelling displacements. The apparent stresses due to core-drilling water appear primarily as hydrostatic tension stresses. Correction to account for the apparent stresses due to core-drilling water can be made using the approach developed herein which requires estimates of the sorptivity and ultimate shrinkage strain of the concrete. Variability in the sorptivity parameter across different concretes means that it must be assessed for interrogated concrete.

The apparent stresses from differential shrinkage also appear as a hydrostatic tension stress. Differential shrinkage stresses will not significantly effect an investigation of in-situ stresses using the core-drilling method except in certain circumstances. Using an estimated value of ultimate shrinkage strain calculated using the GL2000 method (Gardner and Lockman 2001) gave adequate results for the apparent stresses due to differential shrinkage and core-drilling water.

The presence of reinforcement close to a core hole (nearer than 35 mm) and close to the surface of the concrete (nearer than 75 mm) causes a significant under-prediction in stress using the core-drilling method, if the reinforcement is neglected. The effect of proximate reinforcement reduces quickly and significantly with either increasing concrete cover or increasing clear spacing to the nearest bar.

1.4 NOTATION

Below is a detailed list of the notation that is used throughout this dissertation.

\( a \) = hole radius
\( a_{ij} \) = dimensionless calibration coefficients for the incremental core-drilling method
$d_b =$ reinforcing bar diameter
$d_c =$ depth of concrete cover measured from the concrete surface to the surface of the nearest reinforcing bar
$d_x, d_y, \theta_k =$ rigid body horizontal, vertical and rotational displacements
$f =$ porosity
$f_{cm28} =$ 28-day specified concrete strength
$h =$ blind hole depth
$H =$ specimen depth
$i =$ total amount of liquid absorbed during a sorptivity test
$m =$ measurement circle radius
$r =$ distance of any point to center of hole
$r_w =$ wetted radius
$t_s =$ specimen thickness
$r_w =$ wetted thickness
$r_{MAX} =$ measurement radius at maximum apparent stress
$u, v =$ radial and tangential relieved displacements
$A, B, C, F, G, M, J =$ core-drilling material and geometric constants
$A_m, a_k, a'_k =$ Fourier series coefficients
$A_s, A_c, A_g =$ areas of steel, concrete and gross section in transformed section approach
$A_{75}, A_{100}, A_{125}, A_{MAX} =$ apparent stress (due to core-drilling water) at various measurement radii, $m$
$AS =$ apparent stress due to differential shrinkage
$B_{100top}, B_{100bottom} =$ apparent stresses due to core drilling water (from the Chapter 3 Portion B models) on the top and bottom surfaces at a measurement radius of 100 mm
$D =$ capillary diffusivity
$E =$ modulus of elasticity
$E_{cv}, E_{conc} =$ concrete modulus of elasticity
$G_A =$ influence function
$K_x, K_y, K_z =$ stress gradients in the $x$, $y$ and $z$ directions
$K(\theta) =$ generalized, unsaturated permeability function
$M_c =$ moisture content
$N, T =$ normal and tangential stresses around a core hole
$P =$ capillary pressure
$R =$ specimen maximum radius (size)
$RH =$ relative humidity
$S =$ sorptivity
$S_R =$ reinforcing bar spacing
$S_C =$ clear spacing from the edge of a core hole to the nearest reinforcing bar
$U =$ measured displacement
$\alpha =$ angle measured counter-clockwise from the $x$-axis to the point of interest
$\alpha_{ch} =$ degree of cement hydration
$\alpha_i, \alpha_j, \beta, \theta_i, \theta_j =$ geometrical parameters (see Figure 1.1)
$\alpha_w =$ swelling strain
\( \alpha_s \) = shrinkage strain
\( \varepsilon_{ult} \) = ultimate shrinkage strain
\( \chi \) = material constant for plane stress and plain strain
\( \phi \) = Boltzmann variable
\( \mu \) = modulus of rigidity
\( \nu \) = Poisson’s ratio
\( \theta, \theta_r \) = water content, normalized water content
\( \varphi(z), \kappa(z), \psi(z), \Phi(z), \Psi(z) \) = analytic functions of complex variable
\( \rho \) = density
\( \sigma_x, \sigma_y, \tau_{xy} \) = in-plane normal and shear stresses
\( \sigma_{max}, \sigma_{min} \) = maximum and minimum principal stresses
\( \Psi \) = capillary potential
Figure 1.1 – Illustration of the core-drilling method showing displacement measurement between points $i$ and $j$
CHAPTER 2

BACKGROUND

2.1 INTRODUCTION

This chapter presents background information relative to this study. Three distinctly different types of information are presented, all of which are necessary to understand the entirety of the work presented in this dissertation. Section 2.2 reviews several of the previous ways in which researchers have attempted to measure in-situ stress in concrete and other materials. It is provided to relate the context in which the core-drilling method fits within the broader literature of the field of stress testing. The analytical development of the core-drilling method is also reviewed in this section. Section 2.3 describes the measurement techniques considered for use in determining the relieved displacements in the core-drilling method. Included in this section is a description of the preliminary laboratory experiments that were performed as part of the current research to evaluate the most promising of these techniques. Finally, Section 2.4 contains background information on important parameters of concrete and its constituents. This section includes an overview of topics such as moisture movement in concrete materials, shrinkage of concrete, and stiffness properties of aggregates and mortars, and is necessary for further development in these areas in later chapters.

2.2 METHODS OF MEASURING STRESS IN MATERIALS AND STRUCTURES

The following methods for measuring in-situ stress are reviewed in this section:

- ASTM hole-drilling strain-gauge method
- Indirect and direct methods of measuring in-situ stresses in concrete via hole drilling methods developed by Buchner (Buchner 1989) and Mehrkar-Asl (Mehrkar-Asl 1988) at the University of Surrey
- Concrete core trepanning technique
- Core-drilling method
- Other miscellaneous techniques

2.2.1 ASTM Hole-Drilling Strain-Gauge Method

The history of hole-drilling methods for the determination of stresses in objects goes back decades. Mathar (1934) was one of the early pioneers in this field and proposed using a hole small enough to allow for drilled parts to retain their ability to perform their intended function, and coupled hole drilling with strain measurement using a
mechanical tensometer. Soete and Vancrombugge (1950) and Rendler and Vigness (1966) were among the first to apply the method using strain gauges.

Since the inception of these general hole-drilling techniques, many have contributed to advancement and refinement of the method. Hu (1986) gives a good historical review of hole-drilling methods. Because the literature in this area is vast, a list of some areas explored by others is given here, the interested reader is directed to the noted references for further information.

- **Historical** – Mathar (1934), Rendler and Vigness (1966)
- **Accuracy assessment** – Beaney and Proctor (1974), Sasaki et al. (1997)
- **Ring coring (trepanning methods)** – Milbradt (1951), Bohm et al. (1988), Ajovalasit et al. (1996)
- **Overcoring methods** (Bickel 1985)
- **Application to orthotropic materials** – Schajer and Yang (1994), Bert and Thompson (1968), Lake et al. (1970), Prasad et al. (1987a, 1987b)
- **Application to steel reinforcement in concrete** (Owens 1993)

The above list is not exhaustive, as noted hole drilling methods have a rich history. Determination of stresses using a small drilled hole and an accompanying strain gauge rosette has been standardized as the ASTM hole-drilling strain-gauge method which is described below.

The ASTM hole-drilling strain-gauge method (ASTM E837-01e1 2001) is a widely known method primarily used for determining residual and other surface stresses in metals. A detailed overview of the method including its application to stresses that change with depth is provided in Turker (2003), and a brief summary is provided here.

In the method a strain gauge rosette of the type shown in Figure 2.1 is affixed to the surface of the specimen. A small (1-2 mm diameter) hole is drilled into the material at the center of the rosette and the strains that occur as a result of the stress relief are captured. Via elasticity theory the relieved strains are related to the stresses in the specimen. The method is reported to give good results for the following conditions:

- Isotropic, linear elastic material
- Residual stresses do not exceed 50% of the yield strength of the material
- Variation in stress within the boundary of the hole is small
- Stresses do not vary significantly with depth
- Plane stress conditions hold
• The specimen is large compared to the hole size

The derivations involved in relating stress and relieved strain in the method are performed assuming the drilled hole is a through hole in an infinite plate. The equation for the radial strain relaxation in the case of a through hole is

\[ \varepsilon = (A^* + B^* \cos \alpha)\sigma_{\max} + (A - B \cos 2\alpha)\sigma_{\min} \]  

where

\[ A^* = -\frac{1 - \nu}{2E} \left( \frac{a}{r} \right)^2 \]
\[ B^* = -\frac{1 - \nu}{2E} \left( \frac{4}{1 + \nu} \left( \frac{a}{r} \right)^2 - 3 \left( \frac{a}{r} \right)^4 \right) \]

and \( a \) is the hole radius, \( E \) the modulus of elasticity, \( \nu \) Possion’s ratio, \( \alpha \) the angular coordinate measured clockwise from the maximum principal stress and \( r \) the radius at which the strain is measured. The three unknowns in Equation (2.1), \( \sigma_{\max}, \sigma_{\min} \) and \( \alpha \) are found using the strains at the three gauges in the rosette, resulting in

\[ \sigma_{\max}, \sigma_{\min} = \frac{\varepsilon_1 + \varepsilon_3}{4A^*} \pm \frac{\sqrt{(2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_3)^2}}{4B^*} \]
\[ \beta = \frac{1}{2} \tan^{-1} \left( \frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3} \right) \] 

where \( \beta \) is angular direction of the maximum principal stress as measured from gauge 1, and \( \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) are the strain readings at the three gauges.

In most practical applications of the method the hole is a blind hole rather than a through hole. Although a closed form analytical solution is not available for the blind hole case, calibration constants have been developed that allow the method to be applied to the blind hole case. Schajer (1988a, 1988b) has proposed calibration constants that are essentially independent of material properties in the following form

\[ \overline{A^*} = -\left( \frac{1 + \nu}{2E} \right) \overline{a} \]
\[ \overline{B^*} = -\left( \frac{1}{2E} \right) \overline{b} \] 

where \( \overline{a} \) is material independent and \( \overline{b} \) depends only weakly on Possion’s ratio. These dimensionless coefficients are tabulated in the ASTM standard.
Although the standard ASTM hole-drilling strain-gauge method is based on the assumption that the in-situ stresses do not vary through the depth of the object, numerous researchers have worked on methods in which the method is applied incrementally to determine a profile of the in-plane stresses through depth. Three popular methods of analysis of the problem are the power series method (Schajer 1981), the integral method (Schajer 1981, 1988a, 1988b) and the influence function method (Beghini 1998, 2000); all are summarized in depth in Turker (2003). The incremental strain method (Kelsey 1956) and average stress method (Nickola 1986) have been determined by Schajer (1988a) to be simple approximations to the integral method (Schajer 1981, 1988a, 1988b). In each of these methods, finite element analysis is used to determine calibration constants that relate in-situ stresses at particular depths (or as functions of depth) to relieved strains measured at the surface of the interrogated object. In any case, it becomes increasingly difficult to make determinations of in-situ stress the further from the surface of the object that the determination is to be made.

### 2.2.2 Indirect Hole-Drilling Method for Determining Stresses in Concrete

Buchner (1989) presented a method similar to the core-drilling method for measuring in-situ stresses in concrete structures. A hole is drilled into the concrete specimen and then the resulting deformations are related to the in-situ stresses in the specimen. The method was evaluated through laboratory testing of more than 10 concrete plate specimens of various ages. Details of the Buchner tests are given below.

The plates tested by Buchner (1989) measured approximately 1 x 1 x 0.1m. The plates were stripped from the formwork approximately 20 hours after casting and allowed to cure under plastic for 1.5 to 2 days before being moved to a storage room and stored in an upright position that allowed drying from both faces. The plate dimensions and age at testing are contained in Table 2.1; the water cement ratio of the mixture was 0.4. Although Buchner is slightly ambiguous on this point, the shrinkage strain of the tested concrete was estimated previously by Buchner to be 520E-6.

The modulus of elasticity of each plate was determined experimentally by several procedures. Concrete prisms were cast in tandem to the plates to provide compression specimens that were tested in accordance with British Standard BS1881 (BS 1881-121 1983) to measure the elastic constants. The elastic modulus was also calculated using readings from the DEMEC targets across the midsection of the plate spaced at 100 mm, as visible in Figure 2.2. Readings from the DEMEC and vibrating wire strain gauge rosettes were also employed to calculate the modulus of elasticity. The average of all these experimental values of $E$ was reported in Buchner (1989).

Loading of the plates was accomplished in an upright position within a steel testing frame that consisted primarily of a braced cross-beam supported above a concrete reaction floor. A series of 199.3 kN capacity hydraulic jacks suspended from the cross-beam were used to load the plates. To ensure load uniformity, a thick steel load redistribution plate was placed between the plate and the jacks, and plastic padding
was used to level the edges of the plates. Biaxial loading of a plate was accomplished through the use of a self stressing frame consisting of two pairs of threaded rods connected to two 498.2 kN jacks on one side of the plate and a steel load distribution beam on the opposite side of the plate. A pivoting system was incorporated to ensure that load was applied only in the plane of the plates.

The plates were wet cored with two different sizes of coring bits, 150 mm ($a = 75$ mm) and 75 mm ($a = 37.5$ mm) and instrumented around the hole with radial arrays of vibrating wire and demountable mechanical strain (DEMEC) gauges. An orthogonal grid of DEMEC targets with coverage of the entire plate was also used, in addition, deformation readings on the core material itself were performed. The measurement devices employed by Buchner are shown in Figure 2.2. Although Buchner details the results from all of these numerous measurement devices, only the results from the tests with $a = 75$ mm and from the demountable mechanical strain gauges arrayed in a measurement circle with radius $m = 100$ mm are reviewed here.

Buchner calculates in-situ principal stresses from the strains derived from measuring across the hole at the DEMEC target locations. Readings were taken immediately after coring, and repeated for several hours subsequent. The results reviewed here are the average of the reading from immediately after coring and the reading taken 1 – 2 hours after coring. Stresses calculated at these two times (immediately after coring and 1 hour later for instance) differed by less than 0.5 MPa for all of the Buchner plates considered herein except those that were loaded bi-axially (VII, VIII and IX) where the differences between the two times were somewhat larger. Data reported at these early times should minimize the influence of other time dependent phenomena, such as creep. In a different, preliminary plate test, Buchner presented stress results on both faces of a plate (top and bottom), and reported

“...tensile strains created by water absorption on the front face, caused a warping effect which resulted in greater compressive strains measured on the back face. This implied that a bending process was induced due to the coring water.”

Each stress reported by Buchner is the average of principal stresses calculated on the front and back faces of the plates.

Table 2.1 summarizes the important experimental parameters for each of the 9 Buchner plates. The SRSS error column is the square root of the sum of the squared values for the errors in $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$. Where the applied $\sigma_{\text{max}}$ stress was equal to zero, the relative difference in the measured $\sigma_{\text{max}}$ value is referenced to the applied $\sigma_{\text{min}}$ value.

An examination of the measured data in Table 2.1 shows that, for every plate, the calculated stresses appear to differ from the applied stresses primarily by a hydrostatic tension stress. These discrepancies are more fully explored in Chapter 3 and Chapter 4.
Buchner also applied the technique during various site evaluations with some success (Buchner 1989, Buchner 1987). Buchner’s major conclusions were that: (1) the method was feasible and gives repeatable results; (2) larger cores \((a=75\,\text{mm})\) were less affected by shrinkage and coring water than smaller \((a=37.5\,\text{mm})\) cores; (3) it is easier to capture the difference in the principal stresses \((\sigma_{\text{max}} - \sigma_{\text{min}})\) than to capture the principal stresses themselves; and (4) better accuracy was achieved when averaging results calculated from deformations on the concrete core itself with deformations across the hole, rather than using either set of results independently.

### 2.2.3 The Direct Hole-Drilling Method for Determining Stresses in Concrete

Merhkar-Asl (1988) performed research on a technique referred to as the direct hole-drilling method in conjunction with Buchner (1989). The method is similar to that of Buchner except that the passive technique of measuring deformations around a drilled hole is coupled to an active technique involving a hydraulic jack. A circular jack was designed and inserted into the drilled core hole. The jack was energized to provide a restoring force which served to return the measured deformations around the hole to zero. Use of the hydraulic jack also allows for a determination of stiffness properties of the concrete in-situ. Merkhar-Asl has used this technique in conjunction with that of Buchner in a series of field evaluations (Merkhar-Asl 1994, Merkhar-Asl 1996, Brookes et al. 1990).

### 2.2.4 The Concrete Core Trepanning Technique

Kesevan, Ravisankar, Parivallal and Sreeshylam (Kesevan et al. 2005) have proposed an in-situ stress evaluation technique for concrete that involves bonding a strain gauge to the surface of the concrete and then coring a hole around the bonded gauge. They termed their method the concrete core trepanning method. The gauge should be aligned with the direction of maximum stress. An advantage to the method is that the strains released on the core are significantly larger than those released around the periphery of a core hole. The researchers determined that for the 50 mm diameter core hole size investigated, 30 mm long strain gauges gave the best results, and that the maximum strains were released at a drilled depth of approximately 30 mm. An examination of the released strain versus depth plots provided in Kesevan et al. (2005) shows the possibility that differential shrinkage stresses were present in the concrete tested, since drilling beyond a depth of 30 mm actually reduced the relieved strains at the surface, although the reference paper does not address this observation. In the concrete core trepanning technique, the lead wires from the strain gauge must be disconnected from the data recording equipment and carefully bundled in a manner that eliminates damaging of the wires when the core hole is being drilled.

### 2.2.5 The Core-Drilling Method

Turker (2003) presents an analytical development of the core-drilling method which reformulates the ASTM hole-drilling strain-gauge method in terms of displacements measured in specific configurations rather than in terms of strains measured with a rosette of strain gauges. The method is essentially similar to the ASTM hole-drilling
strain-gauge method, except that displacements rather than strains are the measured deformation quantity. The following is a summary of Turker (2003).

2.2.5.1 Development of Relieved Displacement Equations

Figure 2.3(a) shows a core hole drilled in a structure under stress with the hole surface subjected to stresses equal to those that existed before the hole was drilled. The equilibrium of the body thus remains unchanged from prior to the hole drilling. In Figure 2.3(b), equal and opposite stresses to those on the hole surface of Figure 2.3(a) are applied at the core hole surface. The loading of Figure 2.3(b) can be superposed on Figure 2.3(a), resulting in the stress state after the hole is drilled, Figure 2.3(c). Thus the loading and corresponding displacements of Figure 2.3(b) are comparable to the relaxation caused by drilling the hole. In other words, the displacements caused by the loading in Figure 2.3(b) are the relieved displacements.

Elasticity methods treating a small through hole in an infinite, thin plate are used to determine the relationship between the loads and displacements of Figure 2.3(b). Assumptions made in the derivations presented here are that the material is linear elastic, isotropic, homogenous, and that the load is distributed uniformly through the plate thickness. The problem is treated as a two dimensional problem of linear elasticity and solved for plane stress and plane strain assumptions, similar to the approach of the ASTM hole-drilling strain-gauge method except that displacements, rather than strains are the quantities of interest. Turker (2003) incorporates finite elements to investigate the validity or consequences of many of these assumptions, such as the effects of blind holes, the effects of plates of finite size, and the effects of stresses that vary through the thickness of the plate. For the assumptions relating to specimen and hole geometry Turker derives correlation coefficients similar in nature to those provided with the ASTM method. Turker also presents the incremental core-drilling method to evaluate stresses that vary through the depth of an object. The incremental core-drilling method is described in Section 2.2.5.3.

This section treats two related stress states in the plane of the plate, Case 1 and Case 2 of Figure 2.4. Case 2 shows a stress state that is linearly varying in-plane, with constant shear stresses. Case 1 degenerates from Case 2 if the normal stress gradients are taken to be zero. Derivations for Case 1 are presented in detail herein, for Case 2 a summary is presented; the interested reader is directed to Turker (2003).

The two dimensional elasticity problem is solved using the potential function of complex method as outlined by Muskhelishvili (1954). The governing bi-harmonic equation for an isotropic material

\[ \nabla^4 \phi = \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} \]  

\[ (2.4) \]
can be solved by finding a bi-harmonic function, \( \phi(x,y) \) that satisfies the boundary conditions. If \( \phi(x,y) \) is expressed in terms of analytic functions of complex variable

\[
\phi(x,y) = \Re[z\phi(z) + \kappa(z)]
\]  

(2.5)

then Muskhelishvili’s normal and tangential displacement and stress equations for a polar coordinate system are given as

\[
2\mu(u + iv) = e^{-ia}[\chi\varphi(z) - z\varphi'(z) - \psi(z)]
\]  

(2.6)

\[
N - iT = \Phi(z) + \Phi(z) - e^{2ia}[z\Phi'(z) + \Psi(z)]
\]  

(2.7)

where

\[
\varphi(z) = \int \Phi(z) dz
\]

\[
\psi(z) = \int \Psi(z) dz = \frac{d\kappa}{dz}
\]

\[
\chi = \frac{3 - \nu}{1 + \nu}
\]

(2.8)

\[
\chi = 3 - 4\nu
\]

(2.9)

for plane stress and plane strain respectively.

With static equilibrium, it can be shown that for Case 1, the stresses around any circle are

\[
\sigma_x = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha
\]  

(2.8)

\[
\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha
\]  

(2.9)

The tractions, \( N \) and \( T \), around the core hole can also be expressed in complex Fourier expansion as

\[
N - iT = \sum_{-\infty}^{\infty} A_n e^{-ina}
\]  

(2.10)

where the constants \( A_n \) are found by equating terms of like exponents with their counterparts in Equations (2.8) and (2.9), assuming that the tractions applied to the hole are the inverse of the stresses expressed in Equations (2.8) and (2.9). For Case 1, the constants determined are
\[ A_0 = -\frac{\sigma_x + \sigma_y}{2} \]
\[ A_2 = -\frac{\sigma_x - \sigma_y}{2} + i\tau_{xy} \]  
(2.11)

all other \( A_n = 0 \).

With complex Fourier series expansion, \( \Phi(z) \) and \( \Psi(z) \) for a region bounded by a circle are written as

\[ \Phi(z) = \sum_{0}^{\infty} a_k z^{-k} \]  
\[ \Psi(z) = \sum_{0}^{\infty} a'_k z^{-k} \]  
(2.12)

The constants \( a_k \) and \( a'_k \) are determined from the boundary conditions on the core hole circle and at infinity.

Using Equation (2.10) to express the boundary condition on the hole (tractions are equal to the \( N-iT \) derived) and knowing that at infinity the stresses should be zero, the coefficients of Equation (2.12) can be determined by equating terms with like powers of \( z \). The coefficients thus determined are

\[ a_2 = \left[ -\left( \frac{\sigma_x - \sigma_y}{2} \right) - i\tau_{xy} \right] a^2 \]
\[ a'_2 = \left( \frac{\sigma_x + \sigma_y}{2} \right) a^2 \]  
(2.13)

\[ a'_4 = -3a^4 \left[ \left( \frac{\sigma_x - \sigma_y}{2} \right) + i\tau_{xy} \right] \]

All other \( a_k, a'_k = 0 \)

With \( \varphi(z) \) and \( \psi(z) \) now fully defined, Equation (2.6) is applied to yield relieved displacements of

\[ u = \left( \frac{\sigma_x + \sigma_y}{2} \right) A + \left( \frac{\sigma_x - \sigma_y}{2} \right) B \cos 2\alpha + \tau_{xy} B \sin 2\alpha \]  
(2.14)
\[ v = \left( \frac{\sigma_x - \sigma_y}{2} \right) C \sin 2\alpha - \tau_{xy} C \cos 2\alpha \]  
\hspace{1em} \text{(2.15)}

where

\[ A = \frac{a^2}{2\mu r} \]
\[ B = \frac{a^2[r^2(1 + \chi) - a^2]}{2\mu r^3} \]
\[ C = \frac{a^2[r^2(1 - \chi) - a^2]}{2\mu r^3} \]

Relieved displacements give the displacement of a point relative to the center of the through-hole. However, in practice, a displacement measurement might be taken between two points, neither of which is the center of the hole, so as to eliminate the effects of rigid body translations or rotations. Figure 1.1 shows a displacement measurement of this type. The measured displacement between the two measurement points, \( i \) and \( j \) is defined in terms of relieved displacements as follows

\[ U = u_i \cos \theta_{ij} - v_i \sin \theta_{ij} - u_j \cos \theta_{ji} + v_j \sin \theta_{ji} \]  
\hspace{1em} \text{(2.16)}

where

\[ \theta_{ij} = \alpha_i + \beta \]
\[ \theta_{ji} = \alpha_j + \beta \]
\[ \beta = \pi - (\alpha_i + \alpha_j)/2 \]

2.2.5.2 Determination of In-Situ Stress Equations
To solve for the 3 unknown stresses of Case 1 (\( \sigma_x, \sigma_y, \tau_{xy} \)) or the 5 unknowns of Case 2 (\( \sigma_x, \sigma_y, \tau_{xy}, K_x, K_y \)), 3 and 5 measured displacements respectively are required. Either 3 (or 5) equations expressing measured displacements in terms of in-situ stresses must be solved simultaneously for the unknown stress quantities. As an example, the derivations for the two measurement configurations shown in Figure 2.5 are provided below. All measurement points (shown with squares) are located on a fictitious measurement circle (shown dotted) some distance from the edge of the core hole (shown dashed). The measured displacements between two points are shown with solid lines in the figure. Using Equations (2.14) and (2.15), the measured displacements for Case 1, Configuration A are as follows

\[ U_1 = A(\sigma_x + \sigma_y) + B(\sigma_x - \sigma_y) \]  
\hspace{1em} \text{(2.17)}
\[ U_2 = A(\sigma_x + \sigma_y) + 2B\tau_{xy} \]  
(2.18)

\[ U_3 = A(\sigma_x + \sigma_y) - B(\sigma_x - \sigma_y) \]  
(2.19)

Equations (2.17) – (2.19) are solved simultaneously for in-situ stresses resulting in

\[ \sigma_x = \frac{A(U_1-U_3) + B(U_1+U_3)}{4AB} \]  
(2.20)

\[ \sigma_y = \frac{A(-U_1+U_3) + B(U_1+U_3)}{4AB} \]  
(2.21)

\[ \tau_{xy} = \frac{-U_1+2U_2-U_3}{4B} \]  
(2.22)

The process outlined above (Equations (2.10) – (2.22)) for the Case 1 stress state with Configuration A is repeated for the Case 2 stress state in conjunction with Configuration B. The resulting relieved displacement and in-situ stress equations are

\[ u = \left(\frac{\sigma_x + \sigma_y}{2}\right)A + \left(\frac{\sigma_x - \sigma_y}{2}\right)B\cos2\alpha + \tau_{xy}B\sin2\alpha \]  
+ \(K_x(F\sin\alpha + G\sin3\alpha) + K_y(-F\cos\alpha + H\cos3\alpha)\)  
(2.23)

\[ v = \left(\frac{\sigma_x - \sigma_y}{2}\right)C\sin2\alpha - \tau_{xy}C\cos2\alpha \]  
+ \(K_x(M\cos\alpha + J\cos3\alpha) + K_y(M\sin\alpha - J\sin3\alpha)\)  
(2.24)

\[ \sigma_x = \frac{\sqrt{2}}{8AB} \left[ A(U_1-U_2-U_3-U_4+2\sqrt{2}U_5) + B(-U_1+U_2+U_3+U_4) \right] \]  
(2.25)

\[ \sigma_y = \frac{\sqrt{2}}{8AB} \left[ A(-U_1+U_2+U_3+U_4-2\sqrt{2}U_5) + B(-U_1+U_2+U_3+U_4) \right] \]  
(2.26)

\[ \tau_{xy} = \frac{\sqrt{2}}{8C} \left[ U_1+U_2-U_3+U_4 \right] \]  
(2.27)

\[ K_x = \frac{\sqrt{2}}{4(F-G-M-J)} \left[ U_1+U_2+U_3-U_4 \right] \]  
(2.28)
\[ K_j = \frac{\sqrt{2}}{4(F - G - M - J)} [U1 - U2 + U3 + U4] \]  

(2.29)

where \( A, B, \) and \( C \) are as before (Equation (2.13)), and

\[
F = \frac{a^4}{16\mu r^2} \\
G = \frac{a^4}{16r^4} [r^2(\chi + 2) - 2a^2] \\
M = -F \\
J = \frac{a^4}{16r^4} [r^2(\chi - 2) + 2a^2]
\]

The method outlined above is derived assuming that the in-plane stresses do not vary with depth. If the hole is drilled in increments and displacements are recorded after every increment, then a profile of the in-plane stresses may be developed. Turker (2003) describes the incremental core-drilling method in detail, a brief overview is given below.

2.2.5.3 The Incremental Core-Drilling Method

The incremental core-drilling method is essentially an extension of the influence function method (Beghini 2000). To simplify the calculations described herein, the in-situ stresses as a function of depth are written in terms of equibiaxial components as follows

\[
P(H) = \frac{\sigma_x(H) + \sigma_y(H)}{2} \]  

(2.30)

\[
Q(H) = \frac{\sigma_x(H) - \sigma_y(H)}{2} \]  

(2.31)

Each of the in-situ stress components \( P(H), Q(H) \) and \( \tau(H) \) may be treated separately and the results combined using the superposition principle. Only the development for the equibiaxial in-plane stress \( P(H) \) is given here, the interested reader is directed to Turker (2003) for further information. The relieved displacement due to a unit equibiaxial stress placed at a distance \( H \) from the surface when the core hole depth is \( \xi \) may be written as

\[
u R(\xi, H) = \frac{1}{Em} G_A(\xi, H) \]  

(2.32)
where \( H \) is the non-dimensional location of a unit load as measured from the surface and \( \xi \) is the non-dimensional core hole depth. Both are non-dimensionalized by dividing by the measurement circle radius, \( m \). \( G_A \) is termed an influence function and represents the relieved displacement due to a unit equibiaxial stress \( P \) applied at \( H \) when the core hole depth is \( \xi \). By superposition, the relieved displacement due to a loading distribution \( P(H) \) is given by

\[
u R(\xi) = \frac{1}{E} \int_0^\xi G_A(\xi, H)P(H)dH \tag{2.33}
\]

The influence function may be written as a double power expansion as follows

\[
G_A(\xi, H) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \xi^{i-1} H^{j-1} \tag{2.34}
\]

The coefficients \( a_{ij} \) have been found through finite element analysis (Turker 2003). It was determined that \( n=10 \) gives a good balance between accuracy and numerical stability. The coefficients depend on the hole and measurement circle geometry and Poisson’s ratio. Turker (2003) presents coefficients for the case of \( a=50 \text{ mm} \), \( m=75 \text{ mm} \) and \( \nu=0.2 \).

If the function \( P(H) \) is assumed to be in the form of a polynomial of degree \( c \) with unknown coefficients, and the hole is drilled in \( s \) increments such that the relieved displacement at any position is recorded as a vector of length \( s \), then it is possible to recast the constants \( A, B \) and \( C \) in the earlier development (for example Equations (2.20) – (2.22)) as matrices with \( s \) rows and \( c \) columns. The matrix equation for the constants \( B \) and \( C \) may be found in Turker (2003), the equation for \( A \) follows

\[
\frac{1}{A} = (M_A^T M_A)^{-1} M_A \tag{2.35}
\]

where

\[
M_A = \begin{pmatrix}
mA_{11} & \cdots & mA_{1c} \\
\vdots & \ddots & \vdots \\
mA_{s1} & \cdots & mA_{sc}
\end{pmatrix}
\]

\[
mA_{sc} = \frac{1}{E} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{a_{ij}}{i+c-1} \xi_i^{i-1} H_j^{j-2} m
\]

In this case the simple algebraic relationships presented earlier in Section 2.2.5.2 (i.e. Equations (2.20) – (2.22)) become matrix equations, and the measured displacements, \( U1, U2, \) and \( U3 \) are vectors of length \( s \).
2.2.6 Other Miscellaneous Techniques
There exist other proposed methods for determining the in-situ stresses in concrete that also depend on the stress relief principle. Two lines of inquiry are of interest to the current work and are summarized here.

In the stress relief technique presented by Owens (Owens 1993), a 75 mm diameter hole is cut into a concrete specimen to a depth of 50 mm in 10 mm increments and the relieved strains are captured with an array of vibrating wire strain gauges and mechanical strain gauge targets that are essentially similar to that used in the work of Merkhar-Asl (1988) and Buchner (1989). After drilling to a depth of 50 mm, the bottom of the hole is flattened and a strain gauge installed in the direction of maximum determined principal stress. Drilling of the core hole is then continued to a total depth of 100 mm with readings taken on the embedded gauge at 10 mm depth increments. An important distinction between this work and that of Buchner or Merkhar-Asl is that Owens recommends that the drilling always be done dry, i.e. using compressed air or equivalent as a lubricant, rather than the more common water cooled approach. Owens and colleagues have applied similar techniques aimed at determining the modulus of elasticity and Poisson’s ratio of concrete in-situ (Begg et al. 1995), and developing new methods for efficient data processing for the large amount of redundant data developed in these tests (Begg et al. 1997). A derivative technique involving a pattern of closely spaced, smaller core holes has also been proposed (Owens et al. 1994). In all cases, good accuracy (i.e. approximately 3% of existing stresses or 0.3 N/mm², whichever is greater) is reported as being achievable by Owens and companion researchers. It is reported that these techniques have been applied over 200 times (Begg et al. 1998), primarily in the United Kingdom, to evaluate stresses in concrete structures. A major weakness in the publications cited herein is their relative dearth of specifics with regards to experimental validation of the technique. The majority of the noted publications are in volumes produced as compilations of conference proceedings. Although the work is promising and appears to be used in practice, there is simply not enough information available in the publications to allow an objective review of the accuracy of these techniques. In principle however, this work is similar in nature and application to that performed by Buchner and Merkhar-Asl.

A second method involving stress relief has been proposed by Ryall (Ryall 1993) and has been called the inclusion method. In the method a small (42 mm) pilot hole is drilled into the concrete and an instrumented steel inclusion is cemented into the hole. The inclusion is instrumented on planes perpendicular to its embedded length at several locations with stacked tri-axial strain gauges. Overcoring of the inclusion commences which relieves the core from the surrounding stresses in the concrete, but in turn causes stresses in the steel inclusion. Strain readings in the gauges are converted into in-situ stress results using standard theory of elasticity methods, excepting that solutions must be obtained for the case of a hard inclusion embedded in a softer material. The series of stacked gauges allows determination of stresses through the depth of concrete encompassed by the inclusion. Ryall (2001) discusses
this technique, as well as summarizing the techniques of Owens and Merkhar-Asl previously mentioned.

2.3 EVALUATION OF CANDIDATE MEASUREMENT TECHNIQUES

The typical magnitude of displacement that must be measured in a concrete structure subject to testing with the core-drilling method is about 10-20 µm (for a concrete with $f'c = 34.5$ MPa, $v = 0.2$, core hole and measurement circle diameters of 150 mm and 200 mm, and a uni-axial stress state of 5 MPa compression, for example). With this magnitude of displacement providing context, existing displacement measurement techniques were reviewed to evaluate their applicability to the core-drilling method. The measurement techniques that were reviewed are broadly grouped into three categories: (1) contact measurement techniques; (2) full field optical measurement techniques; and (3) discrete point optical techniques. A summary of each category of techniques is given in Section 2.3.1. Section 2.3.2 describes preliminary experiments that were performed to evaluate two of the most promising of these techniques, digital image correlation and photogrammetry, for their suitability for use with the core-drilling method.

2.3.1 Description of Measurement Techniques

2.3.1.1 Contact and Full Field Optical Techniques

Examples of contact measurement techniques include DEMEC (demountable mechanical strain gauges) gauges, LVDT (linear variable differential transformers), and vibrating wire strain gauges, to name a few. Resolution of a DEMEC gauge is reported to be as low as 1-2 µm, depending on the gauge length. Theoretically, the resolution of an LVDT is infinite, but in practice the resolution is limited by the electronic equipment employed in the measurement system. While these and other contact measurement techniques may be able to provide the required measurement resolution, each requires hard mounting of the measurement device to the stressed specimen prior to core hole drilling. These types of approaches may be feasible in a controlled laboratory environment, but under field conditions the difficulty in achieving consistent measurement fidelity is considerably magnified. These types of measurement techniques have been the primary focus of previous researchers using the stress relief principle to evaluate stresses in concrete (Buchner 1989, Merkhar-Asl 1988, Owens 1993, Kenewan et al. 2005).

Optical techniques that take advantage of the interference properties of light waves were included in the category of full field optical measurement techniques. Examples of such techniques include holographic interferometry, speckle interferometry and shearography. While these methods would likely work well in the laboratory with the core-drilling method, these methods are not pursued for this research because the objective is to develop a method that is applicable in the field. The optical techniques cited above can be sensitive to environmental factors such as vibration, and in some cases to rigid body motion, so they may be limited in field use for this particular application.
2.3.1.2 Discrete Point Optical Techniques
Examples of discrete point optical techniques include industrial photogrammetry and 3D digital image correlation. These two techniques were deemed viable and were evaluated with an experimental program described in Section 2.3.2 of this report. A description of the techniques is provided here.

2.3.1.2.1 Photogrammetry
Photogrammetry is a three-dimensional coordinate measurement technique that is widely used in industrial applications, although its origins are based in the field of aerial mapping. Based on triangulation principles, photogrammetry uses a series of photographs taken of the measured object from numerous angles to recreate the three-dimensional coordinates of the targets that are placed on the object. With many different views of each target, the exact location of the target can be triangulated. This triangulation depends on knowledge of the camera’s position and orientation for each photograph that is analyzed. The three major analytical functions that must be performed to analyze photogrammetric data are: (1) triangulation; (2) resection; and (3) self-calibration of the camera to eliminate errors such as those due to lens and camera imperfections, temperature and humidity effects, etc. These three functions are described below.

To triangulate the position of a target in a series of photographs, the \(x, y\) location of the point in each photograph is measured. An object with known scale is included in each photograph for this purpose. In practice this is typically done by placing a simple scale bar or cross on the object. If the camera location and aiming direction for each photograph is known, the theoretical lines from the camera positions to the target can be intersected to produce the target’s three-dimensional location.

The process of determining the camera’s position and aiming direction (collectively hereafter referred to as the camera’s orientation) is called resection. In the resection process, the known \(x, y, z\) coordinates of several well-distributed targets within a particular photograph are used to determine the orientation of the camera for that particular photograph.

