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SEMI-RIGID CONNECTIONS

Analysis and Design

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by
Norzan Mohd. Yusof

A Thesis
presented to the Graduate Committee
of Lehigh University
in candidacy for the Degree of
Master of Science

in
Civil Engineering

Lehigh University

1986

This thesis is accepted and approved in partial fulfilment of the requirements for the degree of Master of Science.

13 May 1986

date

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ABSTRACT

Semi-rigid connections are used in continuous frame construction, primarily for lateral load resistance in office or apartment buildings of moderate height. The concept of semi-rigid connections is to achieve economy in design without sacrificing the safety of the structure in question. At the same time the actual behaviour of the structure, particularly at the joint, can be accurately predicted. A theoretical analysis of the behavior of a top and seat angle connection is carried out by varying the angle size, beam size and column size. Altogether seven cases are analysed. Particular attention will be focussed on the partial restraint afforded by the top and seat angle type of connections. This paper will demonstrate a rigorous analysis of top and seat angle connections using STRUCTR, an application program. The results obtained are compared to an experimental results carried out in 1940's thus verifying the validity of this analysis. Results show that this method underestimate the maximum moment and maximum load obtained through experimental results but is in close agreement with the plastic mechanism method.

Chapter 1

INTRODUCTION

The design of steel frames in multi-story and industrial steel buildings is usually based on the following assumptions:

1. In beam design the beam-to-column connections are assumed to be simply supported or pin connected.
2. In most column design, moments introduced due to frame action are often neglected.
3. For calculating lateral or wind loads, the beam-to-column connections are usually designed as rigid.

Although this approach saves calculation time and results in a safe structure, it overlooks the economic aspect, had the structure been designed in accordance to the way it actually behaves. That is, support conditions usually lie in between simple and rigid(fixed) support. Thus the connection is in actuality a semi-rigid connection. Various types of semi-rigid connection are as shown. (See Figure 6-1) .

In the past, several researches had been carried out in order to determine the savings. It was found that savings in terms of weight of as much as 20% can be achieved had the semi-rigid approach been used. But there remain the question of carrying out the design in a simplified manner. Several attempts had been made to analyse this behaviour by various simplifications but designers were reluctant to use it due to the cumbersome formula that has to be used. Various research for determining an accurate and simple design had been carried out as early as the 1940's in Great Britain and USA.(refer 1, 2) This paper will

also show one of the several ways of analyzing semi-rigid connections. The experimental results which will be used for comparison are obtained from a series of test on top-and-seat-angle connections carried out at Fritz Engineering Laboratory. Building connections may be classified under three different headings with respect to their moment-rotation characteristics. According to the AISC specification, the categories are as follows:

- Type 1 - known as " Rigid Frames " (Continuous Frames), assumes that the beam-to-column connections have sufficient rigidity to prevent any rotation between the intersecting members.
- Type 2 - known as " Simple Framing " (Unrestrained Free-Ended), assumes that the ends of the member, beams and girders are connected for shear only therefore allowing the member to rotate freely under gravity load.
- Type 3 - known as " Semi-Rigid Framing " (Partially Restrained), assumes that the connections of beams and girders possesses a dependable and known moment capacity intermediate in degree between the rigidity of type 1 and the flexibility of type 2.

Chapter 2

THEORETICAL ANALYSIS

The semi-rigid connection may be thought of as a weakened section between the end of the beam and the face of the column to which the connection is made. The effect on analysis is the inverse of the effect produced by end haunches or added cover plates. The typical test behaviour of a bolted or welded connection is as shown in Figure 6-2, which shows the relationship between moment transmitted through the connection and the angle change between the joint center and the end of the beam. In the design range the relationship is assumed to be linear and the inverse slope is termed the connection factor Z , where;

$$Z = Q/M$$

- where Q represents additional angle change due to yield
- and M represents end moment in the beam

The connection factor Z may be defined as " angle change for unit moment " and can be determined experimentally. Thus for a given connection, Z needs to be determined before any analysis could proceed. Z is also that property of a semi-rigid connection which when used in conjunction with E and I of the connected member, becomes the necessary correction factor to make the ordinary exact methods of analysis valid. It varies inversely with the rigidity of the semi-rigid connection, becoming zero for a rigid connection and infinite for a pin connection. The slope of the moment rotation curve of Fig. 6-2 is therefore

the reciprocal($1/Z$).

For the top-and-seat-angle connection, the elastic-plastic load deflection behavior of the member i.e top and seat angles, beam, and column is analysed using STRUCTR, a general purpose FORTRAN program for structural analysis by the *direct stiffness method*. The plastic moment capacity, M_p , of the member must be known at the potential locations of the plastic hinges. As the load increases a plastic hinge, M_p , will form at the location of greatest moment, say B. M_p is in equilibrium on both sides of the hinge. Rotational degree of freedom θ_b is removed from element AB by condensation. The reduced element stiffness matrix now effectively having a row and column of zeroes corresponding to θ_b are assembled into the structure. Therefore the rotation at B in the element and structure are now independent.

Structural analysis in the presence of plastic hinges proceeds in a series of linear steps, the beginning of each step being marked by introduction of another plastic hinge and the consequent reduction of structure stiffness. Collapse is indicated by very large displacements or by the structure matrix not being positive definite. The limiting load that would cause a collapse mechanism can then be determined together with its corresponding deflections.

For this analysis to be carried out the following assumptions were made:-

1. There is no out-of-plane loading or displacement.
2. The depth of beam and column are modelled as a rigid members. Therefore the interior of the joint between the connections is assumed to be infinitely rigid.

3. The angles are simulated as beam elements.
4. Shear deflection is ignored. i.e. deflection is due to the bending effect only.
5. An elastic-perfectly plastic hinge is formed.
6. Moment-curvature is linear until the extreme fiber stress reaches the yield point, after which the moment remains at a value M_p as curvature increases indefinitely.
7. Prying action of the connection is ignored.
8. The effect of axial force on the value of plastic moment capacity, M_p , is ignored.

2.1 STRUCTURE AND MODEL

The actual test set-up of the structure is as shown in Fig 6-3. The test assemblage consists of two beams stub riveted to a column stub and supported in an inverted position. The supports supply shear and moment at the connection approximately equivalent to the shear and moment at the end of a building framed to each end of a column. Due to the symmetrical nature of the structure and its loading condition, the structure is modelled as a line element about its centerline of symmetry. Thus, along the centerline of the column section any chosen node point is restrained against horizontal displacement and rotation but free to move vertically. At the extreme end of the column the support conditions are fixed. The column properties i.e I_{xx} , I_{yy} and Z_{xx} are taken as halved thus the capacities of the column are also half what they actually are. For the model of the structure refer to fig 6-4. The rigid member is modelled by imposing a large modulus of elasticity, a large moment of inertia, and a large area. Thus the rest of the structure is very flexible when compared to the rigid members.

By defining the geometry of a structure, it is thus possible to analyze structures with seat and top angle semi-rigid connections using an ordinary structural analysis computer program with line-type bending members. The analysis concentrates on the bending behaviour of the beam, column, and the flexible angles used to make the connections.

Dummy rigid beams are used to space the bolt lines of the angles at the proper distance from the centerlines of the connected beam and column. Equilibrium of forces are thus invoked at the outer fibers of the beam and column rather than at the centerline. Similarly, compatibility results according to the plane-sections-remain-plane concept. The angles are modelled as a pair of rectangular beams of width and thickness equal to those of the angle placed at right angles to each other. The entire assemblage of columns, beams, and angles is analyzed as a rigid frame.

An elastic analysis identifies the points of maximum stress and therefore plastic hinges may form. After changes in boundary conditions, additional steps of elastic analysis can give increments in the elastic-plastic load-deflection curve of the entire structure up to the point where a mechanism defining ultimate load is defined.

Chapter 3

STRUCTR INPUT

For analysis purposes, a standard format of input is adopted and listed under the following categories.

1. **Overall Dimensions:-** This includes the height of column center to center span of the beam and the beam centerline above the foundation.
2. **Member Sizes:-** These are the member types such as beam and column, their area, and moment of inertia with respect to both axes and torsional constant which are necessary for stiffness calculations. Plastic moment capacity, M_p , and yielding load, P_y , are needed for the plastic capacity program SETUP.FOR. Column and beam depth are required for determining the coordinate locations of angles and dummy rigid members. For the angle leg section, the area, moment of inertia, plastic section modulus, plastic moment capacity, and yield load are calculated from the properties of a rectangle.
3. **Inner fastener gage line for angles:-** The effective part of the angle leg which resists bending lies between its heel and the first gage line of the fasteners. For this solution the angle is assumed to be flat and there is no bending moment from the inner gage line to the toe. Where the angle is non-uniform such as the top angle in Test 20, plastic moment capacity, M_p and yield load, P_y are calculated using the effective area and moment of inertia at the inner gage line for the horizontal leg of the member. Similarly, the same principle applies for the vertical leg. Therefore two values of M_p and P_y are inputted in the plastic capacity program SETUP.FOR.
4. **Node coordinates:-** The following defines the node coordinates:-
 - $X_c = X_{ct}, X_{cb} =$ column top and bottom at the same horizontal location.
 - $Y_{ct} =$ column top Y coordinate.
 - $Y_{cb} =$ column bottom Y coordinate.
 - $Y_b = Y_{bi} = Y_{bj} =$ Y-coordinates of beam centerline.(i and j end)
 - $X_{bj} =$ X-coordinate of right end of beam.

- X_{av} = X-coordinate of angle vertical leg.
- Y_{avt} = Y-coordinate of bolt line in top angle vertical leg.
- Y_{aht} = Y-coordinate of angle horizontal leg in top angle.
- X_{bi} = X-coordinate of bolt line in horizontal leg.
- Y_{ahb} = Y-coordinate of angle horizontal leg in bottom angle.
- Y_{avb} = Y-coordinate of bolt line in bottom angle vertical leg.

In addition the following information is required:

- D_b = depth of beam.
- D_c = depth of column.
- t_a = thickness of angle legs.
- g_a = gage distance in angle legs.

5. **Boundary condition:-** Altogether there are 14 nodal points and 15 members with three degrees of freedom at each joint, i.e, two translations and a rotation. Thus there are 42 equations to be solved. As previously mentioned on page 6, along the centerline of the column except at the support, the boundary condition imposed is known displacement in x-direction, known rotation, and unknown displacement in y-direction. For the support, all displacements and rotations are known and for the rest of the joints, the displacement are unknown. Therefore there are 13 knowns and 29 unknowns. This boundary condition is applied throughout the test. Refer to Fig 6-5 for node and member numbering sequence. With each formation of a plastic hinge, the boundary condition of the particular member in question is changed and the next step of analysis is repeated. Altogether six steps of load-deformation analysis are carried out for each test.

Chapter 4

TEST RESULTS

4.1 NON-DIMENSIONALIZING

The results obtained from STRUCTR output were non-dimensionalized for presentation purposes. This way will allow any unit to be used depending on one's preference. To dimensionalize the moment, M , and load, P , are divided by M_{max} and P_{max} of the test results. Therefore the graph will have an upper limiting value of 1.0. The tabulated results can then be interpreted with ease. Similarly the corresponding deflections from STRUCTR output are divided by the elastic deflection due to the maximum test load, P_{max} .

For a cantilever beam:-

Deflection $\Delta_{max} = P_{max}L^3/3EI$ where,

- P_{max} = Load applied
- L = Span of beam
- E = Modulus of Elasticity
- I = Second moment of Inertia

4.2 RESULTS

Below are the typical results obtained by using STRUCTR on the the seven test cases. These results were then plotted as shown in Figure 6-7 to Figure 6-27.

Test 2

Point	Load	Deflection	Moment	Rotation
1	0.	0.	0.	0.
2	0.1269	0.305	0.2713	0.000528
3	0.1618	0.542	0.3459	0.001150
4	0.1717	4.659	0.3672	0.013096
5	0.1834	52.703	0.3921	0.151180
6	0.4818		1.0300	4.423069

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 2155 \text{ N (46.6 kips)}$
 $M_{max} = 876.6 \text{ Nm (62.2 kipft)}$
 $\Delta_{max} = 2.64 \text{ mm (0.1041 in)}$

Test 5

Point	Load	Deflection	Moment	Rotation
1	0.	0.	0.	0.
2	0.1361	0.4464	0.2876	0.000507
3	0.1676	0.7681	0.3542	0.000991
4	0.1818	5.3550	0.3841	0.008560
5	0.1835	5.7650	0.3877	0.009234
6	0.1849	6.7180	0.3907	0.010840

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 2728 \text{ N (59 kips)}$
 $M_{max} = 1385.6 \text{ N (98.3 kipft)}$
 $\Delta_{max} = 1.825 \text{ mm (0.072 in)}$

Test 9

Point	Load	Deflection	Moment	Rotation
1	0.	0.	0.	0.
2	0.1631	0.3567	0.3446	0.000406
3	0.206	0.6034	0.4351	0.000854
4	0.2196	3.9340	0.4638	0.008054
5	1.02	4.9672	0.3895	0.010970
6	1.095	5.0880	0.3822	0.011250

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 3583 \text{ N (77.5 kips)}$
 $M_{max} = 1820.4 \text{ N (129.2 kipft)}$
 $\Delta_{max} = 2.397 \text{ mm (0.0944 in)}$

Test 10

Point	Load	Deflection	Moment	Rotation
1	0.0	0.0	0.0	0.0
2	0.2189	0.3753	0.4625	0.000341
3	0.2830	0.6184	0.5979	0.000766
4	0.2955	3.4552	0.6243	0.007404
5	1.069	4.4415	0.5531	0.010457
6	1.104	4.4914	0.5498	0.010594

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 3884 \text{ N (84.0 kips)}$
 $M_{max} = 1973.2 \text{ Nm (140 kipft)}$
 $\Delta_{max} = 2.598 \text{ mm (0.1023 in)}$

Test 16

Point	Load	Deflection	Moment	Rotation
1	0.0	0.0	0.0	0.0
2	0.1098	0.4058	0.2351	0.000659
3	0.1370	0.7356	0.2933	0.00134
4	0.1501	6.8463	0.3212	0.014491
5	0.7936		1.697	13.2084
6	0.8178		1.749	14.0

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 1572 \text{ N (34.0 kips)}$
 $M_{max} = 638.9 \text{ Nm (45.33 kipft)}$
 $\Delta_{max} = 1.955 \text{ mm (0.07697 in)}$

Test 20

Point	Load	Deflection	Moment	Rotation
1	0.0	0.0	0.0	0.0
2	0.1516	0.3078	0.3223	0.00054
3	0.1929	0.5216	0.4099	0.00116
4	0.2012	3.3942	0.4275	0.01106
5	0.7839	4.1701	0.3643	0.01451
6	0.8194	4.2214	0.3604	0.01472

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 3352 \text{ N (72.5 kip)}$
 $M_{max} = 1532.7 \text{ Nm (108.75 kipft)}$
 $\Delta_{max} = 3.484 \text{ mm (0.1372 in)}$

Test 22

Point	Load	Deflection	Moment	Rotation
1	0.0	0.0	0.0	0.0
2	0.1522	0.3331	0.3235	0.000459
3	0.1775	0.4766	0.3771	0.000774
4	0.2042	3.5499	0.434	0.008705
5	0.5153	62.7	1.095	0.1596
6	0.7577	108.97	1.61	0.2774

where load = P/P_{max} , moment = M/M_{max} and deflection = Δ/Δ_{max}
 $P_{max} = 3838 \text{ N (83.0 kips)}$
 $M_{max} = 1754.7 \text{ Nm (124.5 kipft)}$
 $\Delta_{max} = 2.618 \text{ mm (0.1031 in)}$

4.3 VERIFICATION OF RESULTS

The limiting load P obtained while using STRUCTR is checked by comparing it with the collapse load obtained from the plastic mechanism concept. In this method, a plastic hinge is assumed to form at nodes 3,5 and 10 since this is the most likely location for the hinge to form. When the structure is loaded until a mechanism occurs, a rigid body motion takes place. The rigid body will rotate about a common point which is node 5. This is the instantaneous center for the rigid body motion. Referring to figure 6-6 and taking equilibrium about node 10 and applying principle of virtual work the collapse load P can be found by using the following equation:-

$$P = \{ Mp1[1 + 2Db/La] + Mp2 \} / \{ Lb + Lc \} \text{ where}$$

- $Mp1$ = plastic moment capacity of top angle
- $Mp2$ = plastic moment capacity of seat angle
- Db = overall depth of beam (including the shims if any)
- La = length of top angle vertical leg (node 3 to node 5)
- Lb = length of beam (node 7 to node 8)
- Lc = length of seat angle horizontal leg (node 10 to node 11)

The collapse load obtained via this method is compared to the one obtained using STRUCTR.

