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ANALYSIS AND DESIGN OF FRAMED COLUMNS UNDER MINOR AXIS BENDING

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Introduction

Columns in a building frame are often subjected to combined axial load and bending moment as a result of the frame action in resisting applied loads. A major concern in the design of framed columns is the effect of instability which may reduce significantly the strength of the column or entire structure. There are two types of instability failure to which careful considerations must be given in design: member instability and overall frame instability. Figure 1(a) shows a typical load versus lateral deflection relationship of an unbraced frame. The gravity load P acting through the lateral deflection Δ produces a secondary overturning moment, called P-Δ moment in the current literature. This additional moment reduces the strength and stiffness of the structure. Failure occurs when the lateral stiffness becomes so small that it is insufficient to resist any increase of the applied load. This is represented by the peak (instability limit) of the load-deflection curve. The member instability effect results from the axial load acting through the deflection δ occurring within the individual columns (Fig. 1b). It is obvious that if a frame is fully braced against sidesway only member instability effect need be considered in the design of its columns. Experience has shown that in a sway frame, frame instability is considerably more important than member

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instability, but the latter could lead to premature local failure. It has been reported that member instability may limit the load-carrying capacity of an unbraced frame even if the structure as a whole still has adequate stiffness to resist frame instability.[1]

Much of the previous work on member instability was on columns subjected to combined axial load and major axis bending. Columns bent about the minor axis have received little attention, although there are several beneficial aspects.[2,3] These columns can usually develop their full in-plane strength without the occurrence of lateral-torsional buckling. Also the shape factor about the minor axis is about 35% larger than that about the major axis. The column formulas contained in most design specifications are based essentially on the studies of columns subjected to major axis bending. A discussion of the development of the currently used design formulas can be found in Ref. 3.

As for overall frame instability, the past work was concerned mostly with building frames in which the columns are oriented for major axis bending.[5,6,7] Although various approaches have been proposed to account for this effect in design,[8-15] specific code provisions are still being developed at this time.

This paper presents a detailed study of the effects of member and frame instability in framed columns under minor axis bending. An important objective is to develop suitable design procedures which will adequately take into account these effects. Specifically, the following are presented in the paper:

1. Ultimate strength analysis of non-sway, pinned-end columns. Numerical solutions for three cases of loading are given.

2. Development of improved interaction formulas for non-sway columns.

3. Analytical study of the behavior of non-sway columns with end restraints.

4. Ultimate strength solutions of two sway frames subjected to combined gravity and lateral loads.

5. A proposed procedure for the design of columns in sway frames. The procedure makes use of a new set of column interaction formulas incorporating the concept of direct moment amplification.

The ultimate strength solutions presented are obtained for wide-flange columns made of A36 steel (yield stress F_y = 36 ksi). The solutions consider the effect of cooling residual stresses. The magnitude and distribution of the residual stresses are the same as those assumed in the previous studies on beam-columns bent about the major axis. Examples are given to illustrate the application of the new design formulas and procedures.

Current Design Procedures

In the allowable-stress method of design, a first-order elastic analysis is performed at the working load, neglecting any effect of instability, and the resulting bending moment and axial force distribution is then used to proportion the members. The formulas used in designing the columns are:

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right)F_b} \leq 1.0 \tag{1}$$

$$\frac{f_a}{0.6\ F_y} + \frac{f_b}{F_b} \leq 1.0 \tag{2}$$

(See Part 1 of the AISC Specification for notation.) The first formula checks the column against possible failure by instability and the second insures that no excessive yielding occurs at the ends of the column. The problem of frame instability was not considered initially in the development of these formulas. They are based on the approximate ultimate strength interaction equations for beam-columns subjected to end moments about the major axis. These equations are given in Part 2 of the AISC Specification.

$$\frac{P}{P_{cr}} + \frac{C_m M_o}{\left(1 - \frac{P}{P_e}\right)M_m} \leq 1.0 \tag{3}$$

$$\frac{P}{P_y} + \frac{M_o}{1.18\ M_p} \leq 1.0 \quad ; \quad M_o \leq M_p \tag{4}$$

in which P_{cr} represents the critical buckling load of the column and is determined from the basic column curve recommended by the Structural Stability Research Council (SSRC):

$$P_{cr} = P_y\left[1 - 0.5\left(\frac{KL/r}{C_c}\right)^2\right] \quad \text{for} \quad \frac{KL}{r} < C_c \tag{5a}$$

$$P_{cr} = P_e = \frac{\pi^2\ EI}{(KL)^2} \quad \text{for} \quad \frac{KL}{r} \geq C_c \tag{5b}$$

For columns subjected to minor axis bending M_m in Eq. (3) is equal to M_p, the full plastic moment about the same axis. The applicability of Eqs. (3)

and (4) to columns bent about the minor axis has not been fully established, even for the case of symmetrical bending.

In Eqs. (1) and (3), the expressions $1/(1 - f_a/F'_e)$ and $1/(1 - P/P_e)$ are called the amplification factors and have the effect of amplifying the computed bending stress f_b or the moment M_o. The factor C_m is to adjust for the shape of the moment diagram. For non-sway columns the AISC Specification gives

$$C_m = 0.6 + 0.4\,\beta \tag{6}$$

in which β is the end moment ratio (β = +1.0 for the case of symmetrical single curvature bending and β = -1.0 for antisymmetrical double curvature bending). A limiting value of $C_m = 0.4$ is specified, which was established from studies on lateral-torsional buckling of columns under major axis bending.[16] Since lateral-torsional buckling is not a problem in the case of minor axis bending, this restriction on C_m could probably be removed.

Equations (1) and (2) are in use in the design of columns in both sway and non-sway frames. To account for the restraining effect offered by the adjacent members and the overall frame action, the actual length of the column is modified by an effective length factor K. F_a, F'_e in Eq. (1) are then calculated using the effective column length, which is smaller than the actual length for columns in a non-sway frame and is greater than the actual length in a sway frame. The use of an effective length greater than the actual length in sway frame design is to recognize, in an indirect way, the effect of frame instability. An additional provision is to use a C_m value of 0.85, which is likely to be greater than that required by Eq. (6).

The reason for this is that double curvature bending (negative values of β) often prevails in framed columns, especially when the frame is also subjected to lateral load. It is apparent that both measures may result in an increase in the sizes of the columns but not the girders. On the other hand, if the design is governed by Eq. (2), then nowhere the effect of frame instability is taken into account. This may lead to unsafe designs.

Several studies have recently been made to examine the adequacy of the current design procedure. It has been reported that the use of K factors greater than 1.0 and $C_m = 0.85$ in column design increases the strength of sway frames only slightly, and this increase is likely to be less than the reduction caused by the P-Δ moment.[6,7] If the P-Δ moment is large in comparison with the lateral load moment, then the present approach could produce designs with a load factor less than 1.30 (this is the load factor specified in Part 2 of the AISC Specification for the case of combined loading). No similar study has been carried out on frames with weak axis column orientation.

