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About the Origin: Is Mathematics Discovered or Invented?

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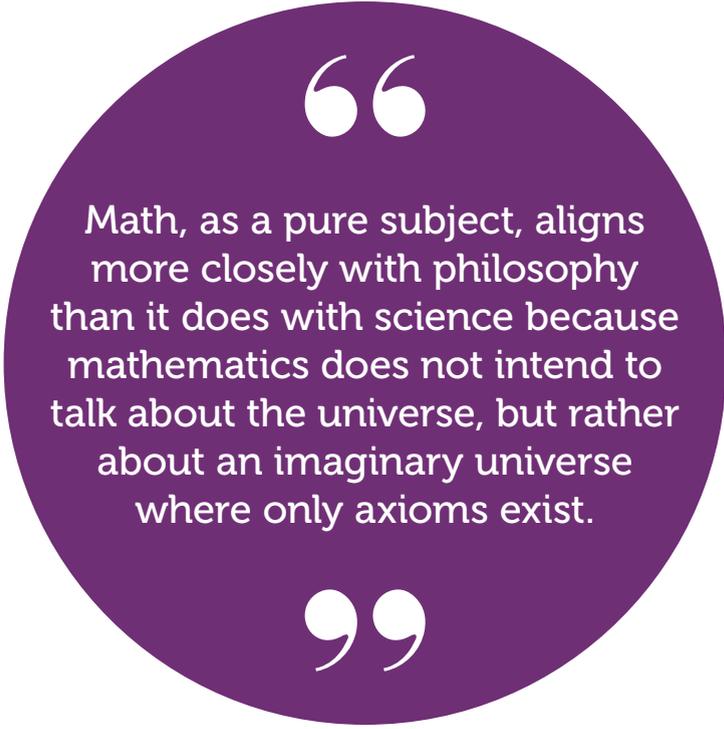
Is Mathematics Discovered or Invented?

Michael Lessel

It's a philosophical question as old as philosophy itself: was the system of math invented or discovered? Did we arbitrarily create fundamental mathematical and logical truths, or did we find them embedded in nature? The answer to such a question has profound implications, including the reality of concepts like logic and truth, which define our understanding of the universe. This centuries-old question has divided philosophers and mathematicians into two main camps: mathematical realists and anti-realists, each with several unique variations. I hope to fairly present and analyze both sides of this existential debate so that we better understand our relationship with truth and the universe around us.

Mathematics, especially in its more advanced forms, can often appear esoteric or unworldly. But regardless of technical fluency, one cannot deny the tremendous influence that the study of mathematics has had on modern civilization. In the process, we have adopted some curious semantic behaviors in our discussions of mathematics. In the same way that words form the human language, we commonly regard mathematics as the language of the universe. We say that when Newton sought to formalize his theories, he invented calculus. A primitive society has not yet created a mathematical logic, but a mathematician discovers results of a theorem. We have many ways of describing how we interact with mathematics, but one must inquire which version best represents reality. Is math an intrinsic property of our universe that humans continually strive to understand or is it an abstract system of logic, carefully formed and methodically developed by mankind? Do we gradually uncover the mathematics underlying our existence or do we invent mathematics to comprehend the universe around us? In plain terms, do we discover math or invent it? The literature on the ontological status of mathematics, starting from Plato all the way through to the modern mathematical logicians, is immense. Rather than drawing my own conclusions in this discussion, I hope to present and analyze both sides of this debate so that one might draw their own.

To understand the complex relationship between humans and mathematics, we must understand how we first used it. In its early forms, math helped us quantify time, make measurements, and take records. Rudimentary math was especially useful during the development of agriculture when surpluses in food allowed trade. Math satisfied the need to keep accurate records and perform basic calculations. Symbols for representing quantities ranged from everyday objects, to geometric shapes. As mathematical techniques evolved, so did our perspective on this new system. Ancient Greeks believed that numbers were both living entities and universal principles; numbers were active agents in nature. Plato pioneered the study of the ontology of mathematical objects, and Aristotle studied logic and issues related to infinity. Philosophers quickly realized that numbers and their operations were very useful in describing our world. The profound convergence of diverse aspects of mathematics, and existent theories in physics, caused many leading minds to ponder the dilemma of invention versus discovery.