Chapter 5

DISCUSSION

The results of the analysis is plotted as moment-rotation, load-rotation and load-deflection graphs.(refer to fig 6-7 to 6-27) From the results tabulated it is observed that the limiting load, P , increases with increase in angle member sizes.

By comparing results of test 2 with test 16 it is observed that test 2 has a larger limiting load, P , value of 57% greater than test 16. This was achieved because of a larger thickness of the top angle by 3.175 mm.(0.125 in). In test 2 the limiting load, P , prior to collapse is 17% of the maximum load obtained experimentally. But for the limiting moment, 37% of the maximum moment is achieved prior to collapse, which is approximately twice the limiting load value. This due to the fact that the limiting load, P , is actually the shear force acting at the support. In reality the load applied to the structure is always double the limiting load, P . For the first two hinges the load, moment, deflection and rotation lie within the test result domain but once the third hinge is formed a large deformation takes place. At collapse the deformation is 4.6 times the maximum deflection due to maximum test load. The maximum rotation is 0.0311 radian. For the test result, at limiting load, P the rotation is 0.0031 radian. Thus by comparing these results it can be said that the contained plastic flow region is approximately ten times the elastic region.

The degree of variation of contained plastic flow is as shown in Table 6-1 for all the tests conducted. For all the tests carried out plastic hinges formed at nodes 3,5 and 10 prior to collapse. Beyond the collapse mechanism, plastic hinge

were formed at node 11 to node 6(test 2,16), node 11 to node 8(test 9,10,20), node 11 to node 7 (test 22) and node 11 to node 10(test 5). For tests 9,10 and 20 the moment-rotation curve shows an instability condition after collapse occurs. These may be due to the formation of hinges at node 8. For tests 2,16 and 5 it is observed that deflection and rotation increases without further increase in load once the collapse mechanism is achieved. This is as expected. For test 22 the deflection and rotation increases with increase in load and moment even after collapse. This indicates a similarity to a strain-hardening effect. Test 10 produces the maximum M/M_{max} ratio of 0.62 which coincides with the largest angle member used.

An interesting observation is that none of the analytical test results reached the maximum load and moment obtained through the experiment. As mentioned above the maximum it ever reached was 62% of the maximum moment achieved via the experiment. This could be due to the following :-

- 1) In our analysis, the strain-hardening effect is neglected. This results in the lowering of the limiting load and moment as compared to the experimental result. Furthermore, after the formation of the first plastic hinge, the modulus of elasticity of the particular member starts to vary. Ideally the effect of these variations should be taken into account before the next step of analysis is carried out. Also an abrupt change in cross-section of the member such as the angle and rigid member makes the analysis more complicated.

- 2) The angle member should be analysed as a short beam. The largest

slenderness ratio of the angle member is $1/12$ (test 2) and the smallest is $1/6.9$ (test 9,10). Therefore, shear deflection can be a significant contribution to the load-deformation characteristic of the connection. According to Timoshenko's theory of short beams, the shear deflection contribution can be as high as 10% of the total deflection of the beam.

Another interesting problem arises when modelling the rigid members. During the initial stage of the analysis the rigid member is assumed to have a very large moment of inertia, I , and cross-sectional area, A . The E value is assumed to be 1000 times the normal E value of steel. Results of the analysis are as shown in Table 2. As can be seen from Table 6-2 the error computed with respect to plastic mechanism concept is very large. For test 10 the error is 201.8%. Only test 5 and 22 seems to provide reasonable accuracy. On checking it was found that equilibrium is never achieved from the first load-step of the analysis. If this is not corrected, further load-step will propagate larger equilibrium error as was found out in test 2 analysis. The error in equilibrium doubles each time the load step is incremented. This is due to the illconditioned effect(numerical instability) of the structural equation matrix. A small change in the stiffness $[K]$ or load vector $\{R\}$ will produce large change in displacement $\{D\}$. The structural equations $[K]\{D\}=\{R\}$ are equilibrium equations which are written in terms of differences in displacement of neighbouring nodes. If these differences are quite small in comparison with the displacements themselves the equations become ill-conditioned. This is what is actually happening to our structure(model) where a condition of a very stiff member surrounded by a flexible member exists. To overcome these inaccuracies

the rigid member is modified by scaling down the E value from a previous value of 1000 times to 10 times the normal E value; other values remaining constant. The final results are shown in Table 6-3. As seen in Table 3 the modification results in a drastic reduction in error; almost negligible. Equilibrium is also maintained throughout the load-step.

In order to avoid future error when carrying this form of analysis the following procedure is recommended:-

1) Equilibrium must be checked for at every load step particularly in the region of sizable change in geometric and material properties. In our case between the rigid member and the flexible member.

2) When introducing boundary conditions avoid introducing a hinge next to an existing hinge, as in node 10. If a hinge is formed in this location, it is better to introduce the hinge at the i th end of member 3 rather than the j th end of member 4.

3) Avoid introducing a hinge at both ends particularly in the region experiencing compression. It was found out that if for example, a hinge is formed at node 11 after formation of a hinge at node 10, it is best to introduce a hinge at the i th end of member 14 rather than introducing hinges at the i th and j th ends of member 3. However this effect does not apply to the members experiencing tension, i.e, member 1 and member 2.

4) A trial and error method is necessary in order to provide the right amount of rigidity to the rigid link member.

Chapter 6

CONCLUSIONS

The results of these analyses indicate that:-

1) When compared to the experimental results, a lower value of limiting load P is obtained. Assuming the experimental value is correct, an improved analytical result can be achieved if the problem is reformulated taking into account the strain-hardening effect and the shear deflection effect.

2) The final mechanism and ultimate load obtained via STRUCTR is in close agreement with the plastic mechanism method.

3) Plastic hinges appear consistently at nodes 3,5 and 10, that is, the first two hinges occur at the top angle and the final hinges occur at the seat angle.

4) The connections(for all tests) on the average, have a reserve of 30% of the ultimate load once they have passed the proportional limit.

5) The load-deflection behavior is a step-by-step piecewise linear function instead of a smooth curve.

6) Deflection at collapse exceeds the elastic deflections, but for the first two hinges, the elastic deflection is not exceeded.

7) The load-deflection behavior of a semi-rigid connection can be reliably

analysed using a linear structural analysis program thus relieving designer's of doing it manually.

Test No.	Rotation at collapse	test rotation	STRUCTR Collapse load
2	0.0131	0.0013	0.1717
5	0.00858	0.0021	0.1818
9	0.00805	0.0021	0.2198
10	0.0074	0.0011	0.2955
18	0.0145	0.0014	0.1501
20	0.0111	0.0021	0.2012
22	0.00871	0.00157	0.2042

Table 6-1: COLLAPSE AND TEST ROTATION

Test No.	STRUCTR	Plastic Collapse	% Error
2	571 N(12.34 kips)	368 N (7.95 kips)	55.2
5	498 N(10.72 kips)	510 N (11.03 kips)	-2.8
9	861 N(18.61 kips)	791 N (17.1 kips)	8.8
10	3489 N(75.03 kips)	1150 N (24.86 kip)	201.8
16	349 N(7.55 kips)	235 N (5.08 kips)	48.6
20	2035 N(44.0 kips)	676 N (14.83 kips)	201.0
22	784 N(16.95 kips)	773 N (16.73 kips)	1.3

Table 6-2: COLLAPSE LOAD WITH ORIGINAL RIGID MEMBER STIFFNESS

Test No.	STRUCTR	Plastic Collapse	% Error
2	370 N(8 kips)	368 N (7.95 kips)	0.6
5	496 N(10.72 kips)	510 N (11.03 kips)	-2.8
9	787 N(17.02 kips)	791 N (17.1 kips)	-0.5
10	1148 N(24.82 kips)	1150 N (24.86 kips)	-0.2
16	236 N(5.1 kips)	235 N (5.08 kips)	0.4
20	674 N(14.58 kips)	676 N (14.63 kips)	-0.3
22	784 N(16.95 kips)	773 N (16.73 kips)	1.3