Self-calibration of the camera relies on the availability of photographs taken from a variety of camera orientations with numerous well-distributed targets captured in each image. These are characteristics that are always present in a well planned photogrammetric survey, as they also aid the mathematical accuracy of the triangulation and resection steps.

It is clear that triangulation and resection are interrelated, i.e. triangulation requires known orientation to calculate location and resection requires known location to get orientation. Thus the simultaneous solution of the governing geometric equations from all three phases is required. Bundle adjustment is the term used to describe the mathematical process whereby triangulation, resection and self-calibration are
performed simultaneously to determine the precise three-dimensional location of the
target points with a minimum amount of error (typically with a root-mean-square
approach).

Accuracy and precision in industrial photogrammetry are related to the size of the
measured object and numerous other factors. Some factors that affect the quality of a
photogrammetric survey include: the resolution of the captured images, camera
calibration, angles between captured photos, redundancy in the appearance of targets
appearing in multiple images, and the placement of the targets. A current guideline
regarding accuracy is that a quality survey (i.e. one that meets accepted standards for
the influencing factors noted above, among others) can yield accuracy in coordinates
of approximately 1 part in 80,000, with 68% probability (one sigma). Thus, as an
example, for a measured object size of 1 m, one would expect accuracy in coordinates
to 13 µm. However, typical surveys are of areas often larger than several square
meters. With all other factors equal, the strong dependence on scale means that
industrial photogrammetry applied with a measured object size substantially smaller
than 1 m can greatly reduce this uncertainty in measured coordinates.

For the study described in Section 2.3.2, a non-metric black and white digital camera
with a 6 mega-pixel charge-coupled device (CCD) was used to capture a series of
approximately 30 photographs of the specimen from many different angles at a
distance of 1 - 2 m. The camera was equipped with a 24 mm manual focus lens.
Custom non-reflective targets were used for this exercise, and as part of the
photogrammetry bundle-adjustment protocol, the camera was self-calibrated on-site.

2.3.1.2.2 Digital Image Correlation
3D digital image correlation (Tyson et al. 2002) combines techniques of image
correlation with the photogrammetric location principles described above, and is
practical only with the advent of high-speed computers. Sample preparation consists
of applying a regular or random pattern with good contrast to the surface of the
measured object. The pattern will then deform with the object under load. The object
is captured in a stereo pair of high quality cameras while it is loaded. These two
cameras are mounted at either end of a base bar such that their relative position and
orientation with respect to one another is fixed and known. In this case, many of the
photogrammetric principles noted above reduce to mathematically simpler forms than
for classical photogrammetry. The optimum total angle between the two cameras is
25 degrees. Lower angles will reduce the accuracy of the triangulation, and thus
reduce the accuracy of the out-of-plane (z-axis) coordinates and displacements. Wider
angles increase the accuracy of the z coordinates, but the increased perspective reduces
the useful field of view.

Thousands of unique correlation areas known as facets (typically 15 pixels square for
the system used herein) are defined across the entire imaging area. The center of each
facet is a measurement point that is tracked, in each successive pair of images, with
accuracy up to one hundredth of a pixel by employing a similarity measure such as the
Normalized Cross Correlation. An image correlation algorithm, as for example, the iterative spatial domain cross correlation algorithm, tracks facets by maximizing this similarity measure. Three-dimensional locations of these facets are calculated before and after each load step, yielding displacements. Tracking the dense cloud of points within the applied pattern provides displacement information that is ‘near’ full field.

3D digital image correlation (DIC) is often more practical than other full-field methods that require interferometric stability between the sensor and test part in order to acquire data. In 3D digital image correlation, significant rigid body motions can first be quantified and then removed. Since strains are calculated from the derivative of displacement, rigid body motion is intrinsically eliminated from strain data in 3D digital image correlation. As long as non-blurred pictures can be captured, 3D coordinates, displacements and strains can be measured. Although not utilized herein, this means that the technology can be tailored to situations involving measurement in the dynamic environment.

In the work described in Section 2.3.2, the 3D digital image correlation system is calibrated using NIST-traceable calibration panels for each field of view. A sequence of pictures of the panel at different distances and orientations is captured and a bundle adjustment used to establish the precise relationship between the two cameras. Each dot on the calibration panel occupies more than 100 pixels on each camera sensor, so dot centers can be interpolated to an accuracy of at least 1/30 of a pixel. The 1/30 figure includes precision in the image correlation algorithm and in the ray-tracing triangulation function which are both critical for the determination of out-of-plane displacements. For displacements in the plane of a set of photographs the displacement accuracy will be better because in this case the precision is governed primarily by the image correlation algorithm and discrepancies introduced during the ray-tracing triangulation function are minimized. The resolution of the technique necessarily follows many of the same precepts noted above for industrial photogrammetry, as well as being influenced by the accuracy of the image correlation algorithm. Since there are 1280 x 1024 pixels on the Vosskuhler CCD-1300 digital cameras used for the tests described in the next section, the overall accuracy of the system used herein can be conservatively stated as 1/30,000 the field of view for out-of-plane displacements and better for in-plane displacements. For a 10 mm field of view, that equates to 0.33 microns displacement sensitivity. The displacement sensitivity scales linearly with the field of view, decreasing to 3 microns for a field of view of 100 mm and 30 microns for a field of view of 1 m, assuming 1280 pixels across the field of view. In the work described in Section 2.3.2, after calibration of the system on-site, the image pairs were captured at a distance near 1 m from the specimen, with a field of view of approximately 150 mm. This corresponds to a sensitivity in out-of-plane displacements of 5 microns, with better sensitivity for in-plane displacements.
2.3.2 Experimental Evaluation of 3D Digital Image Correlation and Photogrammetry

The review of the pertinent literature indicated that two techniques were viable candidates for use in conjunction with the core-drilling method; digital image correlation and photogrammetry. This section describes preliminary experiments that were performed as part of the current work to evaluate the applicability of these techniques to the core-drilling method.

2.3.2.1 Experimental Overview

Three steel plates were tested in tension in these experiments, as shown schematically in Figure 2.6. Plates 1 and 3 were loaded in concentric axial tension with a force \( P \) to generate a uniform stress field, Plate 2 was loaded with a similar axial load in eccentric tension at the kern point to generate a stress field that theoretically varies linearly from zero on one edge of the plate to twice the nominal value at the opposite edge.

The geometry and axial load \( P \) applied to Plate 1 was designed to match the relieved displacements in a hypothetical concrete specimen with an in-situ uni-axial compression stress of 13.8 MPa. Steel was used instead of concrete for three reasons: (1) to provide a specimen with known elastic modulus; (2) to provide a fine-grained, homogeneous specimen, thus eliminating any discontinuities caused by the presence of aggregates; and (3) to allow the test to be performed in tension instead of compression, thereby simplifying the loading needed for the verification experiments. The steel used was HPS 100, with an assumed modulus of elasticity, \( E \) of 200 GPa and an assumed yield stress of 690 MPa. Steel with a relatively high yield point was chosen to ensure that the loads applied to the plate as magnified by the stress concentration around a core hole would not induce yielding in the central region of the specimens. Table 2.2 shows a test matrix with the pertinent geometric, material, and load data for Plates 1, 2 and 3, and the hypothetical concrete structure considered. The applied loads in the table were measured experimentally using a load cell as described in Section 2.3.2.2. The applied stresses in the table were measured using strain gauges as discussed in Section 2.3.2.2 and 2.3.2.3.

Figure 2.7 shows the anticipated radial (\( u \)) and tangential (\( v \)) displacements for the hypothetical concrete structure and the steel test specimen used to represent the hypothetical concrete structure. The figure shows that the expected displacements of the steel test specimen closely match the hypothetical concrete specimen in both magnitude and variation around the respective measurement circles.

2.3.2.2 Experimental Details

An arrangement of bonded wire strain gauges were affixed to each side of each plate to provide verification of the expected in-plane normal stress quantities and stress gradients and to verify that there was not undue out-of-plane bending of the plate. Figure 2.8 shows the layout of these strain gauges for each plate, as well as the numbering scheme for the gauges. The gauges for the front face of Plate 1 are also...
visible in Figure 2.9. Measurements Group Inc. model CEA-06-250UN-350 350 ohm resistance gauges with a gauge factor of 2.05 were used in conjunction with a series of Vishay 2120A strain gauge conditioner and amplifier systems. The gain and excitation voltage of the system were set so that the output readings were analogous to stress readings in the steel plates in units of psi/10; however, stresses in this report are always presented in units of MPa. The gain and excitation voltage settings of the instrumentation system are provided with the output tables for each plate. A load cell was incorporated into the load path to measure load. The load cell was calibrated using a SATEC 600 kip loading machine and a FLUKE 8840A voltmeter. The calibration data for the load cell is presented in Table 2.3 and Figure 2.10.

Figure 2.9 shows the load frame used to test the three plates, with Plate 1 positioned in the frame. The plates were gripped at each end by a clevis with a single load pin and loaded at one end with a hydraulic jack. The length of the plates was chosen to ensure that the load was well distributed in the center test region of each plate. Edge effects were avoided by ensuring that the plate width to core hole diameter ratio was greater than a specified limit (Turker 2003). To remove any hysteresis in the material, prior to testing, each plate was loaded in tension to approximately 125% of the tested load a minimum of 3 times. The residual offset in the strain gauge readings prior to these preliminary loading steps were minimal, and the gauges were re-zeroed prior to final loading. Prior to taking a displacement reading, the hydraulic pump was turned off (but left pressurized to maintain load).

2.3.2.3 Experimental Results
Tables 2.4 – 2.6 contain the strain gauge and load cell data obtained during load application for each of the three plates. In the tables Column (1) provides the load as measured by the load cell, Columns (2) – (7) provide the stresses measured using the gauges installed on the front of a plate, and Columns (8) – (13) provide the stresses from the gauges measured on the back side of the plate. Data is provided at increasing load levels up until the final load at which coring was performed. In the case of Plate 2 (Table 2.5) and Plate 3 (Table 2.6), the last three rows of the table provide the load and stresses in the plate immediately before coring (the row marked ‘Note a’), immediately after coring (the row marked ‘Note b’), and the average of those two rows (the row marked ‘Note c’). In the case of Plate 1 only the average of the data recorded before and after coring is provided as the last row. The last row of each table is used to calculate the applied load and stress for each plate (as provided in Table 2.2).

Figures 2.11 – 2.13 show the stresses at each gauge location plotted versus load for each of the plates (the data upon which the figures are based are provided as the last rows of Table 2.4 – 2.6). Note that the front face gauges are plotted with a dotted line and the back face gauges with a solid line. Further, each set of gauges that were affixed back-to-back (as for example gauges 1 and 7) share the same symbol. The data plotted in this manner allows the presence of in-plane (strong axis) bending and out-of-plane (weak axis) bending to be detected. For Plates 1 and 2, there is little out-
of-plane bending present. Figure 2.13 shows the presence of more significant out-of-plane bending in Plate 3. This is illustrated by the distinct separation between the stress readings on the front face of the plate (shown dashed), and the back face of the plate (shown solid). The difference in stress between front and back face shows classic through-thickness bending and is most prominent in Plate 3. Figures 2.14 – 2.16 show the final stress values for each plate plotted versus the horizontal location across the plate, to clearly show the stress profile across each plate. Again, these plots allow for the detection of strong and weak axis bending. A best-fit linear regression through the strain gauge data of each figure (2.14 – 2.16) results in the stress and gradient values for each plate as reported in Table 2.2, with the exception that the gradient for Plate 1 (0.042 MPa/mm) and Plate 3 (0.015 MPa/mm) have been neglected. For comparison purposes also shown are the stresses computed by dividing the applied loads (from the load cell) by the nominal plate area (and adjusting for the eccentricity in load in the case of Plate 2).

Figure 2.17 shows photographs of two of the plates after coring. Figure 2.17(a) is Plate 1 subjected to traditional photogrammetry, and Figure 2.17(b) is Plate 2 subjected to 3D digital image correlation. Plate 3 was similar in appearance to Plate 2. Shown in these photographs are the manually placed discrete targets used in the traditional photogrammetry, and the ‘spluttered’ spray paint applied for the 3D digital image correlation. Each plate was loaded as described above. A reading was taken with the given displacement measurement technique, and then a core hole was cut in the plate. The coring operation was performed with a magnetically attached drill, as shown in Figure 2.18. After coring, the load was returned to the value immediately prior to coring. It was observed that the load would drop slightly due to coring the plate. Removing the core material slightly reduces the overall stiffness of the plate structure, resulting in a subsequent drop in load. Typically this load drop was less than 2% of the applied load. After this step, a second reading with the given displacement measurement technique was taken to determine the relieved displacements. The load and stress values immediately prior and immediately subsequent to coring along with the averages of the two are presented in Tables 2.5 and 2.6 for Plates 2 and 3. In Table 2.4, only the average values for Plate 1 are shown. As noted, the final row of data in Tables 2.4 – 2.6 was used to determine the applied stresses shown in Table 2.2.

Figure 2.19 compares the measured displacements with the theoretical displacements from all three plates. In the figure, rigid body motions have been removed from each measured displacement quantity. This was accomplished by the subtraction from the displacement output for each technique of terms such as $\gamma_1dx$, $\gamma_2dy$ and $\gamma_3\theta_z$, scaled displacements in the horizontal, vertical and rotational directions respectively. $\gamma_1$, $\gamma_2$, and $\gamma_3$ were varied to minimize the root sum square difference between the measured and theoretical relieved displacements. Note that the subtraction from the relieved displacements of rigid body translations and a rigid body rotation has no impact on the stress solutions, as Equations (2.20) – (2.22) and (2.25) – (2.29) are expressed in terms of measured displacements, i.e. as the difference between two relieved displacements.
For Plate 1, the measurement radius considered was 42 mm, for Plates 2 and 3, 44 mm. 3D digital image correlation provided displacement values for thousands of discrete points on the surface of the plate, only a few are shown here. Further consideration of the richness of this data set beyond that considered here would almost certainly improve the accuracy of the stress measurements presented in the following section. In general, good agreement is obtained between the measured and theoretical displacements. This suggests that the two measurement techniques provide acceptable accuracy for the problem of interest (stresses in concrete).

Equations (2.20) – (2.22) and (2.25) – (2.29) were applied to Plates 1, 2 and 3 as appropriate to determine the in-situ stress in each plate. Table 2.7 shows the measured stress quantities for each plate tested as well as the relative error between the measured quantity and that applied. As noted previously, the applied stress ($\sigma_x$) and stress gradient ($K_x$) were calculated from the output of strain gauges affixed to the plates. The applied values computed in this way compare well to applied values computed by dividing the output of the load cell by the cross-sectional area of the plates (and in the case of Plate 2 adjusting for the applied eccentricity). In each case, the measured values are the results of averaging the stress values obtained from Configuration A or B every 15 degrees around the measurement circle.

2.3.2.4 Discussion of Results
For the Plate 1 test, the calculated stress results are within 17% of the applied stress quantities, encouraging for a first test of the technique. In an actual field test on a concrete structure, the modulus of elasticity, $E$, of the concrete would be determined from the core taken and would likely be determined within 10-15%, so these results are certainly within this uncertainty range. Further, as this was the first use of traditional photogrammetry for this application, the targets were placed manually, resulting in a certain amount of relative positional error that was not accounted for in the equations as currently conceived. A simple, pre-fabricated target array that could be affixed to the specimen would likely improve the accuracy of the technique.

For Plate 2, excellent results (less than 7% error in normal stress) were obtained with 3D digital image correlation. However, the error in the calculation for $K_x$ is significantly higher. In Section 6.2.3.4 this issue is further explored for linear gradients of in-situ in-plane normal stresses in concrete structures.

For Plate 3, the accuracy in stress results was not as anticipated. Plate 3 was the first plate tested, and there were issues with the coring technique. Lubricating oil was not used while coring this plate, resulting in a prolonged drilling time and perhaps exceptional heating of the material immediately surrounding the core hole. For coring the other two plates, lubricating oil was used. Further, as mentioned previously, there was significant out-of-plane bending present in the Plate 3 test that was unaccounted for in the in-situ stress equations as applied. Perhaps application of the incremental formulation of the core-drilling method would yield better results; that effort is left for future study.
In summary, it has been shown in this section that photogrammetry and 3D digital image correlation are robust enough to capture the expected displacements involved in a typical concrete structure subjected to the core-drilling method.

2.4 REVIEW OF IMPORTANT CONCRETE PARAMETERS

This section of the report summarizes many parameters and theories of concrete behavior employed in the study of concrete that are used extensively in subsequent chapters. A summary of moisture movement principles, theory, and in-situ measurement is provided in Sections 2.4.1 and 2.4.2 in preparation for the use of this material in Chapter 3. Additional material regarding expansion of cementitious materials exposed to water is provided in Section 2.4.3. Section 2.4.4 explains a model that has been used to successfully describe moisture movement in initially saturated concrete subjected to drying. The material from that section is used in Chapter 4 to estimate the effect that differential shrinkage has on the core-drilling method.

2.4.1 Moisture Movement in Concrete

Transport of fluids in porous media is a fully evolved field with its own detailed literature and history (Bear 1972). A brief treatment in preparation for the specific properties utilized herein is presented. Although the concepts presented are general, in the current work the porous material of interest is a cementitious material such as concrete or neat cement, and the wetting fluid is water. Hall and Hoff (2002) contains an excellent treatment of moisture movement in building materials. A summary of only those concepts necessary for the current work is presented below.

An important descriptor of porous media in this context is porosity, $f$, a measure of the volume of a material’s void space, $V_v$, in relation to its overall volume, $V_T$

$$f = \frac{V_v}{V_T}$$

(2.36)

Only the void spaces that are not closed from the rest of the pore structure, called the effective porosity, $f_e$, are available for the flow of liquid. Often in the literature, $f$ is assumed to mean only the effective porosity, $f_e$, as will be done here. Porosity may also be written as a ratio of densities

$$f = \frac{(\rho_s - \rho_b)}{\rho_s}$$

(2.37)

where $\rho_s$ is the solid mass divided by the solid volume, and $\rho_b$ is the solid mass divided by the bulk volume.
Another important parameter, the volume fraction water content, $\theta$, of a porous material is defined as the volume of water divided by the total volume. Often, a normalized water content is utilized

$$\theta_r = \frac{\theta - \theta_d}{\theta_s - \theta_d}$$

(2.38)

where $\theta_s$ is the operational saturated state and $\theta_d$ is the operational dry state. The normalized water content has values that vary from $\theta_r = 0$ (completely dry) to $\theta_r = 1$ (completely saturated). Note that $\theta_d$ need not be zero, as in some cases a material may never be completely moisture free.

Movement of water in unsaturated building materials is primarily driven by capillary forces. It is useful to review the concept of surface energy in order to understand the forces that drive capillary action. Each solid and liquid material exhibits a characteristic surface energy, whereby the atoms and molecules at the surface of the material are in less stable positions (and in higher energy states) than those in the interior. Thus, it takes a certain amount of energy to form more surface area, and a further consequence is that the surface is in tension. For example, a liquid drop free from outside forces will form a sphere, as this shape has the least surface area. As summarized in Hall and Hoff (2002), for this basic case, the energy to create new surface, $\sigma dA$, (where $\sigma$ is stress and $dA$ an incremental area) may be balanced against the work done to change the size of the drop, $\Delta P dV$ (where $\Delta P$ is the capillary pressure, and $dV$ an incremental volume), to yield the famous Young-Laplace Law:

$$\Delta P = \frac{2\sigma}{r}$$

(2.39)

wherein the equilibrium excess pressure inside the drop depends on the radius, $r$, and the surface tension of the liquid.

Capillary absorption may be described based on the properties of a wetting liquid coming in contact with a surface. The angle that a drop of wetting liquid makes with a solid surface is based on a combination of three surface energies: the surface energy of the solid in contact with air, the surface energy of the liquid in air, and the surface energy of the liquid in contact with the solid. Under wetting conditions in a porous medium, the liquid will spread into the medium under the action of the Young stress. If the amount of liquid available is less than that required for saturation, the fluid will migrate through capillary action until the system reaches equilibrium wherein the meniscus curvature is constant throughout and in which the menisci meet solid surfaces at the appropriate contact angle. For this case, the meniscus curvature is negative, and thus the pressure in the liquid phase, $P$, is lower than the ambient pressure, $P_0$. As described by the Young-Laplace Law, this pressure difference is a function of the pore radius. Furthermore, this pressure difference is also a function of
the moisture content, $\theta$, of the porous solid, and thus is often expressed as $P(\theta)$, and termed the capillary suction. If the capillary suction function is divided by the density of the liquid, $\rho_L$, and the acceleration due to gravity, $g$, the capillary, or hydraulic potential function is defined

$$\Psi = \frac{P(\theta)}{\rho_L g} \quad (2.40)$$

With this basic background terminology defined, it is possible to examine the flow of moisture through porous media in quantitative terms. The flow may be described locally with the extended version of Darcy’s Law

$$u_f = K(\theta)F \quad (2.41)$$

where $u_f$ is the velocity of the fluid flow, and $K(\theta)$ is the generalized, or unsaturated permeability and is a function of moisture content, $\theta$. Note that the term permeability is reserved for the case of $K(\theta)$, when $\theta = \theta_s$, i.e. the medium is saturated. Flow in saturated materials only occurs due to the presence of a pressure head, while unsaturated flow is driven by capillary suction. The capillary force, $F$, may be replaced with the negative of the gradient of capillary potential so that

$$u_f = -K(\theta)\nabla\Psi \quad (2.42)$$

Combining the equation above with the concept of mass conservation and the continuity equation leads to the famous Richard’s equation

$$\frac{\partial \theta}{\partial t} = \nabla K(\theta)\nabla\Psi \quad (2.43)$$

where $\partial \theta/\partial t$ is the change in moisture content over time. In order to express this equation in terms of moisture content rather than hydraulic potential, one may define the capillary diffusivity as

$$D = K(\partial \Psi / \partial \theta) \quad (2.44)$$

which leads to

$$\frac{\partial \theta}{\partial t} = \nabla D \nabla \theta \quad (2.45)$$

For one dimensional flow, this equation may be rewritten as
\[
\frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x} \left( D \frac{\partial \theta}{\partial x} \right) \tag{2.46}
\]

The boundary conditions on this equation are such that \( \theta = \theta_s \) for \( x = 0 \) and \( t \geq 0 \); \( \theta = \theta_d \) for \( x > 0, t > 0 \). Using the Boltzmann transformation, where

\[
\phi = xt^{\frac{1}{2}} \tag{2.47}
\]

allows this equation to be rewritten as an ordinary differential equation whose solution is

\[
x(\theta, t) = \phi(\theta)t^{\frac{1}{2}} \tag{2.48}
\]

This result indicates that as liquid is absorbed into a porous solid, the liquid content versus distance profile advances as the square root of time and maintains a constant shape, \( \phi(\theta) \) (Hall and Hoff 2002).

The total amount of liquid absorbed \( i \), in a given time, \( t \), may be calculated from Equation (2.48) and is given by

\[
i = \int_{\theta_s}^{\theta_d} x d\theta = t^{\frac{1}{2}} \int_{\theta_s}^{\theta_d} \phi(\theta) d\theta = St^{\frac{1}{2}} \tag{2.49}
\]

The sorptivity, \( S \), is defined by Equation (2.49) above and may be expressed as

\[
S = \int_{\theta_s}^{\theta_d} \phi(\theta) d\theta \tag{2.50}
\]

Sorptivity is often measured through simple testing according to the direct gravitational method, which seeks to engender a situation that is consistent with all the parameters involved in the derivation of Equation (2.48). Figure 2.20 shows a schematic of a typical sorptivity test set-up. In particular, the test is intended to simulate one dimensional flow into a semi-infinite porous material, where the initial and boundary conditions match those noted above. A prismatic concrete specimen, usually cylindrical, is suspended above a pan of water so that its bottom face is in contact with the water. All other faces of the specimen should be previously sealed. The specimen should be initially dry, as sorptivity measurements can be sensitive to initial moisture content as shown in Figure 2.21. The specimen is weighed periodically, and the weight gain divided by the cross-sectional area of the specimen and the density of the water is plotted versus square root of time. The slope of this
plot corresponds to the sorptivity. Figure 2.22 shows a typical sorptivity plot for a limestone specimen.

Deviation from straight line behavior is possible, especially over long time periods and may be attributable to three factors: (1) The material is not homogenous, i.e. the sorptivity of the surface layer differs somewhat from the bulk material; (2) the material is not dimensionally stable during absorption; and (3) the effects of gravity on capillary rise, typically neglected, cannot be ignored. As previously noted, cementitious materials such as concrete swell when exposed to water. Comparisons of sorptivity curves for these materials based on water absorption versus absorption of various organic liquids indicate that this swelling may be most responsible for deviations from straight-line behavior for water absorption in these materials. The sorptivity may be divided into short-term and long term regimes, as separated by the ‘nick-point’ (Bentz, Ehlen, Ferraris and Winpigler 2002), as for example 17 days for Figure 2.22. The ‘nick-point’ is defined as that point in time on the curve where the data no longer essentially follows its initial straight line slope. Typically, the nick point occurs after several hours exposure to water. Most often, the initial, or ‘early’ sorptivity is of interest. A total time of approximately one hour is considered satisfactory for most laboratory measurements of sorptivity (Hall and Hoff 2002).

Table 2.8 provides sorptivity test data summarized from a number of sources. It is apparent that sorptivity values for neat cement tend to be higher than those for concretes, and further, that sorptivity values can vary by orders of magnitude from concrete to concrete, although typically the trends are for values between 0.1 and 5 mm/√min.

The focus of the research presented in Chapter 3 is the advance of the wetted front. For many porous materials, the capillary absorption profiles vary sharply with water content θ and the leading part of the wetting profile is very sharp. These properties mean that the wetted region may be represented by a step function, i.e. the material is considered either saturated (θ = θs), or dry (θ = θd) (Hall and Hoff 2002). In this way, the wetted region is represented by a rectangular profile and thus the Sharp Front Model of absorption (Hall and Hoff 2002). It may be shown that the distance to the wetted front \( t_w \), is given by

\[
t_w = \left( \frac{1}{f} \right) S t^{1/2}
\]

(2.51)

where \( f \) is the porosity. Equation (2.51) is a key equation used throughout Chapter 3 to estimate the distance advanced into a concrete specimen by the wetted front, and consequently, the volume of wetted specimen.

It is apparent then that the distance to the wetted front is dependent on three parameters, the porosity \( f \), the sorptivity \( S \), and the time of exposure to water \( t \).
2.9 shows example values (based on Equation (2.51)) for the distance to the wetted front in concrete exposed to water. The table contains data for concretes with $S = 0.05$ mm/$\sqrt{\text{min}}$ and $S = 0.1$ mm/$\sqrt{\text{min}}$ for exposure to water of 20 minutes and 60 minutes. In the table the porosity, $f$, has been assumed to be $f = 0.1225$, a reasonable value for concrete (Hall and Yau 1987).

The equations presented here were used, as shown in Chapter 3, to generate estimates for the quantity of concrete that is wetted during core drilling.

2.4.2 In-Situ Measurement of Absorption Properties of Concrete

There are various test procedures that have been developed for measuring the water absorption properties of concrete in-situ (Levitt 1970, British Standards Institution BSI 1881-208 1970, Figg 1985, Dhir, Hewlett and Chan 1987, Dhir, Hewlett, Byars and Shaaban 1995). Three different tests are reviewed here: the initial surface absorption test (ISAT) (Levitt 1970, BSI 1881-208 1970), the Figg’s air permeability index test (Figg 1985), and the covercrete absorption test (CAT) (Dhir et al 1987). Of these, only the ISAT is an ASTM or other standard (BSI 1881-208). The tests were primarily developed to assess the ability of cover concrete to resist moisture inflow, and thus of the concrete to resist deterioration. Each is empirically derived, and involves absorption from a different geometry. Historically, the results have not been comparable across tests, or directly related to the material property sorptivity. However, more recent research (see below) has shown analytically that results for the ISAT test may be converted to sorptivity via numerical manipulation.

2.4.2.1 Overview of the ISAT Test

Figure 2.23 shows a schematic of the ISAT test setup. In the ISAT test, a circular or square cap of known cross-sectional dimension is affixed to the surface of a concrete specimen. Water is pumped through the cap at a constant pressure head of 200 mm of water. The initial surface water absorption is defined as the flow of water per unit area into the concrete after a specified amount of time. The flow rate is usually calculated by measuring distance traveled by the inflowing water along a capillary tube in one minute. For example, ISAT$_{10}$ is the absorption from the tenth to the eleventh minute and would be expressed in mL/m$^2$s. The subscript denotes the time at the start of measurement. It has been shown (Wilson, Taylor and Hoff 1998, Lockington, Parlange, Haverkamp, Smettem and Ross 1999, Wilson Taylor and Hoff 1999) that ISAT data may be fitted to the following equation

$$i = St^{\frac{\kappa}{\gamma}} + \frac{g}{rf}St$$

(2.52)

where $r$ is the radius of the circular cap, $g$ is a function of time that increases slowly from 0.6 (Turner and Parlange 1974) to 0.8 (Haverkamp, Ross, Smettem and Parlange 1994), and other variables are as defined previously.
Although laboratory tests usually specify a step wherein the concrete is preconditioned to a known or uniform water content, \( \theta \), in-situ measurements of absorption parameters are particularly sensitive to the prevailing moisture content. If it is desired to compare in-situ tests with laboratory values, Dhir, Shaaban, Claisse and Byars (1993) developed a system of vacuum treating the local area around a test site to evacuate the site of moisture, see Figure 2.24. It was shown to reduce variability in ISAT (Dhir et al 1993), CAT (Claisse, Elsayad and Shaaban 1999) and Figg (Claisse et al 1999) test results to near laboratory levels.

2.4.2.2 Overview of the Figg Air Permeability Index Test

The Figg test (Figg 1985) was developed to assess water and air permeability in concrete. A schematic of the improved test setup is shown in Figure 2.25. In the test, a hole is drilled into a concrete specimen and then capped and sealed off, leaving an air pocket beneath the silicon plug. A hypodermic needle attached to tubing is inserted through the cap and a vacuum is applied. The end of the tubing is then sealed, and the time for the vacuum to decay is measured. Various modifications to specimen geometry, and pressure level (Cather, Figg, Marsden and O’Brien 1984, Dhir et al. 1987) have been shown to improve the repeatability of the test. The permeation index is the time elapsed for the vacuum to decay from 45 kPa to 55 kPa. Note that 1 atmosphere of pressure (1 atm) is equal to approximately 101 kPa.

2.4.2.3 Overview of the CAT test

In the CAT test, a hole 13 mm in diameter and 50 mm deep (the same dimensions as that specified for the Figg test) is drilled into a concrete specimen. The hole is capped and water is fed into the hole at a constant head of 200 mm water. Water inflow into the hole is measured after a specified time, similar to the ISAT test. The covercrete absorption index is defined as the volume of water absorbed per unit area of exposed concrete per second after a given amount of time from the start of the test, as for example CAT\(_{10}\), the index after ten minutes. Figure 2.26 shows a schematic of the CAT test setup. Claisse, Elsayad and Shaaban (1997) correlates the results of standard laboratory sorptivity tests, ISAT tests and CAT tests and gives proposed analytical relationships between them.

2.4.3 Expansion of Cementitious Materials Exposed to Water

It is well known (Neville 1981, Kosmatka and Panarese 1988, MacGregor 1997) that hardened concrete will continue to shrink upon drying. Furthermore, concrete that is subsequently re-exposed to water expands as shown in Figure 2.27. Neville (1981) gives as a guide that the swelling strain, \( \alpha_w \), is on the order of 1/3 to 1/2 of the overall shrinkage strain, \( \alpha_s \).

Note that for any concrete specimen excepting those of the most minimal dimensions, the shrinkage due to aging, or expansion upon water exposure, will increase for some time. This increase is due to the fact that moisture movement in concrete is a relatively slow process. Aging shrinkage is a function of the drying out of a concrete specimen. The interior of a massive specimen will remain at a higher moisture
content, \( \theta \), than the exterior for some time. The resulting moisture content gradient results in differential shrinkage, with the outer surface of the concrete shrinking at a faster rate than the inner portions. This interior region provides a restraint to the free shrinkage that would take place if the entirety of the concrete specimen was at the same water content. Similarly, the interior of a wetted specimen of finite size may remain dry for some time, as the wetting front advances into the specimen. This interior dry region causes a restraint of the free expansion that would take place if the specimen was wetted in its entirety simultaneously. Thus, only values of unrestrained shrinkage or swelling strain are intrinsic to the material itself, other values are necessarily dependent on the test specimen geometry. Reported shrinkage and swelling strain values should therefore be taken from specimens of negligible dimensions, or as ultimate values (i.e. at long enough times such that the interior of the test specimen has reached moisture equilibrium with the exterior).

Neville (1981) reports shrinkage strain values as shown in Table 2.10, based on Lea (1970). These values may be taken as unrestrained shrinkage strain values, based on a review of Lea (1970). The table also shows the theoretical swelling strains calculated by multiplying the shrinkage strains by the 1/3 factor recommended by Neville. These shrinkage strain values for typical concretes were used, as shown in Chapter 3, to estimate the swelling resulting from water exposure during core drilling.

2.4.4 Moisture Movement in Initially Saturated, Drying Concrete

In Chapter 4 moisture profiles in concrete plates of different ages were calculated as part of an investigation of the effects of differential shrinkage on the core-drilling method. This section describes the analytical model that was used for this purpose in Chapter 4.

The moisture profiles in the concrete plates considered in Chapter 4 were calculated using the model proposed by Akita, Fujiwara and Ozaka (1997). This model may be used to calculate the time dependent moisture profiles in initially saturated, drying concrete (Akita et al. 1997). The model is simple to implement and has been shown to give results that correlate well with experimental data (Akita et al. 1997, Aquino et al. 2004). The model has also been shown (Wong et al. 2001) to agree well with the work of several other researchers. A brief overview of that model is presented below.

The governing differential diffusion equation is essentially similar to Equation (2.44) and is written as follows

\[
\frac{\partial M_c}{\partial t} = \nabla(D \nabla M_c) \tag{2.53}
\]

where \( t \) is time, \( M_c \) the relative moisture content, and \( D \) the diffusion coefficient for concrete. The relative moisture content in this case is defined as the ratio of the current moisture content, \( \theta \), and the moisture content at saturation, \( \theta_s \), expressed as a
percentage. The diffusivity coefficient for concrete is typically highly dependent on the moisture content, meaning that this is a non-linear equation.

The equation is solved according to the following temporal and spatial boundary conditions

\[
M_c(x, y, z) = 100\% \quad \text{at } t = 0 \quad (2.54)
\]

\[
D \frac{\partial M_c}{\partial \eta} = -\alpha_m (H_0 - H_s) \quad \text{on any drying surface} \quad (2.55)
\]

where for the convective boundary condition (Equation (2.55)), \( \eta \) is a unit vector normal to the drying surface, \( \alpha_m \) is the mass transfer surface factor, and \( H_0 \) and \( H_s \) are the relative humidities of the environment and at the surface, respectively. Note that to solve Equation (2.53) in this manner, relative humidity is expressed in terms of moisture content, \( M_c \). This is accomplished using isotherms, expressions that relate relative humidity to moisture content at equilibrium for a fixed temperature. Based on laboratory testing of concretes of various mixture proportions, for a fixed temperature of 20° C, Akita et al. reported the following expression for moisture content as a function of relative humidity (RH) and water-cement ratio (Akita et al. 1997)

\[
M_c = a_1 + a_2 RH + a_3 \gamma + a_4 RH^2 + a_5 RH^2 \gamma + a_6 \gamma^2 + a_7 RH^3 + a_8 RH^2 \gamma + a_9 RH \gamma^2 + a_{10} \gamma^3
\]

(2.56)

where \( \gamma \) is the water-cement ratio expressed as a percentage, and the \( a \)’s are constants given in Table 2.11. The constants were defined by taking \( M_c \) as 100% when RH was 100% for arbitrary values of water-cement ratio, and fitting Equation (2.56) to the least squares approximation of experimental data.

In the analysis, it is assumed that most of the hydration in the cement has taken place, and that the small effects of further hydration over time may be neglected. The diffusion coefficient is expressed as follows (Akita et al. 1997)

\[
\frac{D}{D_1} = \frac{1}{\left[29\left(1 - \frac{M_c}{100}\right) + 1\right]^{1.4}}
\]

(2.57)

where

\[
D_1 = \frac{230}{\gamma} + 0.25\gamma - 14.7
\]
Here \( D_1 \) is a constant that is related to the water-cement ratio, and represents the diffusion coefficient when the material is completely saturated, i.e. \( M_c = 1 \). Akita et al. show that, although the volumetric aggregate ratio of varying mixes can influence the value of \( D_1 \), these effects are rather small and have been neglected for simplicity.

The final variable that must be defined in order to solve Equation (2.53) is the surface factor, \( \alpha_m \), which is defined by Akita et al. (1997) as

\[
\alpha_m = \frac{50}{\gamma + 10} + 2.5
\]  

Equation (2.58) was determined to be valid for water-cement ratios varying from 30% to 100%. Variation in the surface factor tends to affect the early (first hours to days) drying regime, after which, changes in the surface factor have little effect on the drying behavior.

The model presented here has been used in Chapter 4 and Chapter 6 to calculate moisture profiles in drying concrete plates.

**2.4.5 Concrete Stiffness Properties**

In this dissertation concrete has been assumed to be a homogeneous isotropic material. The following is provided as a brief summary of the stiffness characteristics of concrete and its constituents.

Concrete is a material that is comprised primarily of hydrated cement paste, aggregate (both coarse and fine), and a small amount of void space. Previous research has approached concrete material behavior from many different directions. A common way to summarize these different approaches was put forth by Wittmann (1982), who proposed a three-level approach, wherein concrete can be treated at the micro-level, the meso-level, and the macro-level. At the micro-level, the structure and chemistry of the hardened cement paste are of primary importance. At the meso-level, the cement paste and fine aggregate are often considered to comprise a mortar, and cracks, pores, and interactions between the paste and included aggregate are considered critical. At the macro-level, concrete is often considered homogeneous and isotropic, and structural element behavior is usually considered.

In this dissertation concrete behavior has been considered at the macro-level. Simplified, homogeneous material properties have been considered.