The effective length factor is usually determined for each individual column using the alignment chart for the "sway permitted case". K values as large as 3 or 4 are not uncommon, and they may differ widely for the individual columns in the same story.[8] For this reason, a "modified effective length" approach has been suggested in which the amplification factors of the various columns are replaced by a single storywise amplification factor.[1] It is given by $1/(1 - \Sigma f_a/\Sigma F'_e)$ or $1/(1 - \Sigma P/\Sigma P_e)$ in which Σ represents summation over all the columns in the story. F'_e and P_e are based on the effective column length.

Methods which permit the direct inclusion of the P-Δ moment in design calculations have been proposed. In one of the methods, known as the P-Δ method, the secondary moment is determined through a series of successive iterations, starting with the moment and deflection from a first-order analysis.[12,13] The secondary moment thus obtained is then included in proportioning the members. In another method the second-order moment is calculated by applying an amplification factor to the first-order moment, much like the procedure used to account for member instability. This method will be referred to as the "direct moment amplification" method. In both the modified effective length approach and the direct moment amplification approach, the columns are treated as non-sway columns and their design is governed by Eqs. (1) and (2). The actual column length (K = 1.0) is used in determining F_a and F'_e and C_m is that given by Eq. (6).

Non-Sway Unrestrained Columns

Ultimate strength solutions of pinned end columns subjected to three types of applied load have been obtained. Results are presented in the form of interaction curve for beam-columns subjected to (1) equal end moments ($\beta = +1.0$), (2) one end moment ($\beta = 0$), and (3) a concentrated lateral load at mid-span. The results in Figs. 2 and 3 are obtained by numerically integrating the moment-thrust-curvature relationships using the column-deflection-curve approach.[17] The curves in Fig. 4 are adapted from the solutions for columns in a sway frame whose height is equal to half of the column length and with an infinitely stiff girder.[18] Intersections on the vertical axis represent the ultimate strength of the columns subjected to pure axial compression.

A comparison of the column strength shown in Fig. 3 with the strength predicted by Eq. (3) is given in Fig. 5 for three column slenderness ratios.* The agreement is not considered satisfactory. For columns of low slenderness ratio, Eq. (3) is very conservative and may underestimate the moment-carrying capacity by more than 100% is some cases. For slender columns, on the other hand, Eq. (3) becomes unconservative. It is recalled that for major axis bending Eq. (3) has been found to give good predictions of column strength.[4]

Since Eq. (3) does not provide good predictions when applied to columns bent about the minor axis, it is highly unlikely that the current design procedure, which is based on this equation and Eq. (4), would yield accurate results. It is also felt that the current procedure can not be significantly improved by merely improving Eq. (3).** A different design procedure is therefore developed. In this procedure, column strength is determined by two new interaction formulas, which retain all the important features of Eq. (3). Included in these formulas is an amplification factor B_1, whose value should always be greater than or at least equal to 1.0

$$B_1 = \frac{C_m}{1 - \frac{P}{P_e}} \geq 1.0 \qquad (7)$$

New coefficients are introduced into the formulas to allow for a more accurate evaluation of the effect of moment amplification.

*To make the comparison consistent, the first term in Eq. (3) assumes the value defined by the intersection on the vertical axis of the theoretical curve of Fig. 3.

**A possible way to improve Eq. (3) is described in the Appendix.

The ultimate strength solutions given in Figs. 2 and 3 are used in "reverse" to develop the new interaction formulas. For each column, B_1 values are first calculated at various levels of axial load. These values are then multiplied to the M_o values given by the interaction curves. Figure 6 shows the resulting $B_1 M_o$ versus P/P_y relationship for a column with $L/r_y = 80$ and subjected to symmetrical bending. Another plot is given in Fig. 7 for the same column but having only one end moment. In the latter case, $B_1 M_o$ is equal to M_o (that is, $B_1 = 1.0$) for P/P_y between 0 and 0.5. Another subject to be noted in the calculation of B_1 is that inelastic column action is considered in determining the parameter P_e. The basic SSRC column curve (Eq. 5a) adopted in this study implies that columns buckle inelastically when the critical load is between 0.5 P_y and P_y. The buckling load may be determined by replacing the elastic modulus E by the tangent modulus E_t given by

$$E_t = E \cdot 4 \frac{P}{P_y}\left(1 - \frac{P}{P_y}\right) \tag{8}$$

or, nondimensionally,

$$\frac{E_t}{E} = \tau = 4 \frac{P}{P_y}\left(1 - \frac{P}{P_y}\right) \tag{9}$$

Equation (8) or (9) is used in computing P_e when $P/P_y > 0.5$.

The relationship between P and $B_1 M_o$ has been found to be approximately linear for $M/M_p < 5/6$ for all slenderness ratios included in this study. Based on this observation, the following set of bilinear equations is proposed for predicting the load-carrying capacity:

$$\frac{P}{P_{cr}} + m_1 \frac{B_1 M_o}{M_p} = 1.0 \tag{10}$$

and

$$\frac{P}{P_{cr}} + n_1 \frac{B_1 M_o}{M_p} = n_1 \tag{11}$$

in which

$$n_1 = 6 - 5\, m_1 \tag{12}$$

The coefficient m_1, which defines the slope of the P versus $B_1 M_o$ plot, can be determined graphically using the available analytical results. The m_1 values thus determined are plotted as a function of L/r_y in Fig. 8. By curve fitting, the following expression for m_1 is obtained

$$m_1 = 0.27 + 0.3\,\beta + 0.61\,\lambda \leq 1.0 \tag{13}$$

in which λ is the normalized slenderness ratio defined by

$$\lambda = \frac{1}{\pi}\sqrt{\frac{F_y}{E}}\,\frac{L}{r_y} \tag{14}$$

The adequacy of the proposed equations may be seen in Figs. 6 and 7 where the predicted moment capacities are compared with the theoretically calculated amplified moment, $B_1 M_o$. Comparisons have also been made for columns bent in double curvature (negative β), and Eqs. (10) and (11) have been found to give good estimates of the ultimate strength.

A similar treatment has also been carried out for beam-columns subjected to a concentrated load at mid-span (Fig. 4). In this case B_1 is given by the approximate expression

$$B_1 = \frac{C_m}{1 - \frac{P}{P_e}} = \frac{1 - 0.2\,\frac{P}{P_e}}{1 - \frac{P}{P_e}} \tag{15}$$

When plotted, the P versus $B_1 M_o$ curves show a similar trend as those given in Fig. 6 for columns subjected to equal end moments, and it is found that the ultimate strength can be closely predicted by Eqs. (10) and (11) with m_1 modified as follows

$$m_1 = 0.85(0.27 + 0.23\,\beta + 0.61\,\lambda) \leq 1.0 \tag{16}$$

A β value of 1.0 is to be used in the above equation.

Non-Sway Restrained Columns

The response of a column with end restraints is considerably different from that of a pinned-end column. When a bending moment is applied to a joint of a restrained column, it is resisted partly by the column and partly by the restraining member. The exact distribution depends on the rotational stiffnesses of the members. An increase in the axial load reduces the stiffness of the column. This results in an increase in the portion of the moment resisted by the restraining member. Figure 9 illustrates the behavior of a simple restrained column. The restraint provided by the beam is defined in terms of the G value.

$$G = \frac{EI_c/L_c}{EI_b/L_b} \tag{17}$$

in which I_c, I_b are, respectively, the moments of inertia of the column and the restraining beam, L_c is the height of the column and L_b the length of the beam. The joint moment M_A is held constant while the axial load P increases from zero to the critical value (corresponding to the Euler buckling load of the column). Elastic behavior is assumed throughout. It is seen that as P increases the column end moment M_{end} decreases and the beam moment M_b increases. At high levels of P, the direction of M_{end} becomes reversed and M_b is equal to the sum of M_A and M_{end}. The restraining beam must therefore be designed for a larger moment capacity.