Table 6-3: COLLAPSE LOAD WITH REVISED RIGID MEMBER STIFFNESS

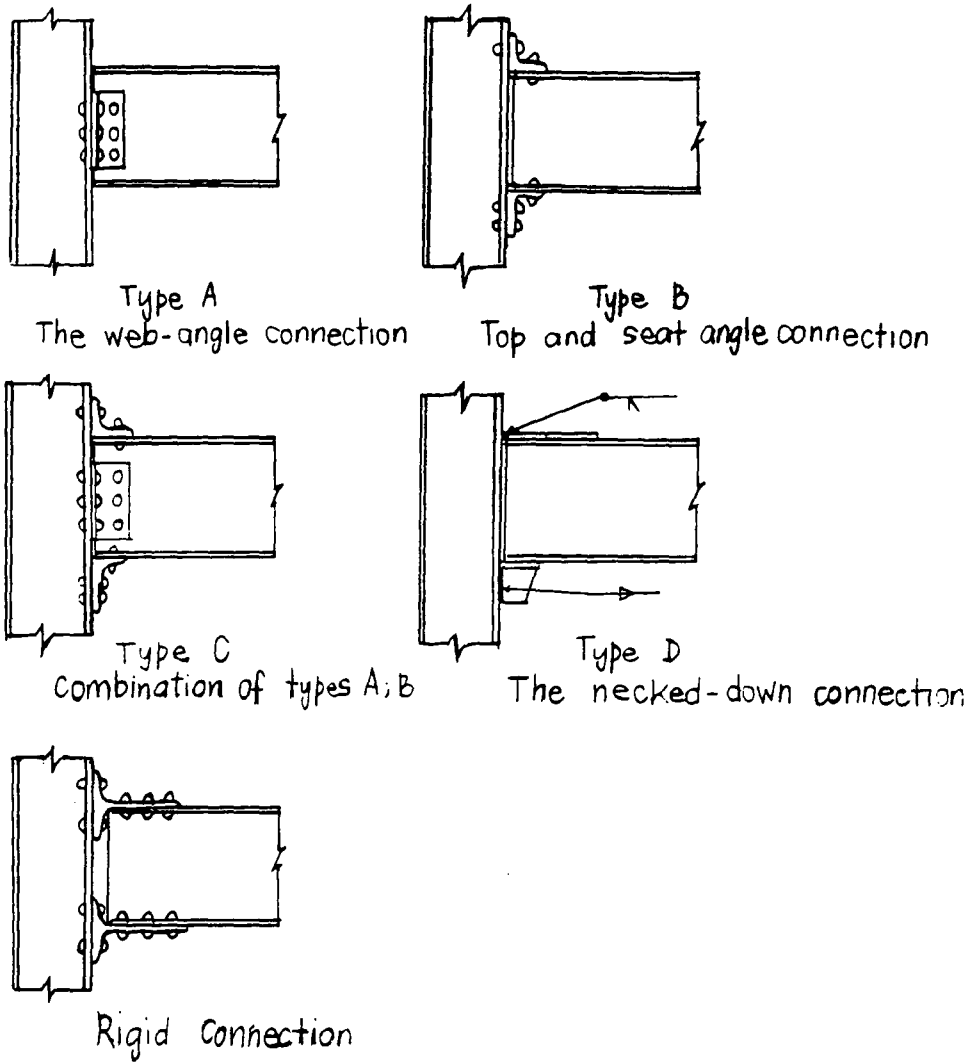


Figure 6-1: TYPES OF SEMI-RIGID CONNECTION

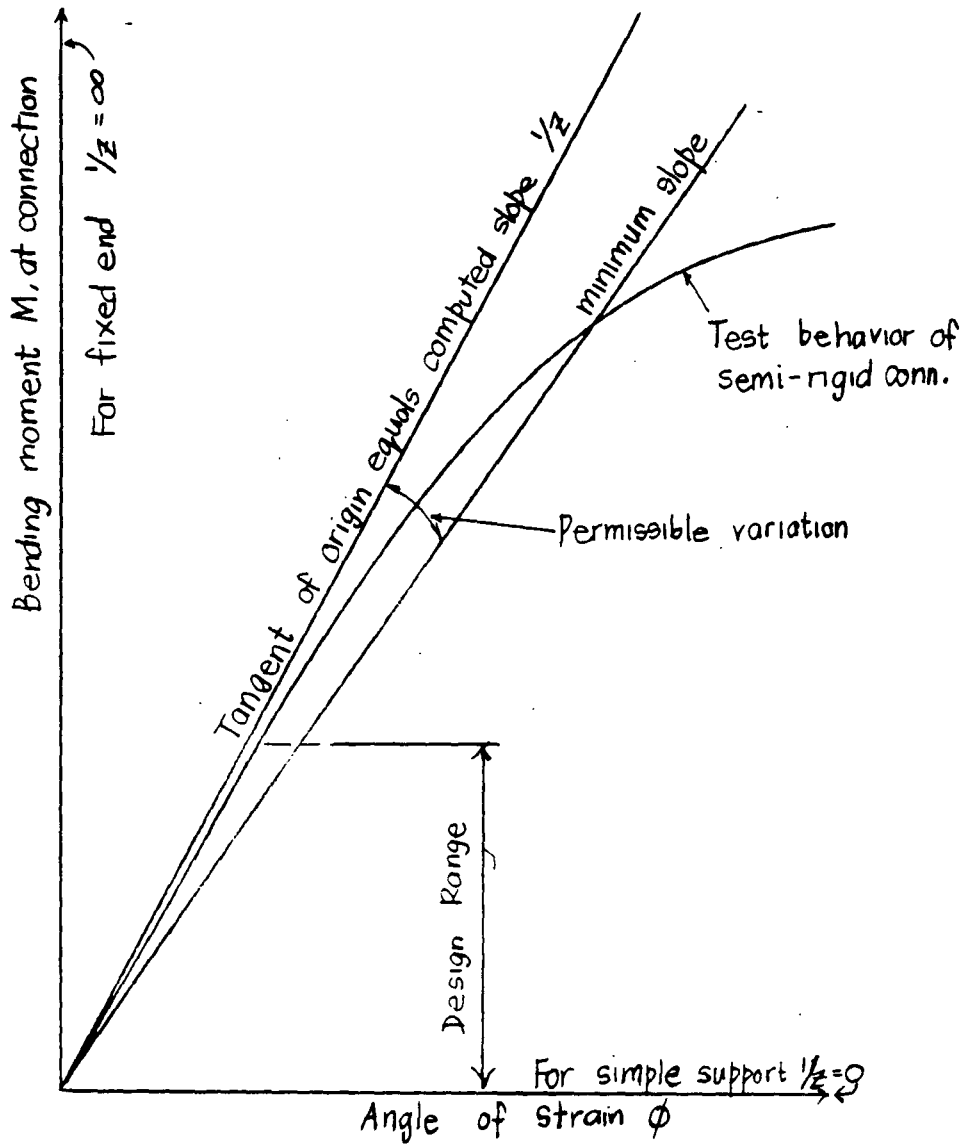


Figure 6-2: MOMENT-ROTATION CURVE-SEMI-RIGID CONNECTION

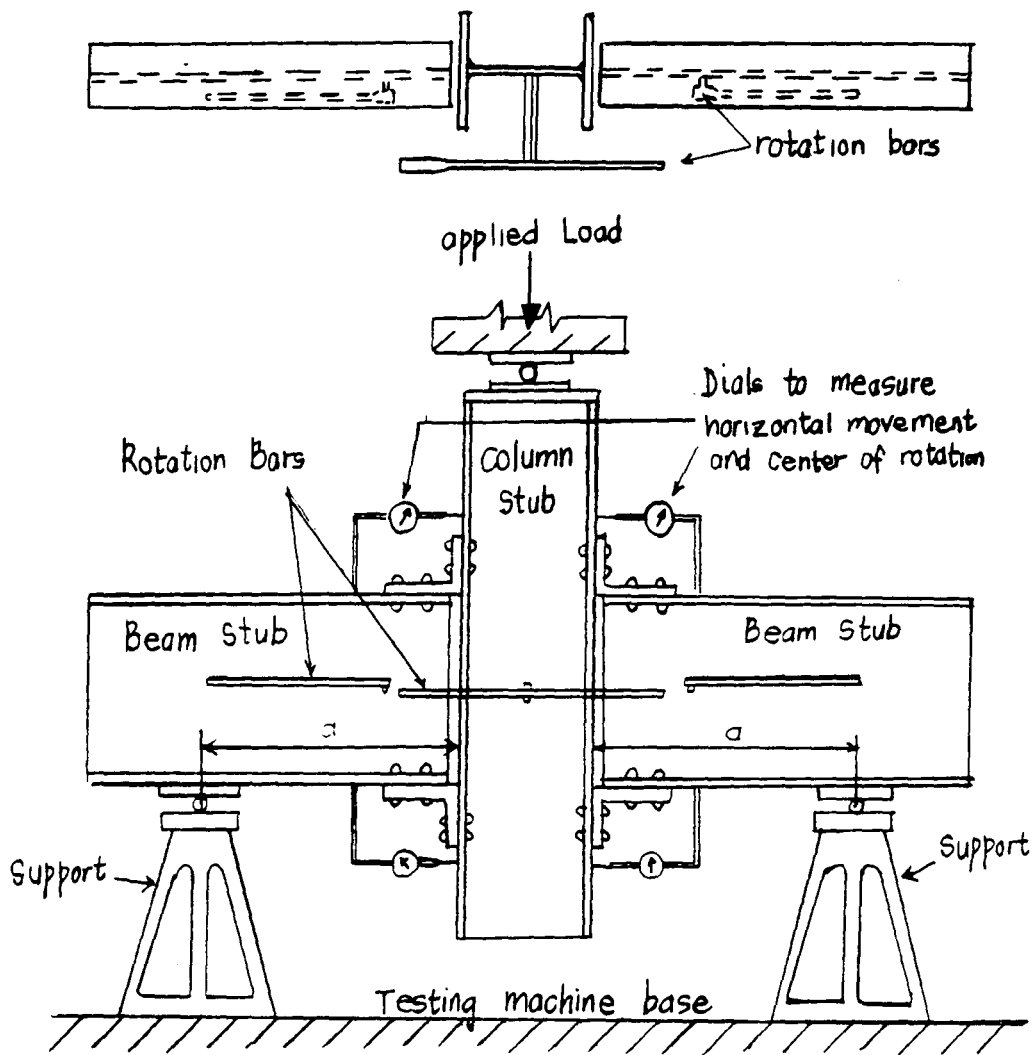


Figure 6-3: TEST ASSEMBLAGES

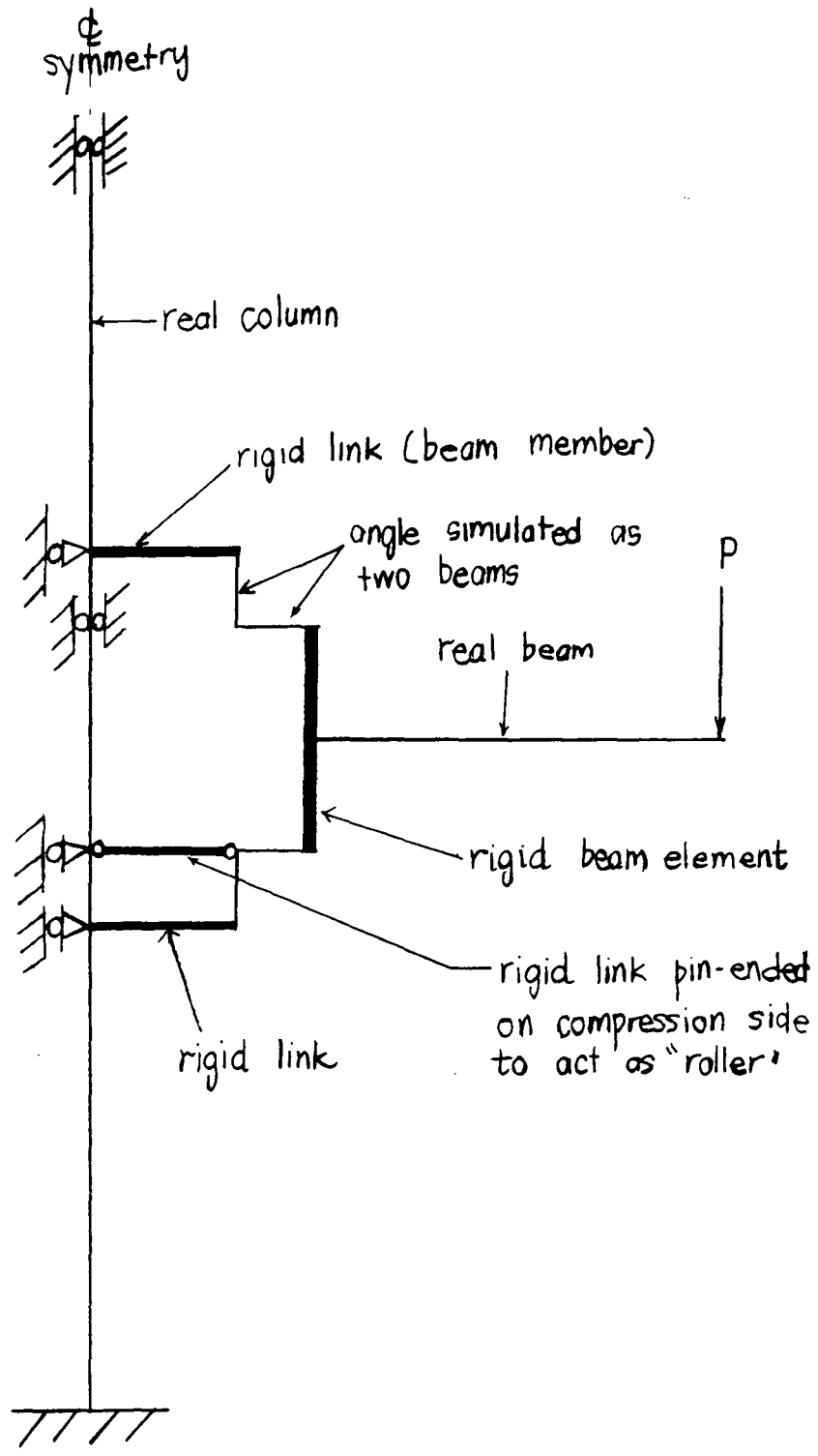


Figure 6-4: TEST MODEL

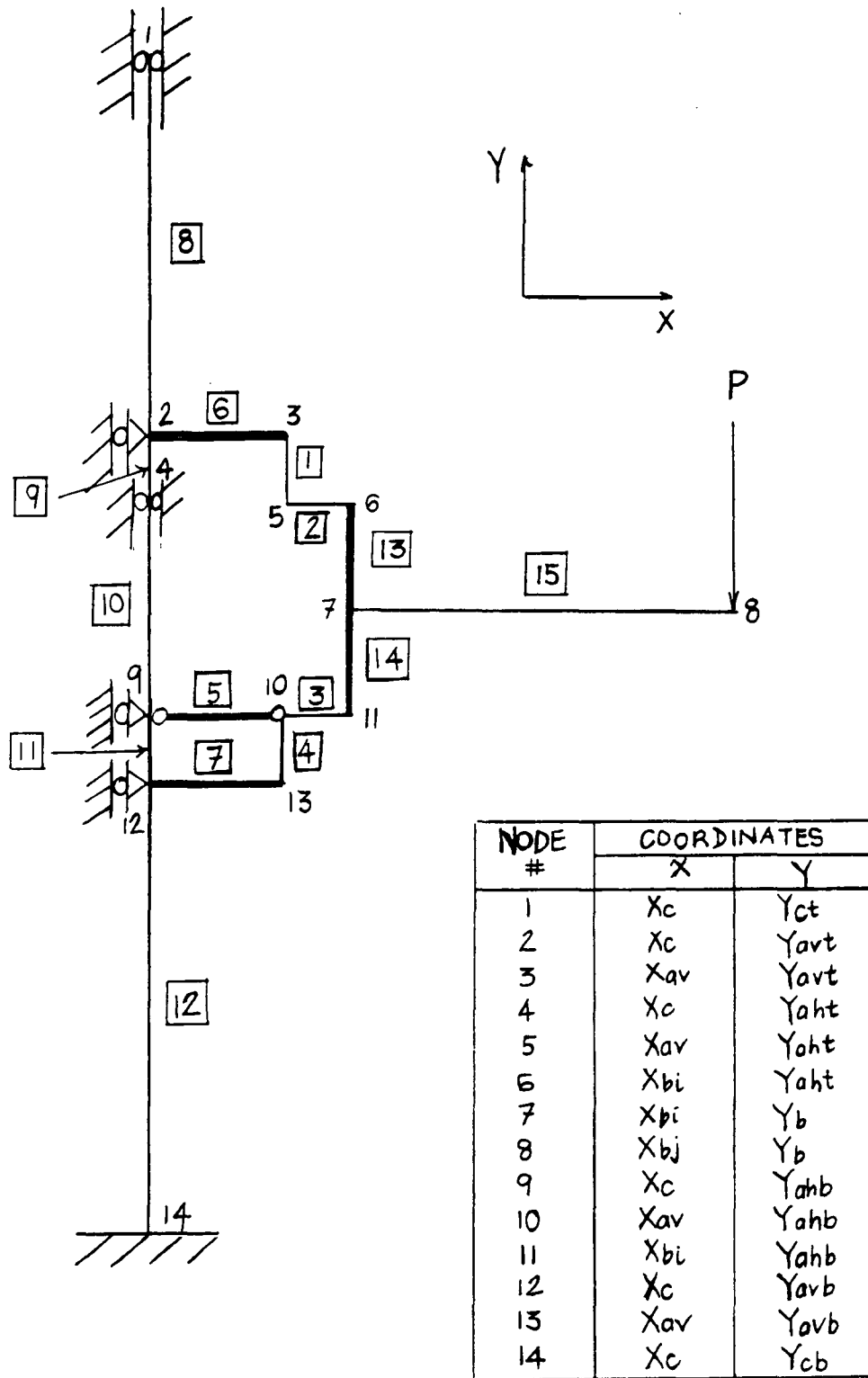
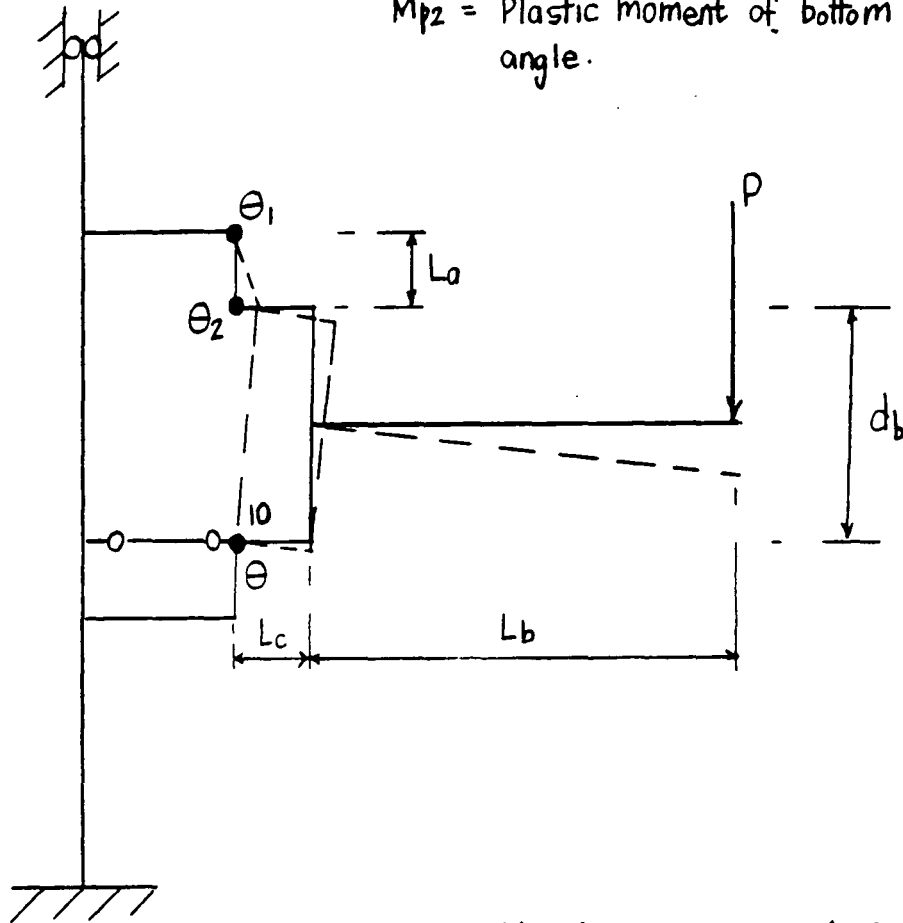


Figure 6-5: NODE AND MEMBER NUMBERING

M_{p1} = Plastic moment of top angle.