Although the derivations of Equations (2.20) – (2.22) assume a material that is linear-elastic, isotropic and homogeneous, concrete is a heterogeneous material. Aggregate is typically broken down into two types, coarse and fine aggregate, with the distinction somewhat arbitrarily made based on grain size. Any aggregate whose basic dimensions are smaller than approximately 5 mm is often considered fine aggregate, and anything larger coarse aggregate. There is a wide body of literature published...
seeking to answer the question of how the overall modulus of elasticity of a concrete specimen is influenced by its mesoscopic composition (Simeonov and Ahmad 1995, Zhao and Chen 1998, Li et al. 1999, Agioutantis et al. 2000). It is generally acknowledged that even within the concrete paste, the paste itself has different properties. In particular, the region of cement paste in immediate proximity to a piece of aggregate differs from that of the bulk cement paste. It is theorized that it is more difficult for cement grains to pack closely against aggregates, and thus the cement paste in the vicinity of aggregates is more porous and of lesser quality and stiffness than the bulk paste (Garboczi 1997). The average thickness of this interfacial transition zone (ITZ) is approximately 10 – 50 micrometers, and the stiffness of the ITZ is as much as 30 – 50% less than the surrounding paste (Lutz et al. 1997). Some have proposed modeling concrete as a three phase composite (aggregate, paste and ITZ), rather than a two phase (aggregate and paste) one (Christensen and Lo 1979). Agioutantis et al. (2000) shows that modeling an ITZ zone with a stiffness as low as 1/10 the stiffness of the surrounding concrete paste does not greatly impact the displacements around and within an aggregate particle.

Typical material properties for various cement pastes, cement mortars and commonly used aggregates are contained in Table 2.12. In general, cement paste is the least stiff of the constituents in a concrete mixture (except void space). When fine aggregate is added to the cement paste to create a mortar, the stiffness increases with the addition to the paste of the stiffer fine aggregate.

Another area of concern is the existence of microcracks. Microcracking has been neglected in the current work. The following paragraphs summarize this phenomenon.

It is known (Nelson 1981) that concrete contains small cracks, even under no load. Seminal work in this area was performed by Slate and Nilson, among others, at Cornell in the 1970’s, 1980’s and 1990’s (Carrasquillo et al. 1981, Smadi and Slate 1989, Smadi et al. 1985). The testing involved high, medium and low strength concretes at several different load levels, including zero load testing that probed cracking only due to curing and shrinkage. Loads were maintained for short duration (less than 1 minute up to 8 minutes), and long duration (30 and 60 days), and the specimens were then examined for cracking. Cracks were divided into bond cracks (those at the interface between aggregate and mortar), mortar cracks, and combined cracks that encompass both bond and mortar cracks. Several conclusions were made that are pertinent to the current work:

- Cracking (exclusively bond cracks) was initiated during the initial curing phase in all of the concretes tested.
- Under short term loading bond cracks increase in number and length with increasing load. The increase was small for loads below 40% of ultimate strength for normal strength concretes and below 60 – 70% of ultimate strength for high strength concretes.
• Under short term loading mortar cracking is negligible for loads up to 75% of ultimate strength for low and medium strength concretes and for loads up to 90% of ultimate strength for high strength concretes.

• Long term loading causes an increase in microcracking over that from short term loading, at all stress levels. However, this increase is negligible at load levels of 40% of ultimate strength for low and medium strength concretes, and 70% of ultimate strength for high strength concretes.

• In normal strength concrete the stress-strain curve is approximately linear up to 30 – 50% of ultimate strength. Higher strength concretes remain linear further into the loading regime.

• In general, high strength concrete exhibits less microcracking under all scenarios than low and medium strength concrete. This is due to the better stiffness match between the mortar and aggregates in these concretes and the higher strength of the bond region in these concretes.

With these conclusions in mind it is apparent that microcracking may have some effect on the results in any investigation performed using the core-drilling method, by influencing the stiffness parameters of the tested concrete, and hence the relieved displacements. However, any effects should be small at service load levels, since, as indicated, micro-cracking at these levels is relatively small, and the stress strain curve at these levels remains essentially linear. The effects of microcracking have been neglected in the current research.
Figure 2.1 – Typical strain gage rosette for residual stress determination (ASTM E837-01e1 2001).

Figure 2.2 – Example instrumentation used on Buchner plates [from Buchner 1989]
Figure 2.3 – Superposition of loading to find relieved displacement caused by drilling a core hole; (a) original stress; (b) relieved in-situ stresses; (c) final stress

Figure 2.4 – Stress states treated in the core-drilling method; (a) uniform normal and shear stress (Case 1); (b) biaxial linear normal stress gradient and uniform shear stress (Case 2)
Figure 2.5 – Measurement configurations; (a) Measurement Configuration A; (b) Measurement Configuration B

Figure 2.6 – Schematic drawings of two test specimens: (a) Plate 1 subjected to industrial photogrammetry; (b) Plate 2 subjected to 3D digital image correlation
Figure 2.7 – Theoretical radial ($u$) and tangential ($v$) relieved displacements for the hypothetical concrete structure and representative steel plate for the uniform stress state.
Figure 2.8 – Strain gauge layout and numbering scheme
Figure 2.9 – Load frame with Plate 1 positioned for testing

Figure 2.10 – Load cell calibration data

\[ y = -0.0318x + 1.0721 \]

\[ R^2 = 0.9995 \]
Figure 2.11 - Stress versus load data for Plate 1

Figure 2.12 – Stress versus load data for Plate 2
Figure 2.13 – Stress versus load data for Plate 3

Figure 2.14 – Stress profile for Plate 1
Figure 2.15 – Stress profile for Plate 2

Figure 2.16 – Stress profile for Plate 3
Figure 2.17 – Photographs of plates after coring: (a) Plate 1 subjected to photogrammetry; (b) Plate 2 subjected to 3D digital image correlation
Figure 2.18 – Coring drill magnetically attached to a spacer plate clamped to Plate 2
Figure 2.19 – Theoretical and measured radial ($u$) and tangential ($v$) displacements from: (a) Plate 1 (photogrammetry); (b) Plate 2 (DIC); [continued]
Figure 2.19 – [continued] Theoretical and measured radial ($u$) and tangential ($v$) displacements from: (c) Plate 3 (DIC)

Figure 2.20 – Schematic detailing the direct gravitational method setup for measuring sorptivity
Figure 2.21 – Variation of $S/S_0$ with initial reduced water content for brick. Points are experimental data on several different brick materials. Solid lines are different theoretical models of the behavior [from Hall, Hoff and Skeldon (1983) as cited by Hall and Hoff (2002)].

Figure 2.22 – Long-term capillary absorption $i$ versus $t^{1/2}$ for water into a Lepine limestone specimen 630 mm high [from Taylor (1998) as cited by Hall and Hoff (2002)].
Figure 2.23 – Schematic arrangement of the ISAT test [from Claisse, Elsayad and Shaaban 1997]

Figure 2.24 – Vacuum drying front for the vacuum apparatus applied to the CAT test [from Claisse, Elsayad and Shaaban 1999]
Figure 2.25 – Schematic arrangement of the Figg Test [from Claisse et al. 1997]

Figure 2.26 – Schematic arrangement of the CAT test [from Claisse et al. 1997]
Figure 2.27 – Moisture movement of a 1:1 cement:pulverized basalt mix stored alternately in water and air at 50 percent relative humidity; cycle period 28 days [from L’Hermite, Chefdeville and Grieu (1949) as cited by Neville (1981)]
Table 2.1 - Description of concrete plate specimens tested by Buchner

<table>
<thead>
<tr>
<th>Plate</th>
<th>Side Length (mm)</th>
<th>Age at Test (days)</th>
<th>Young's Modulus (MPa)</th>
<th>Applied Stress (MPa)</th>
<th>Measured Stress (MPa)</th>
<th>Percentage Difference from $\sigma_{Max}$ or $\sigma_{Min}$</th>
<th>SRSS % Difference</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{Max}$</td>
<td>$\sigma_{Min}$</td>
<td>$\sigma_{Max}$</td>
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<td>-</td>
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</table>

*Plate age, dimensional data, modulus, applied stress and measured stress from Buchner (1989).*

Table 2.2 - Experimental test matrix information from preliminary measurement technique evaluation study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hypothetical Concrete Structure</th>
<th>Specimen</th>
<th>Plate 1</th>
<th>Plate 2</th>
<th>Plate 3</th>
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<td>steel</td>
<td>steel</td>
<td>steel</td>
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<tr>
<td>Measurement Technique</td>
<td>photogrammetry</td>
<td>digital image correlation</td>
<td>digital image correlation</td>
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<td></td>
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<td>$a$</td>
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<tr>
<td>$m$</td>
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<td>42.02 mm</td>
<td>44.45 mm</td>
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</tr>
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<td>-</td>
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<td>267 kN</td>
<td>267 kN</td>
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<td>eccentricity</td>
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<tr>
<td>$\sigma_x$</td>
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Table 2.3 - Load cell calibration data (calibration performed at 10 V excitation)

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<tr>
<th>Load (kN)</th>
<th>Output Voltage (mV)</th>
<th>Load (kN)</th>
<th>Output Voltage (mV)</th>
<th>Load (kN)</th>
<th>Output Voltage (mV)</th>
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Table 2.4 - Plate 1 (photogrammetry) strain gauge and load cell data

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<th>(2)</th>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<th>(10)</th>
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Table 2.5 - Plate 2 (digital image correlation) strain gauge and load cell data

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- \(^a\) Load and stress values *before* coring operation.
- \(^b\) Load and stress values *after* coring operation.
- \(^c\) Average load and stress values from \(^a\) and \(^b\). These values are used elsewhere in this section when final load/stress values for Plate 2 are referenced (for example Figure 2.15 or Table 2.2).
- \(^d\) Gauges failed.
<table>
<thead>
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<th>Load (kN)</th>
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<th>Back Face Gauges (MPa)</th>
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<td>268.2</td>
<td>115.7</td>
<td>111.1</td>
</tr>
<tr>
<td>267.8</td>
<td>115.8</td>
<td>111.4</td>
</tr>
</tbody>
</table>

a  Load and stress values before coring operation.
b  Load and stress values after coring operation.
c  Average load and stress values from a and b. These values are used elsewhere in this section when final load/stress values for Plate 3 are referenced (for example Figure 2.16 or Table 2.2).
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Measured Quantity</th>
<th>Applied $\sigma_x$ or $K_x$ (MPa or MPa/mm)</th>
<th>Magnitude (MPa or MPa/mm)</th>
<th>Percentage Difference from Applied $\sigma_x$ or $K_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate 1</td>
<td>$\sigma_x$</td>
<td>141.2</td>
<td>117.4</td>
<td>-16.9</td>
</tr>
<tr>
<td></td>
<td>$\sigma_y$</td>
<td>0</td>
<td>4.6</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>$\tau_{xy}$</td>
<td>0</td>
<td>6.5</td>
<td>4.6</td>
</tr>
<tr>
<td>Plate 2</td>
<td>$\sigma_x$</td>
<td>135.5</td>
<td>126.3</td>
<td>-6.8</td>
</tr>
<tr>
<td></td>
<td>$\sigma_y$</td>
<td>0</td>
<td>6.9</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>$\tau_{xy}$</td>
<td>0</td>
<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>$K_x$</td>
<td>0.83</td>
<td>0.56</td>
<td>-32.5</td>
</tr>
<tr>
<td>Plate 3</td>
<td>$\sigma_x$</td>
<td>127.6</td>
<td>174.3</td>
<td>36.6</td>
</tr>
<tr>
<td></td>
<td>$\sigma_y$</td>
<td>0</td>
<td>38.9</td>
<td>30.5</td>
</tr>
<tr>
<td></td>
<td>$\tau_{xy}$</td>
<td>0</td>
<td>1.9</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Table 2.8 - Previous laboratory measured values for sorptivity of various cement pastes, mortars and concretes

<table>
<thead>
<tr>
<th>S (mm/√min)</th>
<th>Reference(s)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.072</td>
<td>Vallini and Aldred (2003)</td>
<td>19 year old ordinary portland cement (OPC) concrete w/c = 0.4</td>
</tr>
<tr>
<td>0.015 - 0.125</td>
<td>Vallini and Aldred (2003)</td>
<td>19 year old OPC with various durability enhancing admixtures</td>
</tr>
<tr>
<td>2.79 - 3.6</td>
<td>Bentz et. Al. (2002)</td>
<td>'Early' sorptivity, concrete specimens preconditioned to 50% RH</td>
</tr>
<tr>
<td>3.02 - 8.98</td>
<td>Bentz et. Al. (2002)</td>
<td>'Late' sorptivity, concrete specimens preconditioned to 50% RH</td>
</tr>
<tr>
<td>5.2</td>
<td>Fairhurst and Platten (1999)</td>
<td>7 day old neat cement paste</td>
</tr>
<tr>
<td>3.96 - 4.20</td>
<td>Fairhurst and Platten (1999)</td>
<td>7 day old neat cement paste, with partial silica fume replacement</td>
</tr>
<tr>
<td>0.03 - 0.07</td>
<td>Martys and Ferraris (1997)</td>
<td>Bench dried conventional and high performance concrete</td>
</tr>
<tr>
<td>0.08 - 0.22</td>
<td>MacInnis and Nathawad (1980), Hall and Yau (1987)</td>
<td>Concrete slabs, w/c ratio ~ 0.4 - 0.65</td>
</tr>
<tr>
<td>0.25 - 0.48</td>
<td>Hall and Yau (1987), Hall (1989)</td>
<td>1:2:4 concrete, w/c = 0.4 - 0.9</td>
</tr>
<tr>
<td>0.094 - 0.18</td>
<td>Hall and Yau (1987), Hall (1989)</td>
<td>1:2:4 concrete, w/c = 0.4 - 0.9, prolonged tamping</td>
</tr>
<tr>
<td>0.29 - 0.31</td>
<td>Hall and Yau (1987), Hall (1989)</td>
<td>1:3:4 concrete w/c = 0.6 -0.8</td>
</tr>
<tr>
<td>0.209 - 1.94</td>
<td>Hall and Tse (1986)</td>
<td>Concrete mortars</td>
</tr>
<tr>
<td>0.14 - 5</td>
<td>Hall (1994)</td>
<td>Dense pastes - weak mortars</td>
</tr>
<tr>
<td>~0.2</td>
<td>Lockington and Parlange (2003), Taylor et. al. (2000)</td>
<td>OPC mortar</td>
</tr>
<tr>
<td>~0.09</td>
<td>Lockington and Parlange (2003), Alexander and Mackechnie (1999)</td>
<td></td>
</tr>
<tr>
<td>~0.008 - 0.031</td>
<td>Aldred and Swaddiwudhipong (2001)</td>
<td>w/c ratio 0.4 - 0.6, some specimens admixtures used</td>
</tr>
</tbody>
</table>
Table 2.9 - Distance to wetted front, \( t_w \), for different wetting times, \( t \), and sorptivity, \( S \): (a) for \( t = 60 \) minutes; (b) for \( S = 0.1 \text{mm}/\sqrt{\text{min}} \)

<table>
<thead>
<tr>
<th>( S ) (mm/\sqrt{min})</th>
<th>( t_w ) (mm)</th>
<th>( t ) (min)</th>
<th>( t_w ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015</td>
<td>0.95</td>
<td>5</td>
<td>1.83</td>
</tr>
<tr>
<td>0.03</td>
<td>1.89</td>
<td>10</td>
<td>2.58</td>
</tr>
<tr>
<td>0.1</td>
<td>6.32</td>
<td>20</td>
<td>3.65</td>
</tr>
<tr>
<td>0.3</td>
<td>18.97</td>
<td>30</td>
<td>4.47</td>
</tr>
<tr>
<td>0.5</td>
<td>31.6</td>
<td>45</td>
<td>5.47</td>
</tr>
<tr>
<td>1.0</td>
<td>63.2</td>
<td>60</td>
<td>6.32</td>
</tr>
</tbody>
</table>

\(*f = 0.1225*

Table 2.10 - Typical values for shrinkage and swelling strain for different concretes [from Lea (1970) as cited by Neville (1981)]

<table>
<thead>
<tr>
<th>Aggregate / Cement Ratio</th>
<th>Water / Cement Ratio</th>
<th>Shrinkage Strain ((10^{-6}))</th>
<th>Swelling Strain ((10^{-6}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.4</td>
<td>800</td>
<td>267</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>1200</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>550</td>
<td>183</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>850</td>
<td>283</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>1050</td>
<td>350</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>400</td>
<td>133</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>600</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>750</td>
<td>250</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>850</td>
<td>283</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>400</td>
<td>133</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>550</td>
<td>183</td>
</tr>
<tr>
<td>6</td>
<td>0.7</td>
<td>650</td>
<td>217</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>200</td>
<td>67</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>0.6</td>
<td>400</td>
<td>133</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>500</td>
<td>167</td>
</tr>
</tbody>
</table>
Table 2.11 - Coefficients used in the isotherms defined by Equation (2.56)

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>33.4</td>
<td>$a_6$</td>
<td>4.22E-04</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.46</td>
<td>$a_7$</td>
<td>7.73E-05</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-0.287</td>
<td>$a_8$</td>
<td>1.74E-04</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-1.58E-02</td>
<td>$a_9$</td>
<td>-4.22E-06</td>
</tr>
<tr>
<td>$a_5$</td>
<td>-1.45E-02</td>
<td>$a_{10}$</td>
<td>0</td>
</tr>
</tbody>
</table>
### Table 2.12 - Properties of cement pastes, mortars and aggregates

<table>
<thead>
<tr>
<th>Material</th>
<th>Water Cement Ratio</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Reference(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cement Pastes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paste</td>
<td>0.60</td>
<td>8</td>
<td>Agioutantis (2000)</td>
</tr>
<tr>
<td>Paste</td>
<td>0.40</td>
<td>19.2</td>
<td>Simeonov (1995), Hirsch (1962)</td>
</tr>
<tr>
<td>Paste</td>
<td>0.30 - 0.60</td>
<td>8 - 23</td>
<td>Simeonov (1995), Anson (1966)</td>
</tr>
<tr>
<td>Paste</td>
<td>0.50</td>
<td>12</td>
<td>Reference</td>
</tr>
<tr>
<td><strong>Cement Mortars</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortar</td>
<td>0.43</td>
<td>31.30 - 36.40</td>
<td>Simeonov (1995), Mandel (1963)</td>
</tr>
<tr>
<td>Mortar</td>
<td>0.33</td>
<td>40.5</td>
<td>Simeonov (1995), Counto (1964)</td>
</tr>
<tr>
<td>Mortar</td>
<td>0.50</td>
<td>28.3</td>
<td>Simeonov (1995), Anson (1966)</td>
</tr>
<tr>
<td>Mortar</td>
<td>-</td>
<td>36 - 38</td>
<td>Baalbaki (1991)</td>
</tr>
<tr>
<td><strong>Aggregates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand grains</td>
<td></td>
<td>80</td>
<td>Agioutantis (2000)</td>
</tr>
<tr>
<td>Gravel</td>
<td></td>
<td>61.9</td>
<td>Simeonov (1995), Hirsch (1962)</td>
</tr>
<tr>
<td>Limestone</td>
<td></td>
<td>31.9</td>
<td>Simeonov (1995), Hirsch (1962)</td>
</tr>
<tr>
<td>Diorite</td>
<td></td>
<td>104.7</td>
<td>Simeonov (1995), Mandel (1963)</td>
</tr>
<tr>
<td>Flint Gravel</td>
<td></td>
<td>74.5</td>
<td>Simeonov (1995), Counto (1964)</td>
</tr>
<tr>
<td>Graded Aggregate</td>
<td></td>
<td>74.5</td>
<td>Simeonov (1995), Stock (1979)</td>
</tr>
<tr>
<td>Dolomitic Limestone</td>
<td></td>
<td>49</td>
<td>Baalbaki (1991)</td>
</tr>
<tr>
<td>Quartzite</td>
<td></td>
<td>42</td>
<td>Baalbaki (1991)</td>
</tr>
<tr>
<td>Sandstone</td>
<td></td>
<td>40</td>
<td>Baalbaki (1991)</td>
</tr>
<tr>
<td>Quartzite</td>
<td></td>
<td>80</td>
<td>Alexander (1985), David (1975)</td>
</tr>
<tr>
<td>Dolerite</td>
<td></td>
<td>105</td>
<td>Alexander (1985), David (1975)</td>
</tr>
<tr>
<td>Granite</td>
<td></td>
<td>27.5</td>
<td>Alexander (1985), Thompson (1981)</td>
</tr>
</tbody>
</table>

*Where two references are listed the first is a citing document and the second the original work.*
CHAPTER 3

CORING WATER EFFECTS ON THE CORE-DRILLING METHOD

3.1 INTRODUCTION

The typical current practice for cutting a core hole in concrete involves flushing the core hole with water to cool and lubricate the coring drill bit, flush coring debris from the core hole, and minimize air-borne dust. Hardened concrete typically swells when exposed to water (Neville 1981). However this swelling is not accounted for in the core-drilling method equations for relating relieved displacements to in-situ stresses. This swelling induces displacements around a core hole that are difficult to differentiate from displacements due to stress relief, resulting in errors in in-situ stress predictions when applying the method.

Water moves into porous materials such as concrete in a manner that may be predicted using transport theory. In this study, a simplified prediction of this movement using the concept of sorptivity and a Sharp Front Model (Hall and Hoff 2002) is used to predict the depth of the water penetration during core drilling. Sorptivity is a measure of the ease with which a porous material absorbs liquid and is described in detail in Chapter 2. This is combined with an estimate of the magnitude of concrete swelling strain due to moisture uptake to yield expected displacement values around the core hole measurement circle due to the use of water in the coring process. These displacements are hereafter referred to as moisture displacements, to differentiate them from relieved displacements that arise from stress relief. These moisture displacement values are then converted into apparent stresses using the core-drilling method equations that relate displacements to stresses. These apparent stresses are then removed from the stresses calculated when applying the core-drilling method.

To demonstrate the applicability of the approach used in this chapter, this technique is applied to correct in-situ stress predictions from a hole drilling study of concrete plates performed by Buchner at the University of Surrey (Buchner 1989). Correction of the Buchner results using the procedure outlined herein reduced average errors in in-situ stress prediction over a series of 9 tests from 47% prior to correction to 14% afterwards.

Chapter 3 is organized such that Section 3.2 discusses the analytical approach used, Section 3.3 and 3.4 relate the effects of wetting of the core hole annulus region and surface region respectively, Section 3.5 discusses the influence of blind hole versus through hole drilling, Section 3.6 describes the superposition of results from wetting the annulus and surface regions, and Section 3.7 describes the application of the
techniques related herein to the results of the Buchner study. Section 3.8 provides a summary of research findings and the conclusions for this chapter.

### 3.2 ANALYTICAL APPROACH

The purpose of the study presented in this chapter is to analyze the effect that swelling due to coring water will have on the in-situ stress calculations of the core-drilling method. The two parameters primarily investigated were depth of water penetration, $t_w$, and magnitude of swelling strain, $\alpha_w$.

With an estimation of the time of water exposure (i.e. the time elapsed between the start of coring and the taking of displacement measurements), and the sorptivity of the concrete specimen, a depth of water penetration may be calculated based on Equation (2.51). When coring a hole, the specimen will be wetted primarily at two locations, namely, the top surface of the specimen, where the drilling water is introduced, and around the circumference of the hole, as the core is drilled. For this study, the assumed water exposure will be divided into two portions as shown in Figure 3.1. Portion A is the material around the circumference of the core hole, through the complete depth of the specimen, and Portion B, the material of only the top surface of the specimen. The terms ‘top surface’ and ‘bottom surface’ refer to the initial drilling surface and the opposite surface respectively. The results from these two conditions will be superposed to generate the complete estimation of behavior due to water exposure. Note that the behavior from each Portion will be axi-symmetric about the core hole center, as the model and associated loads for each case comprise an axi-symmetric system.

Figure 3.2 details two paths that may be taken to arrive at a cored concrete specimen that is wetted in the region near the hole. In Path 1, a dry specimen (Figure 3.2(a)) may be wetted to produce a specimen without a hole with a moist central region (Figure 3.2(b)). If this central region is then cored, Figure 3.2(c) results, a cored specimen with a wetted central region. It is also possible to arrive at Figure 3.2(c) along Path 2, where first a core hole is drilled into the dry specimen (Figure 3.2(e), identical to Figure 3.2(a)) in some theoretically dry manner (Figure 3.2(d)), and then the area around the core hole is subsequently wetted to arrive again at Figure 3.2(c). Clearly the moisture displacements in going from the dry reference state (Figure 3.2(a) or 3.2(e)) to the final state (Figure 3.2(c)) will be identical, whether moving along Path 1 or 2. Unlike the relieved displacements (as described in Section 2.2.5 and in particular illustrated in Figure 2.3), the moisture displacements may be calculated directly by analyzing a model with characteristics as Figure 3.2(c). Stated in another way, relieved displacements are caused by the act of hole drilling, moisture displacements are not; rather they are caused by moisture uptake in the material around the hole.

It is important to note here that one of the assumptions in the derivation of the sorptivity parameter (reviewed in Section 2.4.1) is violated during this approach,
namely that the water absorption is one-dimensional. For example, for the water absorbed in Portion B, although the primary direction of water movement is down into the specimen, the wetted front may also advance radially outward from the extreme edge of the Portion B wetted area shown. It is believed that this difference will be very small and any effects it may produce are neglected herein.

The finite element (FE) method was employed to estimate surface displacements due to water induced swelling. Equation (2.51) was used to determine the depth of water penetration for a given $S$ and $t$ value, and then the static swelling case was analyzed. The water induced swelling expansion was modeled by analogy in ABAQUS as swelling due to thermal effects. Moisture movement over time may also be modeled via a heat diffusion analogy (Carlson 1937, Newman 1931, Glover 1934). As mentioned, only the static swelling case was analyzed in this chapter.

In this study, any portion of an FE model that was wetted was assigned a temperature change ($\Delta T$) of 100 degrees Celsius, and the thermal expansion coefficient, $\alpha$, was assigned according to the following expression:

$$\alpha_w = \alpha \Delta T$$  \hspace{1cm} (3.1)

so as to yield an appropriate $\alpha_w$ value.

After completion of a mesh refinement/convergence study, baseline cases for Portion A and Portion B, with appropriate values for $t_w$ and $\alpha_w$, were analyzed to determine the surface displacements caused by the swelling behavior. These surface displacements were evaluated in Equations (2.20) – (2.22) to yield estimations of in-situ stress prediction errors due to swelling in each Portion. This process was repeated for numerous cases in which $t_w$, $\alpha_w$, or both were varied from their baseline values. In this manner, the influence of the assumptions for $t_w$ and $\alpha_w$ was investigated.

### 3.2.1 Baseline Parameters

The parameters of interest for this portion of the current research are the assumed depth of water penetration, $t_w$, and the assumed swelling strain, $\alpha_w$. According to Equation (2.51), $t_w$ depends on three factors, the sorptivity of the specimen, $S$, the porosity, $f$, and the time of water exposure, $t$. Typically, none of the parameters, $S$, $t$, $f$ (from which $t_w$ is derived), nor $\alpha_w$ will be known for a particular test a priori, although $t$ may be measured as the test is performed, and as covered in Section 2.4.2, there are standardized tests that may be used to measure $S$ in situ. In this chapter, values consistent with the literature and with coring practice have been used to present a baseline case.

As shown in Table 2.8, sorptivity values vary greatly for different concrete specimens and types. As a baseline for this study, a value for sorptivity of $S = 0.1 \text{ mm/} \sqrt{\text{min}}$ was assumed. The exposure time, $t$, for the baseline case was assumed to be
approximately 60 minutes, which, in conjunction with the assumed sorptivity, yields a depth of water penetration, \( t_w \), of 6 mm, according to Equation (2.51), for an assumed porosity of \( f = 0.1225 \).

The swelling strain, \( \alpha_w \), is also variable among different concretes, as presented in Table 2.10. The swelling strain for the baseline case was assigned as 167 microstrain.

### 3.3 DESCRIPTION OF PORTION A MODELING AND RESULTS

#### 3.3.1 Model Description and Mesh Refinement

Three dimensional, 8-node tri-linear displacement and temperature solid elements (ABAQUS C3D8T) were used to create a 1/8 symmetry model, as shown schematically in Figure 3.3. The center hole region was actually meshed with similar elements with negligible material and thermal properties and will not be shown. The boundary conditions of the model were full symmetry on Face A, Face B, and Face C, so that one half the thickness of a plate specimen is represented. The core hole diameter modeled was 75 mm, and the thickness of the specimen, \( t_s \), considered was 150 mm. As described previously, those elements within the wetted thickness shown were assigned a temperature change of +100 degrees Celsius, such that the associated free swelling strain in those elements was \( \alpha_w \). The overall radius model, \( R \), was chosen to ensure that edge effects did not influence the displacements near the core hole.

During the coring operation, the material around the inner surface of the core hole is exposed to water only after the drill has reached that depth. Therefore, the profile of the water penetration through the surface for Portion A is likely not a constant linear function as shown in Figure 3.3, but could more accurately be represented by either a linear variation from top to bottom, or some polynomial function of higher order. Two key factors argue for the simplest approximation as shown in Figure 3.3. First, it is known that the influence of interior loading or stress on the surface displacements decreases with increasing depth into the specimen (Turker 2003), with a practical limit being that behavior at a depth below a distance of a core hole diameter is not detectable at the surface. Thus it is of minimal benefit to approximate the profile with higher order functions. Secondly, the profile is dependent on time of exposure and sorptivity of the specimen as noted previously. Only estimates of these quantities are used in this chapter, making an exact, accurate representation of the wetted profile unwarranted.

To determine the appropriate mesh density for the baseline model, a mesh convergence study was performed. A trial mesh was analyzed with the baseline parameters noted in Section 3.2.1, yielding displacements as shown in Figure 3.4. The mesh density of the model was then approximately doubled both in-plane in the core hole region and through-thickness separately, yielding models with significantly more DOF, as summarized in Table 3.1. Each of the mesh refinement models are labeled with ‘MR’ followed by a number that indicates the number of elements in the model. The curves of Figure 3.4 show the effects of the mesh refinement on the
displacements. Table 3.1 also summarizes the peak displacement and the relative peak displacement for each model. It was decided that the mesh of model MR2664, Figure 3.5, provides appropriate balance between accuracy and running time (i.e. number of DOF). Model MR2664 will henceforth be termed the baseline model for Portion A, and labeled ‘BLA’. Table 3.2 contains all the relevant geometric and material information for model BLA.

3.3.2 Displacement and Apparent Stress for Model BLA
Figure 3.6 shows several different displacement quantities for Model BLA. Figure 3.6(a) and 3.6(b) show the scalar magnitude of displacement of the top surface of the global model and core hole region respectively. In Figure 3.6(c) the scalar magnitude of displacement is plotted on the deformed shape. The radial (in-plane) displacements are shown on the deformed shape in Figure 3.6(d), and reflect the expected axi-symmetric behavior. Finally, Figure 3.6(e) shows the out-of-plane displacements. Figure 3.6(e) shows that the out-of-plane displacements are relatively large in the wetted material around the core hole. Also noted in each portion of the figure is the scale, a measure of the amount by which the deformations have been magnified in the displaced shape. The units for Figure 3.6 are mm. Equations (2.20) – (2.22) do not account for any behavior in the out-of-plane direction. Only the in-plane displacements are reported henceforth in this chapter, as for example Figure 3.7, that contains a plot of the in-plane radial displacement versus the measurement radius for the baseline model. It depicts a strong peak in displacement that occurs at a measurement radius, \( m \), of about 82 mm. This coincides closely to the value of the wetted radius, \( r_w \), for this model. Note that \( r_w \) is simply the core hole radius, \( a \), plus the wetted thickness, \( t_w \), and in this case is equal to 81 mm. When core-drilling, the relieved displacements (due to stress relief rather than moisture uptake) attenuate rapidly with distance from the edge of the core hole. Thus, any displacement measurement during a core-drilling method test for in-situ stress will likely be performed in the surface region that is within approximately 1 core hole diameter of the edge of the core hole. Thus, moisture displacements and moisture errors will not be reported here past a distance of 3\( a \) from the core hole center.

The actual stresses in the concrete created by the moisture induced swelling are small and will not be shown, however the moisture displacements are significant. Furthermore, the actual stresses created by the moisture sorption are not related to the in-situ stress in the concrete and do not influence the measurements in the core-drilling method. Equations (2.20) – (2.22) were applied to the moisture displacements as shown in Figure 3.7 to yield the expected in-situ stress reading in the core-drilling method that results entirely from swelling caused by moisture uptake. These calculated stresses are termed apparent stresses in this dissertation, and it is emphasized that they are not related to the in-situ stress in the concrete at the time of the test.

When evaluated within the equations, the moisture displacement field generates apparent in-situ stresses that appear as hydrostatic tension, i.e. \( \sigma_x = \sigma_y \), \( \tau_{xy} = 0 \). This
finding is significant, and serves to explain some of the anomalous results of Buchner (1989), as is shown in Section 3.7. Further, only one quantity needs to be tracked, namely the magnitude of the apparent tension stress. The apparent stress is plotted versus the measurement radius in Figure 3.8. For a core radius, \( a \), of 75 mm, a typical measurement radius considered might be 100 mm, resulting in an apparent stress of prediction of 1.1 MPa for the baseline case. Thus, for example, a concrete structure under a uni-directional stress of \( \sigma_x = -4.0 \) MPa (compression) in-situ would show a bi-directional stress reading of \( \sigma_x = -2.9 \) MPa compression and \( \sigma_y = 1.1 \) MPa tension if the moisture induced deflections are not taken into account.

3.3.3 Portion A Statistical Studies
A total of 25 models were created in which \( t_w \), \( \alpha_w \), or both were varied from the baseline model, Model BLA. The wetted thickness, \( t_w \) was modeled directly, rather than accounting for the individual factors that account for \( t_w \), namely sorptivity, \( S \) and time of testing, \( t \). The magnitudes of the modified quantities were chosen to bound a realistic range of these two parameters. The range of \( t_w \) considered was from 2 mm to 25 mm, corresponding to a sorptivity range of \( S = 0.03 \) mm/√min to 0.40 mm/√min at an exposure time of 60 minutes, or \( S = 0.08 \) mm/√min to 0.97 mm/√min at an exposure time of 10 minutes, for example (with \( f = 0.1225 \)). The range of \( \alpha_w \) considered was 167E-7 to 100E-5, which is nearly plus or minus an order of magnitude from the baseline value, and corresponds to the values summarized in Table 2.10. A scatter-plot summary of the cases investigated is shown as Figure 3.9, and the specific values of \( t_w \) and \( \alpha_w \) for each case are annotated in Table 3.3. For each case considered, a model was created with properties as given in Section 3.2.1, excepting the values of \( t_w \) and \( \alpha_w \). The models are identified with a label of the form \( A_t-w-a-w*10^{-6} \), as for example Model A10-167, a model considered with \( t_w = 10 \) mm, and \( \alpha_w = 167E-6 \). Radial displacements for a representative subset of cases in which only \( \alpha_w \) is varied (Subset 1) are shown in Figure 3.10. Radial displacements for a subset (Subset 2) in which only \( t_w \) is varied are shown in Figure 3.11.

Similar to the baseline case, the displacements from all the cases considered were converted to apparent stresses using Equations (2.20) – (2.22). These results for Subset 1 are shown in Figure 3.12, and for Subset 2 in Figure 3.13. For the Portion A cases, the shape of the stress error versus measurement radius curves for all cases are quite similar, in each case, starting at an intermediate value of error at the core hole radius \( a \), rising to some maximum value of error at an intermediate radius, and then smoothly diminishing from the peak value as the measurement radius increases further. This similarity allows each Portion A curve to be approximated as a combination of linear segments between defined ‘key points,’ as shown in Figures 3.14 and 3.15. In this manner, the apparent stresses for the Portion A models may be defined for any measurement radius via linear interpolation, provided the key point values are known. The values at the ‘key points’ for each case considered are shown tabulated in Table 3.3.
With multiple linear regression techniques (Kennedy and Neville 1976), statistical models were generated that describe each of the ‘key points’ (the dependent variables) in terms of two independent variables, $t_w$ and $\alpha_w$. The results of this procedure are a series of regression coefficients that denote the dependence of the output variable on each of the two input variables. Quadratic dependence on each of the dependent variables was considered, as was cross-dependence. Only those predictor terms that have statistical significance were retained. It is almost always possible to increase the fit of a statistical model to the given data set by either including more dependent variables, or increasing their order. Such a situation results in an over-fitted model that may be adequate to describe existing data, but poor at predicting future behavior. The preliminary regression equation for each of the critical points is given in the equations below

\begin{align*}
A_{75} &= 0.00168\alpha_w + 0.000618\alpha_w t_w - 1.418E - 5\alpha_w t_w^2 \\
A_{100} &= 0.00154\alpha_w t_w - 2.1825E - 5\alpha_w t_w^2 - 0.00185\alpha_w \\
A_{125} &= 0.000730\alpha_w t_w + 8.0912E - 6\alpha_w t_w^2 - 0.000109t_w^2 \\
A_{\text{MAX}} &= 0.00114\alpha_w t_w + 0.00144\alpha_w - 6.5268E - 6\alpha_w t_w^2 - 8.3770E - 5t_w^2 \\
r_{\text{MAX}} &= 73.96 + 1.429t_w
\end{align*}

Note that $\alpha_w$ is expressed in microstrain, $t_w$ in mm such that the units for the resulting apparent stresses are in MPa. A set of 10 cases wherein $t_w$ and $\alpha_w$ were each randomly selected were analyzed to verify the accuracy of these regression equations. Due to mesh limitations, $t_w$ values greater than 10 mm were rounded to the nearest 5 mm, $t_w$ values less than 10 mm were rounded to the nearest mm. These random cases are shown in Figure 3.16. In Table 3.4, for each of these cases, the predicted value calculated from Equations (3.2) – (3.6) is compared with the actual value from the FEA. In general, good agreement is shown between stress errors via FE analysis and regression equations.