Also shown in Fig. 9 is the variation of the maximum moment M_{max} in the column as a function of P. At low levels of P, M_{max} occurs at the column top, and it is equal to M_o, the first-order moment. As P increases, M_{max} gradually moves away from the column top and eventually reaches a value considerably greater than M_o. For a given value of P, M_{max} may be determined by using the amplification factor given by Eq. (7). It is interesting to note that very close agreement with the exact solution may be obtained if P_e is replaced by P'_e which is based on the effective length KL_c ($K < 1.0$) of the column. The reason for this is that the restraining beam tends to delay the development of the second-order moment in the columns. Equations (10) and (11) are therefore applicable to restrained columns if P'_e is used in calculating the amplification factor B_1.

Comparison with Test Results

Equations (10) and (11) have been checked against previously reported tests on wide-flange columns conducted by Johnston and Cheney at Lehigh University.[19] All the columns had pinned ends and were loaded eccentrically with varying amounts of end eccentricities. The essential properties of the test specimens and the results obtained are summarized in Table 1. In Fig. 10, the test results are compared with the proposed interaction equations. Except in the region of low axial load, the proposed equations give good predictions of the ultimate strength.

Design Example 1

The pinned-end column in Fig. 11 is subjected to an axial load of 80 kip and a minor axis bending moment of 48 kip-in. The ends of the column are braced against sway. Design the column by the allowable-stress method, using the proposed interaction formulas. Use A36 steel.

Equations (10) and (11) may be written in terms of the working stresses and the allowable stresses:

$$\frac{f_s}{F_a} + m_1 \frac{B_1 f_b}{F_b} \leq 1.0 \tag{18}$$

$$\frac{f_a}{F_a} + n_1 \frac{B_1 f_b}{F_b} \leq n_1 \tag{19}$$

and B_1 in this case is

$$B_1 = \frac{0.6}{1 - \frac{f_a}{F'_e}} \geq 1.0$$

Try W6x25

$A = 7.34 \text{ in}^2$, $S_y = 5.61 \text{ in}^3$

$r_y = 1.52$ in , $\frac{L}{r_y} = 9.47$, $\lambda = 1.06$

From AISC Manual: $F_a = 13.64$ ksi , $F'_e = 16.65$ ksi

$F_b = 0.75 F_y = 27.0$ ksi

$f_a = \frac{80}{7.34} = 10.90$ ksi $\quad f_b = \frac{48}{5.61} = 8.56$ ksi

$$B_1 = \frac{0.6}{1 - \frac{10.90}{16.65}} = 1.74 > 1.0$$

$$m_1 = 0.27 + 0.61 \times 1.06 = 0.92$$

Check Eq. (18)

$$\frac{10.90}{13.64} + 0.92 \frac{1.74 \times 8.56}{27.0} = 1.31 \quad \text{N.G.}$$

<u>Try W8x28</u>

$$A = 8.25 \text{ in}^2 \ , \ S_y = 6.63 \text{ in}^3$$

$$r_y = 1.62 \text{ in} \quad \frac{L}{r_y} = 88.9 \quad \lambda = 0.998$$

$$F_a = 14.33 \text{ ksi} \ , \ F'_e = 18.89 \text{ ksi} \ , \ F_b = 27.0 \text{ ksi}$$

$$f_a = \frac{80}{8.25} = 9.70 \text{ ksi} \qquad f_b = \frac{48}{6.63} = 7.24 \text{ ksi}$$

$$B_1 = \frac{0.6}{1 - \frac{9.70}{18.89}} = 1.23 > 1.0$$

$$m_1 = 0.27 + 0.61 \times 0.998 = 0.88 \ , \ n_1 = 1.60$$

Check Eq. (18)

$$\frac{9.70}{14.33} + 0.88 \frac{1.23 \times 7.24}{27.0} = 0.97 < 1.0 \quad \text{OK}$$

Check Eq. (19)

$$\frac{9.70}{14.33} + 1.60 \frac{1.23 \times 7.24}{27.0} = 1.20 < 1.60 \quad \text{OK}$$

Use W8x28.

Note that in the above calculations different factors of safety are used for F_a, F'_e and F_b, as specified in the AISC Specification. It has been found that the use of nonuniform factors of safety leads to conservative results.[18] It is, however, not the intent of this paper to discuss this aspect of the design problem.

Restrained Columns in Sway Frames

Analytical work has recently been carried out to study the strength of restrained columns in laterally unbraced frames. The frames selected were simple portal frames having pinned bases, as shown in Figs. 12 and 13. The frame in Fig. 12 is symmetrical and its stiffness to resist lateral load (or sidesway buckling) is provided by both columns. On the other hand, the frame in Fig. 13 has only one column that resists the lateral load and the P-Δ moment. The column with hinged top resists only vertical load.

The solutions given in Figs. 12 and 13 are obtained by following an approach developed in Ref. 20 and the details can be found in Ref. 18. Each curve defines for a given structure and loading condition the relationship between the axial load and the first-order column end moment when failure due to frame instability occurs. The analytical results will be used to develop a proposed design procedure for columns in sway frames.

Before discussing the design procedure, it is useful to examine first the behavior of a sway column and compare it with that of a non-sway column. Shown in Fig. 14 are the axial load versus end moment relationships of a restrained sway column (case b) and an unrestrained non-sway column (case a). Both columns have a slenderness ratio of 40, and, for the sway column, the stiffness of the restrained beam is assumed to be infinite (G = 0). The curve for case (a) is taken directly from Fig. 3. For case (b), two curves are shown: the dashed curve gives the first-order moment at the top of the column and the solid curve shows the second-order moment which includes the contribution of the P-Δ moment. Both curves are for the ultimate load condition. The end moment M_{end} of the sway column is considerably lower than the moment M_o of the non-sway column, except when the axial load is low. This suggests that the interaction equations developed for non-sway columns are not directly applicable to sway columns. Same modifications are necessary.

One of the important considerations in the design sway columns is the effect of frame instability. Several approaches to account for this effect have been proposed, and a brief description of these approaches is given in the section "Current Design Procedures". Two of these, the "modified effective length" approach and the "direct moment amplification" approach, apply an amplification factor (designated as B_2) to the first-order moment. The amplification factors used in these methods are:

$$B_2 = \frac{1}{1 - \frac{\Sigma P}{\Sigma P'_e}} \tag{20}$$

in the modified effective length approach,[1] and

$$B_2 = \frac{1}{1 - \frac{\Sigma P\Delta}{\Sigma HL}} \tag{21}$$

in the direct moment amplification approach.[9,10] In Eq. (21) Σ P and Σ H are, respectively, the total (cumulative) gravity and lateral loads in a story and Δ the first-order story sway (or drift). A comparison of the B_2

values given by Eqs. (20) and (21) and the theoretically computed amplification factors (ratio of the second-order moment to the first-order moment) is given in Fig. 15. Equation (20) gives good predictions of the amplified moment, although for the case $\alpha = 1.0$ the equation is slightly conservative. The B_2 value given by Eq. (21) is generally too low, particularly at high axial loads. A better approximation for B_2 is

$$B_2 = \frac{1}{1 - 1.2 \frac{\Sigma P\Delta}{\Sigma HL}} \tag{22}$$

which, as shown in Fig. 15, agrees very closely with the theoretical results. In Eq. (22), when the column axial load exceeds 0.5 P_y, Δ is to be calculated using the E_t (or τ) value given by Eq. (8) or (9).