M_{p2} = Plastic moment of bottom angle.



$$P(L_b + L_c)\theta = M_{p1}(\theta_1 + \theta_2) + M_{p2}\theta \quad \text{---(1)}$$

$$\theta_1 = \frac{d_b \theta}{L_a} \quad \theta_2 = \theta_1 + \theta$$

$$= \theta \left[1 + \frac{d_b}{L_a} \right]$$

$$\therefore \theta_1 + \theta_2 = \theta \left[1 + \frac{2d_b}{L_a} \right]$$

substitute into (1)

$$P(L_b + L_c)\theta = M_{p1} \left[1 + \frac{2d_b}{L_a} \right] \theta + M_{p2}\theta$$

$$\therefore P = \frac{M_{p1} \left[1 + \frac{2d_b}{L_a} \right] + M_{p2}}{L_b + L_c}$$

Figure 6-6: PLASTIC MECHANISM METHOD

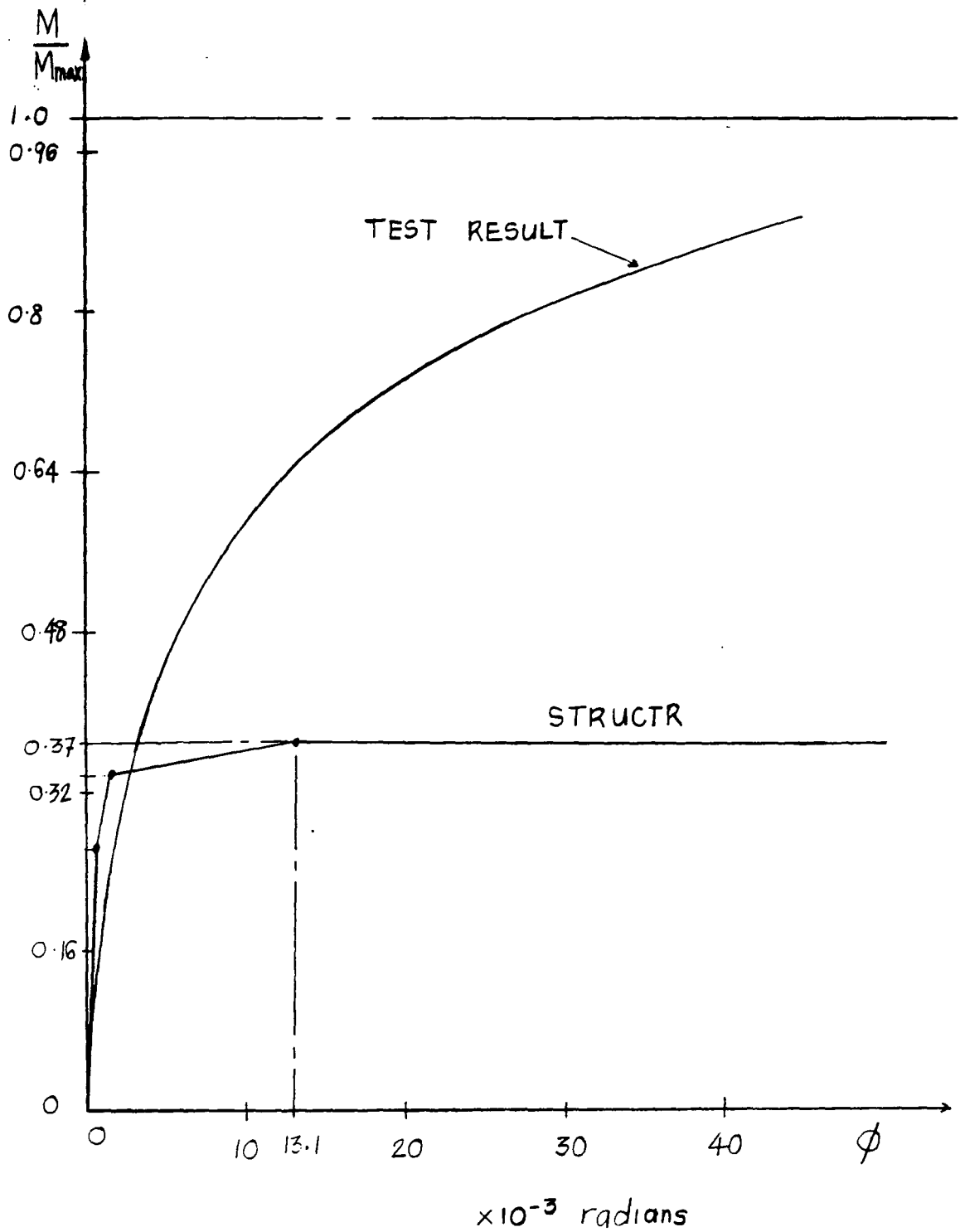