An improvement in the accuracy of the regression equations can be obtained by regenerating the equations with the input data from the original 25 cases considered plus the additional 10 verification cases, resulting in 35 independent data points for each critical point. This procedure was performed, yielding the finalized regression equations as shown below

\begin{align*}
A_{75} &= 0.00138\alpha_w + 0.000678\alpha_w t_w - 1.620E - 5\alpha_w t_w^2 \\
R^2 &= 0.999
\end{align*}
\[ A_{100} = 0.00141 \alpha_w t_w - 1.775E - 5 \alpha_w t_w^2 - 0.00109 \alpha_w \]  
\[ R^2 = 0.997 \]  

\[ A_{125} = 0.000750 \alpha_w t_w + 7.062E - 6 \alpha_w t_w^2 \]  
\[ R^2 = 0.999 \]  

\[ A_{MAX} = 0.00119 \alpha_w t_w - 8.445E - 6 \alpha_w t_w^2 + 0.00126 \alpha_w \]  
\[ R^2 = 0.999 \]  

\[ r_{MAX} = 74.25 + 1.402t_w \]  
\[ R^2 = 0.977 \]  

The units for Equations (3.7) – (3.11) should be the same as those for Equations (3.2) – (3.6). The squared correlation coefficient for each of the finalized equations is also given. Figure 3.17 shows a comparison of the values for each of the key points generated via FEA and via Equations (3.7) – (3.11). The line in each of the figures represents a perfect match between the two methods. Figures 3.18 and 3.19 show a comparison of the apparent stress versus the measurement radius from the FEA compared with the piecewise linear curves generated using the regression equations (Equations (3.7) – (3.11)) for Subset 1 and Subset 2 respectively. Figure 3.20 repeats this for the 10 random test cases. It is apparent that the regression equations generate a curve that closely approximates the FEA response. The piecewise linear curve may be plotted and used to calculate the apparent stress at any measurement radius of interest. Thus, during an actual test, if the parameters \( t_w \) and \( \alpha_w \) can be determined, or at least approximated, the apparent stress may be calculated for the pertinent measurement radius, and this apparent stress may be removed from the results, increasing the overall accuracy of the core-drilling method technique. Table 3.5 summarizes the FEA values and the finalized regression values at the critical points for all 35 cases. Note that any cases considered in the future may be added to the database of available cases to consider when generating the regression coefficients.

### 3.3.4 Other Parameters Considered

In addition to \( t_w \) and \( \alpha_w \), other parameters that can potentially affect the moisture displacement behavior were investigated. Among them were object size and the modulus of elasticity of the concrete, \( E_c \).

The model radius, \( R \), was prescribed in the baseline model (Model BLA) to simulate infinite plate behavior. An additional series of models was created and analyzed to investigate the effect of object size on the results. These models were identical to the baseline model excepting that the overall radius, \( R_c \) of each model was varied from baseline. Figure 3.21 shows stress error versus measurement radius plots for this series and indicates that as long as the overall object size is greater than approximately 4 times the hole radius, \( a \), the object size does not significantly effect the behavior.
It was hypothesized that the modulus of elasticity of the concrete, $E_c$, could potentially effect the displacement behavior of the concrete when exposed to coring moisture. Again, a series of models was examined in which $E_c$ was allowed to vary from the baseline value ($E_c = 28270$ MPa), while other variables were kept constant. Figure 3.22 shows the results of this study and indicates that $E_c$ may vary well beyond realistic limits without influencing the displacement results. It can be shown (using Equations (2.20) – (2.22) that the apparent stresses calculated from a moisture displacement field scale directly with the modulus of the concrete considered. Thus, as the displacement results are unaffected by changes in $E_c$, the apparent stresses are proportional to the modulus considered. For example, the apparent stress value from the baseline case of $A_{100} = 1.06$ MPa may be converted to a value for a concrete with $E_c = 35000$ MPa by multiplying by the ratio $(35000/28270)$ and becomes $A_{100} = 1.31$ MPa.

3.4 DESCRIPTION OF PORTION B MODELING AND RESULTS

3.4.1 Model Description
ABAQUS C3D8T type elements were used to create a 1/4 symmetry model, as shown schematically in Figure 3.23. Unlike the Portion A models, for Portion B, the entire thickness of a concrete specimen was modeled. The boundary conditions of the model were full symmetry on Face A and Face B. The model was constrained in the $z$ direction at the center point. The core hole radius modeled was 75 mm, and the thickness of the specimen considered was 150 mm. As described previously, those elements that were wetted were assigned a temperature change of +100 degrees Celsius, such that the associated free swelling strain in those elements was $\alpha_w$. Unlike in Portion A, where the wetted radius, $r_w$, was simply the addition of the core hole radius and $t_w$, for Portion B, the wetted radius must be chosen to simulate the expanse of surface area subject to wetting during core drilling. For the baseline case this expanse was defined such that $r_w = 2a = 150$ mm. This assumption was investigated as described in Section 3.4.4. The overall model radius, $R$, was chosen to ensure that edge effects did not influence the displacements near the core hole, and was the same as that for Portion A. The baseline configuration for Portion B, model BLB, was defined with material and geometric properties identical to those from Model BLA, as summarized in Table 3.2. The mesh refinement of BLB was similar to that of BLA, except that there were more layers of elements near the surface, to facilitate varying the wetted thickness as discussed in Section 3.4.3. The mesh for model BLB is shown in Figure 3.24.

3.4.2 Displacements and Apparent Stresses for Model BLB
The displacements for the baseline case for Model BLB were axi-symmetric, similar to the Model BLA results. The displacement of the top surface and bottom surface versus measurement radius is shown in Figure 3.25. Unlike Portion A, where the displacements through the thickness of the modeled specimen are similar, the water induced swelling of the top face causes bending of the specimen, and displacements
that vary through the depth of the model. A plot showing the variation through the depth of the specimen at radii of \( r = 75, 85, 100, 150, \) and 225 mm is shown as Figure 3.26. Overall, the top surface expands and the bottom surface contracts slightly. In particular, the wetted region expands in all directions, primarily radially outward, but also upward, and radially inward (especially in the region closest to the core hole, as for example \( r = 75 \) to 100 mm). Figure 3.27 shows a contour plot of the in-plane displacements in the region around the core hole plotted on the displaced shape magnified 2000 times. The figure clearly shows the inward deformation of the wetted region near the core hole.

Equation (2.20) – (2.22) were applied to the moisture displacements shown in Figure 3.25 to yield the apparent stress for model BLB, similar to the procedure described previously for Model BLA. This apparent stress is caused entirely by the moisture induced swelling of Portion B, and is not related to the in-situ stress in the concrete specimen. Similarly to the Portion A results, the moisture displacement field in Portion B evaluates as a hydrostatic stress field in the core-drilling stress – displacement equations, and thus only one apparent stress value need be reported. Figure 3.28 shows the apparent stresses versus measurement radius for the top and bottom surfaces of the plate. Each curve is prepared assuming that the displacements are measured on the respective face (i.e. the displacement field is measured on both the top and bottom surfaces of the concrete specimen). The average of the apparent stresses on the top and bottom surfaces is also shown in the figure.

As noted in Section 3.3.2, for a core hole radius of \( a = 75 \) mm, a typical measurement radius might be \( m = 100 \) mm. The Portion B baseline case values for \( m = 100 \) mm are -0.01 MPa compression on the top face and -0.28 MPa compression on the bottom face, for an average of -0.15 MPa compression.

### 3.4.3 Portion B Statistical Studies

Similar to the Portion A study, additional cases were considered in which \( t_w \) or \( \alpha_w \) or both were varied from the baseline case. The additional cases considered are shown in Figure 3.29. Due to the size and complexity of the Portion B FEA models, fewer cases were analyzed than for Portion A. These models were named in a similar fashion to the Portion A models, that is, the model B6-167 would have \( t_w = 6 \) mm and \( \alpha_w = 167E-6 \). Figure 3.30 shows the apparent stress versus measurement radius for all of these models.

In Section 2.2.2, results from Buchner (1989) are summarized, and it is noted that stresses therein are calculated from displacements measured at \( m = 100 \) mm. Thus, in this exercise, two quantities were deemed ‘key points’, namely, the apparent stress at \( m = 100 \) mm on the top and bottom surface. These values are summarized for each model in Table 3.6. Preliminary regression equations were generated for these two quantities and are as follows
Units for these equations are similar to those for Equations (3.7) – (3.11). Six test cases were analyzed to verify these equations, as shown in Figure 3.31. A comparison between the actual FEA values and the values from the preliminary regression equations for these three cases is contained in Table 3.7. In general good agreement is shown between the techniques. Similar to the Portion A study, a finalized set of regression equations was developed that included the data points from six trial cases plus the original cases. These equations appear below

\[
B_{100\text{top}} = -0.157 \frac{\alpha_w}{100} + 0.04107 \frac{\alpha_w}{100} t_w
\]  

\[
B_{100\text{bottom}} = -0.08309 \frac{\alpha_w}{100} - 0.03297 \frac{\alpha_w}{100} t_w
\]

Units for these equations are similar to those for Equations (3.7) – (3.11). The ‘key point’ data from the Portion B FEA and the finalized regression equations is summarized in Table 3.8. Figure 3.32 shows a comparison of the Portion B ‘key point’ data generated via FEA or Equations (3.14) and (3.15), with the dark line in the figure indicating perfect correlation between the two.

### 3.4.4 Other Parameters Considered

Several other parameters were considered in the investigation of the Portion B models, namely, the thickness of the specimen, \( t_s \), the assumed wetted radius, \( r_w \), and the overall specimen size, \( R \).

To investigate the effect of the specimen thickness, \( t_s \), on the results, an additional series of models was created identical to model BLB except that the thickness was varied from \( t_s = 75 \, \text{mm} \) to \( t_s = 1000 \, \text{mm} \). The mesh for these models was slightly less refined than for model BLB, in order to speed the run time of the models. A comparison of the moisture displacements from the new series (termed MeshLR) with \( t_s = 150 \, \text{mm} \) and BLB (with \( t_s = 150 \, \text{mm} \)) is shown in Figure 3.33 and indicates that the mesh refinement in the new series provides essentially identical results to model BLB. The apparent stresses from the top and bottom surfaces of these models versus the measurement radius are shown in Figure 3.34. It appears that the apparent stresses vary as the inverse of the specimen thickness. For example, Figure 3.35 shows the values for \( B_{100\text{top}} \) and \( B_{100\text{bottom}} \) plotted versus the specimen thickness. The equations of the dashed lines in the figure are
\[ B_{100\text{top}}(t_s) = -0.43 + \frac{60}{t_s} \]  
(3.16)

\[ B_{100\text{bottom}}(t_s) = 0.04 - \frac{45.5}{t_s} \]  
(3.17)

where \( t_s \) is expressed in mm, and the results are in MPa.

The poor fit near \( t_s = 75 \) mm to 150 mm for the curve representing the bottom surface apparent stresses is most likely due to the fact that with thinner specimens, the swelling behavior on the top face strongly influences the bottom face behavior, due to the proximity of the two faces in thinner specimens. It is recommended in this region \((t_s = 75 \text{ mm to } 150 \text{ mm})\) to use

\[ B_{100\text{bottom}}(t_s) = -0.26 \]  
(3.18)

and to use Equation (3.17) for thicker specimens.

The investigation performed here was based on the baseline Portion B case (i.e. \( \alpha_w = 167E-6, \ t_w = 6 \text{ mm} \)). Although unverified, the general inverse \( t_s \) behavior noted should hold for other cases of \( \alpha_w \) or \( t_w \), although the constants noted in Equations (3.16) – (3.18) would change.

Although Equations (3.16) – (3.18) are provided it appears that for specimens with \( t_s > 300 \) mm, it may be assumed that the thickness of the specimen has no influence on the apparent stresses, with little loss in accuracy. Considering the accuracy inherent in predictions for parameters such as sorptivity and swelling strain, it may be possible to neglect the thickness dependence altogether for specimens normally encountered in practice.

To investigate the effect of the assumed wetted radius, \( r_w \), a model was created that was identical to model BLB except that \( r_w = 225 \) mm (rather than the 150 mm of the baseline case). The apparent stress results of this model are shown in comparison to the baseline case in Figure 3.36. The apparent stresses of the new model are similar to the baseline from \( m = 75 \) mm to \( m = 150 \) mm, however after 150 mm the stresses diverge significantly. With more wetted area, the swelling behavior is stronger, and thus higher apparent stresses are noted. However, as previously noted, the area of measurement interest is likely within a radius of \( 2a = 150 \) mm from the center of the core hole, meaning that the divergence in results is not likely to be significant.

Although it was shown in Section 3.3.4 that the object size, \( R \), has little effect on the Portion A apparent stresses (provided \( R > 4a \)) it was theorized that the bending induced in the portion B models might cause \( R \) to play a more significant role. The overall model size, \( R \), of model BLB was varied to ensure that the object size has a
negligible effect on the calculated apparent stresses for the Portion B models. Figure 3.37 shows a comparison of the baseline model BLB results ($R = 750$ mm) with a model with $R = 375$ mm, and indicates that object size does not greatly influence the apparent stresses for the portion B models. The mesh used to create Figure 3.37 was the same as used to create the series of models in which $t_s$ was varied, and does not influence the conclusion drawn, as noted above.

3.5 CHANGES IN BEHAVIOR FOR BLIND HOLES VERSUS THROUGH HOLES

Section 3.3 and 3.4 present the results of studies involving wetting of Portion A and Portion B regions in the case of a through hole. To determine the effect of drilling a blind hole, again the wetted area was subdivided into two portions: Portion A that included the internal portion of the hole, as before, except that the bottom of the blind hole is also wetted, and Portion B, the exposed face region, as before. The mesh termed MeshLR in Section 3.4.4 was used along with the baseline parameters ($t_w = 6$ mm, $\alpha_w = 167E-6$) to investigate the two portions. The model thickness was $t_s = 300$ mm, and the blind hole depth, $h$, was assigned as $h = 150$ mm ($h = 2a$). Figure 3.38 shows the apparent stresses for the blind hole versus the through hole case for Portion A, and indicates that any difference between the two may be neglected for this portion on the top face. On the bottom face, for a blind hole, it may be assumed that the Portion A apparent stresses are zero. Figure 3.39 shows the same for Portion B, with $t_s = 300$ mm and $t_s = 1000$ mm, and shows that the differences between the blind and through hole cases may be neglected for Portion B. In total then, provided $h = 2a$, the only significant difference between a through hole and a blind hole is that the Portion A apparent stresses on the bottom face may be assumed to be zero. It is anticipated that in the majority of cases when drilling a blind hole, no measurements will be made on the bottom face of the specimen, and thus there would be no detectable difference between the through hole and blind hole case.

3.6 SUPERPOSITION OF PORTION A AND PORTION B RESULTS

3.6.1 The Overlap Region
As noted in Figure 3.1, the wetted regions of Portion A and Portion B overlap slightly. If the results of the Portion A and Portion B studies are superposed, this area in effect receives twice the correct allotment of load. A model was created with mesh and properties identical to BLB, wherein only the wetted area is the overlap region shown in Figure 3.1. The apparent stresses for this model are shown in Figure 3.40. Removal of these apparent stresses from the model BLB apparent stresses (shown in Figure 3.28) would allow correct superposition of the BLA and BLB results. Figure 3.41 shows the apparent stresses from BLB and the same with the apparent stresses from Figure 3.40 removed. Although there are minor differences between the two cases, the differences are deemed small enough to neglect. The results for BLA and BLB, as well as all other Portion A and Portion B results will be superposed directly, without correcting for the overlap region.
3.6.2 The Baseline Case

It is possible to superpose the apparent stress results of the baseline portion A and Portion B models in order to generate the complete apparent stress state due to the total moisture displacement field, because the behavior is assumed to be linear elastic. Figure 3.42 shows the apparent stress versus measurement radius on the top and bottom faces for the baseline total case. The average of the top and bottom is also shown. For the baseline case, at $m = 100$ mm, the apparent stresses are 1.05 MPa tension on the top face, and 0.78 MPa tension on the bottom face, for an average of 0.92 MPa tension. It may be noted that the apparent stresses at 100 mm follow directly from

$$AB_{100\text{\,\,top}} = A_{100} + B_{100\text{\,\,top}}$$

$$AB_{100\text{\,\,bottom}} = A_{100} + B_{100\text{\,\,bottom}}$$

$$AB_{100\text{\,\,avg}} = \frac{AB_{100\text{\,\,top}} + AB_{100\text{\,\,bottom}}}{2}$$

as expected. Note that if adjustment is to be made based on specimen thickness, $t_s$, (following Equations (3.16) – (3.18)), it should be performed prior to substitution of $B_{100\text{\,\,top}}$ and $B_{100\text{\,\,bottom}}$ into Equations (3.19) – (3.21).

3.6.3 Other Cases

At $m = 100$ mm, values of apparent stress for cases with $t_w$ and $\alpha_w$ other than as for the baseline case may be calculated by utilizing the regression equations noted in section 3.3.3 (Equation (3.8)) and section 3.4.3 (Equation (3.14) and (3.15)). The values calculated via regression can be substituted into Equations (3.16) – (3.18) noted above to yield the appropriate apparent stresses. Table 3.9 contains examples of total apparent stresses at $m = 100$ mm for $t_w$ ranging from 2 – 20 mm and $\alpha_w$ ranging from 20 – 400 microstrain, with values generated with Equations (3.8), (3.14) – (3.15), and (3.16) – (3.18).

3.7 APPLICATION TO BUCHNER RESULTS

3.7.1 Introduction

Buchner (1989) presents the results of a series of tests using a method similar to the core-drilling method that measured stress in a number of concrete plates. This study is summarized in Section 2.2.2. Although an error in applied stress versus measured stress of approximately +/- 10% was reported as being potentially achievable, for the 9 plates tested with a core hole radius of 75 mm (some others were tested with $a = 37.5$ mm), the average error is considerably higher, especially at measurement times immediately following the coring of the plates. The experimental parameters and results for these 9 plates were summarized in Table 2.1.
An examination of the measured data in Table 2.1 shows that, for every plate, the calculated stresses appear to differ from the applied stresses primarily by a hydrostatic tension stress. As an example, Figure 3.42 shows Mohr’s circle representations of the applied and measured stresses of Plates I and II. For each of the plates, the diameter of each circle is about the same, so they differ only by a hydrostatic stress. The calculation of the adjusted stress Mohr’s circles in the figure is described subsequently. From this observation, and the quote noted in Section 2.2.2, it is reasonable to conclude that coring water significantly affected the Buchner results. The approach outlined in the preceding sections will be applied to the data from these 9 plates to show that the approach reduces errors in predicted stress considerably. A full summary of the Buchner experimental program is provided in Section 2.2.2.

3.7.2 Application of Findings to Buchner Results

As a first pass at correcting the Buchner results, the complete (Portion A and B superposed) baseline case results calculated herein (in Section 3.6.2) may be removed from the Table 2.1 results. Although sorptivity, time of water exposure, porosity, and swelling strain were not explicitly reported by Buchner, some estimates may be made after the fact. As noted in Section 2.2, the shrinkage strain for the Buchner plates was estimated to be 520E-6, this corresponds to a swelling strain, $\alpha_w$, of 173E-6, assuming the Nillson (1982) recommendation (swelling strain approximately 1/3 of shrinkage strain) is followed. This swelling strain is similar to the 167E-6 used for the baseline case herein. Furthermore, for the data summarized in Table 2.1, $t$ is approximately 30 to 60 minutes, corresponding to a sorptivity value of $S = 0.095$ to 0.134 if the baseline case of $t_w = 6$ mm is applied (with $f = 0.1225$). These values are well within the bounds for sorptivity and swelling strain summarized in Table 2.8 and 2.10 respectively.

Table 3.10 shows the effects of removing the baseline case apparent hydrostatic stress from the Buchner data. For each case the baseline Portion B apparent stress value has been translated from the baseline plate thickness ($t_s = 150$ mm) to the Buchner plate thickness ($t_s = 100$ mm) using Equations (3.16) – (3.18). After the Portion A and Portion B baseline results are superposed, the resultant has been scaled by the ratio of the individual Buchner plate elastic modulus divided by the baseline modulus ($E = 28270$ MPa). The apparent stress removed is the average of the values from the top and bottom faces. The removal of these apparent stresses reduces the average RSS error from 47% to 14%, a significant improvement.

Table 3.11 shows the removal of the optimal value of apparent hydrostatic stress (1.64 MPa) from the Buchner data, reducing the average RSS error to 11.8%. This optimal value was calculated by minimizing the average RSS error value across the 9 cases. With Equations (3.8), (3.14) and (3.15), it can be shown that with the average $E$ over the 9 tests of 36790 MPa, this corresponds to $\alpha_w = 209E-6$, $t_w = 6$ mm, or $\alpha_w = 167E-6$, $t_w = 7.4$ mm, for example (although these calculations are not adjusted for plate thickness, $t_s$).
Clearly, if more were known regarding the properties of the concrete that was tested, the appropriate values for $\alpha_w$ and $t_w$ for each plate could be used to generate the correct moisture induced hydrostatic apparent stress for each plate. If the in-situ stress for each plate was corrected individually in this manner, it is possible that the average RSS error would be further reduced. Regardless, even with no other knowledge, the correction with the baseline case apparent stress results reduced the errors by more than a factor of 3, an important finding. Furthermore, the apparent stresses generated in this chapter follow the trends noted in Buchner (1989) regarding bending in the plates introduced as a result of the coring water. A clear bending pattern is present in the Portion B results.

3.8 SUMMARY OF FINDINGS

A method has been presented that accounts for distortions in the predicted core-drilling method in-situ stresses caused by coring water induced swelling. The method uses the parameters $t_w$ (which is derived from sorptivity, $S$, and time of water exposure during the test, $t$) and $\alpha_w$ as input into a finite element investigation that characterized the moisture displacement field. The displacements due to moisture were converted to apparent in-situ stresses, stresses that exist solely due to coring moisture and are unrelated to the in-situ stress present in the structure. To increase the accuracy of the core-drilling method technique, these apparent stresses should be removed from the in-situ stresses so derived.

It was found that these apparent stresses are primarily tension stresses, and furthermore, that they are hydrostatic in appearance, i.e. $\sigma_x = \sigma_y$, $\tau_{xy} = 0$. In addition, the coring water causes apparent stresses that appear as bending through the thickness of a plate specimen.

A baseline case using ‘typical’ values of $t_w$ and $\alpha_w$ was presented. For the baseline case, at $m = 100$ mm, the apparent stresses are 1.05 MPa tension on the top face, and 0.78 MPa tension on the bottom face, for an average of 0.92 MPa tension. Apparent stresses for other values of $t_w$ and $\alpha_w$ may be calculated via the regression equations generated in Section 3.3.3 (Equations (3.7) – (3.11)) and Section 3.4.3 (Equations (3.14) – (3.15)), along with Equations (3.19) – (3.21).

The results from core-drilling type experiments on nine individual plates performed previously by other researchers were reviewed. The new approach was used to show that relative errors in experimental in-situ stress prediction were reduced from 47% to 14% upon its application with baseline values of $t_w$ and $\alpha_w$, and could be reduced to 12% if the optimal values of $t_w$ and $\alpha_w$ were used.

It should be noted here that another potential source of error in the previous experiments is the presence of stresses in the plates caused by differential shrinkage.
Differential shrinkage would cause the faces of the concrete plates tested to be in compression while the interior of the plates would be in compression. This effect was not accounted for in the previous work, and no attempt has been made here to include it. Chapter 4 describes research performed to quantify the effects of differential shrinkage on the core-drilling method.

Other important points to note here are the following:

- Typical values for the properties of moisture and swelling strain have been reviewed in Chapter 2. These may of use during the core-drilling method if approximate values are to be used.
- Methods for measuring sorptivity both in the laboratory and in-situ have been reviewed in Section 2.4.1 and 2.4.2 respectively if more accurate values of $S$ and hence $t_w$ are desired.
- Parameters that were shown to have no significant effect on the apparent stresses were $R$ (the test object size, provided $R > 4a$), and $r_w$ (the wetted radius of the front face).
- The modulus of elasticity of the concrete specimen, $E_c$, was shown to have no effect on the moisture displacements. However, the apparent stresses scale exactly linearly with the modulus. Thus, to generate apparent stresses for a concrete specimen with a modulus of elasticity different from the baseline case herein ($E_c = 28270$ MPa), the baseline case apparent stresses may be scaled by the ratio of specimen modulus to baseline modulus.
- The thickness of the measured specimen, $t_s$, was shown to influence the apparent stresses slightly for $t_s$ less than 300 mm, although with the difficulty in accurately measuring other parameters involved, it may be possible to neglect this dependence. For specimens with $t_s$ greater than 300 mm, the thickness dependence should be ignored.
- It is reasonable to apply these results to objects other than plates, so long as the pertinent dimensions of the tested object are greater than $R = 4a$.  

Figure 3.1 – Regions subjected to core drilling water when drilling a core hole

Figure 3.2 – Evolution of moisture stresses

Path 1

(a)       (b)       (c)       (d)       (e)

Dry Concrete Wet Concrete Hole

Path 2
Figure 3.3 – Portion A 1/8 symmetry finite element model schematic

Figure 3.4 – In-plane (radial) displacement of mesh convergence study models versus measurement radius
Figure 3.5 – Finite element mesh of Model MR2664 showing: (a) an isometric view of the global mesh; (b) a plane view of the mesh around the core hole region
Figure 3.6 – Displacements of the baseline Portion A model (Model BLA): (a) vector magnitude of the global model plotted on un-deformed shape; (b) vector magnitude of the core hole region plotted on un-deformed shape; [continued]
Figure 3.6 - [continued] (c) vector magnitude of the core hole region plotted on the displaced shape; (d) radial of the core hole region plotted on the displaced shape; [continued]
Figure 3.6 – [continued] (e) out-of plane of the core hole region plotted on the displaced shape
Figure 3.7 – Radial surface displacement versus measurement radius for the Portion A baseline Model (Model BLA)

Figure 3.8 – Apparent stress versus measurement radius for the Portion A baseline model (Model BLA)
Figure 3.9 – Cases considered in the statistical study of Portion A parameters, $t_w$ and $\alpha_w$. 

Cases Considered

BLA
Figure 3.10 – Radial surface displacement versus measurement radius for Subset 1

Figure 3.11 – Radial surface displacement versus measurement radius for Subset 2
Figure 3.12 – Apparent stress versus measurement radius for Subset 1

Figure 3.13 – Apparent stress versus measurement radius for Subset 2
Figure 3.14 – Key points for Model BLA

Figure 3.15 – Apparent stress versus measurement radius generated with key point values compared with FEA results
Figure 3.16 – Random 10 cases considered to verify the Portion A regression equations

Figure 3.17 – FEA versus predicted values for Portion A key points: (a) for $A_{\text{MAX}}$; [continued]
Figure 3.17 – [continued] FEA versus predicted values for Portion A key points: (b) for $r_{MAX}$; (c) for $A_{75}$; [continued]
Figure 3.17 – [continued] FEA versus predicted values for Portion A key points: (d) for $A_{100}$; (e) for $A_{125}$
Figure 3.18 – Comparison of apparent stresses from Portion A FEA and from the finalized regression equations – Subset 1

Figure 3.19 – Comparison of apparent stresses from FEA and from the finalized regression equations – Subset 2
Figure 3.20 – Comparison of apparent stresses from FEA and from the finalized regression equations – Random 10 cases

Figure 3.21 – Apparent stress versus measurement radius for models of different size, $R$
Figure 3.22 – Surface displacement versus measurement radius for models with different modulus of elasticity, $E_c$

Figure 3.23 – Portion B finite element model schematic
Figure 3.24 – Finite element mesh of Model BLB

Figure 3.25 – Radial surface displacements versus measurement radius for Model BLB
Figure 3.26 – Displacements through the depth of Model BLB

Figure 3.27 – In-plane displacements of the core hole region of model of Model BLB
Figure 3.28 – Apparent stress versus measurement radius for Model BLB

Figure 3.29 – Cases considered for Portion B
Figure 3.30 – Apparent stress versus measurement radius for the Portion B models: (a) for Model B2-50; (b) for Model B2-167; [continued]
Figure 3.30 – [continued] Apparent stress versus measurement radius for the Portion B models: (c) for Model B2-690; (d) for Model B6-50; [continued]
Figure 3.30 – [continued] Apparent stress versus measurement radius for the Portion B models: (e) for Model B6-690; (f) for Model B12-50; [continued]
Figure 3.30 – [continued] Apparent stress versus measurement radius for the Portion B models: (g) for Model B12-167; (h) for Model B12-690; [continued]
Figure 3.30 – [continued] Apparent stress versus measurement radius for the Portion B models: (i) for Model B26-50; (j) for Model B26-167; [continued]
Figure 3.30 – [continued] Apparent stress versus measurement radius for the Portion B models: (k) for Model B26-50

Figure 3.31 – Verification cases considered for the Portion B regression equations
Figure 3.32 – FEA versus predicted for Portion B key points: (a) for $B_{100\text{top}}$; (b) for $B_{100\text{bottom}}$
Figure 3.33 – Comparison of moisture displacements from BLB and new model with less refined mesh (MeshLR)
Figure 3.34 – Apparent stress versus measurement radius for models with varying thickness, $t_s$: (a) top surface apparent stress; (b) bottom surface apparent stress
Figure 3.35 – Apparent stress at \( m = 100 \) mm versus thickness of specimen, \( t_s \)

Figure 3.36 – Comparison of Model BLB with a model with \( r_w = 225 \) mm
Figure 3.37 – Comparison of the apparent stresses of model BLB with a model with $R = 375$ mm

Figure 3.38 – Apparent stress versus measurement radius for a Portion A model with a blind hole
Figure 3.39 – Apparent stress versus measurement radius for models of different thicknesses with blind holes: (a) $t_s = 300$ mm; (b) $t_s = 1000$ mm
Figure 3.40 – Apparent stresses versus measurement radius for the overlap region

Figure 3.41 – Apparent stresses versus measurement radius for BLB and BLB minus the overlap region
Figure 3.42 – Apparent stresses versus measurement radius for the superposed (BLA plus BLB) baseline case
Figure 3.43 – Mohr’s circle representation of applied, measured and adjusted stresses for two different Buchner plates: (a) Plate I; (b) Plate II
### Table 3.1 - Mesh convergence study information

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### Table 3.2 - Portion A baseline model (Model BLA) parameters

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Table 3.4 - Comparison of preliminary regression predictions versus actual FEA results

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Table 3.5 - Final regression predictions versus actual FEA results - all Portion A cases [continued]

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all Portion A cases [continued]

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### Table 3.7 - Preliminary regression predictions versus actual FEA results - all Portion B test cases

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Table 3.9 - Example calculations using superposition of Portion A and B finalized regression equations

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<td>5.56</td>
<td>0.66</td>
<td>-1.13</td>
<td>6.22</td>
<td>4.43</td>
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<td>400</td>
<td>7.42</td>
<td>0.88</td>
<td>-1.51</td>
<td>8.30</td>
<td>5.90</td>
<td>7.10</td>
</tr>
</tbody>
</table>
Table 3.10 - Buchner data with apparent stresses of the baseline case removed

Baseline Case
Apparent
Stresses
(corrected for
\( t_s = 100 \text{ mm} \))
\( \sigma_{\text{total-top}} \)
1.23 MPa
\( \sigma_{\text{total-center}} \)
1.02 MPa
\( \sigma_{\text{total-bottom}} \)
0.80 MPa

<table>
<thead>
<tr>
<th>Plate No.</th>
<th>Applied Stress</th>
<th>Measured Stress with Baseline Case Apparent Stress Removed</th>
<th>Percentage Difference from ( \sigma_{\text{Max}} ) or ( \sigma_{\text{Min}} )</th>
<th>RSS Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{\text{Max}} )</td>
<td>( \sigma_{\text{Min}} )</td>
<td>( \sigma_{\text{Max}} )</td>
<td>( \sigma_{\text{Min}} )</td>
</tr>
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<td>0</td>
<td>-4.19</td>
<td>1.39</td>
<td>-0.35</td>
</tr>
<tr>
<td>2</td>
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<td>-4.94</td>
<td>1.20</td>
<td>1.04</td>
</tr>
<tr>
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<td>0</td>
<td>-4.96</td>
<td>1.35</td>
<td>-0.11</td>
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<td>0</td>
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<td>1.27</td>
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<td>-0.16</td>
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<td>-4.9</td>
<td>-9.84</td>
<td>1.25</td>
<td>-4.25</td>
</tr>
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<td>-4.92</td>
<td>-9.84</td>
<td>1.38</td>
<td>-5.04</td>
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</tbody>
</table>

Average 14.0
Table 3.11 - Buchner data with optimal apparent stresses removed

<table>
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<tr>
<th>Plate No.</th>
<th>Applied Stress (MPa)</th>
<th>Measured Stress with Apparent Stress Removed (MPa)</th>
<th>Percentage Difference from $\sigma_{\text{Max}}$ or $\sigma_{\text{Min}}$</th>
<th>RSS Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{\text{Max}}$</td>
<td>$\sigma_{\text{Min}}$</td>
<td>$\sigma_{\text{Max}}$</td>
<td>$\sigma_{\text{Min}}$</td>
</tr>
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<td>0</td>
<td>-4.19</td>
<td>-0.60</td>
<td>-4.42</td>
</tr>
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<td>-4.79</td>
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<td>-4.86</td>
</tr>
<tr>
<td>I</td>
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<td>-4.87</td>
<td>0.51</td>
<td>-4.11</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>-4.88</td>
<td>-0.47</td>
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<td>III</td>
<td>0</td>
<td>-4.88</td>
<td>-0.46</td>
<td>-4.60</td>
</tr>
<tr>
<td>VII</td>
<td>-4.9</td>
<td>-9.84</td>
<td>-4.64</td>
<td>-9.01</td>
</tr>
<tr>
<td>VIII</td>
<td>-4.92</td>
<td>-9.84</td>
<td>-5.30</td>
<td>-9.84</td>
</tr>
<tr>
<td>IX</td>
<td>-4.9</td>
<td>-9.81</td>
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<td>-9.365</td>
</tr>
<tr>
<td>Average</td>
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</tr>
</tbody>
</table>
CHAPTER 4

DIFFERENTIAL SHRINKAGE EFFECTS ON THE CORE-DRILLING METHOD

4.1 INTRODUCTION

It is well known that hardened concrete will shrink upon drying. The interior of a massive specimen will remain at a higher moisture content than the exterior for some time. The resulting moisture content gradient causes differential shrinkage, with the outer surface of the concrete shrinking at a faster rate than the inner portions. This interior region provides restraint to the free shrinkage that would take place if the entirety of the concrete specimen was at the same moisture content. This restraint in turn causes a stress gradient to develop through the thickness of the concrete member. Thus any displacements measured around a core hole will reflect not only relief of the stresses due to known applied loads, but also relief of these unknown differential shrinkage stresses.

One way to evaluate the accuracy of the core-drilling method experimentally (as is related in Chapter 6) is to apply a known load to a concrete specimen and compare the theoretical stresses due to this applied load to those calculated upon evaluation of the specimen by the core-drilling method. However, differential shrinkage of concrete causes stresses that complicate accuracy evaluation of the method in this manner.

In Section 4.2, moisture profiles and corresponding differential shrinkage stresses are calculated analytically for concrete specimens, and then these specimens are evaluated numerically with the core-drilling method. In this manner the effects that differential shrinkage stresses have on the core-drilling method are explored. Variables probed herein include the effects of the relative humidity of the storage environment, the age of the concrete at test, and the thickness of the tested concrete specimen.

In Section 4.3 the effect that differential shrinkage stresses had on the Buchner (1989) study is estimated. Buchner’s research is summarized in Section 2.2.2. Buchner’s research involved drilling holes in plates with known loads and comparing stresses calculated from measured deformations around the holes to the theoretical stresses from the applied loads. It was indicated that accuracy of ± 10% in stress calculation was potentially achievable, although for the subset of the Buchner plates examined herein the average relative error in stresses calculated was 47%. It is shown in Section 4.3 that accounting for the differential shrinkage stresses can reduce the error in measured stresses in the Buchner experiments from 47% to 42% or 34% depending on the assumed relative humidity of the plate storage environment.
4.2 ANALYTICAL PROCEDURE

4.2.1 Determination of the Moisture Profile and Restrained Shrinkage Stresses

The movement of water out of initially saturated, curing concrete is governed by the differential diffusion equation, Equation (2.53), with boundary conditions as prescribed in Equations (2.54) and (2.55). The moisture profile within each plate considered in this chapter is calculated using the model proposed by Akita, Fujiwara and Ozaka that is described in detail in Section 2.4.4.

The differential diffusion equation that governs moisture movement (i.e. Equation (2.53)) is of the same form as that which governs heat transfer. Thus, any finite element program that is appropriate for three dimensional heat transfer analysis can be used to solve the moisture movement problem of Equation (2.53). In this study, ABAQUS was used to model the drying of concrete plates via a temperature analogy (Bathe 1982). In this case, a sequentially coupled thermal and stress analysis was performed. The thermal (moisture movement) portion of the analysis is completed first and assumed to be independent of the stress in the object. The results of the thermal analysis are then used as input for a subsequent stress analysis. This leaves only to define the appropriate relationship between moisture content and shrinkage.

Historically, a linear relationship between moisture loss and ultimate shrinkage has often been used (Pickett 1946, Iding and Bresler 1982). Recently, others have proposed more complicated relationships, as for example that proposed by Rahman et al. for the relationship in concrete repair materials (Rahman et al. 2000). In many cases relationships with linked creep and shrinkage have also been proposed. However, in the interest of simplicity, a linear relationship was used in this study, wherein the percentage of moisture lost was directly proportional to the percentage of ultimate free shrinkage experienced. Creep was not considered herein but would likely reduce any differential shrinkage stresses reported.

The environmental relative humidity needs to be defined in order to explicitly define the boundary condition of Equation (2.55). As explained in Section 2.4.4, this relative humidity is then converted into an equivalent relative moisture content through the use of the isotherms of Equation (2.56). In this chapter three different storage environments are considered: 20% RH, 50% RH and 80% RH. These values yield relative moisture contents of 37%, 54% and 75% respectively when evaluated with Equation (2.56) for a water-cement ratio of 0.40.

The analysis of this problem is divided into two phases. In the first phase, the moisture profile and corresponding shrinkage stresses for a particular concrete plate are calculated. In the second phase, the resulting shrinkage stresses are evaluated with the core-drilling method to determine the effect that these stresses have on a core-drilling method investigation.
The finite element model that was used to complete the moisture movement analysis (FEM 1) is shown in Figure 4.1(a). The model contains 504 eight node solid (ABAQUS C3D8T) linear thermal and displacement elements. The analysis is insensitive to the aspect ratio of these elements as the problem considered is essentially one-dimensional. ABAQUS uses an iterative modified Newton scheme to solve nonlinear heat transfer problems, which is implemented in this study with the automatic time increment option, wherein a maximum allowable nodal temperature change per increment is prescribed. Time integration is performed with the backward Euler method. Each model represents 1/4 of a plate in plan and half the thickness of a plate drying from two opposing faces, thus representing an overall 1/8 symmetry condition. Figure 4.1(b) is discussed in the next section.

The FEM1 model requires temperature and displacement boundary conditions as well as prescribed initial conditions on temperature. The convection boundary condition is imposed on the top face of the model (the prominent face in the figure), fully insulated boundary conditions were assigned on all other faces. Spatial boundary conditions were imposed to simulate the aforementioned 1/8 symmetry condition. The moisture content of all elements in the model is assumed to be 100% at the beginning of the analysis.