It is the writers' opinion that the direct moment amplification approach would give more consistent and rational results than the modified effective length approach would, especially for frames carrying heavy gravity loads. Also, in the direct moment amplification approach, the quantities that enter into the calculation of B_2 are those which more truly characterize the problem of frame instability. A view similar to this has been expressed recently for reinforced concrete frame design.[21] Because of these and other observations reported in Refs. 6, 7 and 12, the formulas proposed in this paper for sway columns will be based on the direct moment amplification concept.

Each curve in Figs. 12 and 13 gives the relationship between the gravity load P and the maximum first-order moment at the column top M_o. Multiplying M_o by the factor B_2 according to Eq. (22) gives the amplified moment at the column top. Figure 16 shows two $B_2 M_o$ curves (dashed) for the frame illustrated in Fig. 13. The curve for $\alpha = 0$ resembles closely the M_{end}-P curve in Fig. 14. The two curves should coincide if the exact B_2 values were used to construct the curve in Fig. 16.

The above development suggests that a possible way to include both the member instability and the frame instability effects in column design is to use the amplified moment $B_1 B_2 M_o$. However, for the frames included in this study, the effect of member instability has not been found to affect appreciably the strength of the columns. This is because the effect of frame instability tends to "override" the effect of member instability, as illustrated in Fig. 14. A B_1 value of 1.0 is therefore adopted in the proposed column formulas.

All the available ultimate strength solutions have been carefully analyzed and the following empirical equations are found to represent adequately the column strength:

When $\frac{\Sigma P \Delta}{\Sigma H L} > \frac{1}{3}$

$$\frac{P}{P_{cr}} + \frac{B_2 M_o}{M_p} = 1.0 \tag{23}$$

When $\frac{\Sigma P \Delta}{\Sigma H L} \leq \frac{1}{3}$

$$\frac{P}{P_{cr}} + m_2 \frac{B_2 M_o}{M_p} = 1.0 \tag{24}$$

and

$$\frac{P}{P_{cr}} + n_2 \frac{B_2 M_o}{M_p} = n_2 \tag{25}$$

in which

$$m_2 = 0.85 \tag{26}$$

and

$$n_2 = 6 - 5 m_2 \tag{27}$$

The P_{cr} in Eqs. (23), (24) and (25) is based on the actual length of the column (K = 1.0). Examples of comparing the proposed interaction equations with the analytical solutions are shown in Fig. 17. The proposed equations predict reasonably well the ultimate strength of the frame.

It is important to point out that in the direct moment amplification approach the amplified moment $B_2 M_o$ is to be used also in the design of beams. This may require larger beam sizes.

Design Example 2

Design the columns of the frame in Fig. 18 by the allowable-stress method for the gravity and lateral loads shown. The frame is permitted to sway in its own plane but is adequately braced in the perpendicular direction. The W14x34 beams are oriented for major axis bending and all the columns for minor axis bending. Use A36 steel for the columns.

In the allowable-stress format, Eq. (23), (24) and (25) become:

When $\gamma \frac{\Sigma P \Delta}{\Sigma H L} > \frac{1}{3}$

$$\frac{f_a}{F_a} + \frac{B_2 f_b}{F_b} \leq 1.0 \tag{28}$$

When $\gamma \frac{\Sigma P \cdot \Delta}{\Sigma H L} \leq \frac{1}{3}$

$$\frac{f_a}{F_a} + m_2 \frac{B_2 f_b}{F_b} \leq 1.0 \tag{29}$$

and

$$\frac{f_a}{F_a} + n_2 \frac{B_2 f_b}{F_b} \leq n_2 \tag{30}$$

in which γ is the factor of safety or the load factor and can be taken as 1.67, and

$$B_2 = \frac{1}{1 - 1.20\,\gamma \frac{\Sigma P \Delta}{\Sigma H L}} \tag{31}$$

The need to incorporate the γ factor in the calculation is explained in Ref. 12.

Because of symmetry, it is possible to simplify the frame of Fig. 18(a) to that of Fig. 18(b). Also, each of the exterior columns is assumed to resist half of the applied lateral load, that is 2.5 kip. A first-order analysis gives a column top moment of 30 kip-ft and an axial load (vertical reaction) of 81.25 kip. For combined gravity and lateral loads, AISC Specification permits a 33% increase in the allowable stress. This can be conveniently handled by using 75% of the working values in the calculations.

Interior columns These columns receive lateral support from the exterior columns and their design requirement is that they should not buckle as a pinned end column. The actual column length is therefore used. The design load of the columns is

$$P = 0.75 \times 41.25 = 30.94 \text{ kip}$$

Try W4x13

$$A = 3.83 \text{ in}^2 \qquad r_y = 1.00 \text{ in} \qquad \frac{L}{r_y} = 144$$

$$F_a = 7.20 \text{ ksi}$$

$$f_a = \frac{30.94}{3.83} = 8.08 \text{ ksi} > F_a \quad \text{N.G.}$$

Try W5x16

$A = 4.68\ in^2 \quad r_y = 1.27\ in \quad \frac{L}{r_y} = 113$

$F_a = 11.26$ ksi

$f_a = \frac{30.94}{4.68} = 6.61\ ksi < F_a$ OK

Exterior columns The design loads are (Fig. 18b):

$P = 0.75 \times 81.25 = 60.94$ kip

$M_o = 0.75 \times 30 \times 12 = 270$ kip-in

Try W12x65

$A = 19.1\ in^2 \quad I_y = 174\ in^4 \quad S_y = 29.1\ in^3$

$r_y = 3.02\ in \quad \frac{L}{r_y} = 47.7$

$F_a = 18.55\ ksi \quad F_b = 27.0\ ksi$

$f_a = \frac{60.94}{19.1} = 3.19\ ksi \quad f_b = \frac{270}{29.1} = 9.28\ ksi$

For the frame of Fig. 18(b), the ratio $\frac{\Delta}{\Sigma H}$ (flexibility) is given by

$$\frac{\Delta}{\Sigma H} = (G+1)\frac{L_c^3}{3\ EI_c}$$

Therefore,

$$\frac{\Sigma P \Delta}{\Delta H L_c} = \frac{\pi^2}{3}(G+1)\frac{\Sigma P}{P_e}$$

in which $G = \frac{I_c/L_c}{I_b/L_b}$ and $P_e = \frac{\pi^2 EI_c}{L_c^2}$. The moment of inertia of the W14x34 section is 340 in^4. The following G and P_e values are obtained:

$$G = \frac{174/144}{340/288} = 1.02$$

$$P_e = \frac{\pi^2 \times 29,000 \times 174}{(144)^2} = 2402 \text{ ksi}$$

The total gravity load acting on the frame is

$$\Sigma P = 0.75(80+60) = 105 \text{ kip}$$

Substitution of G, ΣP and P_e gives

$$\frac{\Sigma P \Delta}{\Sigma H L_c} = \frac{\pi^2}{3}(1.02+1)\frac{105}{2402} = 0.291$$

and $$\gamma \frac{\Sigma P \Delta}{\Sigma H L_c} = 1.67 \times 0.291 = 0.486 > \frac{1}{3} \rightarrow \text{check Eq. (28)}$$

The required B_2 factor is given by Eq. (31)

$$B_2 = \frac{1}{1 - 1.2 \times 0.486} = 2.40$$

Equation (28)

$$\frac{3.19}{18.55} + \frac{2.40 \times 9.28}{27.0} = 0.993 \quad \text{OK}$$

The column designs are now complete. Use W5x16 for the interior columns and W12x65 for the exterior columns.