Figure 6-7: TEST 2 MOMENT-ROTATION CURVE

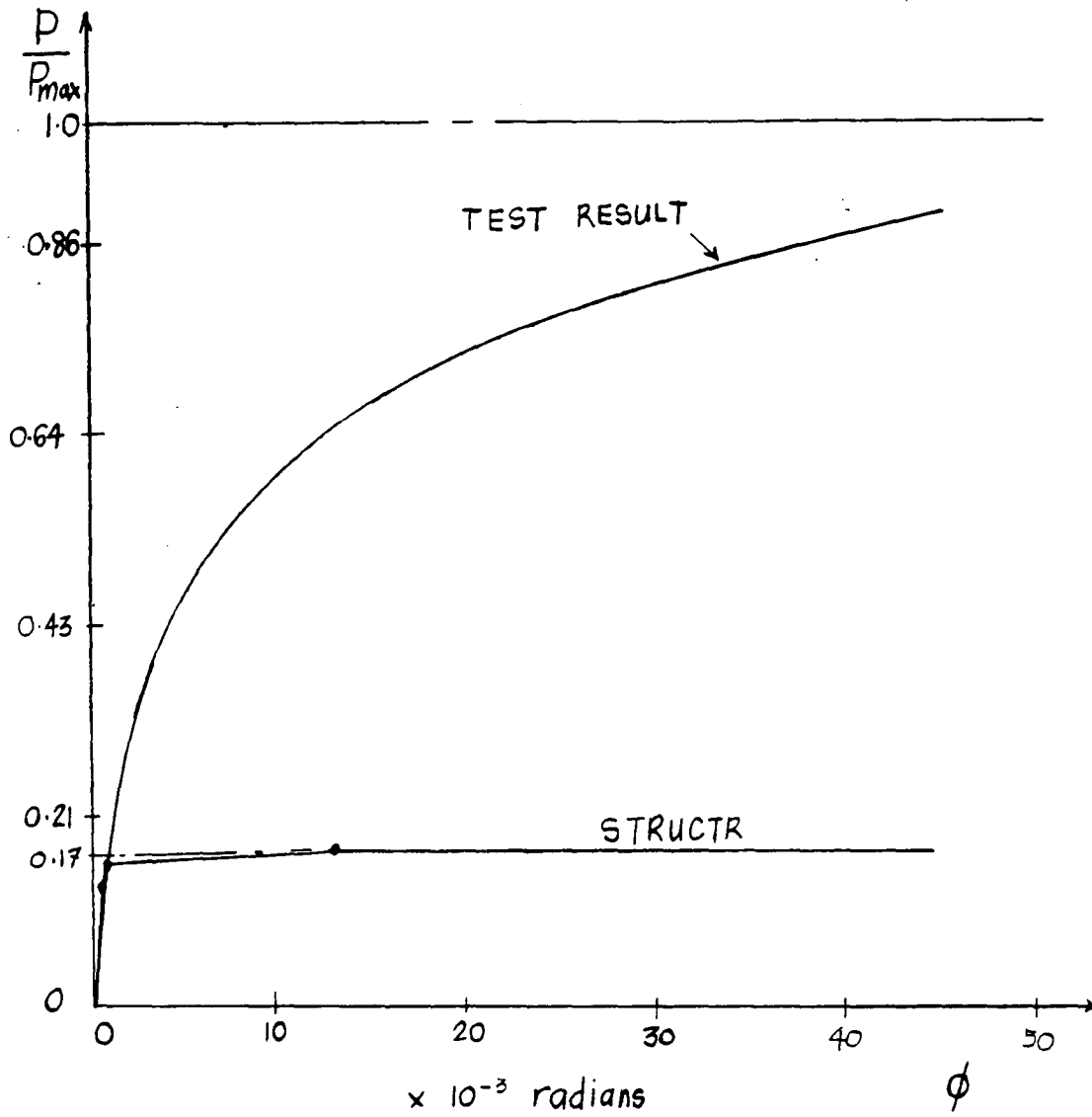


Figure 6-8: TEST 2 LOAD-ROTATION CURVE

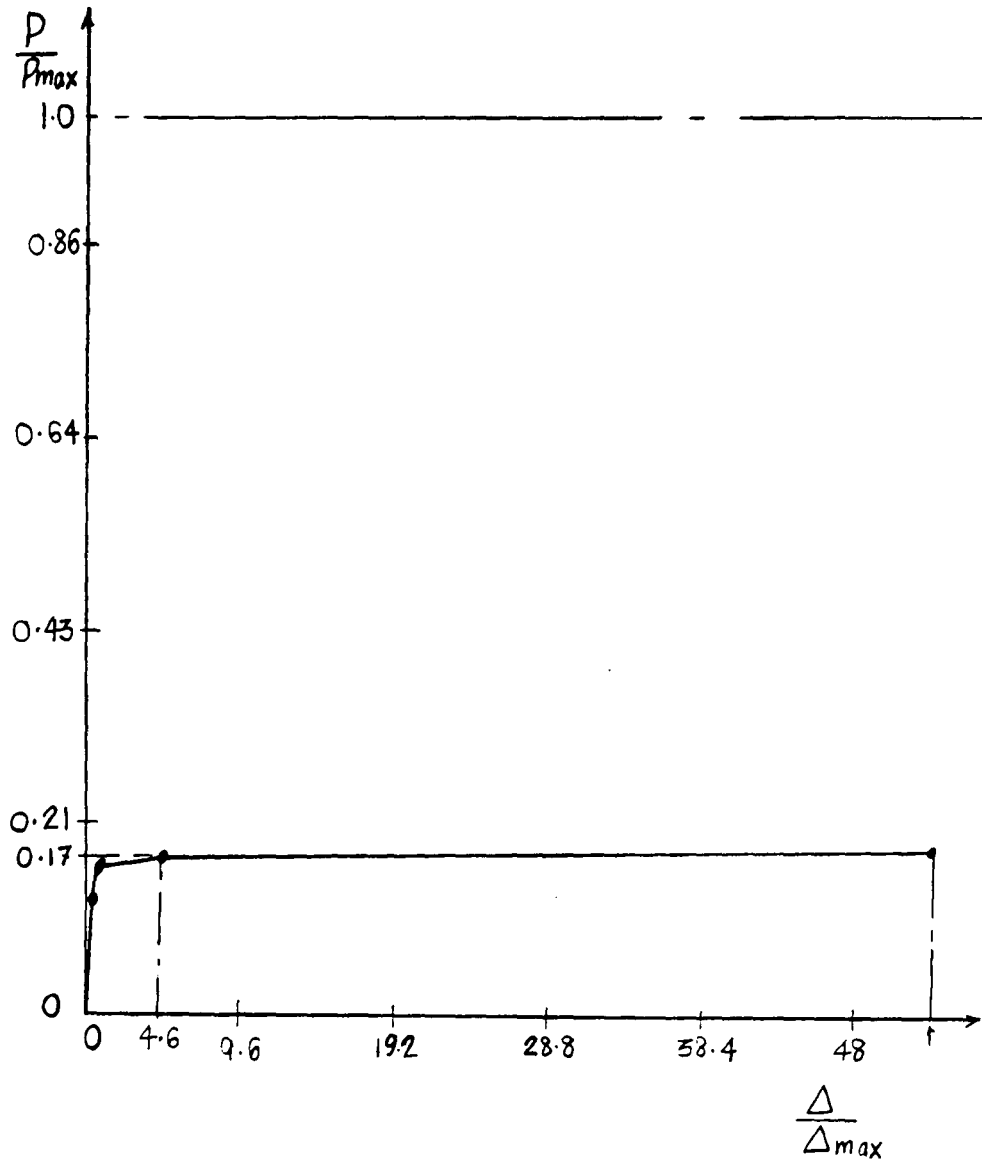


Figure 6-9: TEST 2 LOAD-DEFLECTION CURVE

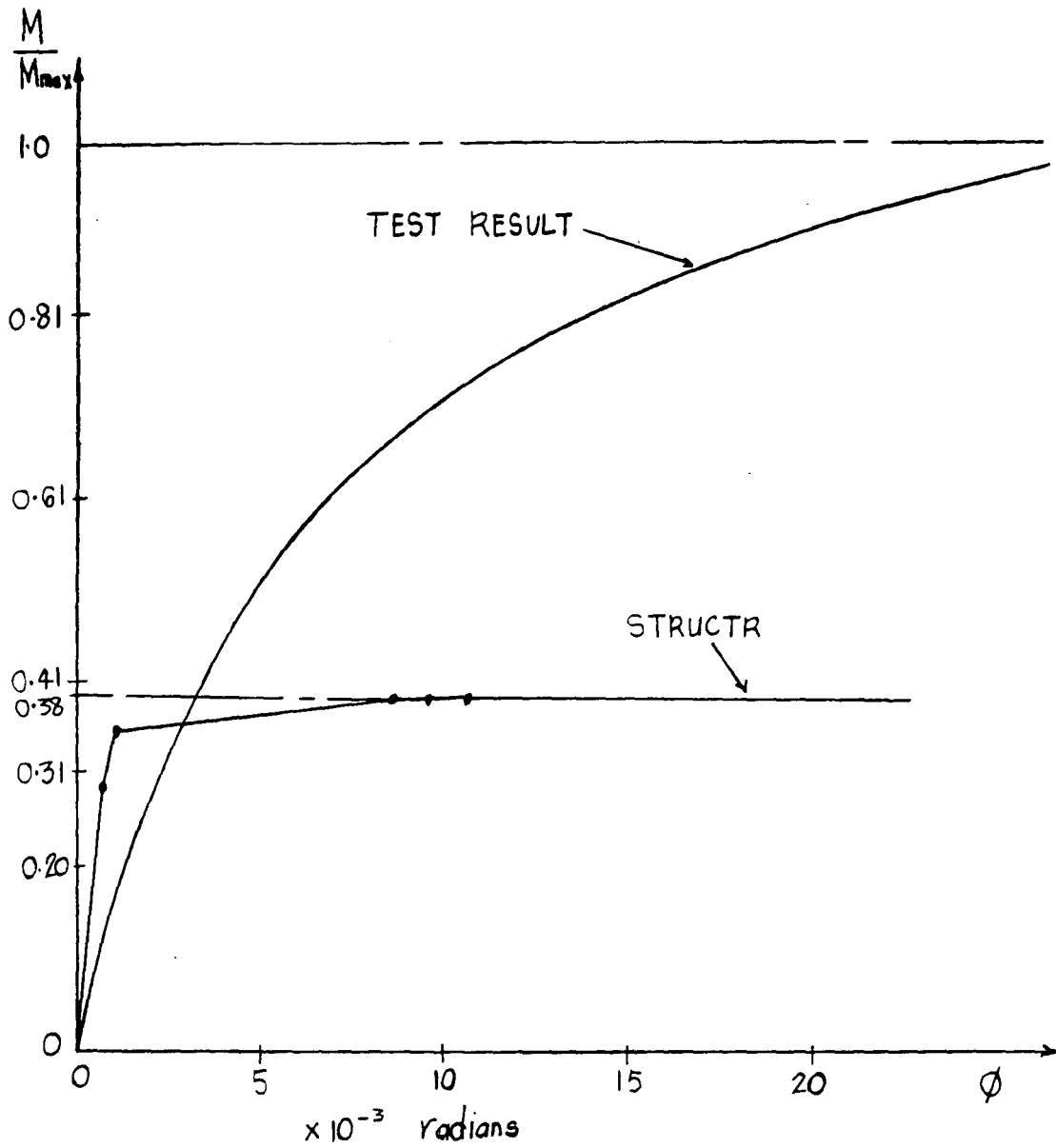


Figure 6-10: TEST 5 MOMENT-ROTATION CURVE

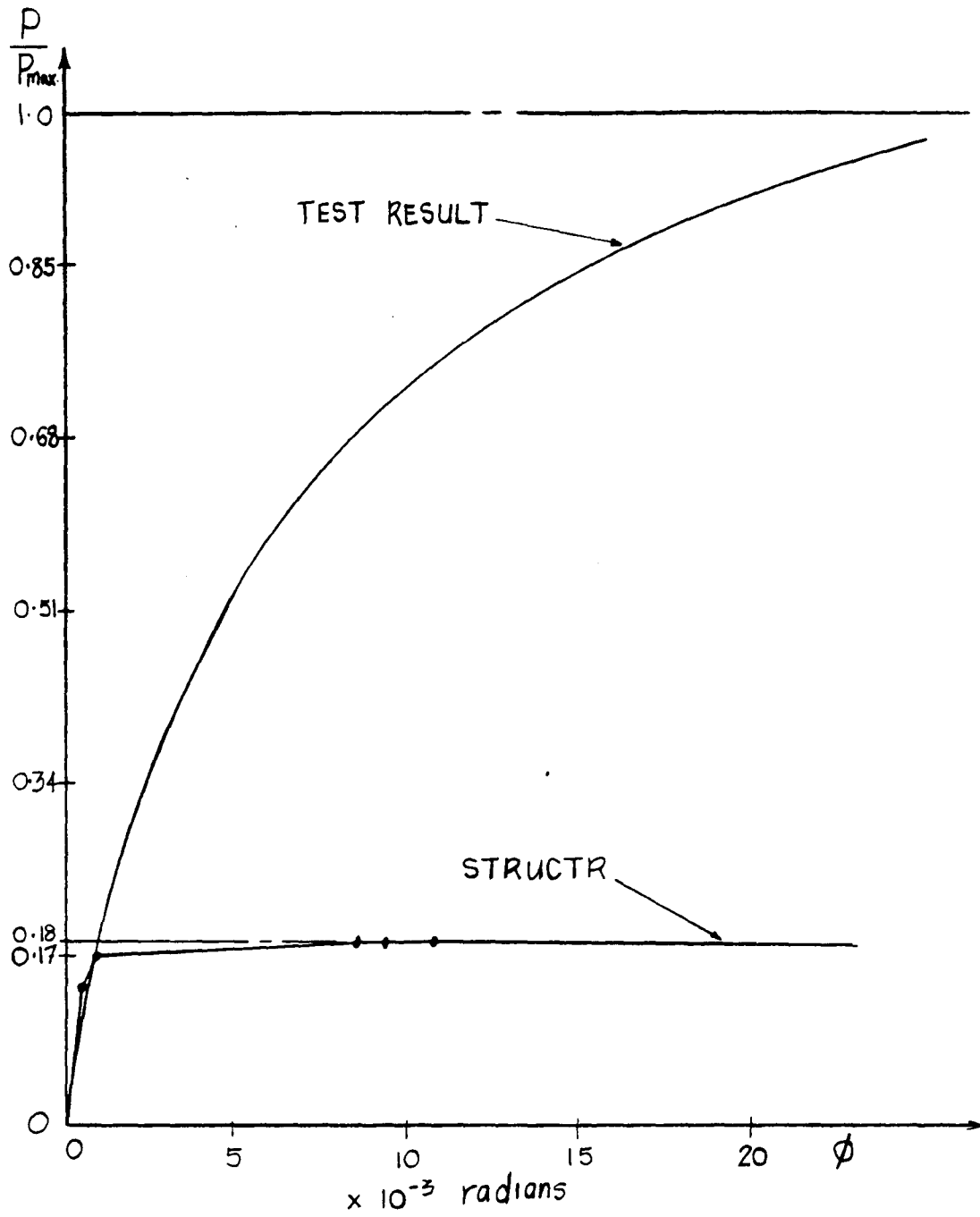


Figure 6-11: TEST 5 LOAD-ROTATION CURVE

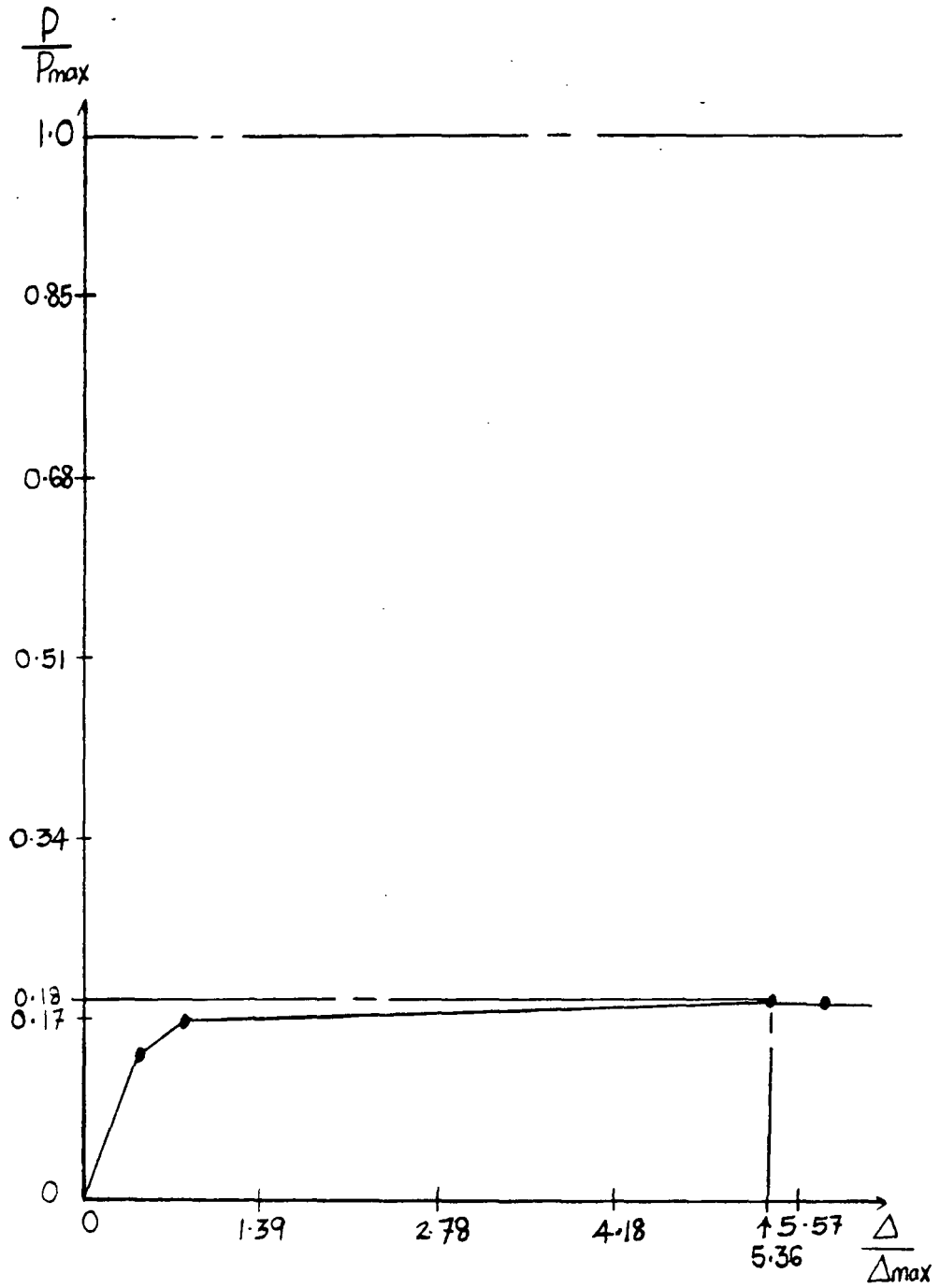


Figure 6-12: TEST 5 LOAD-DEFLECTION CURVE

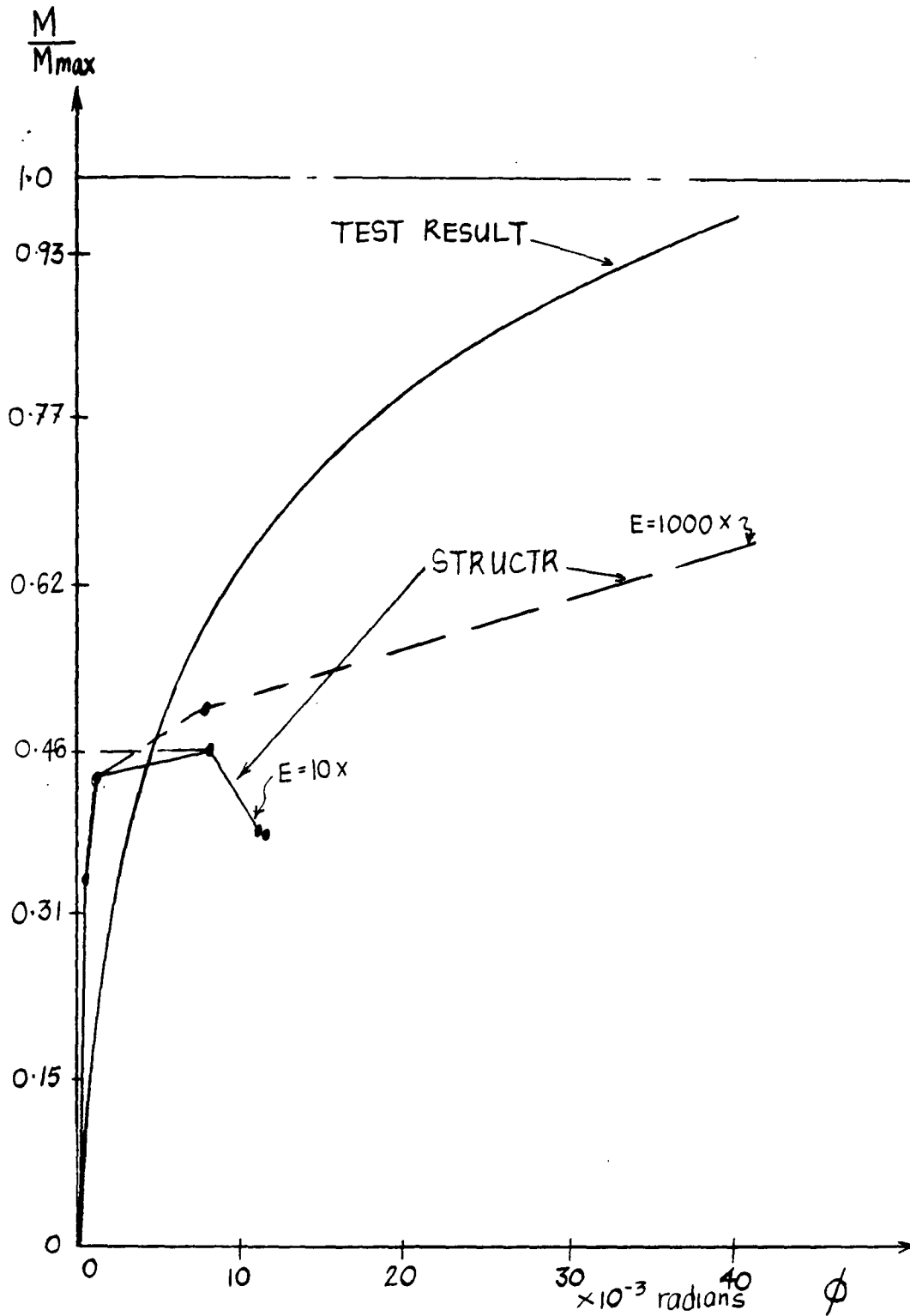


Figure 6-13: TEST 9 MOMENT-ROTATION CURVE

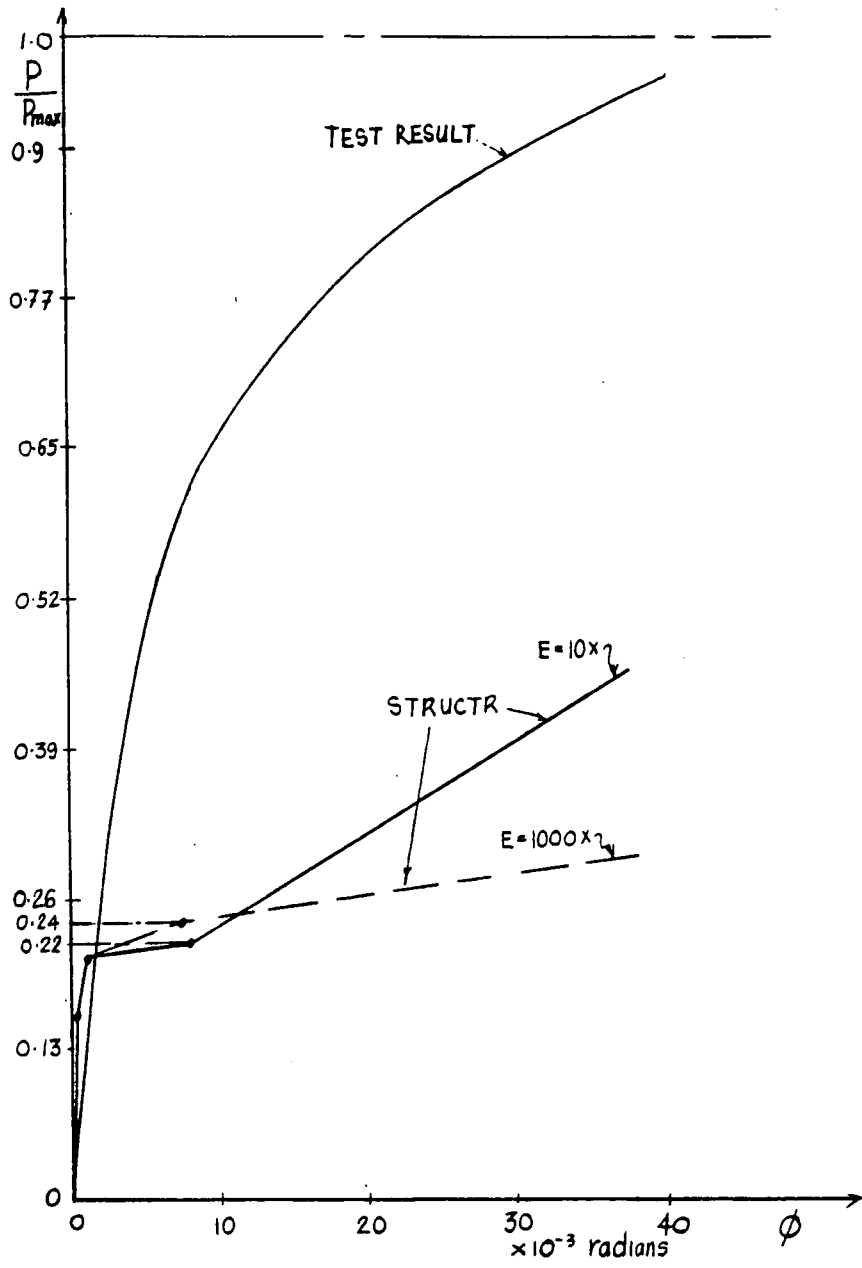