In order to readily apply the findings of this study to the Buchner results discussed in the next section, the modeled plates are square, one meter on a side, and 10 cm thick. The modulus of elasticity and Poisson’s ratio of all finite element models in this work were assigned as 33400 MPa and 0.2 respectively. The modulus of elasticity of the plates is assumed constant through the life of the plates. The water cement ratio of the modeled plates is 0.40, and the ultimate shrinkage strain is assumed to be 520 micro-strain.

The estimated moisture profiles for the 10 cm thick plates stored at 80% and 50% relative humidity are shown in Figure 4.2. The figure shows the moisture content through the depth of the plate after varying amounts of drying time. Plotted times correspond to the age at testing of Buchner plates 3, III, 2 and IX, as described in Section 2.2.2. As expected, the near surface layer in the concrete dries out rapidly, but the internal regions remain at a relatively high moisture content for many months. The corresponding shrinkage stress distributions are shown in Figure 4.3. The stresses reported here are those through the center of the modeled plate, thus avoiding boundary edge effects. As expected, the model shows tension stresses on the surface of the plate, with compression on the interior, thus maintaining equilibrium. Creep was not considered as part of this investigation, but it would likely relieve some of the stresses noted in the figure.

**4.2.2 Evaluation of Shrinkage Stresses in the Core-Drilling Method**

As noted in Section 2.2.5.1, the displacements produced by drilling a hole in a loaded object are equivalent to those generated by applying the inverse of the existing in-situ stresses to the circumference of a hole in an unloaded object. To determine the
relieved displacements that are generated when applying the core-drilling method to objects with differential shrinkage stresses, the inverse of the differential shrinkage stresses (as for example shown in Figure 4.3) was applied to the circumference of a central hole in a second finite element model. The mesh of this second finite element model (FEM 2) is shown in Figure 4.1(b). In this case, 11934 eight node linear, reduced integration (ABAQUS C3D8R) solid elements were employed in a linear-elastic static displacement/stress analysis. The radius of the core hole considered is \( a = 75 \) mm. The aspect ratio of the elements in the vicinity of the core hole ranges from 1 to 10, a mesh refinement study was performed and indicated that this relatively coarse mesh was adequate to accurately capture the displacement behavior of interest. Boundary conditions were applied to the model to simulate \( 1/8 \) symmetry. In Figure 4.4, the calculated differential shrinkage stresses from the first finite element model (Figure 4.1(a)), as well as the stresses applied to the second model (Figure 4.1(b)) are shown for a 10 cm thick concrete plate assumed to be stored at 80% relative humidity for 65 days (this corresponds to Buchner Plate 3 described previously). The resulting displacements for the FEM2 models in the vicinity of the hole were essentially axisymmetric, as expected. The radial displacements for the model representing this plate are shown in Figure 4.5.

These displacements at a measurement radius, \( m \), of 100 mm were converted into in-situ stresses using Equations (2.20) – (2.22). Axisymmetric displacement fields that are evaluated with Equations (2.20) – (2.22) will always generate stresses that appear hydrostatic, i.e. \( \sigma_x = \sigma_y, \tau_{xy} = 0 \). For comparison in the next section with the Buchner results, displacements were evaluated as if the entire depth of the core hole was drilled in one increment. The hydrostatic stress that is calculated (using Equations (2.20) – (2.22)) from the relieved displacements caused by drilling through a differential shrinkage stress field is herein termed an apparent stress. The apparent stress for plates stored at three different relative humidities is shown in Figure 4.6. The solid lines in the figure are generated using the following expression

\[
\ln(AS) = -0.208 - 2.247E - 6RH^3 - 0.00160t + 3.91E - 9t^2RH - 3.817E - 7tRH^2 \quad (4.1)
\]

where \( AS \) is the apparent stress in MPa, \( RH \) the relative humidity expressed as a percentage, and \( t \) the age of the plate expressed in days. In all three cases, the initial apparent stress is high and then decays in an exponential fashion. The drier the storage environment, the higher the initial apparent stress, and the longer it takes for the apparent stress to drop below a given threshold. For example, it takes significantly less than one year for the apparent stress to drop below 0.1 MPa in the 80% relative humidity environment, approximately 2 years in a 50% relative humidity environment, and nearly three and a half years in a 20% relative humidity environment.

Figure 4.6 is based on 10 cm thick plates. Figure 4.7 shows the effect of varying the thickness of the plate specimen, assuming storage at 50% relative humidity. The figure was created using the same approach and finite element models described previously excepting that more elements were used in the thicker models to ensure
appropriate mesh density. For plates thinner than approximately 300 mm, the younger plates \( (t = 100 \text{ days}) \) have higher apparent stresses than the older plates \( (t = 1100 \text{ days}) \), as expected. In the thicker plates however, this trend is reversed. An explanation for this is contained in Figure 4.8. In all cases, the shrinkage tension stress at the surface of a plate is larger at young age than old. At young age these high surface shrinkage stresses dissipate quickly with distance from the surface and become compressive. In older plates, the surface shrinkage stresses are less, and the slope of the shrinkage stress versus distance from the surface plot is more gradual, because the older plates are closer to reaching moisture equilibrium with the surroundings. In thinner plates at any age, the interior compressive stresses are relatively close to the surface and thus relief of these interior compressive stresses serves to partially offset the relieved displacements from the high tension surface shrinkage stresses. For the extremely thick plates at older ages, the interior compressive stresses are so far from the surface that they do not strongly effect the relieved displacements on the surface when they are relieved by hole drilling.

4.3 APPLICATION TO BUCHNER TEST DATA

As described in Section 2.2.2, Buchner (1989) completed a series of tests measuring in-situ stresses in plates using a hole-drilling technique similar to the core-drilling method. Buchner postulated two primary sources for error in measured stresses in that work, (1) swelling caused by hole drilling water, and (2) stresses caused by differential shrinkage (Buchner 1989). The effects of core-drilling water induced swelling are investigated in Chapter 3 and found to be significant. In this section the Buchner data is adjusted to account for differential shrinkage stresses using Equation (4.1).

The relative humidity of the plate storage environment was not reported by Buchner. Several assumptions regarding this value may be made however. The University of Surrey, where the Buchner experiments were performed, is located in Guildford, England. Climate data (United States Department of the Air Force 1996) for several locations in the immediate vicinity of Guildford is contained in Table 4.1. At first glance, it appears that an average relative humidity value of 80% is a reasonable value to assume for the storage conditions of the reference plates. It is also important to note that the temperate climate in England means that the swings in the climactic conditions noted are relatively small.

There is a possible problem with using the recorded climate data to estimate the relative humidity in the interior of a building. It is well known that the heating and cooling of air can affect dramatic changes in the relative humidity. For example air taken in by a heating system at 10° C, 80% relative humidity that is warmed to 21° C will have a relative humidity of only approximately 40% if no additional moisture is added. A relative humidity value of 50% may be considered somewhat typical for the conditions in the interior of a building.
It has been shown (Torrenti et al. 1999) that storage in variable relative humidity conditions has minimal impact on the overall shrinkage of concrete cross-sections compared to storage in a constant humidity environment, for a variation in humidity as high as ± 15%; here the variation is considerably less. A constant value for the environmental relative humidity is used for this study.

Although nothing is known regarding the climate control of the reference storage room, Column (2) of Tables 4.2 and 4.3 shows apparent stresses for each of the Buchner slabs as calculated using Equation (4.1) assuming storage at 80% and 50% relative humidity respectively.

If it is assumed that the Buchner plates were stored at 80% relative humidity, the apparent stress in these plates due to differential shrinkage varies from 0.20 MPa to 0.12 MPa. If it is assumed that the Buchner plates were stored at 50% relative humidity, the apparent stress varies from 0.52 to 0.37 MPa. This seems to indicate that for Buchner’s work, differential shrinkage was not a large source of error in the reported in-situ stresses. In columns (3) and (4) of Tables 4.2 and 4.3, these apparent stresses due to differential shrinkage have been removed from the Buchner results. For the 80% humidity assumption, the average SRSS error drops from 47% prior to making this correction to 42% afterward, a relatively minor change; similarly the error drops to 34% if the 50% relative humidity assumption is used.

4.4 SUMMARY OF FINDINGS
The effects of differential shrinkage of concrete specimens on the core-drilling method have been investigated. The following findings are noted:

1. Differential shrinkage stresses may significantly effect an investigation of in-situ stresses using the core-drilling method in certain circumstances. In particular, high apparent stresses are generated for especially thick concrete specimens (over 300 mm), especially young concrete specimens (less than 3 years old), and concrete specimens stored in very dry ambient conditions. Figures 4.6 and 4.7 give some insight into the ways these parameters affect the apparent stresses. In plates of constant thickness the apparent stresses exponentially decay with increasing plate age. For plates with different thickness, the apparent stresses increase substantially with increasing thickness.

2. The apparent stresses from differential shrinkage appear as a hydrostatic tension stress (i.e. $\sigma_x = \sigma_y$, $\tau_{xy} = 0$).

3. It is likely that differential shrinkage was not a large source of error in the previous Buchner study. The previously studied plates were relatively thin and thus it is estimated herein that the apparent stresses involved in the previous study were small. For the Buchner plates investigated herein, the apparent stress due to differential shrinkage ranges from 0.12 MPa (for the oldest plate assumed to be stored in 80% relative humidity ambient conditions) to 0.52 MPa (for the youngest plate assumed to be stored at 50% relative humidity).
Figure 4.1 – Finite element meshes: (a) Moisture and shrinkage model (FEM1); (b) core-drilling method model (FEM2)
Figure 4.2 – Relative moisture content profile at various times for plates stored at different relative humidities: (a) assumed 80% RH; (b) assumed 50% RH
Figure 4.3 – Differential shrinkage stress profile at various times for plates stored at different relative humidities: (a) assumed 80% RH; (b) assumed 50% RH
Figure 4.4 – Example of stresses applied to FEM 2 for 65 day old 10 cm thick (Buchner plate 3) plate assumed stored at 80% RH

Figure 4.5 – Radial displacements in 65 day old, 10 cm thick plate (Buchner Slab 3) assumed stored at 80% relative humidity
Figure 4.6 – Apparent stress versus age in plates assumed stores at 20%, 50% and 80% relative humidity

Figure 4.7 – Apparent stress versus plate thickness for plates assumed stored at 50% relative humidity
Figure 4.8 – Differential shrinkage stresses for a 300 mm thick plate and a 500 mm thick plate at 100 days and 1100 days assuming a storage environment of 50% relative humidity
Table 4.1 - Climate data for select locations in England

<table>
<thead>
<tr>
<th>Location</th>
<th>Approximate Distance to Guildford, England (km)</th>
<th>Average Yearly Temperature (°C)</th>
<th>Average Monthly Range (°C)</th>
<th>Average Yearly Relative Humidity (%)</th>
<th>Average Monthly Range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northolt</td>
<td>36</td>
<td>11</td>
<td>5 - 18</td>
<td>77</td>
<td>71 - 85</td>
</tr>
<tr>
<td>London / Heathrow</td>
<td>27</td>
<td>11</td>
<td>5 - 18</td>
<td>78</td>
<td>71 - 86</td>
</tr>
<tr>
<td>London / Gatwick</td>
<td>30</td>
<td>10</td>
<td>4 - 17</td>
<td>82</td>
<td>76 - 88</td>
</tr>
</tbody>
</table>

Data is taken from United States Department of the Air Force (1996) and is based on averaging values recorded over a 20 year period from 1973 to 1993.

Table 4.2- Buchner plates adjusted for differential shrinkage assuming 80% relative humidity storage conditions

<table>
<thead>
<tr>
<th>Plate</th>
<th>Apparent Shrinkage Stress Removed (MPa)\textsuperscript{a}</th>
<th>Adjusted Stress (MPa)</th>
<th>Percentage Difference from $\sigma_{\text{Max}}$ or $\sigma_{\text{Min}}$</th>
<th>SRSS % Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{\text{Max}}$</td>
<td>$\sigma_{\text{Min}}$</td>
<td>$\sigma_{\text{Max}}$</td>
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<td>0.86</td>
<td>-2.96</td>
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<td>2.09</td>
<td>-3.30</td>
<td>42.3</td>
</tr>
<tr>
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<td>0.20</td>
<td>1.04</td>
<td>-3.42</td>
<td>21.0</td>
</tr>
<tr>
<td>I</td>
<td>0.19</td>
<td>1.96</td>
<td>-2.66</td>
<td>40.3</td>
</tr>
<tr>
<td>II</td>
<td>0.18</td>
<td>0.99</td>
<td>-3.65</td>
<td>20.3</td>
</tr>
<tr>
<td>III</td>
<td>0.18</td>
<td>1.01</td>
<td>-3.04</td>
<td>20.6</td>
</tr>
<tr>
<td>VII</td>
<td>0.13</td>
<td>-3.13</td>
<td>-7.49</td>
<td>36.1</td>
</tr>
<tr>
<td>VIII</td>
<td>0.12</td>
<td>-3.78</td>
<td>-8.32</td>
<td>23.2</td>
</tr>
<tr>
<td>IX</td>
<td>0.12</td>
<td>-2.89</td>
<td>-7.84</td>
<td>41.1</td>
</tr>
<tr>
<td>Average</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>29.5</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Using Equation (4.1).
Table 4.3- Buchner plates adjusted for differential shrinkage assuming 50% relative humidity storage conditions

<table>
<thead>
<tr>
<th>Plate</th>
<th>Apparent Shrinkage Stress Removed (MPa)$^a$</th>
<th>Adjusted Stress (MPa)</th>
<th>Percentage Difference from $\sigma_{Max}$ or $\sigma_{Min}$</th>
<th>SRSS % Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma_{Max}$</td>
<td>$\sigma_{Min}$</td>
<td>$\sigma_{Max}$</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>0.54</td>
<td>-3.27</td>
<td>13.0</td>
</tr>
<tr>
<td>2</td>
<td>0.43</td>
<td>1.81</td>
<td>-3.58</td>
<td>36.6</td>
</tr>
<tr>
<td>3</td>
<td>0.52</td>
<td>0.72</td>
<td>-3.74</td>
<td>14.5</td>
</tr>
<tr>
<td>I</td>
<td>0.50</td>
<td>1.65</td>
<td>-2.97</td>
<td>33.8</td>
</tr>
<tr>
<td>II</td>
<td>0.50</td>
<td>0.68</td>
<td>-3.97</td>
<td>13.9</td>
</tr>
<tr>
<td>III</td>
<td>0.49</td>
<td>0.70</td>
<td>-3.35</td>
<td>14.3</td>
</tr>
<tr>
<td>VII</td>
<td>0.40</td>
<td>-3.40</td>
<td>-7.76</td>
<td>30.7</td>
</tr>
<tr>
<td>VIII</td>
<td>0.39</td>
<td>-4.04</td>
<td>-8.59</td>
<td>17.8</td>
</tr>
<tr>
<td>IX</td>
<td>0.37</td>
<td>-3.14</td>
<td>-8.10</td>
<td>35.9</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>-</td>
<td>-</td>
<td>23.4</td>
</tr>
</tbody>
</table>

$^a$ Using Equation (4.1).
CHAPTER 5

INFLUENCE OF STEEL REINFORCEMENT ON THE CORE-DRILLING METHOD

5.1 INTRODUCTION

As mentioned in Chapter 2, one of the primary assumptions made in the development of Equations (2.14) – (2.22) is that concrete is an isotropic, heterogeneous material. This simplification is clearly not exact for the case of steel reinforced concrete, as the steel and concrete have different material properties. This paper describes work performed to quantify the effects of steel reinforcement embedded in the concrete in the vicinity of the core hole. Because steel is much stiffer than concrete, steel reinforcement can constrain the displacements that take place during drilling, resulting in an under-prediction in stress when reinforcement is in the vicinity of the core hole. In this chapter the finite element method is used to perform parametric analysis of the case of concrete with embedded steel reinforcement.

5.2 FINITE ELEMENT APPROACH

As mentioned, Equations (2.20) – (2.22) are derived assuming an isotropic medium. In reinforced concrete structures, steel reinforcement can be as much as 10 times stiffer than the surrounding concrete as judged by their respective moduli of elasticity. Reinforcement proximate to a drilled core-hole causes a distortion in the relieved displacement field around the hole, in turn causing an error when applying Equations (2.20) – (2.22) to determine the in-situ stress in the object.

In this chapter the concrete structure was idealized as a concrete plate reinforced in one direction. The plate was loaded in-plane with constant stress in the direction of the reinforcement. The idealized plate is shown in Figure 5.1(a). In results not shown here, plates with reinforcement perpendicular to the loading direction were also analyzed. However, the distortion in the predicted stress results caused by reinforcement for these cases was on average an order of magnitude lower than for the parallel case, and thus those results are not shown.

Figure 5.1(b) shows a section view of the idealized concrete plate and identifies some of the geometrical parameters used to characterize the influence of the reinforcement on stress prediction. In this study, values for each parameter were considered as shown in Table 5.1. In particular, two hole sizes, \(2a = 75\) and \(2a = 150\) mm and three concrete cover dimensions, \(d_c = 18.75\), \(d_c = 37.5\) and \(d_c = 75\) mm were investigated.
All models were analyzed with a bar size, $d_b$, equal to 25 mm, corresponding to a #8 bar in the typical U.S. practice.

The idealized plate was modeled in ABAQUS using 1/8 symmetry (1/4 symmetry in the plane of the plate and 1/2 symmetry through the depth of the plate). The finite element mesh, which contains 45,118 10-node modified tetrahedron elements (ABAQUS C3D10M), is shown in Figure 5.2. The mesh is refined in the region around the core hole (near the coordinate system triad) and around the reinforcement (which is oriented parallel to the $y$-axis). The models were loaded with uniform pressure, $\sigma_{\text{model}}$, in the $y$-direction on the surface noted in the figure. An examination of the stresses in the region of the model closest to the applied loads (within a few millimeters) showed that the uniform pressure applied, $\sigma_{\text{model}}$, redistributed into stress in the concrete, $\sigma_{\text{conc}}$, and reinforcement, $\sigma_{\text{reinf}}$, proportional to their individual moduli of elasticity, and thus it was deemed unnecessary to load the steel and concrete in the model in proportion to their moduli. The $y$-direction stress in the concrete, $\sigma_{\text{conc}}$, can be calculated from the applied stress in the model, $\sigma_{\text{model}}$, using the ratio $E_s / E_c$ via standard transformed section methods (MacGregor 1997). The plan ($x$-$y$) dimensions of the 1/8 symmetry model are 750 x 750 mm.

The concrete and steel reinforcement were assigned moduli of elasticity of 28270 MPa, and 200 GPA respectively. This represents the case of relatively low strength concrete with corresponding low modulus. The model was constructed in such a way as to allow for changing plate depth, amount of concrete cover, size of drilled hole, and spacing of reinforcement.

For each set of bar placement parameters, the model was run twice, once without a core hole and once with a core hole. The relieved displacements due to drilling a hole are taken as the difference between the displacements for the two model runs. The relieved displacements around the measurement circle were converted to in-situ stresses in the concrete using the measurement configuration of Figure 2.5(a) and Equations (2.20) – (2.22). The measurement circle radii for the small and large hole sizes were 50 and 100 mm respectively. Plane stress conditions were assumed, consistent with the findings of Turker (2003). Because this case is neither plane stress nor plane strain however, small errors are introduced when modeling the problem in this manner. Furthermore, there are additional small errors associated with any finite element mesh of reasonable density. To remove these two sources of error, control models without reinforcement (i.e. unreinforced) were analyzed. A control model was created for each plate depth, $H$, and hole size, $2a$, investigated. The in-situ stresses were calculated for each of these models using Equations (2.20) – (2.22) and the $y$-direction stress calculated, $\sigma_y$, was compared to the applied stress. The ratios of applied to calculated $\sigma_y$ for the models with $2a = 75$ mm and $H = 300$, 450, and 600 mm were 1.002, 1004 and 1.005 respectively. For models with $2a = 150$ mm they were 0.958, 0.963 and 0.966. As there was little variability with changing plate depth, the values of 1.004 and 0.963 were used to adjust all further values for calculated $\sigma_y$ in
this report for the hole sizes of 75 and 150 mm respectively. For example, for all models with $2a = 75$ mm, any $\sigma_y$ calculated via Equation (2.21) was then multiplied by 1.004 to minimize modeling error and to isolate only the effect of reinforcement.

5.3 RESULTS

5.3.1 Measurement Configuration Orientation
To determine if there was a preferred orientation of the measurement configuration (Figure 2.5(a)), the measurement configuration was first oriented so that the $x$-axis in Figure 2.5(a) was perpendicular to the load direction and then in-situ stresses were calculated. Subsequently, the configuration was rotated around the core hole in successive 5 degree increments and in-situ stresses recalculated for every new orientation. Figure 5.3(a) shows the error in in-situ stress as a function of the rotation of the measurement configuration for the plates with $h = 300$ mm and $2a = 75$ mm. The errors shown are derived by comparing the $\sigma_y$ calculated via Equation (2.21) (after rotation via a Mohr’s Circle approach into the correct $x$-$y$ system) with the applied stress in the concrete, $\sigma_{conc}$. For $d_c = 18.75$ mm, the values range from -13.7\% to -20.6\%; the average error is -17.8\%. The other lines in the figure (and all other cases examined as part of this research) exhibit significantly less variability in error as the measurement configuration orientation changes. In each case herein (including those in the previous section), the rotation procedure described was performed and the resulting errors averaged to generate a single error number for each case.

5.3.2 Effect of Plate Depth
Figure 5.3(b) shows the effect of varying the depth of the concrete plate. It is apparent that the plate depth is unimportant as a parameter in this investigation. All subsequent results are reported based on a plate depth, $H$, of 450 mm.

5.3.3 Effect of Concrete Cover
The values of cover considered in this study, $d_c = 18.75$, $d_c = 37.5$ and $d_c = 75$ mm, correspond to the ACI-318-05 (2005) provisions for minimum cover associated with various reinforcement geometries and concrete exposure levels. Figure 5 shows, for the two hole sizes investigated, the effect of varying the amount of cover, $d_c$. As expected, with increasing cover the effect of the reinforcement diminishes, doing so in a non-linear manner. Trends are shown for each of the bar spacings, $S_R$, investigated. For any bar spacing, the error is less than 5\% so long as the cover provided is approximately 75 mm. Schajer (1988a, 1988b) and others have shown that, for hole drilling methods, stress changes in the interior of an object become increasingly difficult to detect at the surface (where the displacements are measured) with increasing depth to the stress change.

5.3.4 Effect of Bar Spacing
Figure 5.5 shows, for the two hole sizes investigated, the effect of varying the bar spacing, $S_R$. As expected, the impact of the reinforcement decreases significantly with increasing bar spacing. It is tempting to attribute these results in some way to the
reinforcement ratio, $\rho_s$, the percentage of steel in any given cross-section. However, changing the plate depth with a constant bar spacing results in variable $\rho_s$ but does not change the error due to reinforcement (as noted previously). It appears then that $\rho_s$ is not an important parameter. Further work, investigating the effect of the diameter of reinforcement considered, $d_b$, may provide further insight to this issue.

5.3.5 Effect of Clear Spacing

For the same bar spacing, $S_R$, models with different hole sizes, $2a$, will have differing amounts of clear spacing, $S_C$, the distance from the edge of the core hole to the edge of the nearest reinforcement. To this point, all results for the two different hole sizes, $2a = 75$ mm and $2a = 150$ mm have been plotted on separate graphs, as they seem to be at separate scales. However, graphing errors versus clear spacing allows all the data to be plotted on the same graph, and clear trends emerge, as shown in Figure 5.6. Relatively smooth behavior is noted for the differing amounts of concrete cover, with the error due to reinforcement for all cases dropping below 5% when clear spacing exceeds approximately 35 mm. An examination of Figure 7 suggests that the basic influence of reinforcement in the core-drilling method can essentially be quantified by two parameters, namely concrete cover, $d_c$, and clear spacing to the nearest bar, $S_C$. This neglects of course the ratio of the moduli of the concrete and steel and the effect of the bar diameter, $d_b$, work that remains to be accomplished. It remains to quantify the effect of changing the bar diameter on these results, all results herein are for $d_b = 25$ mm.

5.4 SUMMARY OF FINDINGS

The presence of reinforcement close to a core hole (nearer than 35 mm) and close to the surface of the concrete (nearer than 75 mm) causes a significant under-prediction in stress using the core-drilling method, if the reinforcement is neglected. However, this effect reduces quickly and significantly with either increasing concrete cover or increasing clear spacing to the nearest bar.
Figure 5.1 - Idealized concrete plate: (a) plan view; (b) section view
Figure 5.2 - 1/8 symmetry finite element model: (a) schematic; (b) mesh
Figure 5.3 – Effect of measurement configuration orientation and plate depth in tests with $2a = 75$ mm and $S_R = 100$ mm: (a) error versus measurement configuration orientation ($H = 300$ mm); (b) error versus plate depth
Figure 5.4 – Effect of cover on the error due to reinforcement: (a) with $2a = 75$ mm; (b) with $2a = 150$ mm
Figure 5.5 – Effect of bar spacing on the error due to reinforcement: (a) with $2a = 75$ mm; (b) with $2a = 150$ mm
Figure 5.6 – Effect of clear spacing to the nearest bar on the error due to reinforcement
<table>
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<th>Values in Current Work</th>
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<tr>
<td>$d_b$</td>
<td>bar diameter</td>
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CHAPTER 6

CORE-DRILLING EXPERIMENTS

6.1 INTRODUCTION

This chapter describes the experiments that were conducted to verify the applicability of the core-drilling method to actual concrete specimens. A series of concrete plates were loaded and subjected to the core-drilling method to evaluate the in-situ stresses. In addition to validating the core-drilling method for use in concrete, the experiments were designed to probe the effects of steel reinforcement, core-drilling water, and differential shrinkage on the method. Section 6.2 provides the experimental details including an overview, a description of the specimens tested, and a description of the test procedure followed. Section 6.3 presents the results of the tests including the measured relieved displacements and the corresponding in-situ stresses. Note that the in-situ stresses provided in Section 6.3 are not corrected for the phenomena influencing these results discussed in Chapters 3 – 5. Section 6.4 details the correction of in-situ stresses for these factors including correcting for the presence of steel reinforcement (6.4.1), correcting for core-drilling water (6.4.2) and correcting for differential shrinkage (6.4.3). The final corrected evaluation of the in-situ stresses for each of the tests is provided and compared to the actual applied stresses. Section 6.5 discusses the experimental results and summarizes the findings of this chapter.

6.1.1 Experimental Goals

The goals of the experimental research were as follows:

- To confirm the suitability of the digital image correlation measurement technique to the application of the core-drilling method performed on concrete structures.
- To evaluate the effectiveness of the core-drilling method in measuring in-situ stresses on simplified concrete structures.
- To evaluate the applicability of the analytical formulations presented in Chapters 3 – 5 that account for various factors which tend to complicate measurements of in-situ stress on more realistic structures. In particular the approaches detailed in Chapters 3 – 5 for accounting for the effects of steel reinforcement, differential shrinkage and core drilling water were examined and validated.
- To determine if in-plane gradients in applied normal stresses prevent determination of the average (constant) value of the applied normal stresses and furthermore to determine if capturing a gradient in normal stresses was feasible.
6.2 EXPERIMENTAL DETAILS

6.2.1 Experimental Overview
Three concrete plate specimens subjected to uniform or linearly varying in-plane uniaxial stresses were tested in compression in these experiments. A schematic of one of the plates is shown in Figure 6.1. Each of the plates was cored at multiple locations resulting in a total of eight core-drilling experiments. Across the eight tests several variables were probed as noted in Table 6.1. The experiments were designed to evaluate the core-drilling method experimentally in its simplest form and then to increase in complexity and introduce additional experimental variables with each new test. First, tests were performed in saturated, plain concrete (Plate 1) as the simplest cases considered. Plate 1 was wet-cured from the time of concrete placement until the core-drilling tests were performed. It was supposed that testing saturated concrete minimizes issues associated with core-drilling water absorption by the concrete during testing and also minimizes differential shrinkage effects. Next, tests were performed in saturated, steel-reinforced concrete (Plate 2), in order to probe the effects of steel-reinforcement (Chapter 5). In each of the saturated plates two tests were performed with uniform in-plane normal stresses and one with linearly varying normal in-plane stresses with the objective to determine if in-plane stress variation prohibits the average in-plane stress value from being determined. Finally, tests were performed in drying concrete (Plate 3) to probe the effects of core-drilling water (Chapter 3) and differential shrinkage (Chapter 4). The tests of drying concrete were first performed in plain concrete and finally in reinforced concrete to combine both the drying and reinforcement variables in a single test.

Figure 6.2 summarizes the types of specimens tested. The dashed lines in the figure represent steel reinforcing bars. The plates were cast with embedded conduits and loaded in compression via post-tensioning of Dywidag bars positioned within these conduits. As mentioned, each plate was designed to allow a core-drilling test to be performed at multiple locations along its length, spacing the locations so that the behavior at each was unaffected by prior drilling at one or more of the other locations. Plates 1 and 2 were each designed to be drilled at three locations, Plate 3 was designed to be drilled at 2 locations. These specimens thus result in eight independent applications of the core-drilling method, termed herein as Holes 1 – 8. Each core hole drilled was 150 mm in diameter.

6.2.2 Specimen Description
Three specimens were constructed. The specimens were designed to simulate simple conditions that allowed for experimental exploration of the core-drilling method on real concrete. The specimens are concrete plate structures loaded primarily in the plane of the plate.

6.2.2.1 Specimen Dimensions
A schematic of one of the plates is shown as Figure 6.1. The design dimensions of each of the plates were 488 cm x 864 mm x 152 mm. After testing, the width and
height of each plate was measured at the cored locations. This data is shown in Table 6.2. The widths were measured on the top and bottom of the plates at each hole location and averaged. Plate heights were calculated by measuring the heights of the measured cores. To calculate applied stresses (see for example Section 6.2.3.3), the dimensions of the plates at the hole locations are used. To calculate modulus of elasticity (see Section 6.2.4.1), the average dimensions for each plate are used.

Additional details of the plates are shown in Figure 6.3. Figure 6.3(a) shows a plan view of Plates 1, 2 and 3 and provides the locations of each core hole. Figure 6.3(b) shows a section view of Plate 2 and provides the reinforcement and conduit locations. The details of the reinforcement are described more fully in Section 6.2.2.2.2.

6.2.2.2 Materials
6.2.2.2.1 Concrete
The concrete used to construct the plates was obtained from a ready mix supplier. The concrete had a water cement ratio of 0.42 and a maximum aggregate size of 10 mm. A relatively small maximum aggregate size was chosen to make the concrete mixture more homogeneous minimizing issues associated with the stiffness mismatch between coarse aggregate and the surrounding mortar. The effect of coarse aggregate on the core-drilling method is an area of proposed future research noted in Section 8.6. Constituent ingredients of the concrete were obtained from the supplier, with important mixture proportions provided in Table 6.2. The fineness modulus of the sand used was 2.40. The specific gravities of the coarse and fine aggregates were 2.75 and 2.61 respectively. In addition to the constituents listed in the table, approximately 50 ounces of Daracem-55, a mid-range water reducer, was added to the concrete mixture. The design compression strength ($f'_c$) of the concrete was 41.4 MPa. A slump test was performed on a sample of the fresh, wet concrete and a slump of approximately 10 cm was measured. The age of the concrete at core-drilling from the date of casting was 126 days (Plates 1 and 2) and 127 days (Plate 3). Additional testing performed as part of this research to further characterize the material properties of the hardened concrete is discussed in Section 6.2.4.

6.2.2.2.2 Steel Reinforcement
All steel reinforcement used in the specimens was Grade 60 with a yield stress in excess of 400 MPa. The modulus of elasticity of the steel reinforcement was assumed to be 200000 MPa. The steel reinforcement in the plates has been divided into three types, primary reinforcement, instrumentation reinforcement and loading point reinforcement. The primary reinforcement is four 36 mm diameter (#11) bars in Plates 2 and 3. This results in a reinforcement ratio ($A_r/A_g$) for the reinforced plates of 0.031. Figure 6.3 shows the location of the primary steel reinforcement in each of the plates tested. Note that Plate 1 does not contain primary steel reinforcement. In Plate 2 the primary reinforcement runs the entire length of the plate. In Plate 3 the primary reinforcement bars span only approximately half the length of the plate. Thus one end of Plate 3 can be considered reinforced concrete and the other end unreinforced (plain) concrete.
The second type of steel reinforcement is that provided as part of the instrumentation system for the plates. Each of the plates contains four 9.5 mm diameter (#3) instrumentation reinforcement bars. These bars (shown in Figure 6.3(b)) are located in the corners of the plates and are instrumented with bonded wire strain gauges. A description of the instrumentation used in these tests is provided in Section 6.2.2.4.

The structural effects of the instrumentation reinforcement have been neglected. In Chapter 5 the effects of steel reinforcement in close proximity to a core hole were evaluated. The results shown in Chapter 5 for analyses of plates reinforced with 25 mm diameter bars indicated that steel reinforcement farther than approximately 35 mm from a core hole has a minimal effect on the in-situ stresses calculated using the core-drilling method. The instrumentation reinforcement is significantly smaller (9.5 mm diameter), and is located more than 300 mm from the edge of the core holes. Thus neglecting the instrumentation reinforcement should have minimal impact on the results of the tests.

The third type of reinforcement in the plates is that comprising the reinforcement cages at each end of all of the plates. A cage of 9.5 mm diameter reinforcing bars is provided at the ends of the plates to aid in carrying the bearing stresses at each of the loading points. The cage consisted of three sets of two #3 bars spaced at approximately 75 mm on center along the length of the plate. One of the reinforcement cages is depicted in Figure 6.4.

6.2.2.3 Construction, Curing and Environmental Conditions

The forms for the plates were constructed using plywood reinforced with timber as shown in Figure 6.5. The bottom of the forms was lined with a double layer of plastic film. The forms were filled with concrete (see Figure 6.6) which was then vibrated using a handheld vibrator. The tops of the plates were screeded flat and then the top surface of the concrete was finished with a steel trowel. Figure 6.7(a) shows the plates poured and surface finished.

After hardening, Plates 1 and 2 were kept in a saturated condition until the core-drilling tests. In contrast, Plate 3 was allowed to dry naturally after a period of moist curing. These conditions were achieved in the following manner.

After the concrete had set the top surface of all of the plates was shrouded with burlap which was then liberally wetted with water. The plates were then wrapped completely in a double layer of plastic sheeting. For the next 12 days at least once per day the plates were unwrapped and liberally watered before being rewrapped.

After 12 days, the formwork on the sides of all of the plates was removed (see Figure 6.7(b)). Soaker type hoses were then placed on the top face of Plates 1 and 2 (on top of the burlap shroud), as visible for Plate 1 in Figure 6.8(a). Plates 1 and 2 were then rewrapped in the double layer of plastic sheeting, allowing access to the inlet port of the soaker hoses. These two plates were then watered via the soaker hoses for
approximately 10-20 minutes an average of once per day until they were load tested and cored. This was to ensure that the concrete in these two plates remained in an essentially saturated state.

The plastic sheeting and burlap shroud were also removed from Plate 3 at 12 days. Plate 3 was then turned on its side to allow drying from two faces, as shown in Figure 6.8(b). Relative humidity and temperature data was recorded from the age of 12 days until the plate was cored to quantify the storage environment of this plate. A White Box CT485RS series High Performance Microprocessor-Based Temperature and Humidity Recorder was used for this purpose. This device is a rotary style temperature and humidity recorder. An example of the output from this device is shown as Figure 6.9. The circular data sheet shown in Figure 6.9 contains recorded data for a 32 day period. The circumferential lines on the chart are the divisions in the relative humidity and temperature scales. The dark circumferential lines represent 5 degree increments in temperature (in Celsius) and 10% increments in relative humidity. Inscriptions have been provided for two of these dark circumferential lines for clarity. Lines depicted on the rotary chart that are purely in the radial direction indicate that electrical power was lost by the data recorder. Over the course of approximately 115 days, approximately 10 days of data were lost due to electrical power losses to the machine. The average relative humidity over the period of storage for Plate 3 was 17%. This average was calculated by visually determining the average for every 12 hour period recorded and then averaging all of these values.

As a check on the reliability of the White Box system, the environmental data from the White Box CT485RS was compared with measurements recorded by a Campbell Scientific CS500 Temperature and Humidity Probe attached to a Campbell Scientific CR5000 data logger that recorded data every five minutes. This comparison was performed over the course of several days to ensure that accurate data was captured by the White Box CT485RS. The two systems showed good agreement over the compared time period.

6.2.2.4 Instrumentation
Strain gauges and load cells were employed as part of the test protocol. The load cells were used to measure the applied loads during testing. These loads were then divided by the measured cross-sectional area of each plate at each hole location to determine the applied stress for each test as described in Section 6.2.3.3. The strain gauges were used to determine the strains in the plates during tests. These strain measurements were used in the following ways:

- The strains were used to estimate the horizontal and vertical eccentricity in load.
- The strains were converted to stresses and used as a secondary check on the in-plane stress state in each plate during testing (the primary method for determining the stress state in each plate was to divide the applied loads from the load cells by the measured area of the plate).
- In Section 6.2.4.1 the strains were used along with the output from the load cells to calculate an estimated modulus of elasticity for each plate.
The rest of this section describes the strain gauge and load cell equipment used and the placement of the instrumentation.

Measurements Group Inc. model CEA-06-250UN-350 350 ohm bonded wire resistance gauges with a gauge factor of 2.05 were used in conjunction with a series of Vishay 2120A strain gauge conditioner and amplifier systems. The strain gauges were affixed to the 9.5 mm diameter reinforcing bars at the corners of each of the plates, at approximately the third points along the length of the plates. The location of each of the strain gauges is shown in Figure 6.10. The strain gauges were affixed to the bars with epoxy and waterproofed with a system involving protective putty, aluminum tape and caulk. Figure 6.11 is a photograph of one of the gauges with its protective covering clearly visible.

The load cells were incorporated in the load path of each Dywidag bar during every test. The load cells were calibrated using a 2670 kN capacity SATEC loading machine and a FLUKE 8840A voltmeter. Figure 6.12 shows the calibration data for the two load cells and depicts a graph of load versus output voltage.