In an actual design, the frame must also be checked for the gravity load alone case. To apply the proposed design procedure to this case, a small fictitious lateral load, say equal to 0.5% of the gravity load, may be assumed for the stability check. In this example, the gravity loading condition controls the design of the interior columns but the W5x16 section is still adequate. The combined loading condition controls the exterior columns.

Summary

This paper deals with the analysis and design of framed columns subjected to minor axis bending. Both sway and non-sway columns are included in the study. A review is presented of the current design procedures which are based largely on the previous studies on columns bent about the major axis. For non-sway columns the interaction formulas given in the AISC Specification have been found to give results which do not agree well with the theoretical solutions. For sway columns specific design provisions need be developed to account for the effect of frame instability.

Ultimate strength solutions for non-sway columns have been obtained for three loading cases and numerical results are presented in the form of interaction curves. Based on these curves, a new set of column design formulas, Eqs. (10) and (11), has been developed and its application is illustrated in Design Example 1. The formulas are applicable to columns subjected to symmetrical and nonsymmetrical end moments and to lateral load. The strength predicted by these formulas compares favorably with the available test results.

The elastic behavior of a simple restrained column has been studied in detail. It is shown that the column may "shed" its entire resisting moment when axial load exceeds a certain value.

Ultimate strength solutions of two unbraced frames subjected to combined gravity and lateral loads have been presented and new design formulas for sway columns, based on the direct moment amplification approach, are proposed. These formulas have essentially the same appearance as the formulas for non-sway columns, except that they use a different set of amplification

factors and empirical coefficient. Design Example 2 illustrates the application of the new formulas.

Although the formulas and design procedures presented in this paper are for columns subjected to minor axis bending, the basic concepts and approaches adopted in their development are also applicable to the case of major axis bending. A follow-up paper will present a more complete discussion of the design requirements and provisions for columns in both braced and unbraced frames.

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The theoretical solutions given in Figs. 9, 12, 13 and 15 were first presented in the senior writer's Ph.D. dissertation submitted to the University of Texas at Austin in 1977. This dissertation was prepared under the supervision of Joseph A. Yura. The interaction curves shown in Figs. 2 and 3 were obtained by Francois Cheong-Siat-Moy using a computer program prepared by Lee C. Lim.

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Notation

The symbols used in this paper are defined in the AISC Specification except the following:

B_1 = amplification factor accounting for member instability effect;

B_2 = amplification factor accounting for frame instability effect;

E_t = tangent modulus;

H = lateral load;

M_o = first-order moment;

m_1 = empirical coefficient in interaction formulas for non-sway column;

m_2 = empirical coefficient in interaction formulas for sway column;

n_1 = $6 - 5\,m_1$;

n_2 = $6 - 5\,m_2$;

α = proportionality constant for vertical load;

β = end moment ratio;

γ = factor of safety or load factor (1.67);

Δ = first-order story sway;

δ = deflection of column;

λ = normalized slenderness ratio;

τ = E_t/E

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Appendix - A Possible Modification to the Present Column Formula

The following equation has been proposed in Ref. 22 as an improvement of Eq. (3) for non-sway columns:

$$\frac{P}{P_{cr}} + \frac{C_m}{1 - \mu \frac{P}{P_y}} \frac{M_o}{M_p} = 1.0 \qquad \text{(A1)}$$

in which μ is an empirical coefficient depending on λ. For H or I section members subjected to minor axis bending, μ is given by

$$\mu = 2.47\,\lambda - 1.47 \quad \text{for} \quad \lambda \leq 1.0 \qquad \text{(A2)}$$

$$\mu = \lambda^2 \quad \text{for} \quad \lambda > 1.0 \qquad \text{(A3)}$$

In Eq. (A2) μ takes on values between -1.47 and 1.00. For $\lambda > 1.0$ the amplification factor in Eq. (A1) becomes $C_m/\left(1 - \frac{P}{P_e}\right)$ which is the same as that in Eq. (3).

Figures A1, A2 and A3 show comparisons between Eq. (A1) and the analytical solutions presented in Figs. 2, 3 and 4 for a column with L/r_y = 40. The equation is quite accurate for the case of equal end moments, but becomes conservative for lateral loading. Also, as shown in Fig. A2, the equation gives unconservative results when the axial load is low. A cut-off at M_o/M_p = 1.0 should be specified.

Table 1 Summary of Johnston and Cheney Tests

Column	L in	$\frac{L}{r_y}$	F_y ksi	λ	e^a in	P kip	M_o kip-in	$\frac{P}{P_y}$	$\frac{M_o}{M_p}$
C22	12.59	23.7	40.8	.283	0.35	46.6	16.1	.69	.55
C23	12.59	23.7	40.8	.283	0.47	38.9	18.3	.58	.62
C24	12.59	23.7	40.8	.283	0.71	29.6	21.0	.44	.71
C25	12.59	23.7	40.8	.283	1.18	19.1	22.5	.28	.76
C28	25.86	48.8	40.8	.583	0.35	36.6	12.8	.55	.43
C29	25.86	48.8	40.8	.583	0.47	30.8	14.5	.46	.49
C30	25.86	48.8	40.8	.583	0.71	23.5	16.7	.35	.56
C31	25.86	48.8	40.8	.583	1.17	14.9	17.4	.22	.59
C32	25.86	48.8	40.8	.583	1.65	11.8	19.4	.18	.66
C34	39.12	73.8	40.8	.881	0.35	27.2	9.5	.41	.32
C35	39.12	73.8	40.8	.881	0.47	23.6	11.1	.35	.37
C36	39.12	73.8	40.8	.881	0.71	19.0	13.5	.28	.46
C37	39.12	73.8	40.8	.881	1.18	14.9	17.6	.22	.60
C38	39.12	73.8	40.8	.881	1.65	10.6	17.5	.16	.59