Figure 6-14: TEST 9 LOAD-ROTATION CURVE

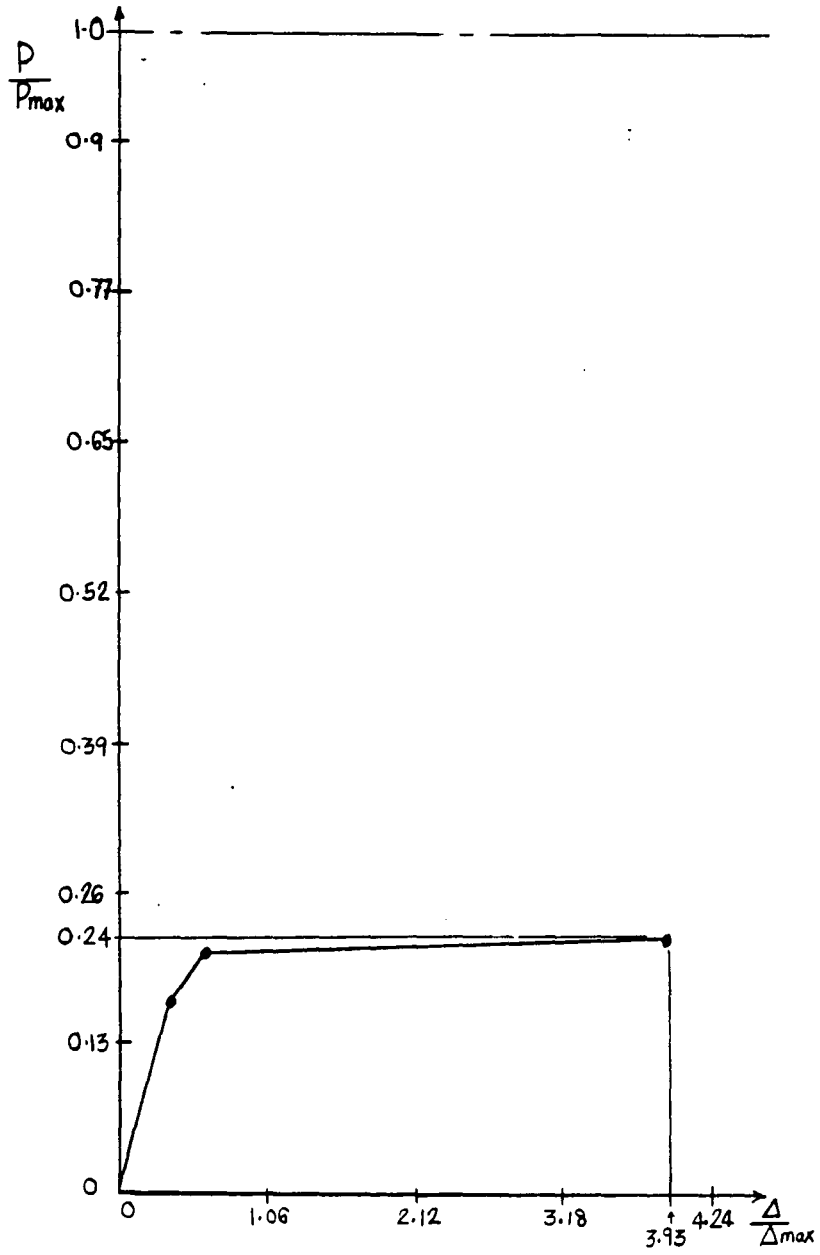


Figure 6-15: TEST 9 LOAD-DEFLECTION CURVE

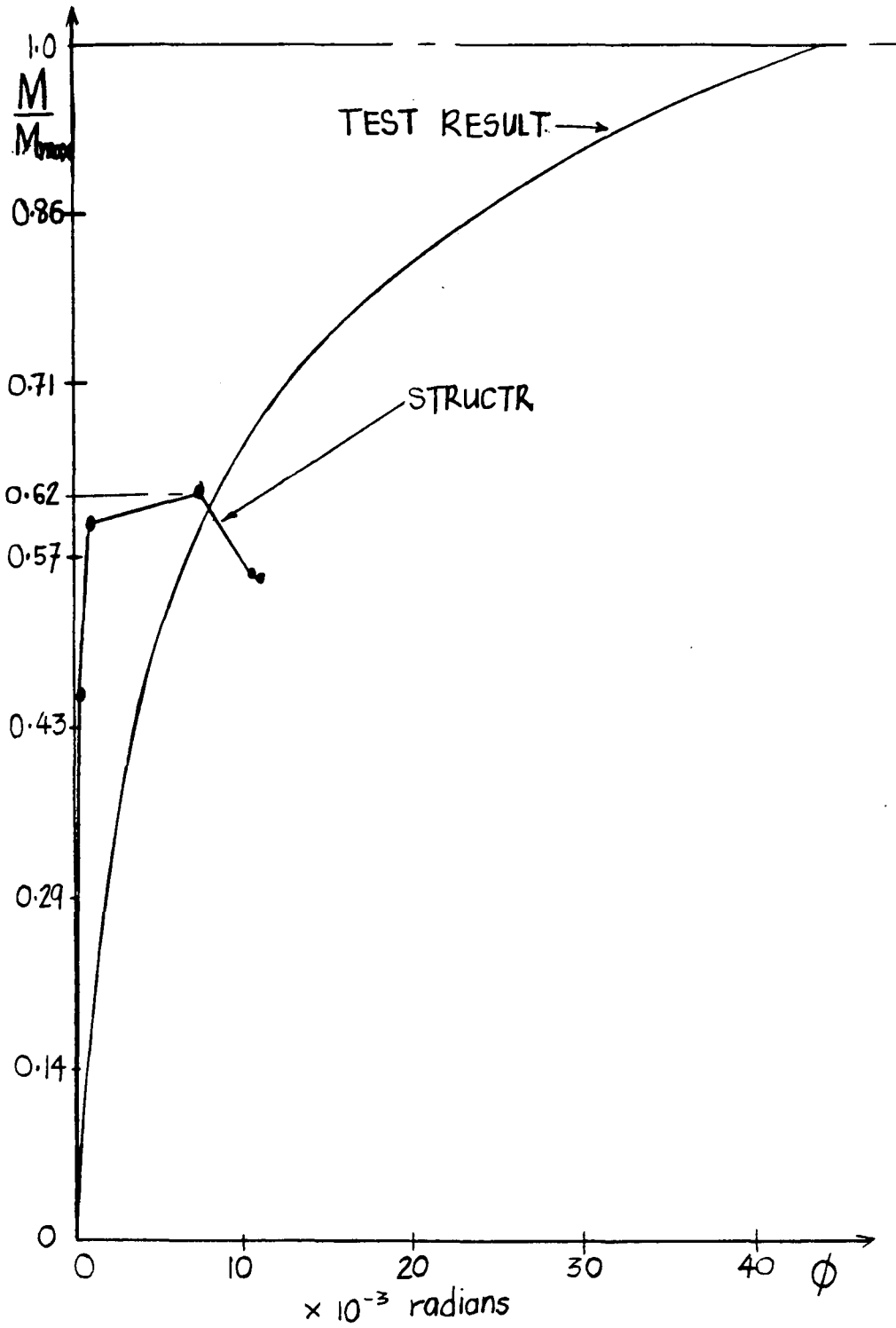


Figure 6-16: TEST 10 MOMENT-ROTATION CURVE

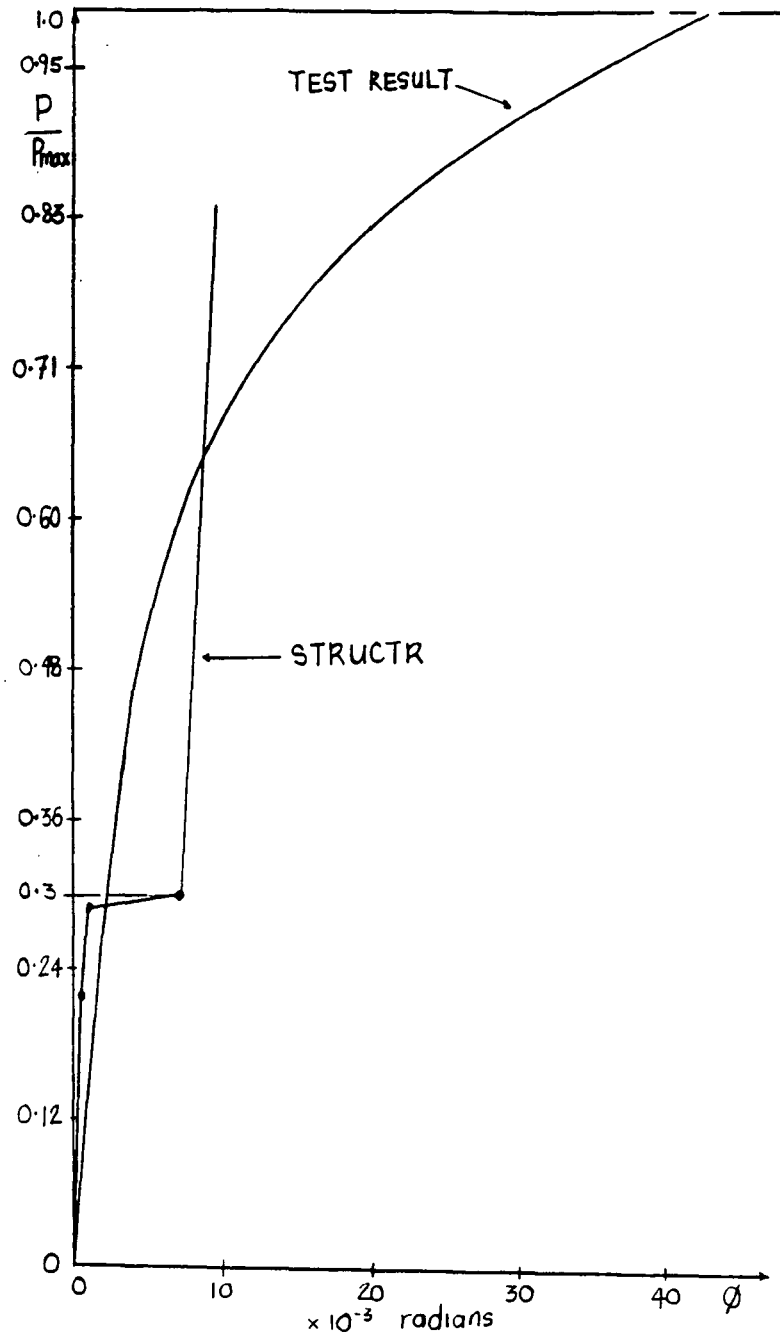


Figure 6-17: TEST 10 LOAD-ROTATION CURVE

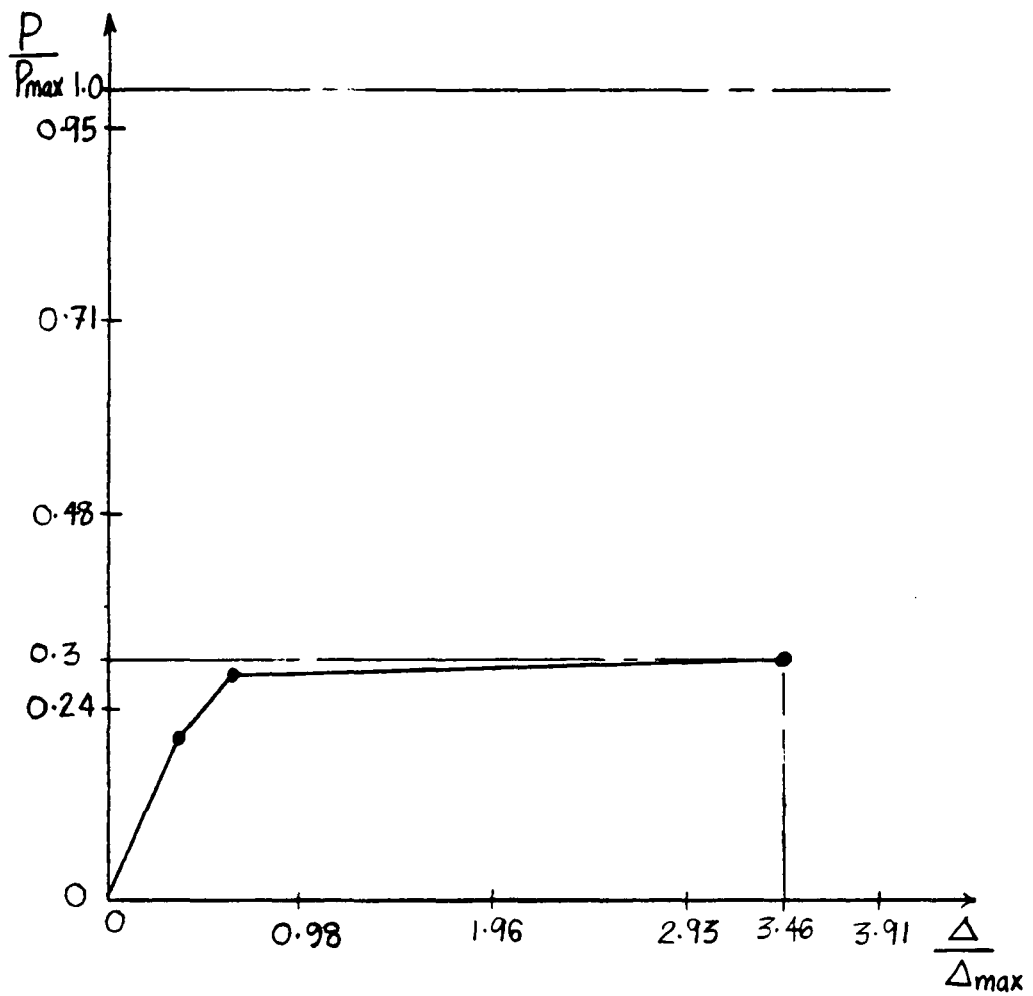


Figure 6-18: TEST 10 LOAD-DEFLECTION CURVE

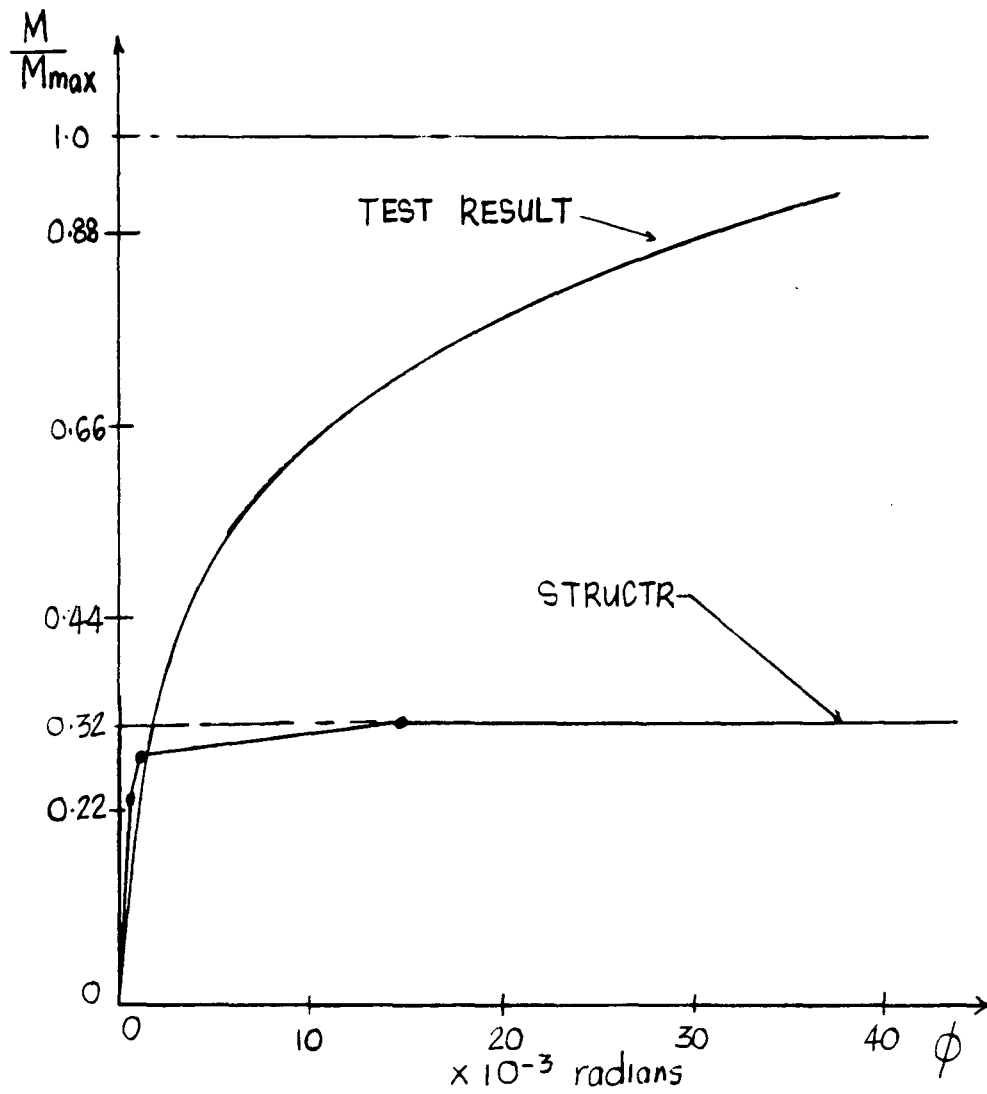


Figure 6-19: TEST 16 MOMENT-ROTATION CURVE

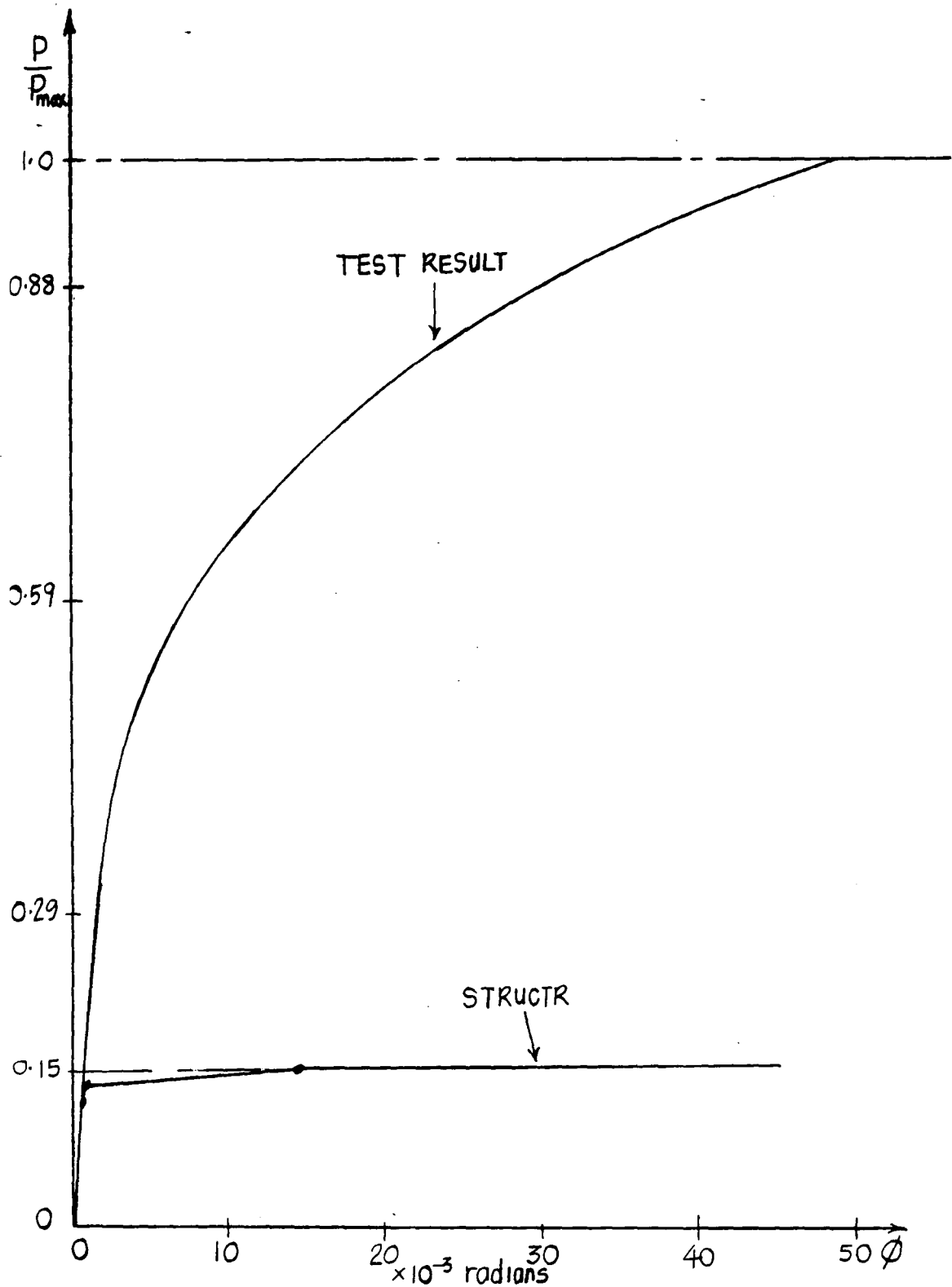


Figure 6-20: TEST 16 LOAD-ROTATION CURVE

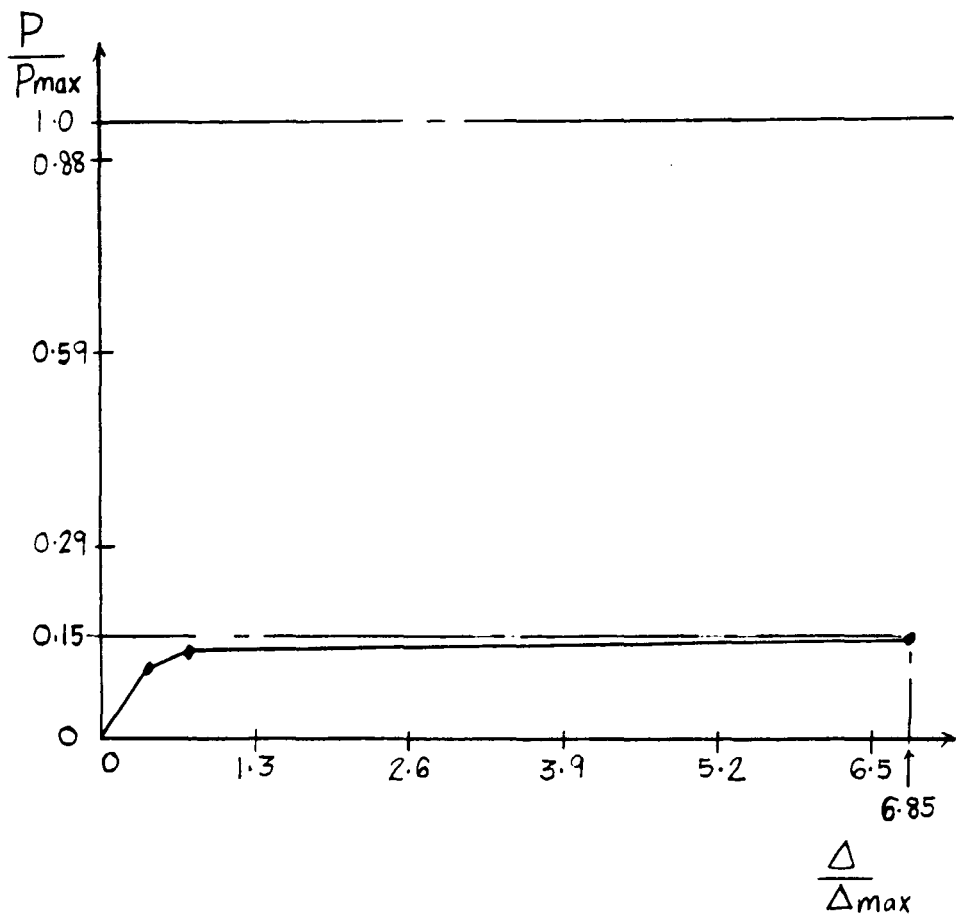