6.2.2.5 Digital Image Correlation System
The relieved displacements were measured during each test using digital image correlation. As described in detail in Chapter 2, this technique involves applying a pattern to the surface of the measured object and then photographing the object in a pair of digital cameras before and after a loading event. Individual sub-portions of the applied pattern are tracked using photogrammetric triangulation principles from one set of photographs to the next and thus displacement information may be derived. Chapter 2 contains a discussion of the theoretical background of the technique, as well as an accounting of experiments performed as part of the current work to assess the applicability of this technique to the magnitude of displacements associated with application of the core-drilling method to concrete structures. The digital image correlation system used in the experiments described in the current chapter is identical to that described in Section 2.3. The cameras and processing software were the same as that used in the preliminary experiments discussed in Section 2.3. The field of view in the images captured in this chapter was approximately 250 mm wide. Following the guidelines summarized in Section 2.3, this means that the displacement resolution of the system was approximately 8 microns for out-of-plane displacements and better for the in-plane displacements.

6.2.3 Description of Test and Loading Procedures
As mentioned previously, the intended stress state in each plate during a core-drilling test was either constant in-plane normal stress or linearly varying in-plane normal stress, depending on the test. The loads in Tests 1, 3, 4, 6, 7 and 8 were intended to provide a constant stress state and the loads in Tests 2 and 5 were intended to provide a linearly varying stress state. The loading system and protocol were specified in order to achieve these stress states. Section 6.2.3.1 provides the details of how the loads were applied to the plates to accomplish this.
As noted in Chapter 2 and Section 6.2.2.5, digital image correlation involves the comparison of photographs of an object taken before and after a loading event in order to determine displacements. In the case of the core-drilling method the loading event is the drilling of a core-hole. The experimental procedure was tailored specifically to acquire photographs of the surface of the concrete specimens with the digital image correlation system before and after drilling a core hole. The procedure is detailed in Section 6.2.3.2.

6.2.3.1 Loading
The loading system was designed to create either constant or linearly varying in-plane normal stress within each plate. Each of the plates was loaded with a pair of hydraulic jacks. A photograph of the load hardware is shown in Figure 6.13(a) and a corresponding schematic is shown in Figure 6.13(b). At one end of each Dywidag bar a steel anchorage plate (127 x 203 x 38 mm) and Dywidag nut were provided. On the opposite end, a similar anchorage plate was provided, along with two additional steel plates, a hydraulic jack, a load cell, and another anchorage plate and Dywidag nut. The jacks are hydraulically operated via a pump and a system of hoses and have a capacity of 890 kN each. The hydraulic hoses were connected in a manner that allowed for independent load control for each of the jacks. For each test the jacks were pressurized to provide approximately 311 kN or 622 kN in each of the Dywidag bars. In some tests (Hole 1, 3, 4, 6, 7 and 8) the load in the two bars was the same (approximately 622 kN), inducing an approximately uniform compressive stress state of 9.5 MPa in the plates. In others (Hole 2 and 5) the loading in one bar was approximately 622 kN while the load in the other was approximately 311 kN, inducing a state of approximately linearly varying normal compressive stress in the plates, with the stress varying from 13.3 MPa on one side of the plate to 5.6 MPa on the other side. Note that the design $f'_c$ for the concrete was 41.4 MPa. Loads were held essentially constant during the drilling phase of testing by closing needle valves within the hydraulic path of each jack after the loads in each bar were as specified.

6.2.3.2 Test Procedure
The test procedure was specified to allow for the determination of relieved displacements due to core-drilling using 3D digital image correlation (DIC). Plates 1 and 2 were unwrapped and the burlap covering removed approximately 30 minutes prior to testing each plate. During this time the digital image correlation system was calibrated as described in Section 6.2.2.6, and surface preparation at each planned core location was performed. White spray paint was applied to each future hole location after the surface in those areas was deemed dry enough for the paint to adhere in these regions. Black spray paint was then applied in these regions in a random pattern using a spluttering technique. An example of the pattern applied is shown in Figure 7.14. The large markings on the ruler shown denote centimeters.

Three holes were drilled in Plates 1 and 2, respectively, and 2 holes were drilled in Plate 3. Photographs of the specimen surfaces are required immediately before and
after coring the holes, and additional photographs taken during testing are used for informational or other purposes, rather than for the determination of relieved displacements. The procedure shown in the flowchart in Figure 6.15 was followed for Holes 1-5 and 7-8. Provided below is a detailed description for each step in the procedure. The numbered steps below correspond to the circled numbers in the flowchart.

1. Begin testing
2. Prior to loading the plate, photograph the hole surface area with the digital image correlation system. The photographs taken at zero load are referred to as Stage 0 photographs.
3. Energize the hydraulic system and load both Dywidag bars to a load of approximately 311 kN.
4. Photograph the surface in the hole region with the digital image correlation system at this load level. These are referred to as Stage 1 photographs.
5. For holes 1, 3, 4, 7, 8 continue loading both Dywidag bars to a load of approximately 622 kN.
6. For holes 2 and 5 a valve is closed that locks the load in one of the Dywidag bars at the 311 kN level.
7. For holes 2 and 5 the pump is then energized further to bring the load in the second Dywidag bar to the 622 kN level.
8. For all holes, photograph the surface in the hole region with the digital image correlation system at the given load level. These are referred to as Stage 2 photographs.
9. For each hole, affix the coring drill to the face of the concrete using the vacuum assisted mounting platform. Liberally wet the plate in the area of drill attachment to increase the seal necessary for vacuum attachment of the coring drill.
10. Drill a through hole through the concrete. Water was used to lubricate the drill and flush coring debris from the hole during this process. Figure 6.16(a) shows a photograph taken during coring of Plate 2 at the Hole 5 location.
11. Remove the coring drill from the surface of the concrete and wash away the coring debris around the hole using additional water. Figure 6.16(b) shows Plate 2 at the Hole 5 location after coring but before washing away the coring debris. Blot the surface in the vicinity of the hole dry with paper towels.
12. Photograph the surface in the hole region with the digital image correlation system at this load level. These are referred to as Stage 3 photographs.
13. Unload the specimen.
14. Terminate testing.

This procedure was followed for each of the 8 holes except Hole 6. In Hole 6 the hole was drilled in four increments and the surface of the specimen was photographed after every increment. Data recorded in this way allows for the application of the incremental version of the core-drilling method, as outlined in Chapter 2. The procedure followed for Hole 6 is shown in the flowchart of Figure 6.17. The steps
prior to those outlined in the flowchart are identical to those from the other tests. The increments were of unequal depth, as shown in Table 6.4. Photographs of the surface were captured after each of the drilled increments, as noted in the flowchart. The interpretation of the incremental core-drilling results is left for future work. In this work only the data recorded after the final increment (for example the Stage 6 photograph) will be used to calculate in-situ stresses, similar to all the other holes.

6.2.3.3 Load and Stress State for Each Test
The procedure described above was followed for the loading and testing at each hole location. For comparison with the stresses determined using the core-drilling method (Section 6.3 and 6.4) the applied stresses for each test were determined by dividing the applied loads (as measured by the load cells) by the cross-sectional area of the plate at the hole location. The plates soften slightly during coring and thus the applied loads change slightly from before drilling to after drilling. The applied loads from before and after drilling each hole are shown in Table 6.5. The averages of the loads from before and after coring are used to calculate the in-situ stresses in Table 6.6. The stresses in Column (4) of Table 6.6 are calculated by dividing the average loads applied by the cross sectional areas of the plates as given in Table 6.1. Column (5) shows the linear variation stress parameter, $K_y$, for each test. This parameter is calculated by taking the moment about the centroid of the plate created by the applied loads and dividing by the in-plane moment of inertia of the plate noted in Table 6.1.

As mentioned, the stress values from Column (4) of Table 6.6 are used for comparison with the stresses calculated via the core-drilling method (in Section 6.3 and 6.4).

The stress state in each plate was also determined using the output from the strain gauges. This was done for two primary reasons: (a) as a comparison with the stresses determined using the load divided by area approach described above; and (b) to allow for the in-plane and through thickness variation in stress to be determined. Table 6.7 contains the stress data derived from the strain gauges in each plate for each test. Portions of the table marked with a dash (-) indicate that particular gauges malfunctioned prior to or during testing. Strain from each gauge is converted to stress using the modulus of elasticity noted. Determination of the elastic modulus of each plate is discussed in Section 6.2.4.1.

As noted above, the stresses computed using the strain gauge output (Table 6.7) were also examined to provide information regarding stress variation within the plates during tests:

- Comparing the stress values on the east end of a plate with the values on the west end gives insight into the variation of stress within the plates from end to end and shows that this variation is relatively small in every test.
- Comparing the values from the north side of a plate with those from the south side gives insight into the in-plane variation in stress for each test. For Holes 1, 3, 4, 6,
7 and 8 the in-plane stress variation is small. This is expected since the loading in these tests was intended to simulate a uniform in-plane stress state. For Holes 2 and 5 the in-plane variation is much larger, again as expected since these tests were intended to simulate in-plane linearly varying stress.

- Comparing the stress values that are both at the same plane location gives insight into the through-thickness variation of stress within a plate. The through-thickness variation of stress for each test is discussed in detail in Section 6.3.2.2.

6.2.3.4 Estimation of Displacements due to Linear Gradients

One of the objectives of the current research was to determine if it is feasible to determine in-plane stress gradients using the core-drilling method. It was determined that it was not possible to determine gradients in normal stress and thus the tests performed herein are treated as if the imposed stress states are entirely comprised of constant normal stresses, and the portion of the stress state due to the linear variation term is neglected. The reason for this is given in the following three paragraphs.

Equations (2.25) – (2.29) give expressions for the relieved displacements due to a stress state that is linearly varying in the plane of the plate. As discussed in Section 6.2.3.1, some of the tests were performed such that a linear gradient stress state was induced in the tested plate. Column (5) of Table 6.6 details the linear gradient parameter, \( K_y \) for each test. For example, Hole 2 has the largest \( K_y \) value of all the tests, with \( K_y = 0.008 \) MPa/mm.

Testing in this manner exposes the disparity in the relative magnitude of displacements due to the linearly varying portion of the stress state versus those from the normal stress state. An example of this is shown in Figure 6.18 wherein the theoretical relieved displacements from Hole 2 (at a measurement radius of \( m = 100 \) mm) are shown in two portions, those due to the applied \( \sigma_y \), and those due to the applied \( K_y \). It can be seen that the displacements due to the normal stress are many times larger than those due to the \( K_y \) portion of the loading. In fact the displacements due to linear variation are barely discernable on the Figure 6.18(a) plot and thus have been plotted separately in Figure 6.18(b). The maximum theoretical radial displacement due to the \( K_y=0.00 \) shown is approximately 0.4 microns. This is below the threshold of displacements that can be reliably detected with the digital image correlation equipment used in these tests. A comparison of the maximum absolute value of displacement from the normal and linear portion of the stress state shows that the normal displacements are nearly 60 times the linear variation displacements for the Hole 2 test.

The maximum reasonable theoretical linear gradient that can be considered in the core drilling method involves a stress state wherein the stress on one edge of the hole is zero and at the other edge of the hole is some maximum value in relation to the compressive strength of the concrete. A stress state of this nature is shown in Figure 6.19, with a maximum stress of 20.7 MPa which is approximately 50% of \( f'_c \) for 41.4 MPa strength concrete. This results in a \( K_y \) for this stress state of 0.138 MPa/mm, and
is a reasonable upper bound for the maximum $K_y$ that might be encountered in practice. The relieved displacements for this maximum hypothetical $K_y$ stress state are shown in Figure 6.20, similar to those for Hole 2 shown in Figure 6.24. Even for this maximum $K_y$ case, the relieved displacements from the normal stress portion of the stress state are approximately 5.4 times those from the linear gradient portion and as such dominate the measured displacement response. The conclusion in this case is that it is not feasible with the current digital image correlation technology to capture accurately the displacements due to the linear variation portion of a stress state. This also means that future tests should not be sensitive to unintended small variations in applied in-plane normal stresses. Since it was not possible to differentiate displacements due to the normal portion of the applied stresses from those due to the linear variation portion, it was not reasonable to attempt to determine the linear variation term using the core-drilling method and hence the tests performed herein were treated as if only constant in-plane normal stress was applied.

### 6.2.4 Tests Performed for Material Property Characterization

Solution of Equations (2.20) – (2.22) requires knowledge of the material properties of the concrete. Section 6.2.4.1 discusses the tests that were performed to determine the modulus of elasticity and Poisson’s ratio of the concrete. Sorptivity testing was also performed in order to aid in the determination of the effect of core-drilling water on the tests of Hole 7 and Hole 8. This work is described in Section 6.2.4.2.

#### 6.2.4.1 Calculation of Concrete Modulus of Elasticity and Poisson’s Ratio

The Poisson’s ratio of the concrete was determined by cylinder testing similar to ASTM C469-94 (ASTM C469-94 1994) and is described later. The modulus of elasticity of the concrete was calculated several ways:

a) using cylinder testing similar to ASTM Standard C469-94;
b) using the strain gauge and load cell output recorded during each test;
c) using the ACI 318 (ACI 318-05 2005) equation that relates elastic modulus to concrete compressive strength
d) using the method of Alexander (1996) that includes the effects of aggregates with different stiffnesses.

Method (b) noted above was deemed the most appropriate and is the method finally selected. However, all of these ways of finding $E_{conc}$ are described in order below and the reasons behind the final selection of method (b) are given.

For method (a) noted above, Several 152 diameter x 305 mm height cylinders were cast at the same time as the concrete plates. The cylinders were cast in three layers and rodded 25 times between each layer, following standard methods. These cylinders were then sealed by placing a plastic cap on the top surface. No special curing regime was used for the cylinders, they were unsealed and de-molded immediately prior to testing. Two of these cylinders were de-molded and tested for modulus of elasticity and Poisson’s ratio after 123 days, the approximate age of the plates at core-drilling. The cylinders were capped with a sulfur mortar capping compound to ensure that the ends were flat and parallel. The test for modulus and Poisson’s ratio were essentially
similar to those specified in ASTM standard C469-94. The necessary displacements were measured in a fixture similar to that shown in Figure 1 of the ASTM standard, the fixture used is shown in Figure 6.21. Unlike the fixture shown in the ASTM standard, in the fixture that was used both the lateral and dilatational displacements are measured on opposing sides of the cylinder and averaged, to increase accuracy in the overall measurements. The cylinders were tested in a 2670 kN capacity SATEC loading machine, and the load values during the test were recorded by the instrumentation associated with this machine. A stress versus strain curve for both cylinders is shown as Figure 6.22. The modulus of elasticity can be measured from these plots as the slope of a chord line at any level of stress. At the 21 MPa stress level the chord moduli for Cylinder 1 and 2 are 33240 and 32675 MPa respectively, values that differ from each other by less than 2%. The average modulus from the two tests is 32958 MPa. The Poisson’s ratio of these two cylinders was also calculated during these tests and was 0.196 and 0.192 respectively for each of the cylinders.

Although the values for modulus obtained from the cylinders showed good repeatability, there are issues with using the modulus from these companion cylinders as a proxy for the modulus of the concrete plates themselves. The next three paragraphs detail these issues and indicate why the modulus from the cylinder testing was not used as the final determination of $E_{conc}$.

Although the cylinders were the same age as the plates, the curing conditions of the cylinders and plates are not identical. The plates were either kept completely saturated over time (for Plates 1 and 2), or allowed to air dry from two faces after moist curing for 12 days (Plate 3). In contrast, the cylinders were sealed until testing and only dry out through self desiccation, which is possible for concretes with a water cement ratio below 0.5 (Neville 1981). Different curing conditions means that the modulus from the plates and cylinders may be slightly different.

A second point of concern involves the modulus of wet concrete versus that of dry concrete. It is well known (Neville 1981) that the modulus of elasticity of wet concrete is generally somewhat higher than that of otherwise identical dry concrete. A difference of between 12 and 25 percent was reported by Davis and Troxell (1929). A reasonable explanation for this phenomenon has been put forth by Guo and Waldron (2001). They have postulated a simplified elasticity model involving a spherical inclusion that is either modeled as a completely compressible air filled inclusion or one filled with incompressible fluid. Evaluation of displacements and stresses in these two models shows an increase in modulus of between 7 and 22 percent for concretes with porosity within normal ranges. The moisture content condition of the cylinders is not known exactly but is likely between the saturated state of Plates 1 and 2 (due to self dessication of the cylinders) and the drying state of Plate 3 (because the cylinders were sealed).

A final point of note concerns the hydration of cement within the specimens. The saturated condition of Plates 1 and 2 likely leads to further hydration of cement in
these two plates and thus higher stiffness as compared to the cylinders and Plate 3. It is to be expected then that the modulus of elasticity of Plate 1 and 2 is higher than that of Plate 3 or of that measured from the cylinders and that the modulus of Plate 3 may be closest to the modulus measured during the cylinder testing.

In method (b) noted above, the strains measured by the bonded wire strain gauges along with the load data recorded using the load cells has been used to calculate a modulus of elasticity during each hole test. The moduli found using this method has been used in Sections 6.3 and 6.4 and also to convert strain gauge output to stress in Section 6.2.3.3. In this method the strains at all the gauges at full load prior to drilling the hole have been averaged, and the stress calculated using the load prior to drilling the hole divided by the average area of the plate. The modulus is then calculated as the ratio of the stress to strain. Table 6.8 shows the values of the modulus for each of the hole tests calculated in this manner. In the case of the plates with steel reinforcement (Plate 2 and one half of Plate 3), standard transformed section techniques have been used to calculate the modulus of elasticity of the concrete rather than the effective modulus of the concrete and steel reinforcement composite (Leet 1991). The differences in the modulus of the plates and cylinders are within the expected variability, with the plate that may be closest to the moisture condition of the cylinders, Plate 3, exhibiting the most similar modulus. For the calculations of in-situ stress using Equations (2.20) – (2.22) that are performed for each hole and described in Sections 6.3 and 6.4, the average modulus of each plate shown in Table 6.8 is used. These moduli are also used to convert the strain data from the strain gauges to stresses as for example in Table 6.7.

For comparison purposes, the modulus calculated by the standard ACI 318 equation \( (4730\sqrt{f'_{c}}) \) is 30443 MPa. Alexander (1996) presents a method of calculating \( E_c \) to include the effect of variations in aggregate stiffness. Using the average stiffness for limestone found in Table 2.21 and Alexander’s method of calculation yields an \( E_c = 33551 \) MPa. The values for modulus found in Table 6.9 are within the variation found in the literature.

6.2.4.2 Sorptivity Experiments
The sorptivity is used to determine the approximate depth of wetting of the concrete during the drilling of the core hole. This parameter is important for Holes 7 and 8 in Plate 3 which was allowed to dry. The sorptivity value for the cast concrete was determined using the direct gravitational method as described in Section 2.4.1. A detailed treatment of this method is available in Hall and Hoff (2002). Two different curing conditions of the concrete were investigated. Oven dried cylinders were tested, as well as the concrete cores removed from Holes 7 and 8 of Plate 3. The age of the concrete cylinders and cores was approximately (within 7 days) the same as the age of the plates at core-drilling. Differences in sorptivity measured from the cores and cylinders were expected due to curing differences and other factors. The sorptivity values determined from the cylinders were ultimately used in Section 6.4 as the
sorptivity for the experimental concrete. The rest of this section describes both sets of tests and relates why the values from the cylinders were used.

The three cylinders (S1, S2 and S3) were oven dried at approximately 65 degrees Celsius for 8 days and the mass loss measured after each day until approximately constant mass was achieved. Oven drying in this manner may induce some microcracking which can influence sorptivity. Mass was measured using an Adamslab scale model CBW35a with a capacity of 15000 grams and accuracy and repeatability ratings of 0.5 grams. The sides and top of these cylinders were then sealed with a commercial grade concrete sealer that was applied with a paint brush. After waiting 24 hours for the sealer to cure, each cylinder was balanced on a set of steel ball bearings in a pan of water such that 2 – 3 mm of the bottom of the cylinder were submerged. Here the bottom of the cylinder refers to the smooth face of the cylinder cast against the bottom of the cylinder mold. Sorptivity tests of additional cylinders that were inverted (so that the top of the cylinder was submerged) gave essentially similar results. Figure 6.23(a) shows the pan and ball bearing set up used, and Figure 6.23(b) shows a photograph of one of the cylinders being tested. Also visible in both photographs is the Adamslab scale used. The cylinders were then removed from the water periodically and weighed after wiping the bottom surface with a moist cloth to remove excess water. The weight gain divided by the cross sectional area of the cylinder and the density of the water was plotted versus the square root of time. The sorptivity of each cylinder was calculated as the best fit slope of this plot. Figures 6.24 shows sorptivity plots for each of the cylinders tested. The calculated sorptivity for the three cylinders is 0.145, 0.144 and 0.139 mm/√mm for cylinders S1, S2 and S3 respectively. There is little variability between the values for each of the three cylinders, with the average value being 0.142 mm/√mm and none of the cylinders differing from the average by more than 3%.

Sorptivity testing of the cores followed a similar procedure except that the cores were not oven dried. The sorptivity plots for the two cores are shown as Figures 6.25 and 6.26. At first glance these plots look significantly different than those of Figure 6.24. The curves in this case are not the simple linear curves shown in the previous figure. This has a simple explanation and is related to the moisture conditioning differences between the cylinders and the cores. As mentioned in Section 2.4.1, sorptivity measurements can be sensitive to the initial moisture content of the tested specimen. The cores were removed from Plate 3 that was drying from two faces and as such the moisture content within the cores is expected to be variable, with a relatively low moisture content on the exterior faces and a higher moisture content in the interior region. An additional source of complication regarding the moisture content is the fact that the coring the plates is a wet process and the cores likely absorbed water during drilling. The sorptivity tests on the cores were performed approximately 24 – 48 hours after core-drilling and so some of the water absorbed may have evaporated or redistributed into the interior concrete of the cores. Ultimately the exact moisture content and profile within each of the cores is unknowable although likely they are similar to that predicted in Section 6.4 for Plate 3. The plots in Figures 6.25 and 6.26
begin with a relatively steep slope during the first few minutes of time (compared to later times) and then the slope of the plots diminishes as the time of testing increases. This logically follows from the progression of water into the cores. At first the water encounters the relatively dry outer region of the core and the water moves in somewhat rapidly. As time passes the advancing wetted front encounters regions with ever increasing moisture contents, causing the rate of water ingress to decrease with time.

As mentioned the sorptivity parameter is used to estimate the water absorption in the Hole 7 and Hole 8 tests of Plate 3. The average time that elapsed during the drilling of these two holes was 12 minutes. If the sorptivity in Figures 6.33 and 6.34 is measured over the first 10 minutes, the values for Core 7 and Core 8 are 0.110 and 0.147 mm/√min, in good agreement with the 0.142 value measured on the cylinders. The average sorptivity value from the cylinders (0.142 mm/√min) is used in Section 6.4.2 for calculating the depth of wetting in the calculations for the apparent stress due to core-drilling water during the Hole 7 and Hole 8 tests.

6.3 RESULTS

Figure 6.27 shows a photograph of all of the plates after the drilling of Holes 1 – 8. Figure 6.28 shows a photograph of Plate 2 at the Hole 5 location and shows the cored plate and associated core. Visible in the photograph is the speckled paint pattern applied as part of the digital image correlation process. The cores from the other tests look similar. Figure 6.29 shows Plate 1 at the cored Hole 1 location, without the core being shown.

6.3.1 Relieved Displacements

As mentioned digital image correlation was used to determine the relieved displacements from each hole test. The coordinate system used for each test is shown in Figure 6.30. Relieved displacements for Holes 1, 2, 3, 4, 5, 7, and 8 were computed by comparing Stage 2 and Stage 3 photographs in all cases. Relieved displacements for Hole 6 were computed by comparing Stage 2 and Stage 6 photographs as discussed in Section 6.2.3.2. Figures 6.31 – 6.34 show plots of the relieved displacements for Holes 1 and 2 in the $x$ and $y$ directions as provided by the digital image correlation system.

Relieved displacements are reported herein at a measurement radius of $m = 100$ mm and are reported in the radial and tangential directions. Digital image correlation outputs displacements at many different points within the field of view, at irregular intervals depending on the random pattern applied to the surface of the object. To simplify later calculations, linear interpolation is used to convert the irregular spaced displacements measured with digital image correlation to displacements regularly spaced at one degree increments. Figure 6.35 shows the relieved displacements versus measurement angle for the Hole 3 test as measured by digital image correlation and as linearly interpolated to one degree increments. There is no substantive difference in

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the displacements directly from the digital image correlation output versus those that have been interpolated. Additional relieved displacements presented are those that have been interpolated to one degree increments. Figures 6.36 – 6.43 show the relieved displacements for Holes 1 – 8.

6.3.2 In-Situ Stresses
The relieved displacements shown in Figures 6.36 – 6.43 can be converted into in-situ stresses using the measurement configuration shown in Figure 2.5(a) and Equations (2.20) – (2.22). These calculations were performed for the tests at each hole. There are two issues that should be discussed prior to a description of the stress results: (1) any adjustments that are made to the calculated in-situ stresses to account for the test specimen geometry; (2) any adjustments that are made to the calculated in-situ stresses due to through thickness variation in the applied in-situ stresses. These two issues are discussed below.

6.3.2.1 Adjustment for Specimen Geometry
Equations (2.20) – (2.22) were derived assuming a small hole in an infinite, thin plate. However the plates tested as part of the current research have geometric aspects not considered in the derivations. This section discusses these geometric aspects and covers the procedure used to account for them.

Turker (2003) performed finite element calculations to demonstrate the applicability of Equations (2.20) – (2.22) to objects of finite size and showed that finite element calculations can be used to account for the effects of the finite dimensions on the stress calculations. There are two geometric issues with Plates 1 – 3 that are addressed here. The width of the plates (864 mm) is approximately 5.8 times the diameter of the core hole and thus may cause a small (less than approximately 3%) error in stress results. In addition, the conduits embedded in the plates may cause an error in the measured stresses because the conduit interior space essentially has zero stiffness and thus may affect the relieved displacements. To calculate the influence of these two factors on the experimentally determined in-situ stresses a finite element investigation was performed. Finite element models with an without these geometric aspects were analyzed and the stress results compared to determine the impact of the geometric aspects.

A schematic of the model with the geometric aspects is shown as Figure 6.44(a). It is a 1/8 symmetry representation of Plate 1 with 1/4 symmetry in the plane of the plate and 1/2 symmetry through the thickness. The dimensions of the model match the design dimensions of Plate 1, and the core hole diameter is 150 mm. Thus the width of the model is 431 mm (one half of the design dimension for Plate 1). The finite element mesh is shown in Figure 6.44(b) and contains 20336 10 noded tetrahedral quadratic solid elements (ABAQUS C3D10M). The conduits were modeled as void spaces and have elements that were assigned negligible stiffness properties. The model was loaded with pressure in the loaded area noted that corresponds to the area of the bearing plate in the experimental tests.
The schematic of the model without geometric aspects is shown as Figure 6.45(a). It is similar to the previous model except that concrete stiffness properties were assigned to the elements in the conduit region and an ‘extra’ region has been added so that the overall width of the model is 2440 mm, more than 5 times the width of the previous model and resulting in a width to core hole diameter ratio of more than 30. The mesh of this model is shown in Figure 6.45(b). The mesh of the ‘extra’ region contains 832 8 noded hexahedral linear solid elements and has been tied to the plate region with tie constraints in ABAQUS. The mesh of the rest of the model is identical to the previous model.

Relieved displacements at a measurement radius of \( m = 100 \) mm from the two models were converted into in-situ stresses using Equations (2.20) – (2.22). The results of this investigation are shown in Table 6.10. Differences in the stresses from the two models are due to the geometric effects of testing a plate with a finite width and embedded conduits. The narrow width and conduits cause an over-prediction in both \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) of approximately 0.5 MPa as compared to the model without them. In the Turk (2003) investigation, similar trends were noted, although the effect was somewhat smaller in the previous study. However, in that study, the results were examined for a measurement radius of \( m = 112.5 \) mm rather than the \( m = 100 \) mm used herein, and furthermore the conduit effects were not within the scope of the previous study. The procedure of the current investigation was repeated with a mesh density considerably finer (with 37986 ABAQUS C3D10M elements in the plate region) than those shown in Figures 6.44(b) and 6.45(b) and the results did not appreciably change.

The geometric effects cause a slight over-prediction in \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) in-situ stresses. The over-prediction should be essentially constant among the eight hole tests that were performed and can be removed by subtracting the stresses noted in Row (3) of Table 6.10 from the in-situ stresses calculated for each hole test. The removed values must be scaled by the ratio of the applied stress in each test versus the applied stress in the finite element model (-9.655 MPa). All reported in-situ stresses herein have been modified in this manner to account for the geometric effects involved with the tested specimens.

6.3.2.2 Adjustment for Through-Thickness Eccentricity

Slight installation of the Dywidag post-tensioning bars away from the mid-plane of the tested plates can result in a stress profile through the thickness of the plates that is non-constant. Figure 6.46 shows theoretical stress profiles for a plate with the same nominal dimensions as those experimentally tested under various amounts of through-thickness eccentricity in load and indicates that a relatively modest eccentricity in loading (5 mm) can result in a stress on the top surface of the plate that is 16.1% of the nominal stress. It is reasonable then to quantify the effect that this through thickness eccentricity in the loading has on the calculated in-situ stresses using the core-drilling method.
A finite element model of one of the tested plates was created to investigate this issue. The model is a 1/8 symmetry model that is loaded with a point load and point moment at the end of the plate to simulate a nominal compressive stress and any through thickness gradient in stress that might arise due to eccentricity in loading. Relieved displacements from this model were then converted into in-situ stresses using Equations (2.20) – (2.22). The model was run several times with varying amounts of eccentricity and the stress results from each case were compared to a run in which the eccentricity was zero. The relative difference in the stress results is plotted versus the amount of eccentricity in Figure 6.47.

A new variable, $K_z$, may be defined that is similar to the $K_x$ and $K_y$ defined in Chapter 2. In this case $K_z$ is defined as the stress variation with distance in the through thickness direction of the plate. It may be calculated by taking the applied moment in the model and dividing by the out-of-plane moment of inertia of the plate. A plot of relative difference in stress results versus $K_z$ is shown as Figure 6.48 and shows linear behavior in this regard. The equation of the line in the figure is

$$\% \text{Diff} = 532.9K_z$$  \hspace{1cm} (6.1)

If the amount of eccentricity in load is known for a specific test then Figures 6.47 and 6.48 may be used to account this eccentricity on the in-situ stress calculations.

The strain gauges were located in each plate in order to facilitate determination of the eccentricity in load for each test. Strain gauges were located in the top and bottom of each plate so that the through thickness eccentricity could be calculated. Unfortunately, the wiring connecting each of the strain gauges to the monitoring equipment was not properly labeled and it was not possible to determine the elevation location of each strain gauge within each plate. For example in Table 6.7 although it is known that in the Hole 4 test the stress output for the East-North strain gauges (7.04 and 8.55 MPa respectively for the output from before core-drilling) occur at the same plan location in Plate 2, information regarding which reading is from the top of the plate and which is from the bottom of the plate was lost.

To quantify the maximum possible effect of the through thickness eccentricity in load, it may be assumed that the higher reading in each pair of gauges is the top gauge within the slab such that all higher readings from a slab are located on the top surface. This is shown for the Hole 4 stresses in Table 6.11. In this case any gauge that is not part of a top-bottom pair is neglected. If this gauging arrangement is assumed, the assumed eccentricity in load may be calculated as 2.0 mm. Equation (6.1) is used to calculate an apparent affect in the in-situ stress results for this test of 4.3%. Thus coring a through hole in a plate with top surface stresses slightly higher than nominal and bottom surface stresses slightly lower will result in slightly higher relieved displacements than expected from the nominal stress state and hence higher calculated measured stresses than nominal. The reverse is true if the higher stresses are on the
bottom surface of the plate. This process was repeated for the other hole tests and results in the data found in Table 6.11. The largest discrepancy noted is for Hole 7 and shows a potential 16.9% discrepancy in calculated stresses due to the through thickness eccentricity issue. Again it is noted that the values in the table are for the worst case assumption that all of the extreme values in stress are on one face of the plate. It is possible that the extreme values are distributed some on either side of the plate, in which case the through thickness variation in stress is much less. Because the effects noted are small and because it is impossible to determine which gauge is the top gauge in each pair of strain gauges, no correction for through thickness eccentricity effects on in-situ stresses has been attempted for the current work.

6.3.2.3 In-situ Stress Results
The calculated in-situ stresses for each hole test are shown in Table 6.12. In all cases in this chapter where experimental stress results are calculated using the Measurement Configuration shown in Figure 2.5(a) and Equations (2.20) – (2.22), the in-situ stress results ($\sigma_x$, $\sigma_y$, $\tau_{xy}$) are calculated numerous times around a measurement circle by rotating the configuration. In the experiments the relieved displacements were determined in one degree increments so the rotation is performed 179 times (further rotation simply provides duplicate calculations). The resulting stress calculations are averaged to report a single set of values for the experimentally determined in-situ stresses. Figure 6.52 shows the calculated in-situ stresses for Hole 3 versus the measurement configuration orientation. The calculations are performed using Equations (2.20) – (2.22) and with the data at one degree increments around the measurement circle. The $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ from the equations are then converted into principal stresses. There is very little variation in in-situ stress as the measurement configuration is rotated around the measurement circle. The average values for Hole 3 are $\sigma_{max} = 0.31$ MPa, $\sigma_{min} = -8.91$ MPa. For all other hole tests average values using this procedure are reported. Note that the average in-situ stresses reported are adjusted for geometric effects as noted in Section 6.3.2.1. The stresses in Table 6.13 have been determined in this manner.

An examination of Table 6.13 shows two interesting trends. The four tests that involved testing of reinforced concrete (Holes 4, 5, 6 and 7) show a marked under-prediction in in-situ stress. For Holes 4, 5 and 6 the average under-prediction in the $\sigma_{min}$ stress is 21.2%. This is expected and is addressed in Section 6.4.1. The two drying specimens show a marked increase in measured tension stress coupled with a reduction in measured compressive stress. The average measured tension stress for the two drying tests was 3.59 MPa, a 17-fold increase from the average measured tension stress in the saturated specimens (0.20 MPa). This is also expected and reflects the additional apparent stresses induced due to core-drilling water effects and differential shrinkage. These phenomena are addressed in detail in Sections 6.4.2 and 6.4.3 respectively.
6.4 CORRECTION OF IN-SITU STRESSES FOR THE EFFECTS OF STEEL REINFORCEMENT, SWELLING DUE TO CORE DRILLING WATER AND DIFFERENTIAL SHRINKAGE

As mentioned previously, the in-situ stress calculations from the core-drilling method are subject to distortions by several influencing factors. The effects of some of these factors are examined in Chapters 3 – 5. In the experimental portion of the current work the effects of three of these factors were probed: (1) the effect of steel reinforcement in close proximity to a drilled core hole, (2) the effect of core-drilling water induced swelling displacements around a core hole, and (3) the effects of differential shrinkage. Each of these factors serves to bias the in-situ stresses that are calculated using the core-drilling method. Steel reinforcement in close proximity to a drilled core hole causes an under-prediction in the in-situ stresses in an object. Water-induced swelling displacements and differential shrinkage induce additional hydrostatic tension apparent stresses that are added to the actual stresses measured in the object. This section addresses these influencing factors and details the procedures followed to reduce the effects of them. Section 6.4.1 discusses the calculations performed to account for the effects of the primary steel reinforcement present in the tested plates. Section 6.4.2 discusses the calculations performed to account for the effects of core-drilling water. Finally, Section 6.4.3 discusses the calculations performed to account for the effects of differential shrinkage of the test specimens.

6.4.1 Correction for Steel Reinforcement

Chapter 5 details the investigation into the effects of steel reinforcement on the core-drilling method. In that chapter it is noted that steel reinforcement in the near proximity to a drilled core-hole causes a reduction in the relieved displacements and hence an under-prediction in in-situ stresses. The finite element method was used to show that the reduction due to the steel reinforcement can be as high as 18% depending on the nearness to the rebar and the amount of cover to the outmost reinforcing bar. The work explained in Chapter 5 was performed assuming a 25 mm diameter steel reinforcing bar.

As explained in Section 6.2.2.3, Plate 2 contains 36 mm diameter steel reinforcement bars at a nominal spacing of 216 mm on centers. The layout of the reinforcement was shown in Figure 6.3. This diameter and bar spacing coupled with a drilled core hole 150 mm in diameter means that the nominal clear spacing to the nearest reinforcing bar in the Plate 2 experiments was approximately 15 mm. The nominal cover provided in the experiments was 57 mm. This case is not covered in the detailed investigation of Chapter 5. However, it is possible to use the same methodology presented in Chapter 5 to investigate the experimental reinforcing bar lay-out. A finite element model similar to that shown in Figure 5.2 was created in which the reinforcing bar diameter and spacing were as in Plate 2. An analysis similar to that described in Chapter 5 was used to determine that the effect of the steel reinforcement in the experimental plates was to reduce the measured in-situ stresses in the Plate 2 experiments (Holes 4, 5 and 6) by 19.4% and in the Hole 7 test by 21.1%. The
The difference in the two values is due to the difference in concrete modulus between Plates 2 and 3. The measured $\sigma_{min}$ values for the Hole 4, 5 and 6 tests were adjusted to account for this 19.4% reduction in calculated in-situ stress in order to increase accuracy in the in-situ stress calculations.

In the Hole 7 test a large apparent hydrostatic tension stress is induced due to the drying of the slab and the associated issues with core-drilling water and differential shrinkage. These issues are detailed in Chapters 3 and 4. The addition of this large hydrostatic tension stress to the measured stress results causes a shift in the measured stresses in Mohr’s Circle stress space. Thus for Hole 7 both the measured $\sigma_{min}$ and $\sigma_{max}$ values were adjusted to account for the 21.1% under-prediction in stress in the reinforced portion of Plate 3 caused by the steel reinforcement.

Table 6.14 shows the results for Holes 4, 5, 6 and 7 as adjusted for the effects of steel reinforcement in the plate. A comparison with the uncorrected results contained in Table 6.13 shows a significant improvement in the results for Holes 4, 5 and 6. The Hole 7 results have not been adjusted for drying effects (see the subsequent sections) and thus still reflect large discrepancies between measured and applied stresses. The uncorrected and corrected stress results for these tests are provided in Mohr’s circle form in Section 6.4.4.

As an aside, and to provide information that may be of use in further experiments, additional models were analyzed in which the amount of cover and bar spacing were modified from the experimental values. The effects of changing these variables are contained in Table 6.15. Similar to the values in Chapter 5, the values in Table 6.15 are calculated assuming a concrete with a modulus of elasticity equal to 28270 MPa and thus must be adjusted based on the modulus of the tested plates to arrive at the figures (19.4% and 21.1%) quoted above.

### 6.4.2 Correction for Core-Drilling Water Effects

Chapter 3 discusses the effect of core-drilling water on stresses measured with the core-drilling method. This section provides the correction of the experimental in-situ stresses due to core-drilling water effects.