[a]Eccentricity of applied load

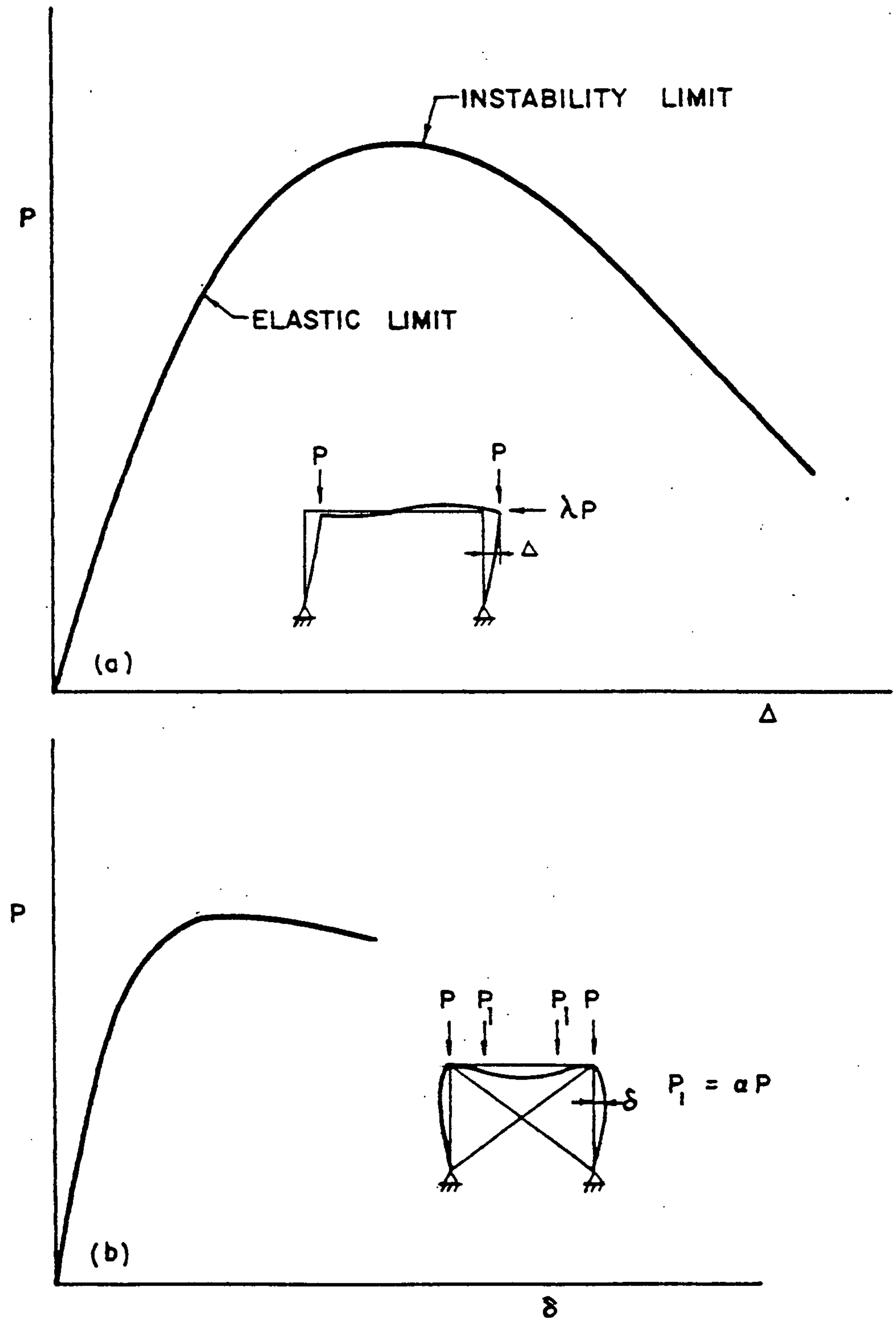

Fig. 1 Frame instability and member instability

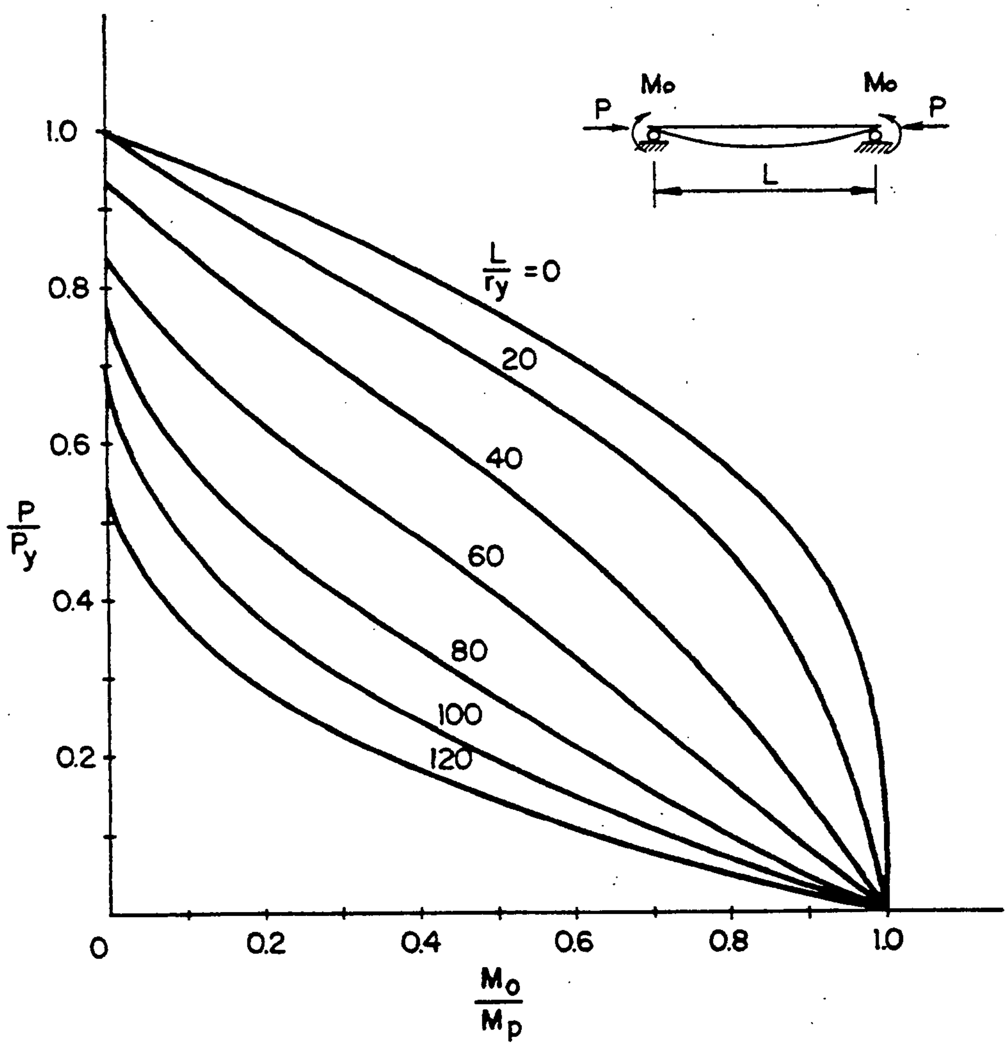

Fig. 2 Ultimate strength of columns subjected to equal end moments