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The major themes in the philosophy of mathematics are mathematical realism and anti-realism. We will introduce and compare them once we define our terms.

Terminology

Mathematics

It is useful to start with a definition of mathematics, but one does not seem as readily available as other fields, like science. Life defines biology as chemicals do chemistry, and matter does physics. Math lacks an empirical component. The fundamental units of mathematics are elements and sets, which given their abstract rather than tangible nature, seems to suggest that mathematics defines itself. Math, as a pure subject, aligns more closely with philosophy than it does with science because mathematics does not intend to talk about the universe, but rather about an imaginary universe where only axioms exist. More than any other discipline, mathematics concerns itself with logic and truth. By definition, math is the abstract study of how the structures of systems relate and operate. Sometimes we discover principles and then try to build a rigorous theory around them. Other

times we just lay down an arbitrary formal definition and see what follows. Sometimes we try to model real-world behavior of computational systems and other times we discover that different theories have a strikingly high level of resemblance to nature. When we talk about something that changes the way we look at math, we are more likely to admit that we were doing math wrong than that we found something new. The system of math analytically contains any and all new discoveries.

Invention & Discovery

In this discussion, we shall utilize these terms as they are commonly used. In the vernacular, we say that one invents a clever excuse, a fictional story, or a new technology. One might also discover a solution, a new species, or a problem in a design. Merriam-Webster¹ generalizes these uses as follows:

Invent — to create or produce (something useful) for the first time; to devise by thinking

Discover — to see, find, or become aware of (something) for the first time

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Discovery seems to imply that the thing in question was there beforehand, while invention implies an original concoction. If you sail to an uninhabited continent, you would likely claim that you discovered it. In order for Christopher Columbus to have invented the New World, by general understanding, he would have had to physically construct the land mass, bringing it into existence. If you were the first to run electricity through a wire, under given circumstances, you can claim to have invented the light bulb. Electricity was certainly around before we had anything to do with it, but a light bulb only existed once humanity brought it into existence. The materials that went into making the light bulb already existed, but the inventor does not claim to have invented them; rather, they were discovered. The light bulb itself didn't exist as a functional object until someone made it out of those materials. Whether one discovers or invents the abstract principles of mathematics is the topic of this discussion.

Realism

“Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it...”²

These words, from Galileo Galilei in *The Assayer*, perfectly embody the sentiment of realism. Mathematical realism holds that the universe operates based on a strict set of equations that governs all of its behaviors and that humans simply reveal them. According to realism, math exists objectively and independent of human thought. Mathematical concepts are disembodied in the universe and available for us to uncover and bring into practical use. In realism, discovering a new theorem is like discovering a new species of animal. The animal exists without our observation of it and probably has for a long time, but we have the ability to discover it, name it, and add it to our set of knowledge of animals. Mathematical realists believe that beyond the math that we currently know, there is more math. They believe in an objective mathematical universe that contains concepts we might one day discover or might

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never be able to discover. In this sense, realism regards mathematics as almost ethereal. In the absence of direct observation, mathematical realism boils down to faith in a mathematical entity, or set of entities, which contain all mathematical knowledge. This set is believed to be inherent in the universe and patiently waiting to be uncovered.

The belief in an external mathematical universe, that can't be perceived or directly interacted with, departs from the normal realm of science and encroaches upon the territory of religion. Thus, one might conclude that mathematical realism is based on a belief in a supernatural creator of the universe who decided on these governing equations. Similarly, one might believe that an all-knowing mind laid the groundwork for mathematics, from which humans have extrapolated mathematics. The Prussian mathematician and logician, Leopold Kronecker, famously declared “God made the integers; all else is the work of man.” He believed that math is a language and a tool but it's one that we discovered. We did not invent arithmetic; adding two and two will always give you four, say realists. We just invented the language to describe that result. Take gravity for example: the apple fell to the ground at 9.81

m/s^2 before humans discovered that number. Therefore, the idea of gravity was discovered and just the theory of gravity was invented. Mathematics still worked before we started using it; realism argues it belongs with gravity.