Core-drilling water causes apparent hydrostatic tension stresses that are superposed on any stresses evaluated using the technique. These are caused by the swelling of initially dry concrete around the core hole that is wetted during the course of drilling a core hole. Equations (3.7) – (3.11) and (3.14) – (3.15) allow the calculation of the apparent stress due to core-drilling water for concrete with variable properties. The apparent stress at a particular measurement radius can be calculated with knowledge of the depth of water penetration during a test and the swelling strain of the tested concrete. The depth of water penetration depends on three parameters according to Equation (2.51): the sorptivity, $S$ of the concrete, the porosity, $f$ of the concrete, and the time of water exposure, $t$.  

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Plates 1 and 2 (Holes 1-6) should not absorb significant amounts of water during coring as these plates have been kept in a saturated state prior to testing. The Hole 1-6 in-situ stress results have not been adjusted for core-drilling water effects. The calculated in-situ stress results for Hole 7 and Hole 8 (drilled in the drying specimen, Plate 3) must be adjusted for core-drilling water effects. The equations of Chapter 3 are used to accomplish this, and require the appropriate input information. Section 6.4.2.1 below details the appropriate input information either measured or estimated during experimental testing, and Section 6.4.2.2 describes the adjustment of the measured in-situ stress results for the Hole 7 and Hole 8 tests.

6.4.2.1 Input Parameters for Core-Drilling Water Effects
As mentioned previously, evaluation of Equation (3.8) and (3.14) requires the following information: sorptivity, time of water exposure, porosity, and swelling strain. As described in Section 6.2.4.2 the sorptivity of the concrete in the experiments was measured via the direct gravitational method to be
\[
S = 0.14 \text{ mm}/\sqrt{\text{mm}.}
\]
The time of water exposure for the Hole 7 test was measured during the test to be 12 minutes, for the Hole 8 test the time was measured as 14 minutes. The porosity of the concrete was not measured but has been estimated using the process outlined in Pann, Yen, Tang and Lin (2003). Pann et al. (2003) give a relationship for the porosity of concrete in relation to its water cement ratio and degree of cement hydration as follows

\[
f = -0.255\ln(\alpha_{ch}) + \frac{0.334}{0.307^{w/c}} - 0.621 
\]

(6.2)

where

\[
\alpha_{ch} = -44.028 + 621.42(w/c) - 1144.053(w/c)^2 + 696.890(w/c)^3
\]

(6.3)

Pann et al. compare porosity calculated via Equation (6.2) with porosity measured experimentally across data sets from several other researchers and indicate that the correlation coefficient (R²) for Equation (6.2) is 0.9762. For the concrete in the current experimental program the water cement ratio is 0.42 which yields an estimated porosity of \(f=0.0981\) using Equation (6.2).

The foregoing sorptivity, porosity and time of testing have been used along with Equation (2.51) to calculate a depth of water penetration, \(t_w\) of 4.94 mm for Hole 7 and 5.34 mm for Hole 8. These values are rounded to 5 mm for both tests.

The swelling strain of the concrete in the experimental program was not measured. As mentioned in Chapter 2, Neville (1981) gives as a guide that the swelling strain is approximately \(1/3\) of the ultimate shrinkage strain for typical concrete. The ultimate shrinkage of the concrete used in the experiments was not measured but is estimated according to the procedure outlined by Gardner and Lockman (2001). Gardner and
Lockman (2001) provide an equation for the ultimate shrinkage of a concrete mixture in the following form

\[
\varepsilon_{sha} = 1000 \cdot K_{GL} \cdot \left( \frac{30}{f_{cm28}} \right)^{1/2} \cdot 10^{-6}
\]  

(6.4)

where

\[ K_{GL} = 1 \text{ for Type I cement} \]

\[ f_{cm28} = 1.1 f_{ck28} + 5 \text{ MPa} \]

and \( f_{cm28} \) is the 28-day specified concrete strength. The method has been termed by Gardner and Lockman (2001) as the GL2000 method. Gardner and Lockman demonstrate that the GL2000 method shows good agreement with experimentally determined shrinkage values from a RILEM databank of 115 data series. Al-Manaseer and Lam (2005) used statistical methods to evaluate several methods for estimating concrete shrinkage and concluded that the GL2000 method was a reasonable technique for estimating shrinkage. The design strength of the concrete used in the experiments was 41.4 MPa, and the cement used was Type I. Using Equation (6.4) this results in an estimated ultimate shrinkage strain of 770 microstrain. Using the 1/3 factor recommended by Neville (1981) yields an estimated swelling strain of 257 microstrain.

6.4.2.2 Correction for Core-Drilling Water Effects

The work presented in Chapter 3 allows for the calculation of the effects of core-drilling water by looking at two separate wetted regions, namely the annulus region around the core hole (termed Portion A) and the surface region (termed Portion B) on the side of the concrete from which drilling commences. The next several paragraphs describe the calculations of apparent stresses due to wetting these two areas and discuss why ultimately the surface wetting (Portion B) effects were neglected.

Equation (3.8) is provided in Chapter 3 for estimating the apparent stress due to annulus wetting at a measurement radius of 100 mm and Equations (3.14) – (3.15) for estimating the apparent stresses due to surface wetting at a measurement radius of 100 mm for a 75 mm radius core hole. It is noted in Chapter 3 that the calculated results from Equations (3.8) and (3.14) – (3.15) should be adjusted based on the ratio of the Chapter 3 analytical modulus of elasticity (28270 MPa) and the modulus of elasticity of the tested concrete. In the experimental tests, displacements were only measured on the top face of the plates and hence for the surface wetting, only Equation (3.14) is applicable. Equation (3.16) is also provided in Chapter 3 to allow the results of Equation (3.14) (calculated based on a specimen thickness of 150 mm) to be adjusted for application to objects of different thicknesses. The thickness of Plates 1 – 3 was approximately 150 mm and thus no adjustment based on thickness is required.
Calculations of apparent stresses via Equations (3.8) and (3.14) using the input parameters for the concrete used in the experiments and the modulus of elasticity of Plate 3 from Table 6.9 (31476 MPa) yield apparent stresses of

\[
A_{\text{top}} = 1.58 \text{ MPa} \quad (6.5)
\]

\[
B_{\text{top}} = 0.11 \text{ MPa} \quad (6.6)
\]

It is possible that the specimen may also be wetted on the bottom side during testing. Plate 3 was lying on a plywood sheet resting on the laboratory floor while being cored, and this situation may have led to water seeping under the plate during testing. Equation (3.16) – (3.18) can be solved assuming the wetted and dry faces are reversed and results superposed from assuming both faces are wetted. Solving Equation (3.16) – (3.18) in this manner yields an apparent stress on the measured face of

\[
\sigma_{\text{apparent}} = -0.36 \text{ MPa} \quad (6.7)
\]

(if adjusted by the ratio of moduli, 31476/28270 as recommended in Chapter 3). This value (as calculated via Equation (3.16) – (3.18)) assumes \(\alpha_w=167\) microstrain and \(t_w=6\) mm, and must be further adjusted for the actual parameters \(\alpha_w=257\) microstrain and \(t_w=5\) mm) of the concrete in Plate 3. Equations (3.16) – (3.18) are not formulated to account for a difference in these parameters. As an approximation however, Equation (3.8) may be calculated for the two sets of parameters and Equation (6.7) scaled by the resulting ratio. This calculation results in an apparent stress of -0.47 MPa. This value is hypothetical and is not used to adjust the measured stresses in the Hole 7 and Hole 8 tests, because as mentioned, there is no way to ascertain whether water was seeping under the plate in significant quantities during testing.

During experimental testing, an interesting phenomenon was observed regarding wetting of the surface region. The paint that was applied to the surface (as part of the digital image correlation measurement process) appears to be somewhat water resistant and hence likely serves to resist moisture ingress through the surface of the concrete. This effect is visible in Figure 6.50 which depicts wetting of the painted region of one of the plates and shows water beading on the surface of the paint. The scale of the DIC pattern in the figure is similar to that shown in Figure 6.14. Although the paint may not have provided a complete water barrier, it is clear that some reduction in water ingress through the surface of the concrete was affected. This would serve to further reduce the apparent stress noted in Equation (6.6) due to wetting of the surface region. Because the apparent stress due to the surface wetting is small as calculated, and because it is likely that the paint served to further reduce this value, the apparent stress due to surface wetting has been neglected.
Table 6.16 shows the experimental in-situ stress results for hole tests 7 and 8 as modified to account for core-drilling water effects. Note that the results shown in Table 6.16 for hole test 7 have already been adjusted to account for steel reinforcement as described in Section 6.4.1. It is clear that the accuracy in the measured stress results in hole tests 7 and 8 has been improved, however there is still a substantial discrepancy in these tests between the measured and applied stresses. The next section addresses estimation of the effects that differential shrinkage had on the measured stress results of the hole 7 and 8 tests. The uncorrected and corrected stress results for these tests are provided in Mohr’s circle form in Section 6.4.4.

6.4.3 Correction for the Effects of Differential Shrinkage

Chapter 4 presents an investigation that was performed to determine the effects of differential shrinkage on the core-drilling method. Differential drying results in differential shrinkage which in turn creates a profile of in-plane stresses through the thickness of concrete objects. If a core hole is drilled into a loaded object, relieved displacements will reflect relief of the stress due to the applied loads but also relief of these stresses due to differential shrinkage. This section presents calculations that were performed to correct the experimentally determined in-situ stresses for the effects of differential shrinkage.

In Chapter 4 the differential stress profiles for plates stored in several different relative humidity environments were estimated using the methods of Akita et al. (1997) to estimate moisture profiles and a simplified relationship between moisture loss and shrinkage. The core drilling method equations were then used to calculate stresses from displacements that resulted from coring holes through these stress profiles. These calculated stresses were termed apparent stresses due to differential shrinkage. Finite element modeling was the primary tool used in Chapter 4 to estimate the apparent stresses due to differential shrinkage, with the following important input parameters in the investigation: the age of the tested concrete, the environmental relative humidity, the water cement ratio of the tested concrete, the thickness of the interrogated object, and the ultimate shrinkage of the tested concrete. These parameters for the concrete used in the experimental work are listed in Table 6.17. It is clear that this exact scenario is not covered in the efforts documented in Chapter 4. However, the same methodology presented in Chapter 4 may be used with the input parameters noted in Table 6.16 to estimate the apparent stress due to differential shrinkage in the Hole 7 and Hole 8 tests.

Finite element models similar to those shown in Figure 4.1 were created and analyzed with the input properties noted in Table 6.17 and the methodology detailed in Chapter 4. The resulting moisture and differential shrinkage stress profiles are shown in Figures 6.54 and 6.55 respectively. According to Figure 6.54, due to the relatively dry storage environment ($RH = 17\%$), the exterior faces of the plate experience significant drying, but the internal regions of the plate remained at a high moisture content at test. This resulted in an estimate of the apparent stress due to differential shrinkage in the
Hole 7 and 8 tests of 2.55 MPa. As noted in Chapter 4 this apparent stress is a hydrostatic tension stress.

Table 6.18 shows the measured in-situ stresses for Holes 7 and 8 adjusted for differential shrinkage effects. Note that these tests have already been adjusted for the effects of core-drilling water (see Section 6.4.2), and in the case of Hole 7 for the effects of proximate steel reinforcement (see Section 6.4.1). The measured stress results now show excellent agreement with the applied stresses. The average measured $\sigma_{\text{max}}$ stresses in the table are now small and in general agreement with those from the other hole tests. The uncorrected and corrected stress results for these tests are provided in Mohr’s circle form in Section 6.4.4.

### 6.4.4 Summary of Corrected Stress Results

Table 6.19 shows the results for all eight hole tests as adjusted for the various influencing factors noted in the preceding sections. In particular, Holes 4, 5, 6 and 7 have been adjusted for the effects of proximate reinforcement, and Holes 7 and 8 have been adjusted for the effects of core-drilling water and differential shrinkage. The average SRSS error over the 8 tests is 9.5%. This can be contrasted with the average error of 28.4% before these effects were considered. It is clear that these effects must in fact be considered, 28.4% error is likely insufficient accuracy during a structural assessment.

One way to present the applied and measured stress comparisons is through the use of Mohr’s circle. Figures 6.53 – 6.60 show the applied, uncorrected measured and corrected in-situ stresses presented in this fashion for all the hole tests. Figures 6.53 – 6.55 show only the applied and measured stresses as no correction is necessary for the Plate 1 tests. Figures 6.56 – 6.58 involve correcting for steel reinforcement effects. It is clear from an examination of these figures that the correction for steel reinforcement involves increasing the diameter of the Mohr’s circle. In Figure 6.59 the ‘Corrected’ curve includes only the correction performed regarding proximate steel reinforcement, and the ‘Final’ curve includes this correction as well as those due to core-drilling water effects and differential shrinkage effects. In Figure 6.60 the results have been corrected for only core-drilling water and differential shrinkage effects. It is clear from an examination of Figures 6.62 – 6.63 that correction for core-drilling water and differential shrinkage effects involves shifting the location of the Mohr’s circle rather than changing it’s diameter. These trends have been noted previously in Chapters 3 – 5.

Two other issues are addressed here. The first issue concerns the linear gradients applied in the Hole 2 and Hole 5 tests. It was stated that the objective of this was to determine if linear variation in the in-plane applied normal stresses compromises the ability of the core-drilling method to discern the average applied in-plane normal stresses. The SRSS errors for Holes 2 and 5 in Table 6.19 are approximately the same as for any of the other tests. This indicates that it is possible to determine the average in-plane normal stress even in the presence of an in-plane normal stress gradient.
The second issue concerns testing of saturated plates. It was stated that testing saturated plates should avoid issues with core-drilling water induced swelling and with differential shrinkage. Only the tests in the drying plate (Holes 7 and 8) have been adjusted in Table 6.19 for core-drilling water and differential shrinkage. The tests in the saturated plates have not been adjusted for these effects. Yet the $\sigma_{\text{max}}$ stresses noted in Table 6.18 for all the 8 tests are all relatively small (with magnitudes less than approximately 1 MPa). This indicates that testing saturated plates achieved its purpose; the saturated plates did not develop significant apparent stresses from the absorption of core-drilling water or from differential shrinkage.

6.5 SUMMARY OF FINDINGS

One of the primary goals of the experiments described herein was to evaluate the effectiveness of the core-drilling method in measuring in-situ stresses on simplified concrete structures. To that end it would seem that the overall average error across all of the core-drilling tests depicted in Table 6.19 of 9.5% indicates that the core-drilling method can be sued to assess stresses in concrete. The following additional findings are noted:

- Digital image correlation is validated for use in measuring the displacements involved in an investigation of the core-drilling method. In particular, there were no negative effects noticed on the displacements derived from the digital image correlation system wrought by the substantial soiling and wetting of the applied pattern during drilling of the core hole. Additionally, patting the applied pattern dry with paper toweling following testing seemed sufficient to ensure consistent displacement measurements. No detectable effects were noticed due to any residual water remaining on the pattern surface.

- The analytical formulations relating in-situ stresses to relieved displacements explored in Turker (2003) appear to accurately describe the behavior noted in the tests. In particular the relieved displacements (as for example shown in Figures 6.36 – 6.43) follow the expected trigonometric dependence on measurement angle and are generally of the expected magnitude.

- Accuracy in uncorrected calculated in-situ stress results is not high. The overall SRSS error across the eight holes noted in Table 6.13 is 28.4%. This is a substantial amount of error and leads to the conclusion that it is imperative to consider correcting calculated in-situ stresses to account for the influencing factors noted in Section 6.4.

- The analytical techniques presented in Chapters 3-5 for dealing with proximate steel reinforcement, core-drilling water and differential shrinkage were successful at reducing error in measured stress. The average error across the eight tests in corrected measured in-situ stresses noted in Table 6.19 is only 9.5%.

- As expected, the uncorrected in-situ stress results in the steel reinforced plates reflect an under-prediction in stress of approximately 21.9% (the SRSS average across Holes 4-6). This same quantity drops to 6.9% following the correction procedure outlined in Section 6.4.1.
• As expected, the uncorrected in-situ stress results in the drying plate showed a large hydrostatic tension stress as compared to the saturated plate tests. The uncorrected measured $\sigma_{\text{max}}$ stress across the six saturated specimen hole tests (Holes 1-6) ranged from -0.46 to +0.58 MPa, while in the drying specimen tests (Holes 7 and 8) the average uncorrected $\sigma_{\text{max}}$ stress was 3.55 MPa (if Hole 7 is first adjusted for the proximate steel reinforcement effects). The average SRSS error in the drying tests is 61% prior to adjusting for core-drilling water effects and differential shrinkage (if Hole 7 is first adjusted for proximate steel reinforcement effects). This same quantity is 9.6% after adjusting for these phenomena.

• As expected, saturated plates (Plates 1 and 2) did not develop significant apparent stresses due to core-drilling water induced swelling or differential shrinkage.

• In-plane linear stress gradients did not prohibit determination of the average normal stress in the Hole 2 and 5 tests.

• As noted in Section 6.2.3.4 it was not possible to detect linear variation in in-plane stresses during testing.
Figure 6.1 Schematic of typical plate specimen

Figure 6.2 – Types of specimens tested

Figure is not to scale
Figure 6.3 – Specimen details (dimensions are in mm): (a) plan view; (b) section view
Figure 6.4 – Loading point reinforcement cage
Figure 6.5 - Specimen forms

Figure 6.6 – Placing concrete into the forms
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Figure 6.18 – Theoretical relieved displacements for Hole 2: (a) normal and linear gradient stress state components; (b) linear gradient stress state components only.
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Figure 6.29 – Plate 1 cored at the Hole 1 location
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Figure 6.31 – Relieved displacements in the $x$-direction for Hole 1 measured by comparing Stage 2 and Stage 3 photographs

Figure 6.32 – Relieved displacements in the $y$-direction for Hole 1 measured by comparing Stage 2 and Stage 3 photographs
Figure 6.33 – Relieved displacements in the $x$-direction for Hole 2 measured by comparing Stage 2 and Stage 3 photographs

Figure 6.34 – Relieved displacements in the $y$-direction for Hole 2 measured by comparing Stage 2 and Stage 3 photographs
Figure 6.35 – Relieved displacements for Hole 3 – Digital image correlation (DIC) versus values interpolated to even one degree increments
Figure 6.36 – Radial and tangential relieved displacements for Hole 1

Figure 6.37 – Radial and tangential relieved displacements for Hole 2
Figure 6.38 – Radial and tangential relieved displacements for Hole 3

Figure 6.39 – Radial and tangential relieved displacements for Hole 4
Figure 6.40 – Radial and tangential relieved displacements for Hole 5

Figure 6.41 – Radial and tangential relieved displacements for Hole 6
Figure 6.42 – Radial and tangential relieved displacements for Hole 7

Figure 6.43 – Radial and tangential relieved displacements for Hole 8
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Figure 6.45 – Finite element model without geometric aspects: (a) schematic; (b) mesh
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Figure 6.47 – Percentage difference in measured stress versus amount of eccentricity
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Figure 6.49 – Measured principal stresses versus measurement angle orientation for Hole 3
Figure 6.50 – Water beading on the painted surface of a tested plate
Figure 6.51 – Calculated moisture profile in Plate 3

Figure 6.52 – Calculated differential shrinkage profile in Plate 3
Figure 6.53 – Mohr’s circle plot of Hole 1 stresses (saturated plain concrete)

Figure 6.54 – Mohr’s circle plot of Hole 2 stresses (saturated plain concrete)
Figure 6.55 – Mohr’s circle plot of Hole 3 stresses (saturated plain concrete)

Figure 6.56 – Mohr’s circle plot of Hole 4 stresses (saturated steel reinforced concrete)
Figure 6.57 – Mohr’s circle plot of Hole 5 stresses (saturated steel reinforced concrete)

Figure 6.58 – Mohr’s circle plot of Hole 6 stresses (saturated steel reinforced concrete)
Figure 6.59 – Mohr’s circle plot of Hole 7 stresses (drying steel reinforced concrete)

Figure 6.59 – Mohr’s circle plot of Hole 8 stresses (drying plain concrete)
Table 6.1 - Experimental specimen test matrix

<table>
<thead>
<tr>
<th>Plate</th>
<th>Hole</th>
<th>Saturated</th>
<th>Drying</th>
<th>Reinforcement</th>
<th>Loading</th>
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<td>✓</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>2</td>
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<td></td>
<td></td>
<td>Linear</td>
</tr>
<tr>
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<td>3</td>
<td>✓</td>
<td></td>
<td></td>
<td>Constant</td>
</tr>
<tr>
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<td>4</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>Constant</td>
</tr>
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<td>5</td>
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<td></td>
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<td>7</td>
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<td>8</td>
<td>✓</td>
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</tbody>
</table>

Table 6.2 - Measured plate dimensions

<table>
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<tr>
<th>Location</th>
<th>Width (mm)</th>
<th>Height (mm)</th>
<th>Calculated Area (mm²)</th>
<th>Calculated In-Plane Moment of Inertia (mm⁴)</th>
<th>Calculated Through Thickness Moment of Inertia (mm⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole 1</td>
<td>873</td>
<td>162</td>
<td>141381</td>
<td>8.982E+09</td>
<td>3.089E+08</td>
</tr>
<tr>
<td>Hole 2</td>
<td>867</td>
<td>161</td>
<td>139493</td>
<td>8.733E+09</td>
<td>3.011E+08</td>
</tr>
<tr>
<td>Hole 3</td>
<td>867</td>
<td>161</td>
<td>139149</td>
<td>8.712E+09</td>
<td>2.988E+08</td>
</tr>
<tr>
<td>Average Plate 1</td>
<td>869</td>
<td>161</td>
<td>140006</td>
<td>8.808E+09</td>
<td>3.029E+08</td>
</tr>
<tr>
<td>Hole 4</td>
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<tr>
<td>Hole 5</td>
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</tr>
<tr>
<td>Hole 6</td>
<td>867</td>
<td>157</td>
<td>136033</td>
<td>8.514E+09</td>
<td>2.793E+08</td>
</tr>
<tr>
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Table 6.3 - Concrete mixture volumetric proportions

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<th>Constituent</th>
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<tr>
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<td>Water</td>
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<td>Air</td>
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<td>Fine Aggregate (Sand)</td>
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<td>Total</td>
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### Table 6.4 - Increment depths for Hole 6

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<td><strong>Total</strong></td>
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### Table 6.5 - Applied loads for each test

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<td>N (kN)</td>
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### Table 6.6 - Applied loads and stresses for each test

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<th>Applied Stress</th>
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<td>K_y (MPa/mm)</td>
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Table 6.7 - Stresses computed from strain gauge output for each test (MPa)

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<th>(6)</th>
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<td>-10.77</td>
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<td>-9.45</td>
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* 'Before' and 'After' refer to readings taken before and after coring a hole.
### Table 6.8 - Comparison of applied average stresses calculated by different methods

<table>
<thead>
<tr>
<th>Test</th>
<th>Average Applied Stress from Load Divided by Area (MPa)</th>
<th>Average Applied Stress from Strain Gauge Output (MPa)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>-8.91</td>
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<tr>
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<td>-6.74</td>
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<td>-8.64</td>
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<td>-6.09</td>
<td>-6.26</td>
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<td>-7.71</td>
<td>-7.71</td>
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### Table 6.9 - Modulus of elasticity for each test calculated using the strain and load data from each test

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<th>Modulus (MPa)</th>
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</tr>
<tr>
<td>3</td>
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<td>34462</td>
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<td>5</td>
<td>33024</td>
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<td>6</td>
<td>35386</td>
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<td>32100</td>
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<td>Average Plate 3</td>
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### Table 6.10 - Stresses from geometric effects modeling

<table>
<thead>
<tr>
<th>Model</th>
<th>Width (mm)</th>
<th>Applied Stress (MPa)</th>
<th>$\sigma_{min}$ (MPa)</th>
<th>$\sigma_{max}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Exact (with conduit)</td>
<td>864</td>
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<tr>
<td>(2) Large (no conduit)</td>
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<td>(3) Difference</td>
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Table 6.11 - Assumed stress distribution for Hole 4 (Stresses in MPa)

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<tr>
<th>Quantity</th>
<th>East - North Pair</th>
<th>West - North Pair</th>
<th>West - South Pair</th>
<th>Average</th>
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<tbody>
<tr>
<td>Assumed Top Stresses</td>
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<td>Assumed Bottom Stresses</td>
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<td>7.19</td>
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Table 6.12 - Maximum assumed difference in measured stresses due to through-thickness eccentricity in load

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<th>Hole</th>
<th>Assumed Maximum Eccentricity (mm)</th>
<th>Assumed Maximum $K_z$ (MPa/mm)</th>
<th>Assumed Maximum % Difference in Measured Stress</th>
</tr>
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Table 6.13 - Uncorrected in-situ measured stress results

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<th>Primary Experimental Variable</th>
<th>Hole</th>
<th>SR</th>
<th>CDW</th>
<th>DS</th>
<th>Applied Stress (MPa)</th>
<th>Uncorrected Measured Stress (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
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<td>6.5 $\sigma_{\text{max}}$ 5.5 $\sigma_{\text{min}}$ 8.5</td>
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<td>-0.18 $\sigma_{\text{max}}$ -7.87</td>
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<td></td>
<td>-5.91</td>
<td>-0.46 $\sigma_{\text{max}}$ -4.85</td>
<td>-7.8 $\sigma_{\text{max}}$ -18.0 $\sigma_{\text{min}}$ 19.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>X</td>
<td></td>
<td></td>
<td>-7.92</td>
<td>0.26 $\sigma_{\text{max}}$ -6.18</td>
<td>3.2 $\sigma_{\text{max}}$ -22.0 $\sigma_{\text{min}}$ 22.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-7.79</td>
<td>2.30 $\sigma_{\text{max}}$ -3.57</td>
<td>29.6 $\sigma_{\text{max}}$ -54.2 $\sigma_{\text{min}}$ 61.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-9.00</td>
<td>4.17 $\sigma_{\text{max}}$ -4.85</td>
<td>46.4 $\sigma_{\text{max}}$ -46.1 $\sigma_{\text{min}}$ 65.4</td>
<td></td>
</tr>
</tbody>
</table>

Average 28.4

---

In this table SR represents steel reinforcement, CDW represents core-drilling water swelling and DS represents differential shrinkage.
Table 6.14 - Measured stress results adjusted for the effects of steel reinforcement

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Stress Corrected for Reinforcement Effects (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-8.14</td>
<td>0.91</td>
<td>-7.49</td>
<td>11.2</td>
</tr>
<tr>
<td>5</td>
<td>-5.91</td>
<td>-0.46</td>
<td>-6.01</td>
<td>-7.8</td>
</tr>
<tr>
<td>6</td>
<td>-7.92</td>
<td>0.26</td>
<td>-7.67</td>
<td>3.2</td>
</tr>
<tr>
<td>7a</td>
<td>-7.79</td>
<td>2.92</td>
<td>-4.53</td>
<td>37.5</td>
</tr>
</tbody>
</table>

Average 20.6

* Hole 7 test results have not been corrected for core-drilling water and differential shrinkage effects.

Table 6.15 - Percentage underprediction in measured stresses for 36 mm diameter steel reinforcement

<table>
<thead>
<tr>
<th>Case</th>
<th>Spacing (mm)</th>
<th>Clear Spacing (mm)</th>
<th>Cover (mm)</th>
<th>Percentage Underprediction in Measured $\sigma_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>216</td>
<td>15</td>
<td>38.5</td>
<td>25.1</td>
</tr>
<tr>
<td>2</td>
<td>216</td>
<td>15</td>
<td>58</td>
<td>23.5</td>
</tr>
<tr>
<td>3</td>
<td>216</td>
<td>15</td>
<td>76</td>
<td>22.3</td>
</tr>
<tr>
<td>4</td>
<td>648</td>
<td>231</td>
<td>58</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 6.16 - Measured stress results adjusted for core-drilling water effects

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Stress Corrected for Core-Drilling Water (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>7a</td>
<td>-7.79</td>
<td>1.58</td>
<td>-6.11</td>
<td>17.2</td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>1.58</td>
<td>-6.43</td>
<td>28.8</td>
</tr>
</tbody>
</table>

Average 34.1

* Hole 7 test results reflect previous correction (as noted in Table 7.15) for steel reinforcement effects but have not been corrected for differential shrinkage effects.
Table 6.17 - Parameters used in differential shrinkage apparent stress assessment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ultimate shrinkage</td>
<td>770 microstrain</td>
</tr>
<tr>
<td>water cement ratio</td>
<td>0.4</td>
</tr>
<tr>
<td>relative humidity</td>
<td>17%</td>
</tr>
<tr>
<td>time of exposure</td>
<td>115 days</td>
</tr>
<tr>
<td>modulus of elasticity</td>
<td>31476 MPa</td>
</tr>
<tr>
<td>diameter of core hole</td>
<td>150 mm</td>
</tr>
<tr>
<td>diameter of measurement radius</td>
<td>200 mm</td>
</tr>
</tbody>
</table>

Table 6.18 - Measured stress results adjusted for differential shrinkage effects

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Apparent Stress Due to Differential Shrinkage (MPa)</th>
<th>Stress Corrected for Differential Shrinkage (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ_{max}</td>
<td>σ_{min}</td>
<td>σ_{max}</td>
<td>σ_{min}</td>
<td>Difference</td>
</tr>
<tr>
<td>7a</td>
<td>-7.79</td>
<td>2.40</td>
<td>-1.06</td>
<td>-8.51</td>
<td>9.1</td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>2.40</td>
<td>0.19</td>
<td>-8.83</td>
<td>-1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td></td>
<td>9.6</td>
</tr>
</tbody>
</table>

a Hole 7 test results reflect previous correction for steel reinforcement effects (as noted in Table 7.15), and core-drilling water swelling effects (as noted in Table 7.16).
Table 6.19 - Final corrected measured in-situ stress results

<table>
<thead>
<tr>
<th>Primary Experimental Variable&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Applied Stress (MPa)</th>
<th>Final Corrected Measured Stress (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole</td>
<td>SR</td>
<td>CDW</td>
<td>DS</td>
<td>$\sigma_{\text{max}}$</td>
</tr>
<tr>
<td>-----</td>
<td>----</td>
<td>-----</td>
<td>---</td>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>-8.91</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>-6.74</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>-8.92</td>
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<tr>
<td>4</td>
<td>X</td>
<td></td>
<td></td>
<td>-8.14</td>
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<td>X</td>
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<td>-5.91</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
<td></td>
<td></td>
<td>-7.92</td>
</tr>
<tr>
<td>7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-7.79</td>
</tr>
<tr>
<td>8</td>
<td>X</td>
<td>X</td>
<td></td>
<td>-9.00</td>
</tr>
</tbody>
</table>

Average: 9.5%

<sup>a</sup> In this table SR represents steel reinforcement, CDW represents core-drilling water swelling and DS represents differential shrinkage.
CHAPTER 7

SENSITIVITY STUDY

7.1 INTRODUCTION

It is apparent from a review of Chapters 1-6 that determination of the in-situ stresses in a concrete structure via the core-drilling method can be complicated by several factors. Depending on the details of the structure under investigation, the investigation may require determining or estimating the values for numerous concrete physical parameters as well as accounting for several different influencing phenomena that can affect the final calculated stress results. The core-drilling method experiments described in Chapter 6 were designed so that knowledge regarding some of these factors and parameters was known a priori. However, in a field investigation of a structure the determination of many of these factors can be complicated.

In no particular order, it has been determined or noted in the preceding chapters that the following important tasks must be completed during a core-drilling investigation for in-situ stresses:

1. Determination of the concrete modulus of elasticity. The procedure for determining the modulus of elasticity of the concrete during the Chapter 6 experiments is described in Section 6.2.4.1, in a field investigation this value would likely be determined through compression testing of the removed core.
2. Application of the digital image correlation pattern.
4. Accounting for any steel reinforcement in the vicinity of the core-hole
5. Accounting for the effects of core-drilling water if water is used as part of the drilling process. As detailed in Chapter 3, this requires knowledge of the time of water exposure during core-drilling, the sorptivity of the concrete, the porosity of the concrete and the swelling strain of the concrete. In the experiments described in Chapter 6, the sorptivity of the concrete was measured using companion cylinders and cores removed from the test specimens. The porosity was estimated using the procedure outlined in Pann et al. (2003). The swelling strain was calculated as a fraction (Neville 1981) of the ultimate shrinkage strain estimated using the method of Gardner and Lockman (2001).
6. Accounting for the effects of differential shrinkage. As detailed in Chapter 4, this requires knowledge of the ultimate shrinkage strain of the concrete, the humidity of the concrete storage environment, and the time of concrete exposure. In the experiments described in Chapter 6, the relative humidity was measured and the time of exposure recorded. The ultimate shrinkage strain
was estimated using the method of Gardner and Lockman (2001). In a field investigation the relative humidity and ultimate shrinkage strain would likely be estimated. Furthermore, Figure 4.6 indicates that differential shrinkage is likely not a significant factor for concrete that is older than three or four years.

The purpose of this chapter is to address the relative importance of each of these phenomena. A further goal is to provide guidance toward testing procedures for future field investigations of structures in-service, indicating which of the measured parameters (if any) need be experimentally measured versus estimated, and which of the influencing factors (if any) necessitate special experimental procedures or approaches to sufficiently account for their impact on accuracy in calculated stresses.

Section 7.2 describes a sensitivity study performed to investigate the dependency of measured stress results on accurate values of concrete material properties. This is accomplished by varying material properties to note their influence on the measured stresses of the Chapter 6 experiments that are summarized in Table 6.19. For each parameter that is varied a range of variation is chosen based on the amount of variability in this parameter amongst different concretes as well as the precision with which the parameter can be estimated or experimentally measured during field testing.

Section 7.3 discusses the relative importance of proximate reinforcement effects, core-drilling water effects and differential shrinkage effects. Insight is given on their relative importance in previous work (i.e. Buchner (1989)), current work (i.e. the experiments detailed in Chapter 6), and in anticipated future field investigations.

The implications of the findings of Sections 7.2 and 7.3 to field evaluation are summarized in Section 7.4 which provides recommendations for treatment of many of these parameters and factors in future tests.

7.2 VARIATION OF CONCRETE MATERIAL PROPERTIES

7.2.1 Modulus of Elasticity
Determination of the modulus of elasticity is one of the most important aspects of a core-drilling method investigation. The in-situ stresses computed from Equations (2.20) – (2.22) depend linearly on the concrete modulus and thus any discrepancy in this value will be directly reflected in reported in-situ stresses. Section 6.2.4.1 describes the method used in this research to determine the modulus of elasticity. The method involved determining stress based on loads measured via load cells, and strain measured via strain gauges.

One way to assess the impact of accuracy in measured $E_{conc}$ in the Chapter 6 experiments is to vary the assumed $E_{conc}$ in the tests and recalculate the stresses reported in Table 6.19. Tables 7.1 and 7.2 show modified in-situ stresses. Table 7.1 shows calculated in-situ stresses based the assumption that the associated $E_{conc}$ values are 10% lower those used in Chapter 6. Table 7.2 provides similar results for the
assumption that the experimental concrete has $E_{concrete}$ values that are 10% lower than those used in Chapter 6.

Ten percent variation in $E_{concrete}$ was deemed a reasonable expected range in measurement of this parameter in field testing. The estimated precision noted in the ASTM standard covering the determination of elastic modulus (ASTM C469-94 1994) is approximately 5% for the testing of cylinders. It can be surmised that variability of concrete material properties determined from concrete cores will typically be higher than that determined from cylinders due to many factors, especially the variability in curing conditions in the concrete in an actual structure compared with companion cylinders.

Similar to Chapter 6, the applied stress values for the reinforced plate tests (Holes 4, 5, 6, and 7) are reported as the stress in the concrete and computed through a standard transformed section approach (Nilson, Darwin and Dolan 2003) based on the section properties and concrete modulus. This means that the applied stresses in Table 7.1 and 7.2 are slightly different from each other and from those in Table 6.19. The average SRSS error across the eight tests reported in Table 6.19 is 9.5%; in Table 7.1 (reduced $E_{concrete}$) it is 10.5%, and in Table 7.2 (increased $E_{concrete}$) it is 14.4%. The values of $E_{concrete}$ used in Chapter 6 lead to the lowest SRSS error amongst the three cases considered. This provides evidence supporting the validity of the Chapter 6 values.

For each of the eight individual tests the SRSS difference quantity in Tables 7.1 and 7.2 is sometimes larger and sometimes smaller than the quantity for that test shown in Table 6.19, although usually larger. Another way of viewing the data is in chart form as shown in Figures 7.1 and 7.2. These figures show the percentage differences in $\sigma_{min}$ and $\sigma_{max}$ for the Chapter 6 $E_{concrete}$ values ('Test') and for $E_{concrete}$ plus and minus 10% (Tables 7.1 and 7.2). In this format it is readily apparent that the measured $\sigma_{min}$ stresses (Figure 7.2) in the Chapter 6 tests are relatively evenly distributed between being too large and too small. The values for the reduced $E_{concrete}$ case are consistently too small, and the values for the increased $E_{concrete}$ case are consistently too large.

Changing $E_{concrete}$ from the Chapter 6 values does increase the overall error, however this increase is small, from 9.5% to either 10.5% (Table 7.1) or 14.4% (Table 7.2), at most a 4.9% increase in the average overall error. For any individual one of the Chapter 6 tests an error in the estimate of $E_{concrete}$ is directly reflected in the measured in-situ stresses. However, the average error across all of the tests does not depend as strongly on the values of the estimated moduli.

In any future field investigations it can be expected that $E_{concrete}$ will be determined within approximately 5-10%. The ASTM standard gives as a guide that the modulus of elasticity from “tests of duplicate cylinders from different batches should not depart more than 5% from the average of the two” (ASTM C469-94). The variability of material properties measured from cores rather than cylinders is often significantly higher, as for example the results reported by Bloem (Bloem 1968) where the
variability in experimental strength measurements taken from cores was approximately twice that taken from cylinders. Since calculated in-situ stresses depend directly on the estimated modulus of elasticity, it may not be possible to determine in-situ stresses with more precision than noted above within 5-10% without special methods of determining $E_{\text{conc}}$.

7.2.2 Sorptivity
As investigated in Chapter 3, absorption of water during a core-drilling method test can cause the concrete immediately around the core hole to swell and thus influence the calculated in-situ stress results. In that chapter the wetted concrete regions were divided into two parts (the Portion A annulus region and the Portion B surface region) whose effects were superposed to generate apparent stresses that are an artifact of the wetting process and hence must be removed from stresses calculated during a structural assessment. Typically these apparent stresses due to core-drilling water manifest as hydrostatic tension stresses. In Chapter 6, the parameter sorptivity was used along with other variables to estimate the depth of wetting in these regions during testing (as described by Equation (2.51)) and hence to establish an estimate of the distortion in measured stresses.