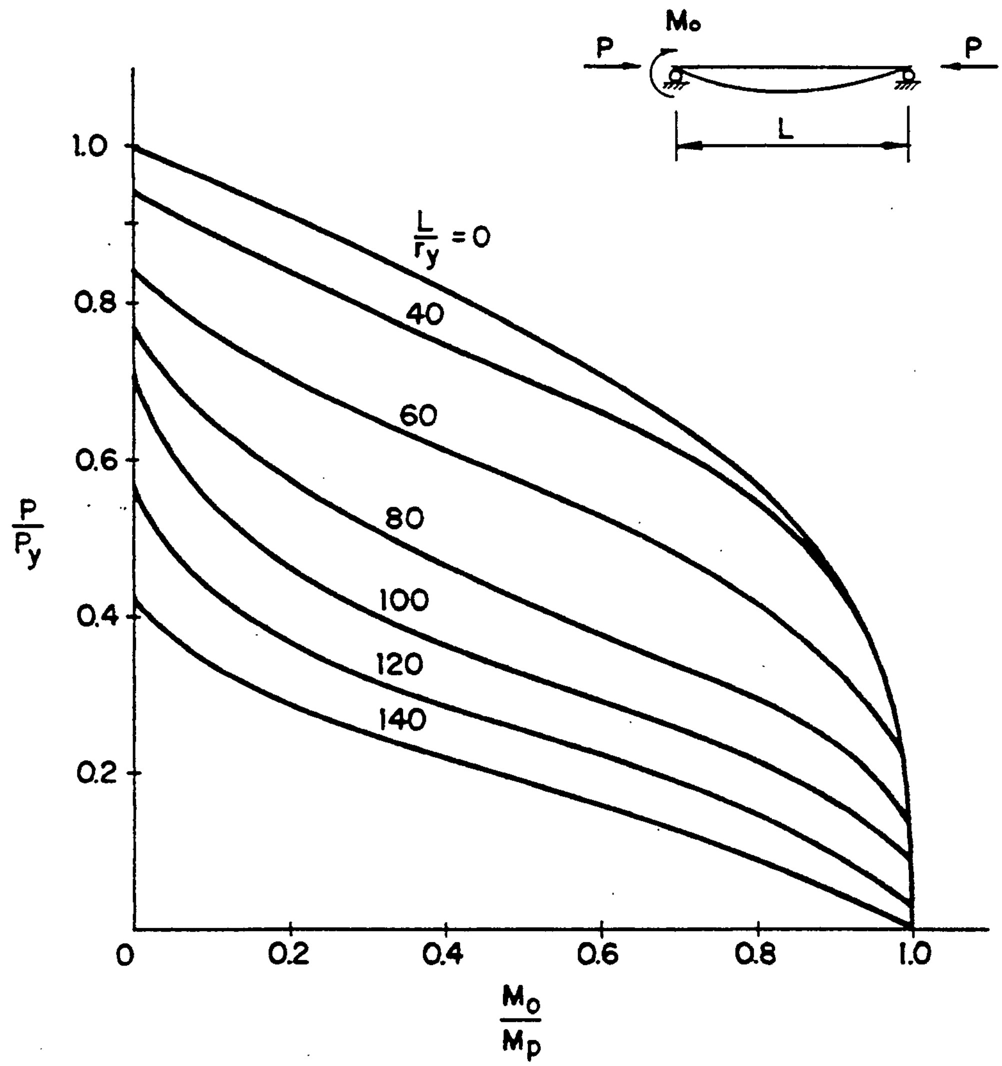

Fig. 3 Ultimate strength of columns subjected to moment at one end

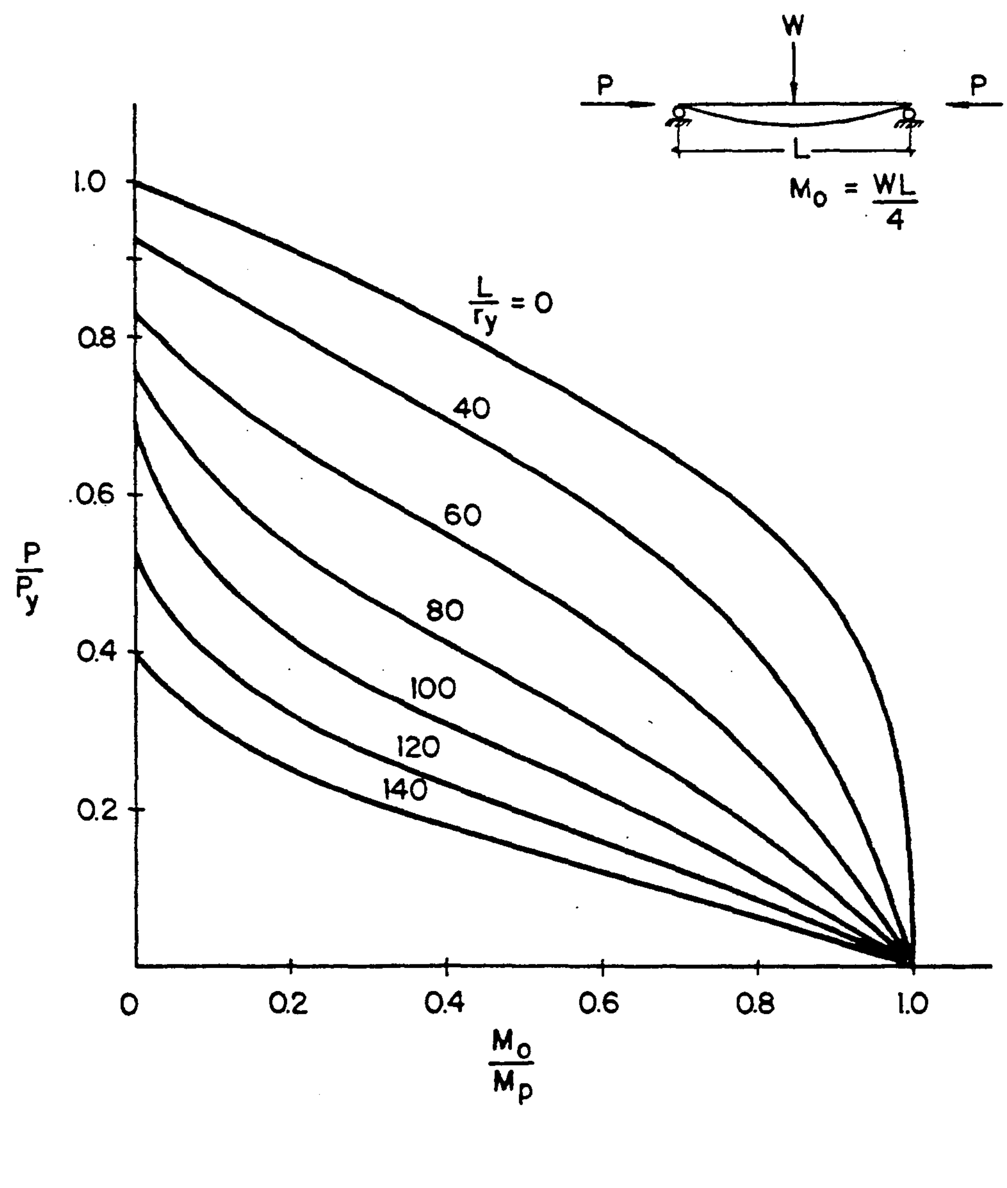

Fig. 4 Ultimate strength of columns subjected to concentrated load at midspan