The tremendous accuracy with which our mathematical system models the universe lures many into the realist position. Eugene Wigner noted that our particular math seems unnaturally natural and thus boldly titled one of his works *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*.³ Wigner noted that many mathematical equations, often aimlessly developed in a vacuum, have later appeared in nature or proved critical to developments in physics by showing how the universe has been working all along. The most profound example comes from Fibonacci's sequence, which first emerged as part of an idealized model of rabbit population growth. It was later seen in sunflower seeds, flower petal arrangements, the structure of a pineapple, and the branching of bronchi in the lungs. Knot theory, from the late 18th century, aided our understanding of DNA and string theory three centuries later. While new physics sometimes demands developments in mathematics, math often tells the story first.

There are many variations on realism, but the most popular form is Platonism: a metaphysical position which offers that mathematical entities are abstract, have no spatiotemporal or causal properties, and are eternal and unchanging.⁴ Plato argued there are three real worlds: a physical world of appearances, an astral plane of thought and emotion, and the platonic plane of numbers and logic. To a certain degree realism adheres to concepts of a platonic plane of existence. This idea was widely spread thousands of years ago and continues to influence modern philosophers today. Of course, concepts evolve according to what we find out, but sometimes we also go in circles. If a platonic plane exists, then it does so in the universe and inside consciousness, meaning that man also has these planes of body, mind, and reason. On an ultimate level, everything would actually just be information. But we formulate mathematics in a similar way to how we use language; both are interrelated in how the mind functions.

In the *Republic*, Plato illustrates the mysterious nature of mathematics using the “allegory of the cave.” In this metaphor, there is a group of prisoners who have spent

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their whole lives chained in an underground cave facing a blank cavern wall. As things pass in front of a fire behind them, the prisoners watch the shadows projected on the wall, and begin to give names to the shadows. Viewing the shadows is as close to reality as the prisoners get, and they believe they understand their world. Analogously, humans can only observe and understand a cross-section of the fundamental truths of the universe. According to Plato, the everyday world can only imperfectly approximate an unchanging ultimate reality.

Anti-Realism

Mathematical anti-realism holds that mathematics is a product of the human imagination and is carefully engineered to make formal statements about nature in order to aid our understanding of the behavior of the universe. If conscious beings never existed, anti-realists assert, neither would mathematics. Anti-realists claim that mathematical statements have truth values, but that they do not do so by corresponding to a special realm of immaterial or non-empirical entities. Math is our way of creating models of what we observe in reality, making predictions, and discerning truth to a higher degree. The

mathematical system we are most familiar with is actually a set of axioms, truths, and their logical consequences. These operational symbols, and the rest of our mathematical language, have become part of the human condition and are therefore worthy of the title invention.

Many scholars have voiced opinions related to the anti-realist position. Immanuel Kant held that geometry is an abstraction of space, and numbers are an abstraction of time. Physicist and philosopher Ernst Mach held the instrumentalist view that mathematics is just a calculation procedure, and one can claim nothing about the reliability of its model of the universe. This follows from the fact that mathematics is not a direct translation of the content of the universe. Rather than discover the raw data of the universe, we observe it. We then use those observations to create data, from which we formulate theories and equations in a language that we have created. Though mathematical objects obey sets of rules, anti-realism argues that they are rules ultimately created by us, maintained by us, and often reevaluated by us. These rules are not the law of the land. They are our best attempt to interpret the behaviors of the universe. Cognitive linguists Lakoff and Núñez say that humans are great at inventing systems to help us do what we naturally desire to do: describe, discover, and probe.⁵ In this case, the invented system includes numbers, operations, and even measurements. They say that math is a system of human pattern recognition, able to model regularities in the universe that are not inherently mathematical, but are understood through math by humans. If there existed something better than math at explaining and modeling the world, then we would use that instead.