Two ways to vary the sorptivity parameter will be considered here. Each will be described and then the effects of the variation using both methods will be compared.

One way to vary the sorptivity parameter in the experimental tests is to simply use the sorptivity values measured on the cores removed from the plates. In Chapter 6 the sorptivity value used to determine the apparent stresses due to core-drilling water was measured using companion cylinders cast in tandem with the plates as described in Section 6.2.4.2. The value used in Chapter 6 was 0.14 mm/$\sqrt{\text{mm}}$. It was noted that the sorptivity values measured on the cores removed from Hole 7 and Hole 8 (0.110 and 0.147 mm/$\sqrt{\text{min}}$ respectively) were not used because it was impossible to ascertain the exact moisture condition in these cores at the time of sorptivity testing, although it is likely that the moisture condition in these cores was similar to the moisture condition in Plate 3 at these locations at the time of core-drilling testing. Table 7.3 shows the final corrected stresses for Holes 7 and 8 if the sorptivity measured from the core removed at each location is used to calculate the apparent stress due to core-drilling water induced swelling at each location. The values in the table are termed ‘final’ because similar to those summarized in Table 6.19, they have been corrected for all three influencing factors probed in this study, i.e. core-drilling water induced swelling, differential shrinkage and proximate steel reinforcement.

Another way to assess the effects of variation in the sorptivity parameter in the Chapter 6 experiments is to change the value used by a percentage, similar to the approach used in the previous section (Section 7.2.1) to assess the effect of variability in the concrete modulus of elasticity. As explained in Chapter 2, sorptivity can vary by orders of magnitude between different concretes. The method used in the current experiments to measure sorptivity (Section 6.2.4.2) is essentially the direct
gravitational method. In future field testing this would likely involve sorptivity testing of the removed core. As noted previously only the stress results of Holes 7 and 8 need to be corrected for core-drilling water, the other tests involved saturated specimens. Table 7.4 presents the results for Tests 7 and 8 assuming a value for sorptivity 50% lower ($S = 0.7 \text{ mm/}\sqrt{\text{min}}$) than that used in Chapter 6 ($S = 0.14 \text{ mm/}\sqrt{\text{min}}$), Table 7.5 the same with sorptivity 50% higher ($S = 0.21 \text{ mm/}\sqrt{\text{min}}$) than the Chapter 6 value.

The value for the variation in sorptivity (50%) was chosen to reflect a compromise between the variability in this parameter across different concretes, and the variation in the determination of this parameter for a particular concrete. As noted in Chapter 2, sorptivity can vary widely between different concretes. Determining the sorptivity of a particular concrete via the direct gravitational method (Hall and Hoff 2002) gives little variability between specimens of the same concrete, provided that the conditioning of the concrete is the same. This can be observed in the tests described in Section 6.2.4.2, where the sorptivity across the three cylinder tests varied within 3% across tests, and is also apparent in the data presented by Hall and Tse (1986) for different cement based mortars where the average relative standard deviation in measured sorptivity is approximately 3%.

Changing the sorptivity serves to change the apparent stresses due to core-drilling water that are removed from the uncorrected calculated in-situ stresses. Note that the other corrections of the measured stress results (for steel reinforcement and differential shrinkage) have been performed exactly as in Chapter 6, only the calculations involving the sorptivity parameter have been changed in Tables 7.3, 7.4 and 7.5. Also as in Chapter 6, only the annulus wetting (Portion A, Equation (3.8)) apparent stresses due to core-drilling water are considered. In Chapter 6 it was noted that the Portion B (surface wetting) apparent stresses in the experiments were small and therefore neglected. That approach has been continued here.

Table 7.6 is provided to compare the variation in sorptivity by the different methods. It shows the $\sigma_{\text{max}}$ stresses for each variation in sorptivity (using the values from the cores, using a value 50% higher than that in Chapter 6 and using a value 50% lower than in Chapter 6). Ideally the $\sigma_{\text{max}}$ value for Hole 7 and Hole 8 would be zero since no tension stresses were applied. Since it is unlikely that the $\sigma_{\text{max}}$ stresses in both tests will be zero, a secondary goal is that the average of the $\sigma_{\text{max}}$ stresses across the Hole 7 and Hole 8 tests is zero. This would indicate that the proper amount of apparent tension stress was removed from these tests. The last row of the table shows the average $\sigma_{\text{max}}$ stress for each case considered. The average $\sigma_{\text{max}}$ value that is closest to zero is for the case where the sorptivity values determined from the cores are used to determine the apparent stresses due to core-drilling water. Determining the most appropriate way to use the sorptivity information derived from removed cores is an avenue for future research proposed in Section 8.6. The average $\sigma_{\text{max}}$ from the Chapter 6 calculations and from assuming a value of sorptivity 50% lower than in Chapter 6 are each approximately the same magnitude away from zero, at-0.44 and
+0.48 respectively. This indicates that likely the actual sorptivity parameter is in between the value used in Chapter 6 (0.14 mm/√min) and the value 50% less (0.07 mm/√min). The average $\sigma_{\text{max}}$ stress calculated assuming a value of sorptivity 50% larger than in Chapter 6 is the farthest from zero by a significant margin.

Figure 7.3 presents the data in graphical form and shows the $\sigma_{\text{max}}$ stress for the Chapter 6 sorptivity value (‘Test’) and the high sorptivity and low sorptivity values, as well as the $\sigma_{\text{max}}$ stress for the cases assuming the sorptivity is determined from the cores. Again it is noted that the average $\sigma_{\text{max}}$ stress closest to zero is that from the calculations assuming that the sorptivity is determined from the removed cores.

An examination of Equation (2.51) shows that $t_w$ is dependent on sorptivity, porosity and time of water exposure. Another way to gain insight into the core-drilling water behavior is to plot the apparent stress (due to core-drilling water) against the wetted thickness as shown in Figure 7.4. Equation (3.8) defines the pertinent relationship. The values are calculated using the swelling strain determined in Chapter 6 ($\alpha_w = 257$ microstrain) and normalized by the values for $t_w = 5$ mm. Although Equation (3.8) indicates that the apparent stresses due to core-drilling water have a quadratic dependence on $t_w$, the figure shows that this quadratic dependence is slight as the curve is nearly linear. $t_w$ depends linearly on sorptivity, and thus the figure indicates that if sorptivity doubles, the apparent stress due to core-drilling water essentially doubles.

As mentioned, sorptivity can be quite variable amongst different concretes. This means then that for future field studies it is imperative that the sorptivity of the interrogated concrete is determined. Fortunately, the direct gravitational method of determining sorptivity is relatively straightforward, fairly rapid, and does not require particularly expensive equipment (Section 6.2.4.2).

7.2.3 Porosity
Equation (2.51) also indicates that $t_w$ is dependent on the concrete porosity, in particular on its reciprocal. As compared to sorptivity however, the variation in porosity from one concrete to the next is rather small. In Chapter 6, the work of Pann et al. (2003) was used to estimate the porosity of the concrete in the Chapter 6 tests to be 0.0981. In order to estimate the effect of variation in the concrete’s porosity, the wetted thicknesses for Holes 7 and 8 (from Chapter 6) have been recalculated assuming variation in porosity.

In this case the porosity was changed by plus and minus 10% from the value determined in Chapter 6. The 10% variation was assumed based on an examination of the variation in the porosity in the concretes in the database considered by Pann et al. (2003) and from the variability of porosity in the specimens measured by Hall and Yau (1987). Equation (6.2), proposed by Pann et al. to estimate concrete porosity, was derived with regards to an extensive experimental database and showed good agreement with the experimental results considered by those researchers (Pann et al.).
The porosity estimated via this equation for concretes with water cement ratio of 0.3 to 0.6 vary from 0.18 to 0.22. Hall and Yau present porosities measured in concretes with water cement ratios between 0.4 and 1.0 with porosity values spanning from 0.113 to 0.141.

As noted in Chapter 6, the wetted thickness for Hole 7 and Hole 8 were estimated to be 4.94 mm and 5.34 mm respectively. The two values were rounded to 5 mm for use in the Chapter 6 calculations. When the calculations were performed the new wetted thickness values for a porosity parameter increased by 10% for Hole 7 and Hole 8 are 4.49 mm and 4.85 mm respectively with an average of 4.67 mm. For a porosity parameter decreased by 10% the wetted thickness values are 5.49 mm and 5.93 mm for Hole 7 and Hole 8 respectively with an average of 5.71 mm. In either case, the value used in Chapter 6 (5 mm) is still a reasonable estimate for the wetted thickness. This means that the apparent stresses due to core-drilling water and hence the final calculated in-situ stresses for Holes 7 and 8 for these cases with modified porosities are identical to those presented in Chapter 6 (Table 6.19). Hence the results of Chapter 6 are unchanged for this amount of variability in porosity.

Provided that porosity may be estimated with reasonable precision (as for example using the method of Pann et al. (2003)) it seems that an experimental determination of porosity is not required in future investigations with the core-drilling method. The calculations involving porosity of apparent stresses due to core-drilling water that involve porosity are not sufficiently sensitive to expected variations in this parameter to necessitate special procedures to provide for its measurement.

### 7.2.4 Ultimate Shrinkage Strain

The final parameter investigated in this section is ultimate shrinkage strain. In Chapter 6 the ultimate shrinkage strain of the concrete was estimated using the method of Gardner and Lockman (2001) to be 770 microstrain. The ultimate shrinkage strain affects the measured stresses reported in Table 6.19 in two ways: (1) as input to the calculation of the core-drilling water apparent stresses (where $\alpha_w$ is assumed to be one third of $\varepsilon_{ult}$ Neville (1981)); and (2) as input to the calculation of the differential shrinkage apparent stresses. The apparent stresses due to core-drilling water and due to differential shrinkage both depend linearly on the ultimate shrinkage strain. Changing the estimate ultimate shrinkage strain of the tested concrete has effect essentially similar to those resulting from changing the sorptivity, i.e. a decrease in the estimate of $\varepsilon_{ult}$ results in a direct decrease in the total apparent stress (in this case due to both core-drilling water and differential shrinkage) and an increase in the estimate results in a direct increase in total apparent stress. Tables 7.7 and 7.8 show the results for the Chapter 6 Hole 7 and 8 tests corrected for an assumed ultimate swelling strain that is 30% lower and 30% higher than that used in Chapter 6. Figure 7.6 compares the relative difference in $\sigma_{max}$ stress for the Chapter 6 calculations (‘Test’) and those for the assumed increased and decreased $\varepsilon_{ult}$ values.
The variation in this parameter of 30% was based on the findings of Gardner and Lockman (2001). Gardner and Lockman reported that their model estimated concrete shrinkage within 30% of experimentally determined values in all but a small minority of cases.

As with sorptivity, it appears that the value for ultimate shrinkage strain used in Chapter 6 is the most reasonable of the three cases. The decreased $\varepsilon_{ult}$ case results in consistently positive $\sigma_{max}$ measured stresses, the increased $\varepsilon_{ult}$ case results in consistently negative $\sigma_{max}$ measured stresses. In contrast, the value for $\varepsilon_{ult}$ used in Chapter 6 results in $\sigma_{max}$ stresses that are closer to zero.

It will likely not be possible to measure $\varepsilon_{ult}$ in future field investigations. However it does appear that the method of Gardner and Lockman (2001) was adequate for estimating this parameter in the experiments. Measurement of $\varepsilon_{ult}$ in future experimental testing would be an interesting exercise that could generate further evidence supporting the validity of the estimation of $\varepsilon_{ult}$ using the Gardner and Lockman (2001) approach.

7.3 RELATIVE IMPORTANCE OF CORE-DRILLING WATER INDUCED SWELLING, DIFFERENTIAL SHRINKAGE AND PROXIMATE STEEL REINFORCEMENT

Chapters 3-5 detail the ways in which the following three factors influence the calculated in-situ stresses in the core-drilling method: (1) core-drilling water; (2) differential shrinkage; and (3) steel reinforcement. Section 7.2 details the ways that measurement or estimation of various concrete properties affects the measured in-situ stresses. This section comments on the relative importance of these factors. This is done in terms of previous testing (Buchner 1989), current testing (Chapter 6) and with regard to future field investigations.

In Chapters 3 and 4 analytical procedures were developed to account for the effects of core-drilling water and differential shrinkage respectively. These procedures were then used to correct the results of the Buchner (1989) experiments to account for these phenomena, and similarly in Chapter 6 to correct the uncorrected results of the current experiments. In each case the phenomena in question results in fictitious apparent stresses that are measured in addition to the in-situ stresses and that generally manifest as hydrostatic tension stresses. These apparent stresses must therefore be removed from calculated in-situ stresses in order to achieve acceptable accuracy in these measured quantities. An examination of Tables 3.10 and 4.3 indicates that for the Buchner experiments, core-drilling water was the factor that likely caused the greater effect in those experiments (i.e. caused greater apparent stresses). However, an examination of Table 6.15 and 6.17 shows that in the current experiments this trend is reversed and the apparent stresses due to differential shrinkage are larger than those due to core-drilling water. This observation has three main causes:
1. A difference in thickness between the two types of experimental specimens. The Buchner (1989) specimens were 10 cm thick, whereas the current (Chapter 6) specimens were 15 cm thick. Figure 4.7 makes it clear that the apparent stresses due to differential shrinkage increase substantially with increasing specimen thickness. For the core-drilling water effects, only the Portion B (surface wetting) apparent stresses change with increasing thickness, and this change is slight as shown in Figure 3.35.

2. A substantial difference in the relative humidity of the storage environment between the two types of specimens. The Buchner (1989) relative humidity is unknown, however in Chapter 4 calculations were performed assuming 80% and 50% relative humidity environments. The relative humidity of the storage environment for the current specimens was measured to be approximately 17%. Figure 4.6 shows that such dramatic differences in relative humidity can have significant impacts on apparent stresses due to differential shrinkage, with lower relative humidity environments having significantly higher apparent stresses. The environmental relative humidity does not factor into the core-drilling water apparent stresses.

3. A difference in the estimated ultimate shrinkage strain between the two types of specimens. The Buchner ultimate shrinkage strain (520E-6) results in lower predicted apparent stresses due to differential shrinkage than the ultimate shrinkage strain estimated in the Chapter 6 experiments (770 E-6).

In future testing the relative importance of these two factors will likely be determined by the following issues:

- **Structure age** – Figure 4.6 clearly shows that specimen age is of tremendous importance in evaluating apparent stresses due to differential shrinkage. Age was not a significant source of difference between the Buchner (1989) plates and the current ones as the two types of specimens were approximately the same age at time of test. However, regardless of relative humidity, all of the apparent stresses shown in Figure 4.6 for 10 cm thick plates are very low (less than 0.1 MPa) once the age of the specimen is greater than 4 years. Structures subjected to field investigations are likely to be even older than this and thus the apparent stresses due to differential shrinkage will likely be low. For older structures subjected to field evaluations, core-drilling water effects may be of greater import than differential shrinkage effects. For newer structures that are being evaluated, the opposite may be the case.

- **Environmental conditions** – Again, Figure 4.6 provides insight into the effects of relative humidity on the apparent stresses due to differential shrinkage. Concretes may have different relative humidity exposure levels due to factors such as the location within a building (i.e. interior/exterior), local climate conditions, building climate control provisions, etc. A judgment should be made regarding the environmental relative humidity experienced by the interrogated object in order to assess the likely importance of differential shrinkage effects. The apparent stresses due to core-drilling water should be relatively insensitive to environmental relative humidity.
• Thickness of structural element – As mentioned, the thickness of the concrete specimen plays a large role in the apparent stresses due to differential shrinkage, but does not in regards to core-drilling water effects. As noted above however, regardless of thickness apparent stresses due to differential shrinkage tend to diminish with age, so past a certain age even relatively thick specimens will likely have low apparent stresses due to differential shrinkage.

• Sorptivity – As shown in Section 7.2 the apparent stresses due to core-drilling water are nearly linear with respect to measured sorptivity. It was also noted that because sorptivity can be highly variable from one concrete to another it is important to measure the sorptivity of the tested concrete. Concretes with higher sorptivities will tend to have higher apparent stresses due to core-drilling water.

The presence of steel reinforcement in close proximity to a core-hole was not a factor in the Buchner (1989) experiments. The Buchner specimens did not have steel reinforcement in close proximity to the drilled holes. In contrast, the experiments involving Plate 2 (Holes 4, 5 and 6) and the reinforced portion of Plate 3 (Hole 7) described in Chapter 6 showed marked effects of proximate steel reinforcement. In those tests the nearby reinforcement causes an under-prediction in uncorrected in-situ stresses of approximately 20%. In future experiments it is therefore imperative that reliable location of steel reinforcement within an interrogated structure is ascertained. Numerous devices exist to probe the exact location of steel reinforcement in existing structures if design drawings are unavailable or deemed unreliable. Figure 5.6 indicates that for steel reinforcement 25 mm in diameter (or less) as long as the reinforcement is less than 35 mm from the edge of the core hole and/or has at least 75 mm of concrete cover than neglecting the effects of the reinforcement should cause minimal error in calculated in-situ stresses.

Table 7.9 is provided to summarize the preceding discussion regarding the relative importance of the influencing factors noted for the previous testing, the current testing, and for future field investigations. In the table the relative importance of each of the phenomena is listed. The table ranks the influencing factors or measurement parameters deemed most important in past present and future testing. The terms are listed in the table in order of decreasing importance.

7.4 RECOMMENDATIONS FOR FUTURE TESTING

The preceding sections provided insight into the ways that the influencing factors and measurement of concrete parameters affect testing using the core-drilling method. The following list provides summary commentary regarding these measurement parameters and influencing factors in light of future testing:

• Measurement of concrete modulus ($E_{conc}$) – Critical because any discrepancy in $E_{conc}$ is directly reflected as uncertainty in calculated in-situ stresses.

• Measurement of sorptivity ($S$) – Variability in this parameter across different concretes means that it must be assessed for interrogated concrete. This value directly affects apparent stresses due to core-drilling water.
• Estimation of ultimate shrinkage strain ($\varepsilon_{ult}$) – Method of Gardner and Lockman (2001) appears to result in sufficiently accurate calculated in-situ stresses.

• Estimation of porosity ($f$) – Method of Pann et al. (2003) appears to result in sufficiently accurate calculated in-situ stresses.

• Core-drilling water effects – Need to be considered as apparent stresses due to core-drilling water of approximately 1.6 MPa have been estimated for the Chapter 6 tests and higher apparent stresses are possible for other concretes.

• Differential shrinkage effects – Although relatively high apparent stresses due to differential shrinkage (2.40 MPa) were estimated for the Chapter 6 tests, field evaluations are likely to involve structures greater than 5 years old and hence differential shrinkage effects are likely not of concern.

• Proximate reinforcement effects – The location of reinforcement in the interrogated structure must be ascertained as under-prediction in the in-situ stresses of the reinforced specimens in the Chapter 6 tests was approximately 20% if these effects were neglected. Without knowledge of the reinforcement placement within a structure this large under-prediction might go unnoticed.
Figure 7.1 – Comparison of final corrected $\sigma_{\text{max}}$ assuming different values for $E_{\text{conc}}$

Figure 7.2 – Comparison of final corrected $\sigma_{\text{min}}$ assuming different values for $E_{\text{conc}}$
Figure 7.3 – Comparison of $\sigma_{\text{max}}$ assuming different values for sorptivity

Figure 7.4 – Plot of relative apparent stress versus relative wetted thickness
Figure 7.5 – Plot of relative apparent stress versus relative shrinkage strain

Figure 7.6 - Comparison of $\sigma_{\text{max}}$ assuming different values ultimate shrinkage strain

Hole Test

Figure 7.6 - Comparison of $\sigma_{\text{max}}$ assuming different values ultimate shrinkage strain
Table 7.1 - Adjusted stress results assuming $E_{conc}$ reduced by 10%

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Final Adjusted Measured Stress (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-8.91</td>
<td>$\sigma_{\text{max}}$ -8.43, $\sigma_{\text{min}}$ 0.48</td>
<td>5.3, -5.4, 7.6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-6.74</td>
<td>-0.20, -7.05</td>
<td>-3.0, 4.7, 5.6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-8.92</td>
<td>0.23, -8.23</td>
<td>2.6, -7.7, 8.1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-8.01</td>
<td>0.77, -6.88</td>
<td>9.6, -14.1, 17.1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-5.82</td>
<td>-0.45, -5.53</td>
<td>-7.7, -5.0, 9.2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-7.79</td>
<td>0.18, -7.04</td>
<td>2.3, -9.6, 9.9</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-7.66</td>
<td>-1.32, -8.14</td>
<td>-17.2, 6.4, 18.3</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>-0.27, -8.31</td>
<td>-3.0, -7.7, 8.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average 10.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2 - Adjusted stress results assuming $E_{conc}$ increased by 10%

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Final Adjusted Measured Stress (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-8.91</td>
<td>$\sigma_{\text{max}}$ -10.38, $\sigma_{\text{min}}$ 0.69</td>
<td>7.7, 16.5, 18.2</td>
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<tr>
<td>2</td>
<td>-6.74</td>
<td>-0.17, -8.68</td>
<td>-2.5, 28.8, 29.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-8.92</td>
<td>0.39, -10.14</td>
<td>4.3, 13.7, 14.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-8.26</td>
<td>1.05, -8.11</td>
<td>12.7, -1.8, 12.8</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-5.99</td>
<td>-0.47, -6.51</td>
<td>-7.8, 8.6, 11.6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-8.03</td>
<td>0.33, -8.30</td>
<td>4.1, 3.3, 5.3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-7.91</td>
<td>-0.80, -8.87</td>
<td>-10.1, 12.2, 15.8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>0.66, -9.35</td>
<td>7.3, 3.9, 8.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average 14.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.3 - Final corrected measured in-situ stresses for Holes 7 and 8 if the sorptivity values measured from Core 7 Core 8 are used

<table>
<thead>
<tr>
<th>Sorptivity of Core</th>
<th>Applied Stress</th>
<th>Apparent Stress Due to Core-Drilling Water</th>
<th>Final Corrected Measured Stress (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole (mm/√min)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>SRSS %</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.110</td>
<td>-7.79</td>
<td>1.19</td>
<td>-0.67</td>
<td>9.4</td>
</tr>
<tr>
<td>8</td>
<td>0.147</td>
<td>-9.00</td>
<td>1.79</td>
<td>-0.01</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Average 4.9

Table 7.4 - Measured and final adjusted stresses assuming sorptivity is reduced by 50%

<table>
<thead>
<tr>
<th>Applied Stress</th>
<th>Total Apparent Stress</th>
<th>Corrected Stresses</th>
<th>% Difference from Applied Stress</th>
<th>SRSS %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole (MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>SRSS %</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-7.79</td>
<td>3.06</td>
<td>-1.15</td>
<td>3.2</td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>3.06</td>
<td>1.11</td>
<td>17.3</td>
</tr>
</tbody>
</table>

Average 10.2

Table 7.5 - Measured and final adjusted stresses assuming sorptivity is increased by 50%

<table>
<thead>
<tr>
<th>Applied Stress</th>
<th>Total Apparent Stress</th>
<th>Corrected Stresses</th>
<th>% Difference from Applied Stress</th>
<th>SRSS %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole (MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>SRSS %</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-7.79</td>
<td>4.83</td>
<td>-1.91</td>
<td>31.6</td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>4.83</td>
<td>-0.65</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Average 21.0

Table 7.6 - $\sigma_{max}$ stresses determined with different values of sorptivity

<table>
<thead>
<tr>
<th>Sorptivity (mm/√min)</th>
<th>$\sigma_{max}$ (MPa)</th>
<th>$\sigma_{max}$ (MPa)</th>
<th>$\sigma_{max}$ (MPa)</th>
<th>$\sigma_{max}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>from Ch. 6 (S from Ch. 6)</td>
<td>from cores (S from cores)</td>
<td>50% lower from Table 7.4</td>
<td>50% higher from Table 7.5</td>
</tr>
<tr>
<td>7</td>
<td>0.14</td>
<td>0.110</td>
<td>0.07</td>
<td>0.21</td>
</tr>
<tr>
<td>8</td>
<td>0.14</td>
<td>0.147</td>
<td>0.07</td>
<td>0.21</td>
</tr>
<tr>
<td>Average</td>
<td>-0.44</td>
<td>Average</td>
<td>Average</td>
<td>Average</td>
</tr>
</tbody>
</table>
Table 7.7 - Measured and final adjusted stresses assuming $\varepsilon_{ult}$ is reduced 30%

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Total Apparent Stress (MPa)</th>
<th>Corrected Stresses (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-7.79</td>
<td>2.79</td>
<td>0.13 -7.31 1.7 -6.2</td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>2.79</td>
<td>1.39 -7.64 15.4 -15.2</td>
<td>21.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>14.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.8 - Measured and final adjusted stresses assuming $\varepsilon_{ult}$ is increased 30%

<table>
<thead>
<tr>
<th>Hole</th>
<th>Applied Stress (MPa)</th>
<th>Total Apparent Stress (MPa)</th>
<th>Corrected Stresses (MPa)</th>
<th>% Difference from Applied Stress</th>
<th>SRSS % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-7.79</td>
<td>5.18</td>
<td>-2.26 -9.70 -28.9 24.5</td>
<td>37.9</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-9.00</td>
<td>5.18</td>
<td>-1.00 -10.03 -11.1 11.4</td>
<td>15.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>26.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.9 - Relative importance of influencing factors and measurement parameters

<table>
<thead>
<tr>
<th>Bucher (1989)$^1$</th>
<th>Current</th>
<th>Future Investigation$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Differential shrinkage effects</td>
<td>2. Reinforcement effects</td>
<td>2. Reinforcement effects$^3$</td>
</tr>
<tr>
<td>4. Measurement of $E_{conc}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Measurement of $S$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Estimation of $\varepsilon_{ult}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^1$ Note: Reinforcement not present, $S$ not measured.

$^2$ Assumes a structure greater than 5 years old @ 50% RH.

$^3$ If sufficiently close and/or near surface.
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 INTRODUCTION

The primary objective of the current work was to probe the effects of core-drilling water induced swelling, differential shrinkage and proximate steel reinforcement on the core-drilling method and thus enable the core-drilling method to be used to provide an accurate assessment of the in-situ stresses in in-service concrete structures. The research involved analytical, numerical and experimental thrusts designed to probe each of these issues.

Chapter 2 provides background information necessary to complete this objective. Chapters 3-5 discuss the analytical work performed to investigate these issues. Chapter 6 describes the experiments that were conducted to probe these issues. Core-drilling experiments were performed in saturated plain concrete, saturated reinforced concrete, drying plain concrete and drying reinforced concrete. Displacements in the experiments were measured with digital image correlation and in-situ stresses were determined with and without considering the effects of core-drilling water induced swelling, differential shrinkage and proximate steel reinforcement. Chapter 7 describes a sensitivity study that was performed to evaluate the relative importance of the effects of these phenomena.

The following sections present the conclusions drawn from the current work. Section 8.2 provides general conclusions regarding the core-drilling method. Section 8.3 provides conclusions regarding core-drilling water induced swelling. Section 8.4 provides conclusions regarding differential shrinkage. Finally, Section 8.5 provides conclusions regarding proximate steel reinforcement. Recommendations for future research are provided in Section 8.6.

8.2 GENERAL CONCLUSIONS

The following general conclusions regarding the core-drilling method are made:

1. The analytical formulations relating in-situ stresses to relieved displacements explored in Turker (2003) accurately describe the behavior that occurs in the core-drilling method. In particular the relieved displacements (as for example shown in Figures 7.40 – 7.47) follow the expected trigonometric dependence on measurement angle and are generally of the expected magnitude.
2. **Digital image correlation** is an acceptable measurement technique for application of the core-drilling method to concrete. In particular, there were no negative effects noticed on the displacements derived from the digital image correlation system wrought by the substantial soiling and wetting of the applied pattern during drilling of the core hole. Additionally, patting the applied pattern dry with paper toweling following testing seemed sufficient to ensure consistent displacement measurements. No detectable effects were noticed due to any residual water remaining on the pattern surface.

3. **The effects of core-drilling water induced swelling, differential shrinkage and proximate reinforcements** must be considered in the calculation of in-situ stresses using the core-drilling method to obtain acceptable accuracy. The average SRSS error across the eight experiments in the Chapter 6 tests was 9.5 percent if these issues were addressed but only 28.4% if they were neglected.

4. Of the three influencing factors, either core-drilling water induced swelling, differential shrinkage or proximate steel reinforcement can be the most influential on calculated in-situ stress results, depending on the condition and history of the concrete structure being investigated. The most important parameters that control which of these aspects has the greatest influence are the age of the concrete at test, the relative humidity of the concrete storage environment, the thickness of the structural element being considered, the sorptivity of the concrete and the proximity and diameter of the nearest reinforcing bar.

### 8.3 CONCLUSIONS REGARDING CORE-DRILLING WATER EFFECTS

An analytical method was presented in Chapter 3 that accounts for distortions in calculated in-situ stresses caused by coring water induced swelling. The method uses the parameters wetted thickness $t_w$ (which is derived from sorptivity, $S$, and time of water exposure during the test, $t$) and swelling strain $\alpha_w$ as input into a finite element investigation that characterized the moisture induced displacement field. The displacements due to moisture were converted into apparent in-situ stresses. To increase the accuracy of the core-drilling method technique, these apparent stresses should be removed from the in-situ stresses so derived. The approach was used to show that the results of a previous hole-drilling investigation (Buchner 1989) and the current core-drilling experiments could be significantly improved if core-drilling water effects were considered. The following conclusions are made regarding the effects of core-drilling water induced swelling:

5. **Absorption of water by the concrete around a core hole causes swelling of this concrete and swelling displacements.** Superposition of these displacements with those due to stress relief induces apparent stresses that are unrelated to the in-situ stresses in the cored object.

6. **The apparent stresses due to core-drilling water appear primarily as hydrostatic tension stresses** (i.e. $\sigma_z = \sigma_y$, $\tau_{xy} = 0$).
7. The apparent stresses also appear as a small amount of bending through the thickness of a plate specimen. The apparent stresses on the top face of a plate are larger in tension than those on the back face of a plate and in fact the back face of a plate may actually exhibit hydrostatic compression stresses in some cases.

8. Correction to account for the apparent stresses due to core-drilling water can be made using the approach developed in Chapter 3. In Chapter 3 a baseline case using ‘typical’ values of $t_w$ and $\alpha_w$ was presented. For the baseline case, at $m = 100$ mm, the apparent stresses are 1.05 MPa tension on the top face, and 0.78 MPa tension on the bottom face, for an average of 0.92 MPa tension. Apparent stresses for other values of $t_w$ and $\alpha_w$ may be calculated via the regression equations generated in Section 3.3.3 (Equations (3.7) – (3.11)) and Section 3.4.3 (Equations (3.14) – (3.15)), along with Equations (3.19) – (3.21). The new approach was used to show that relative in-situ stress errors in a previous experimental hole-drilling study (Buchner 1989) were reduced from 47% to 14% upon its application with baseline values of $t_w$ and $\alpha_w$.

9. The plan dimensions of an object do not significantly influence the apparent stresses due to core-drilling water induced swelling and hence the analytical formulations developed in Chapter 3 should be applicable to objects other than plates.

10. The thickness of an object has minimal impact on apparent stresses due to core-drilling water for objects thicker than 300 mm and has a slight effect on thinner objects.

11. Variability in the sorptivity parameter across different concretes means that it must be assessed for interrogated concrete. This value directly affects the apparent stresses due to core-drilling water.

### 8.4 Conclusions Regarding Differential Shrinkage Effects

The effects of differential shrinkage of concrete specimens on the core-drilling method were investigated. In Chapter 4, moisture profiles through the depth of drying concrete plates were estimated using the approach of Akita et al. (1997) and corresponding shrinkage stress profiles were calculated using the finite element method. The effects of drilling through the resulting differential stress profiles were quantified numerically. Drilling a core hole through these stress profiles relieves these stresses and results in surface displacements in addition to those caused by relief of stresses due to applied loads. Similar to coring water effects these surface displacements have been converted into apparent stresses. The analytical findings of Chapter 4 were used to correct the results of the Buchner (1989) experiments and the experiments discussed in Chapter 6. The following conclusions regarding differential shrinkage effects on the core-drilling method are made:

12. The apparent stresses from differential shrinkage appear as a hydrostatic tension stress (i.e. $\sigma_x = \sigma_y$, $\tau_{xy} = 0$).
13. Differential shrinkage stresses do not significantly effect an investigation of in-situ stresses using the core-drilling method except in certain circumstances. In particular, high apparent stresses are generated for especially thick concrete specimens (over 300 mm), especially young concrete specimens (less than 3 years old), and concrete specimens stored in very dry ambient conditions. Figures 4.6 and 4.7 give some insight into the ways these parameters affect the apparent stresses. In plates of constant thickness the apparent stresses exponentially decay with increasing plate age. For plates with different thickness, the apparent stresses increase substantially with increasing thickness.

14. Using an estimated value of ultimate shrinkage strain calculated using the GL2000 method (Gardner and Lockman 2001) gives adequate results for the apparent stresses due to differential shrinkage (and core-drilling water) in the Chapter 6 tests and is an appropriate procedure in future testing.

8.5 CONCLUSIONS REGARDING PROXIMATE STEEL REINFORCEMENT EFFECTS

Chapter 5 investigates the effects of steel reinforcement on the core-drilling method. The equations relating relieved displacements to in-situ stresses (as for example Equation (2.20) – (2.22)) are derived assuming isotropic material properties and as such do not account for the stiffness mismatch in reinforced concrete between concrete and embedded steel reinforcement. The method discussed in Chapter 5 was used to correct the measured in-situ stress results from the steel reinforced specimens in Chapter 6. The following conclusions regarding steel reinforcement are made:

15. The presence of reinforcement close to a core hole (nearer than 35 mm) and close to the surface of the concrete (nearer than 75 mm) causes a significant under-prediction in stress using the core-drilling method, if the reinforcement is neglected.

16. The effect of proximate reinforcement reduces quickly and significantly with either increasing concrete cover or increasing clear spacing to the nearest bar.

17. For the bar size and placement parameters treated in this study, the analytical method discussed in Chapter 5 gives reasonable estimates for the effect of steel reinforcement on the experiments. The experiments in Chapter 6 treated plates with large diameter bars closely positioned to the core holes. The effects of smaller bars or bars further away from core holes would presumably be less. The uncorrected in-situ stress results in the steel reinforced plates in Chapter 6 reflected an under-prediction in stress of approximately 21.9% (the SRSS average across Holes 4-6). This same quantity drops to 6.9% following the correction procedure outlined in Section 6.4.1.
8.6 RECOMMENDATIONS FOR FUTURE RESEARCH

This section provides recommendations for future research on the core-drilling method. The following avenues of research are recommended:

1. Core-drilling experiments on more realistic structures in the laboratory and in the field should be performed. In the Chapter 6 experiments the specimens tested were idealized concrete plates less than 6 months old. Testing of reinforced concrete beams, prestressed elements such as box-type structures and other actual concrete elements is desirable. Testing of these sorts of structures might entail investigating objects whose applied loads cause stresses that vary through the depth. Testing structures with significant age would allow the probing of aspects such as the effects of creep and carbonation on the core-drilling method and would provide further insight into structures whose differential shrinkage stresses have essentially dissipated. Testing field structures would allow the effects of estimating the relative humidity of the concrete environment to be addressed.

2. Further research should be performed that is designed to address the issue of determining in-plane gradients in normal stress with the core-drilling method. It was determined that it is not possible to determine a gradient in normal stresses in practical concrete structures with the linear gradient equations (for example Equations (2.23) – (2.29)) as currently conceived. The relative magnitude of relieved displacements due to gradients in normal stress is below the practical digital image correlation displacement detection threshold. A source of displacement information that has heretofore not been probed is the displacement behavior of the core itself. To date the displacements examined have only been those of the material around the core hole. However, the displacements on the top surface of the removed core itself can theoretically be of greater magnitude than those in the material surrounding the hole. Some others involved in hole-drilling methods for determining in-situ stresses in concrete (for example (Buchner 1989) and Kesevan et. Al 2005) have installed a strain gauge on the core in the direction of presumed maximum stress and used results from that strain gauge in an attempt to further refine calculated in-situ results for normal stresses. Digital image correlation allows for a much more robust determination of the displacements on the surface of the interrogated object, including the surface of the removed core. It is possible that the displacements from the surface of the removed core could be captured and used in some manner to improve the accuracy of the measured stress results as well as provide a determination of normal stress gradients.

3. Further numerical manipulation of the full field displacement results available when digital image correlation is used to measure displacements as part of a core-drilling method investigation. At present only the displacement information in a measurement circle around the core hole is utilized to calculate in-situ stresses. Digital image correlation provides displacement
results for a full field around the hole however. The data around the measurement circle provided approximately 360 individual measures of displacement in the tests performed in Chapter 6, many more than the minimum 3 required to determine in-situ stresses. This redundancy was used advantageously in Chapter 6 in a simple averaging process. However, approaches that take advantage of the full field of displacement information available (as for example that proposed by Schajer and Steinzig (2005)) can potentially increase accuracy in computed in-situ stress results.

4. The effects of coarse aggregate on the core-drilling method should be addressed. In the work discussed in this dissertation the concrete is considered as an isotropic, homogeneous material, however actual concrete is a heterogeneous mixture consisting of stiff aggregates embedded in a relatively less stiff mortar matrix.

5. Experiments should be designed to probe the effects of differential shrinkage and core-drilling water individually and directly. Large apparent tension stresses were noted in the tests performed in Chapter 6 in drying concrete, as expected. These apparent stresses were removed using the procedures outlined in Chapters 3 and 4 for core-drilling water induced swelling and differential shrinkage effects. The net result of this removal was satisfactory results in in-situ stress predictions. However, each of these aspects should be probed on specimens designed to exhibit only one type of behavior, to ensure that accurate results for each of the individual facets are achieved, rather than results that are verifiable only in total.

6. In Chapter 6 the sorptivity value used to calculate the apparent stresses due to core-drilling water was calculated through sorptivity testing of cylinders cast in tandem with the plates. In Chapter 7 it was noted that if the sorptivity values determined from the cores removed from Plate 3 at Hole 7 and Hole 8 were used the error in final calculated in-situ stresses was lower than that provided in Chapter 6 in Table 6.19. The proper value to use for the sorptivity in future testing should be addressed in future research.
CHAPTER 9

REFERENCES