Figure 6-21: TEST 16 LOAD-DEFLECTION CURVE

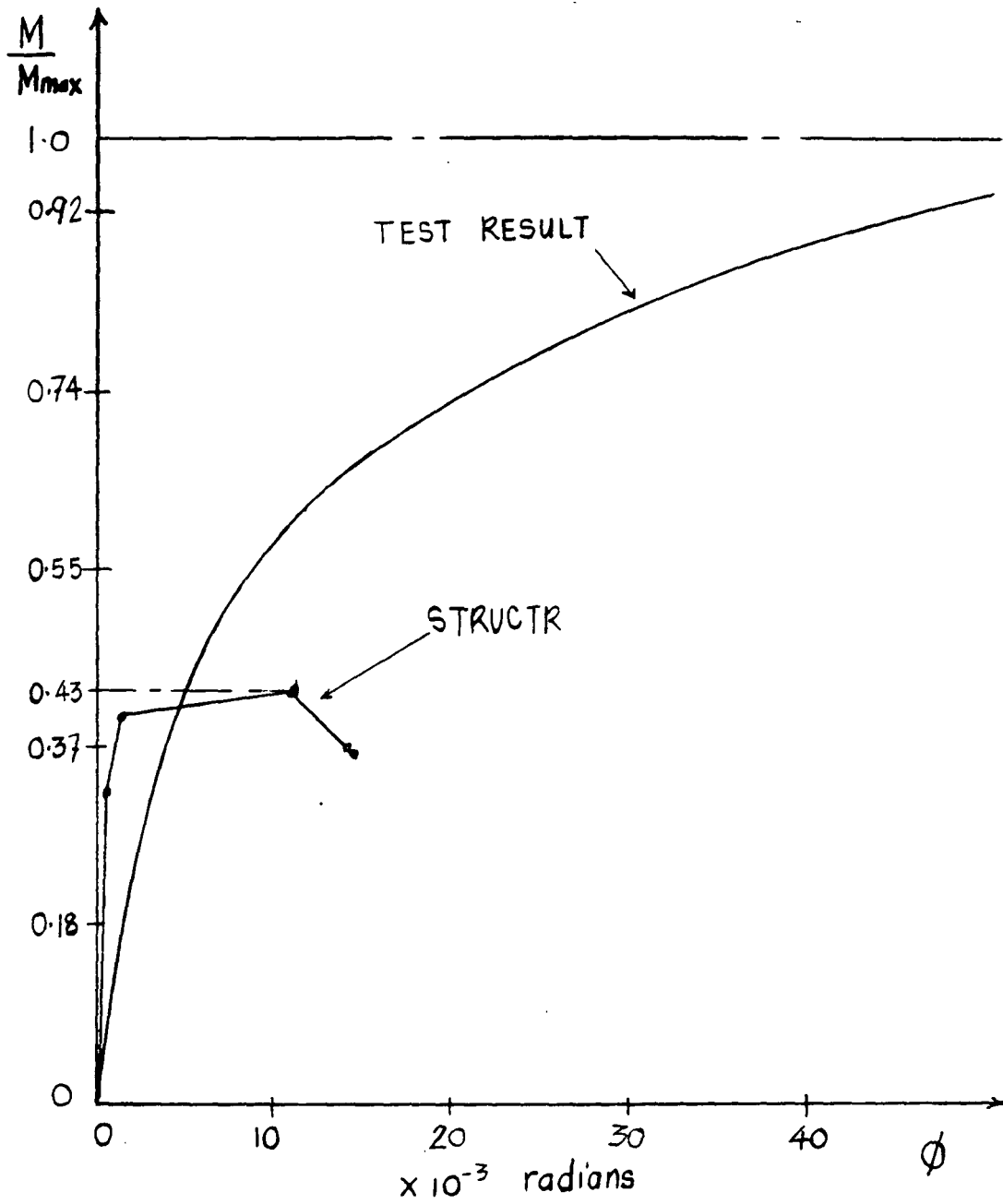


Figure 6-22: TEST 20 MOMENT-ROTATION CURVE

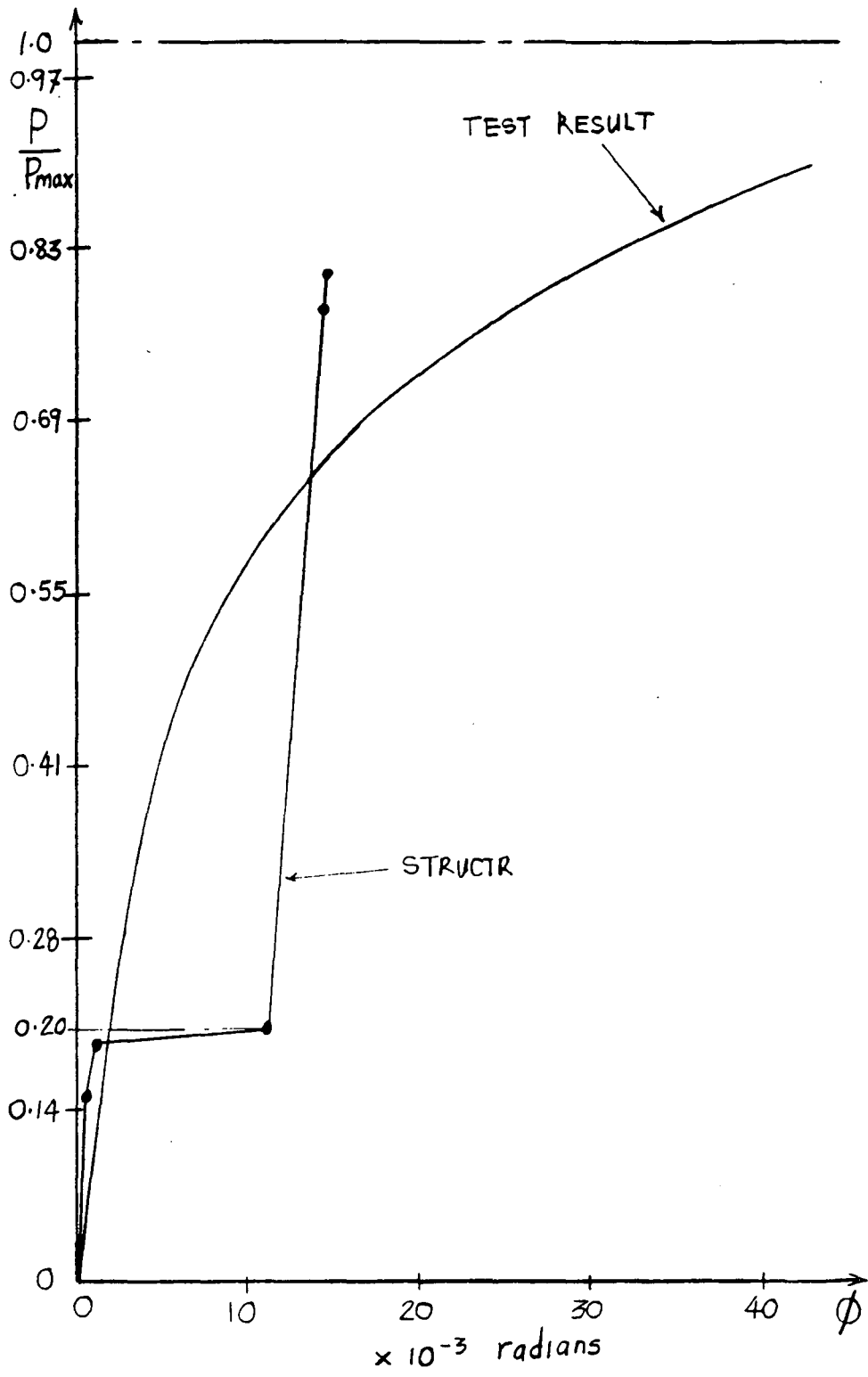


Figure 6-23: TEST 20 LOAD-ROTATION CURVE

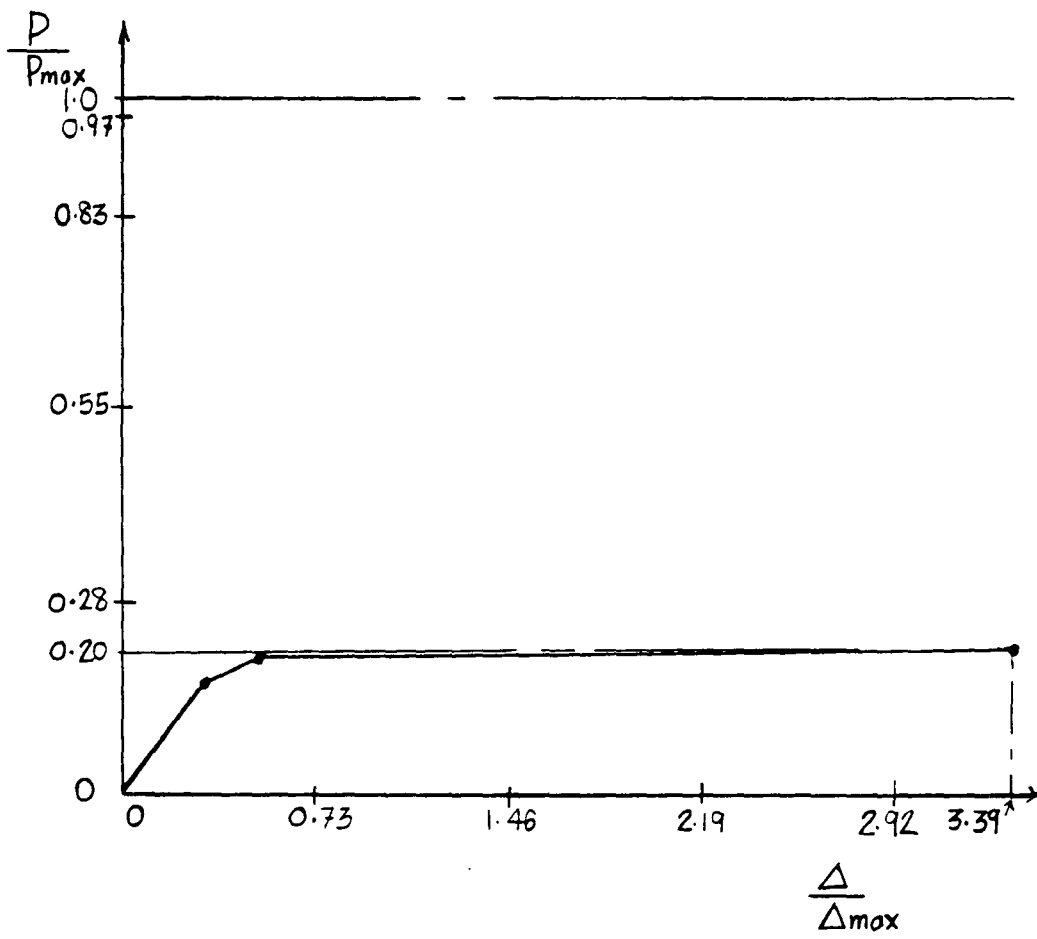


Figure 6-24: TEST 20 LOAD-DEFLECTION CURVE

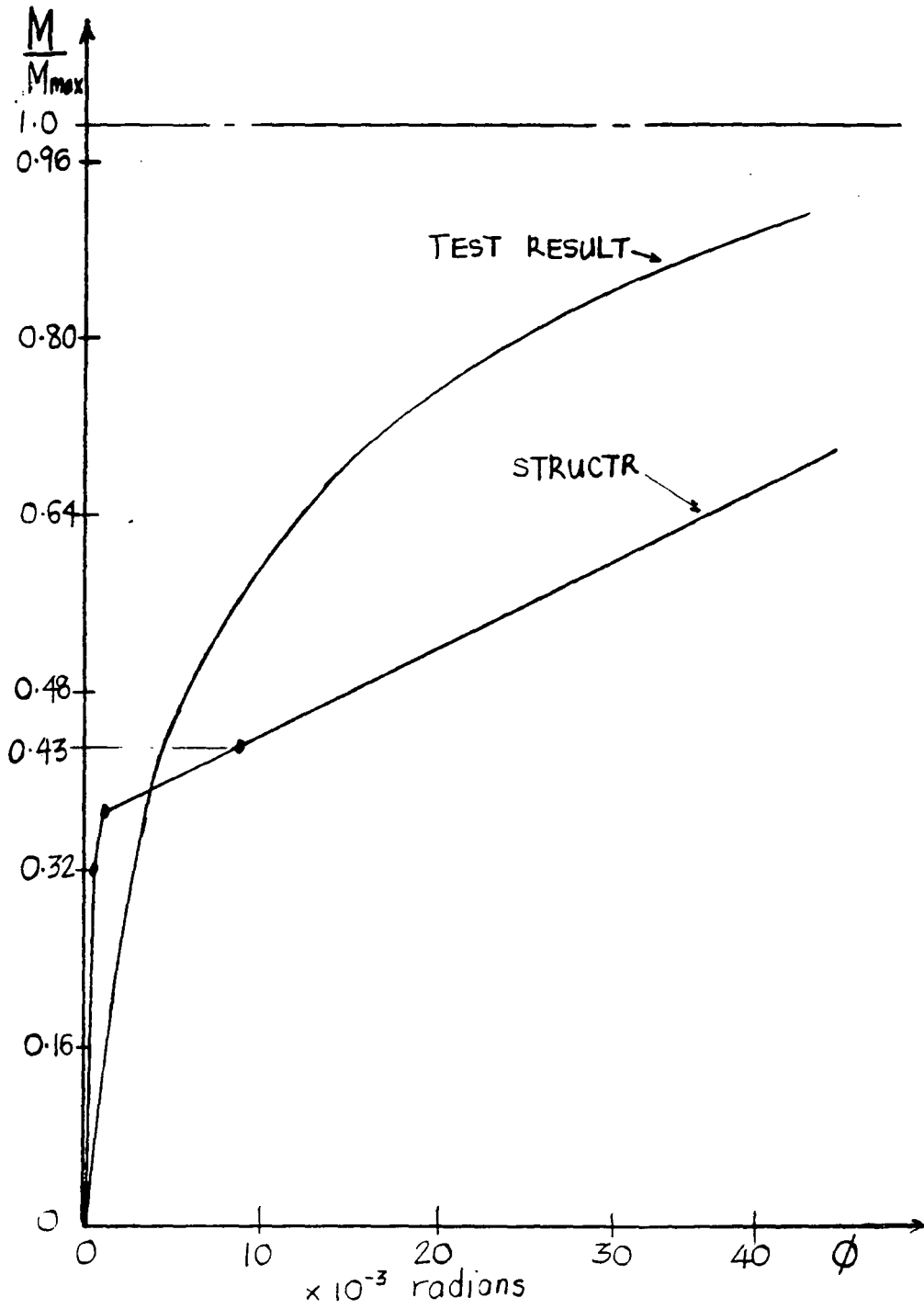


Figure 6-25: TEST 22 MOMENT-ROTATION CURVE

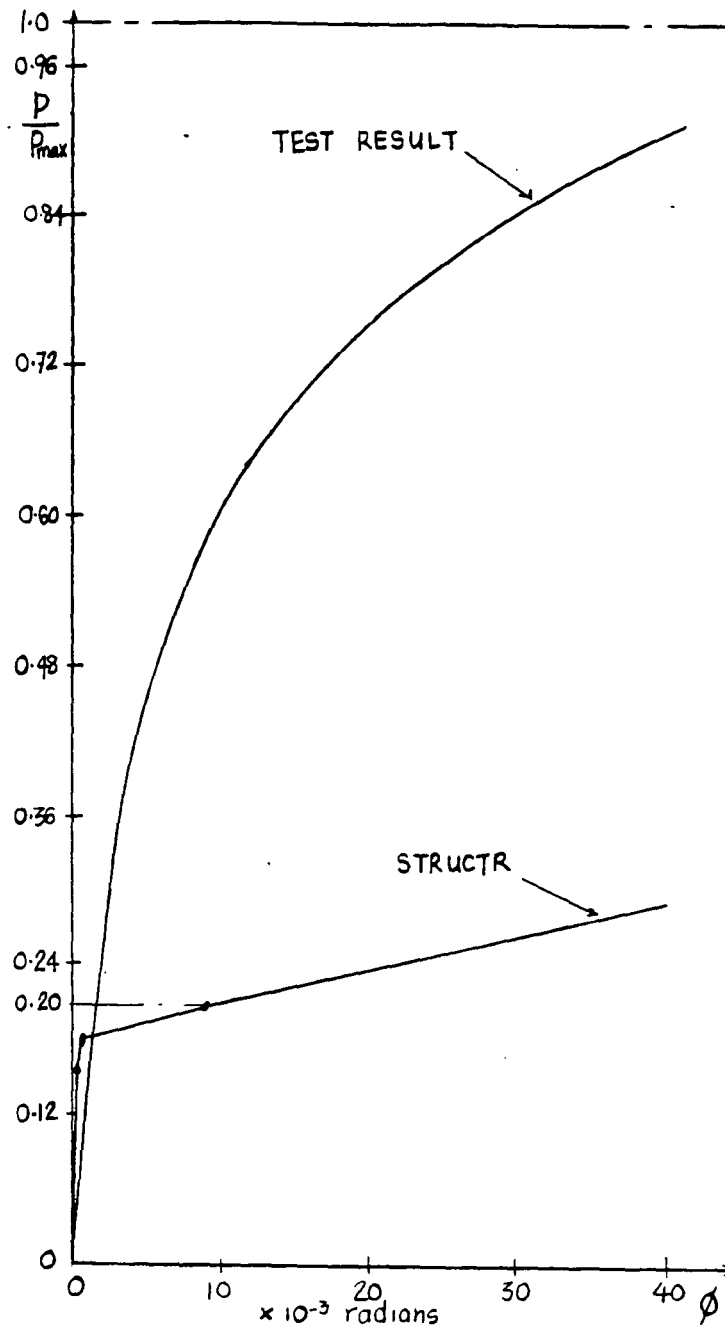


Figure 6-26: TEST 22 LOAD-ROTATION CURVE

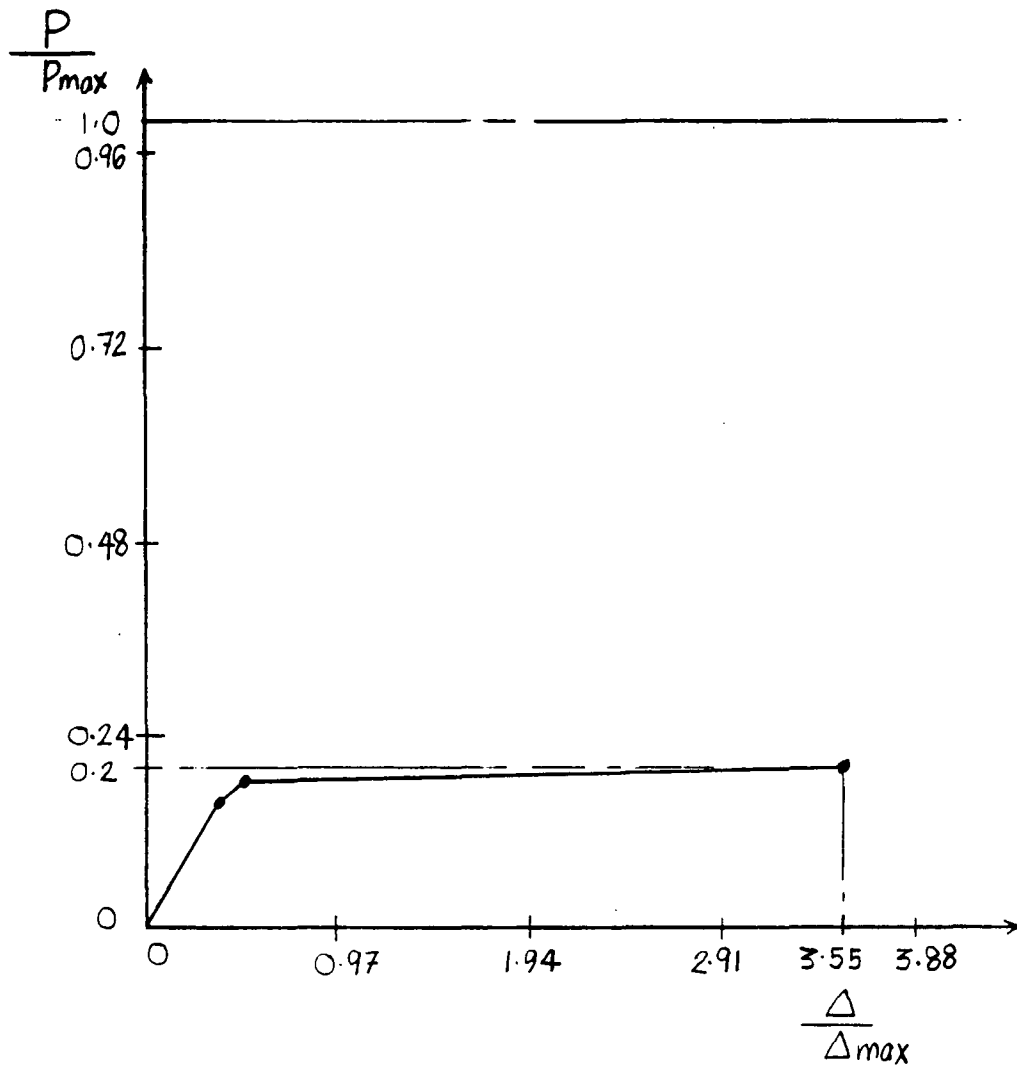


Figure 6-27: TEST 22 LOAD-DEFLECTION CURVE

NOMENCLATURE

- **Z**- Connection Factor, defined as the angle change per unit moment
- **Q**- Additional angle change due to yielding
- **M**- End moment in the beam
- **1/Z**- Slope of the moment rotation curve
- **M_p**- Plastic moment capacity
- **P_y**- Yield load
- **E**- Modulus of Elasticity
- **I**- Moment of Inertia
- **P**- Collapse load or Limiting load
- **P_{max}**- Maximum load of test result
- **M_{max}**- Maximum moment of test result
- **Δ_{max}**- Maximum displacement due to P_{max}

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VITA

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