Like in the case of realism, there are many variations on anti-realism. The most prevailing is Formalism, which suggests that all of mathematics can be derived from a set of axioms or self-evident assumptions.⁶ Axioms are typically so basic that they cannot be proven true and thus must be assumed. We do not discover axioms in any real sense; we decide upon them. Axiomatic systems are invented with tools and a notation, and the resulting consequences, or theorems, are discovered. Of course we don't automatically know what those consequences will be. Even though we create the system when we invent the axioms, we have to discover what's true when the axioms hold. This concept is analogous to making a new recipe. You know what ingredients you want to include, so you

should have a general idea of how it will taste. But there is an interaction between different flavors that is not always obvious, so you have to take a bite to see how it actually tastes. You invented the dish by putting it all together, but there are things about it that you now have to discover. Just because you like ice cream and you also like marinara sauce does not mean you would care for marinara sauce on your ice cream. Further, consider the game of checkers: a man-made board game with man-made rules. Checkers was recently “solved,” meaning we now have computers that are unbeatable at checkers. Even though the rules to checkers are arbitrary, the solution to checkers was a natural consequence of the invented rules. The original designers of the game did not invent the solution, and were neither aware of the solution nor the fact that one could even exist. We discover results, but only because we invented the rules.

One of the other interesting versions of anti-realism is Fictionalism. It holds that all mathematical concepts like numbers, infinity, or limits, are not aiming at a literal truth but are better regarded as part of a fiction.⁷ Within the constructs of the story they make perfect sense, but outside of the story, mathematical concepts exist to the

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extent that characters in a movie can be said to exist. French philosopher Alain Badiou holds that math is a “rigorous aesthetic.” It tells us nothing of real being, but forges a narrative of logical consistency.

Further Discussion

In Euclidean space we define a triangle as a planar object with three straight sides and three angles. But just because we defined it does not mean we created it. In the same way, we define the laws of physics without actually creating the laws by which physical things operate. Because of the fundamental rules of geometry the triangle already existed as a possible shape, and that shape has 180 degrees. The fundamental rules of geometry and trigonometry exist independent of us. Without us they are not in a symbolized form, but they exist just like the universe would obey the same laws without us. This seems to support realism, but issues arise.

We have already established that different systems arise depending on the assumptions made. Under a certain set of axioms we get Euclidean geometry which at first modeled the universe sufficiently well. Euclidean geometry results from invented axioms not deeply embedded into every single thing. As physics became more sophisticated, Euclidean geometry became more inept at describing what space in this universe is like. Einstein’s theory of general relativity had to use other geometry that was also invented to describe curved space. He disregarded one of the axioms that establishes Euclidean geometry in favor of a new geometry. Can his equations be said to be discovered, if his discovery was dependent on a personal choice? Further, according to our definitions, discovery implies that the subject in question exists before it is determined. If one holds that mathematics concepts are discovered, then in what sense do they exist? Are mathematical theorems in some abstract sense embedded in physical entities? Can we really juxtapose abstraction with existence without losing the meaning of either?

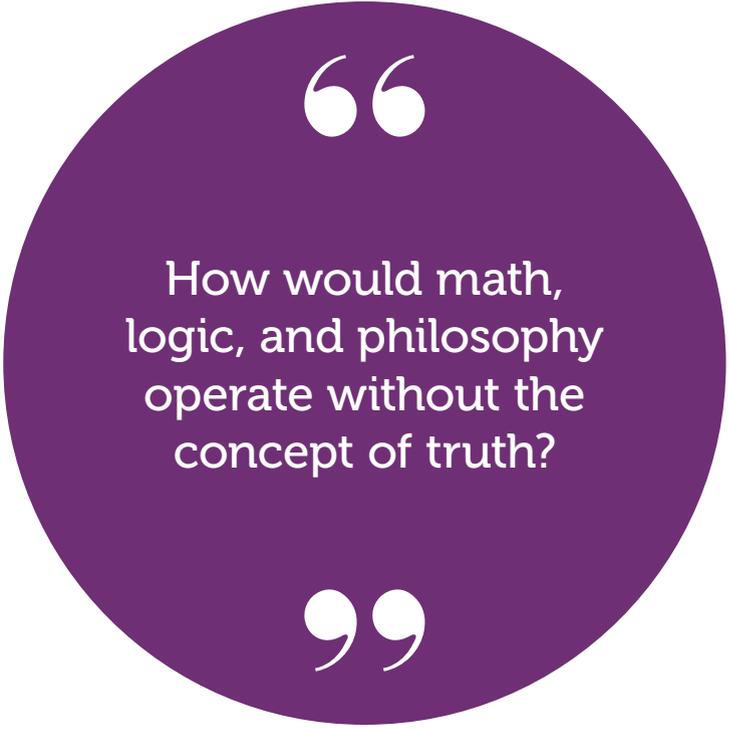
Once we laid the groundwork about what mathematics is, a whole set of tautologies followed, and we have slowly revealed them to ourselves. We decided to call a pair of prime numbers whose difference is two “twin primes.” Truths about twin primes lie embedded in our mathematical system, but we have yet to discover all of

these truths, including whether the list of twin primes is infinite, or what the largest pair is. Perhaps we will someday show that an answer cannot be proven. To a realist, this endeavor may seem more like an invented logic exercise than an attempt at understanding reality. They may hold that one can find all the results of invented rules that they want, but writing something down doesn't guarantee usefulness, or presence in a model. One cannot derive meaningful results on axioms that were not handed to us by the universe. Perhaps only those mathematical principles which reveal fundamental truths of our universe can be truly said to be discovered.

Some may contest the way we have defined our terms. If math is defined as the set of behaviors of the universe, it might be prudent to concede that math, in this sense, is discovered according to the definition of discovered. But if we extend to other areas, our definitions do not sound so reliable. Was *Huckleberry Finn*, the great novel, invented or discovered? Most people would claim that it is invented by general understanding. But is it fair to say, as a Formalist might, that once the rules of English and all its variations were invented, every English text that followed thereafter is simply discovered? Is it fair to say that Twain and Shakespeare merely looked for texts that already existed, in some abstract sense, by virtue of the existence of English? Such a paradox might make Formalism seem less valid or at least less applicable to other areas. Or perhaps it illustrates how liberally vocabulary can be applied.

Closing Thoughts

To attempt to resolve this debate naturally is a seemingly fruitless endeavor. One can regress tirelessly into definitions and into what counts as truth, knowledge, or existence. This does not mean, however, that we should cease all attempts to understand and ponder mathematical or other abstract concepts. The process of questioning and debating is the cornerstone of philosophy and can be very powerful when done properly. But why does an answer seem so evasive? It is entirely possible that this question creates a false dilemma between invention and discovery. Perhaps the ideas necessary to express meaning are limited by our language and the concepts of invention, discovery, and mathematics are not yet well-defined. With enough semantic gymnastics, one could make the case for either side. Others may



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feel this is too much of a semantic distinction to make in the first place or that this is a distinction without a real difference. Perhaps the belief that mathematics is discovered or mathematics is invented is just a belief and cannot be said to be right or wrong.

One must ponder what kind of evidence it would take to resolve this debate. No type seems obvious for conclusive proof, but one might imagine a scenario where we get strong evidence. As far as we know, only humans necessitate a mathematical system. Modern studies in animal cognition have shown that concepts like quantity, magnitude, and configuration are not unique to us, but this hardly constitutes math. As we are the only intelligent life on Earth, we would need to look elsewhere for other types of math. If we ever meet an alien race, they will probably have a completely different system of their own that bears no resemblance to our mathematics or the human notions of logic or numbers. But what happens if they have the same mathematic systems as us? Does that mean we were right all along? If they visit us, they will certainly be more advanced than us, and likely have a more complete description of the universe. But if their system in any way resembles ours, or includes ours as a

subset, it would seem to act as very strong evidence that math is indeed the native language of the universe. The biggest problem is that we have no raw data on this issue. We have no other intelligence with which to compare notes. We have only the amalgamation of opinions from experts of our own species in math and philosophy. There's not really a definitive argument one way or the other. The belief may be representative of how a person wishes to think of their own work: exploring the unknown or designing new innovations.

Under this scenario, or another scenario where we arrive at a definitive answer, what are the ramifications? What would change? Perhaps not much and we would still continue to do mathematics because it is both useful and elegant. But perhaps everything. For mathematical realists, math is about truth. If we determine that math and its underlying logic don't exist, would we conclude that truth, or at least some version of it, doesn't exist? How would math, logic, and philosophy operate without the concept of truth? In the absence of absolute realities, would we lose motivation in our pursuit of the most fundamental questions of our universe? The philosophy of mathematics and its consequences are indeed daunting for even the best scholars. I hope, however, that from this discussion, one can at least gain a greater appreciation for the complexity and intricacy of mathematics and philosophy. One might appropriately conclude that in general, mathematical historians behave like odd functions: they both persistently reflect about the origin.