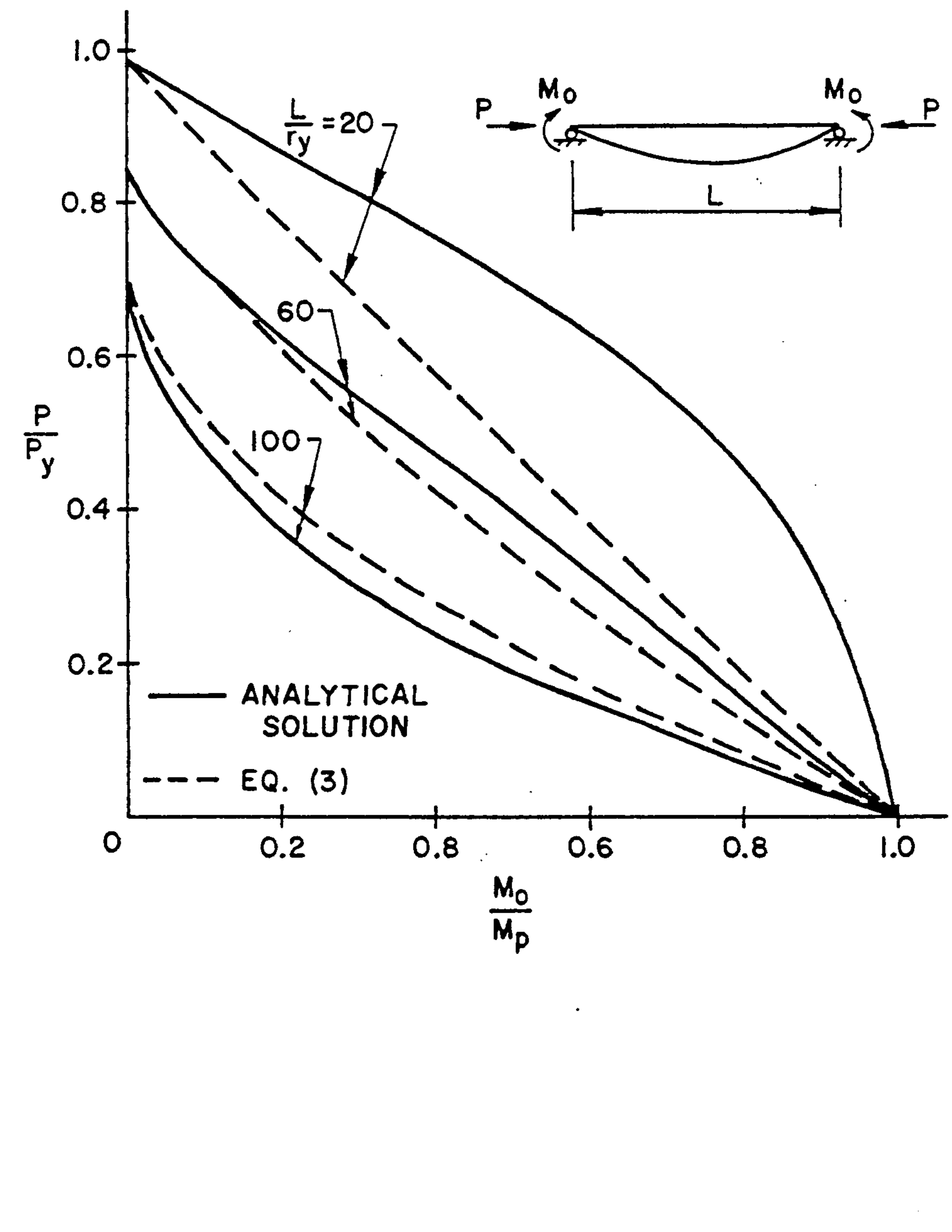

Fig. 5 Comparison of analytically determined column strength with predicted strength by Eq. (3)

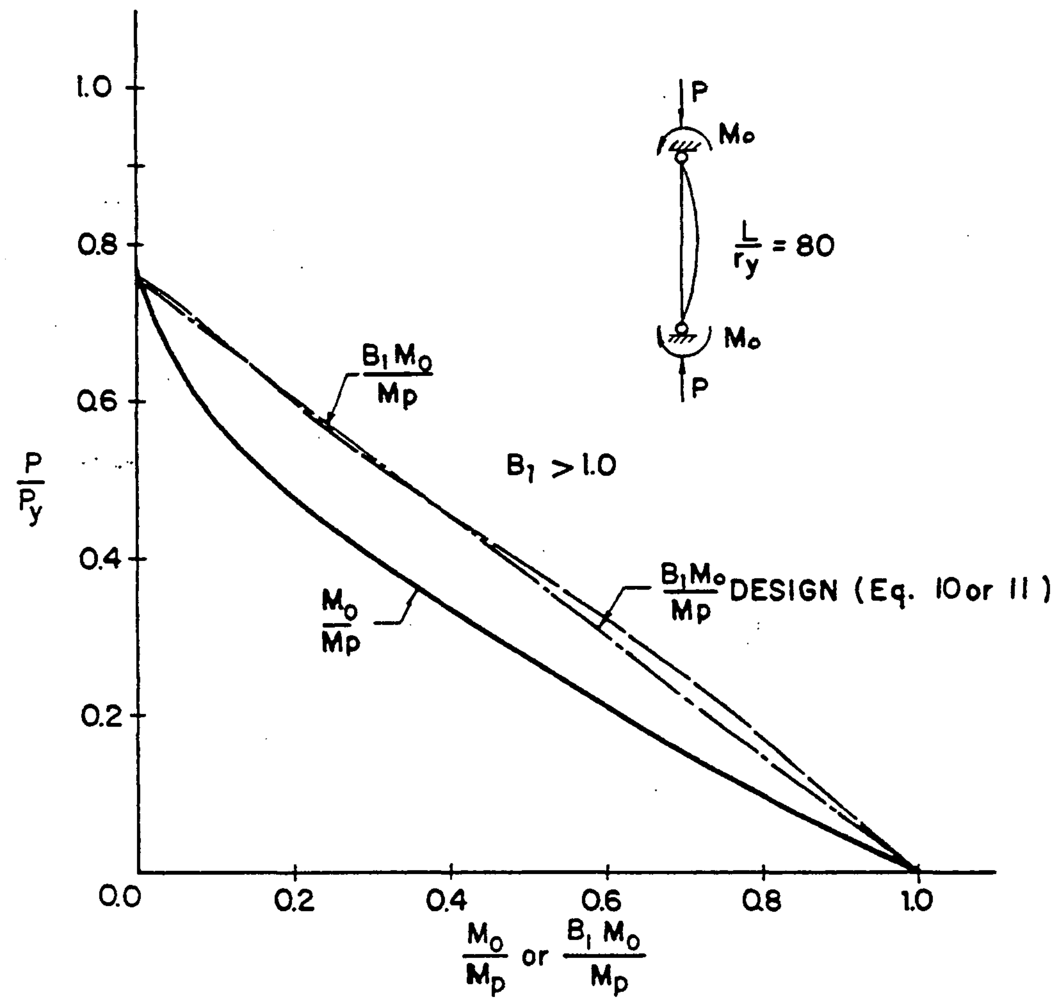

Fig. 6 Second-order moment in column subjected to equal end moments

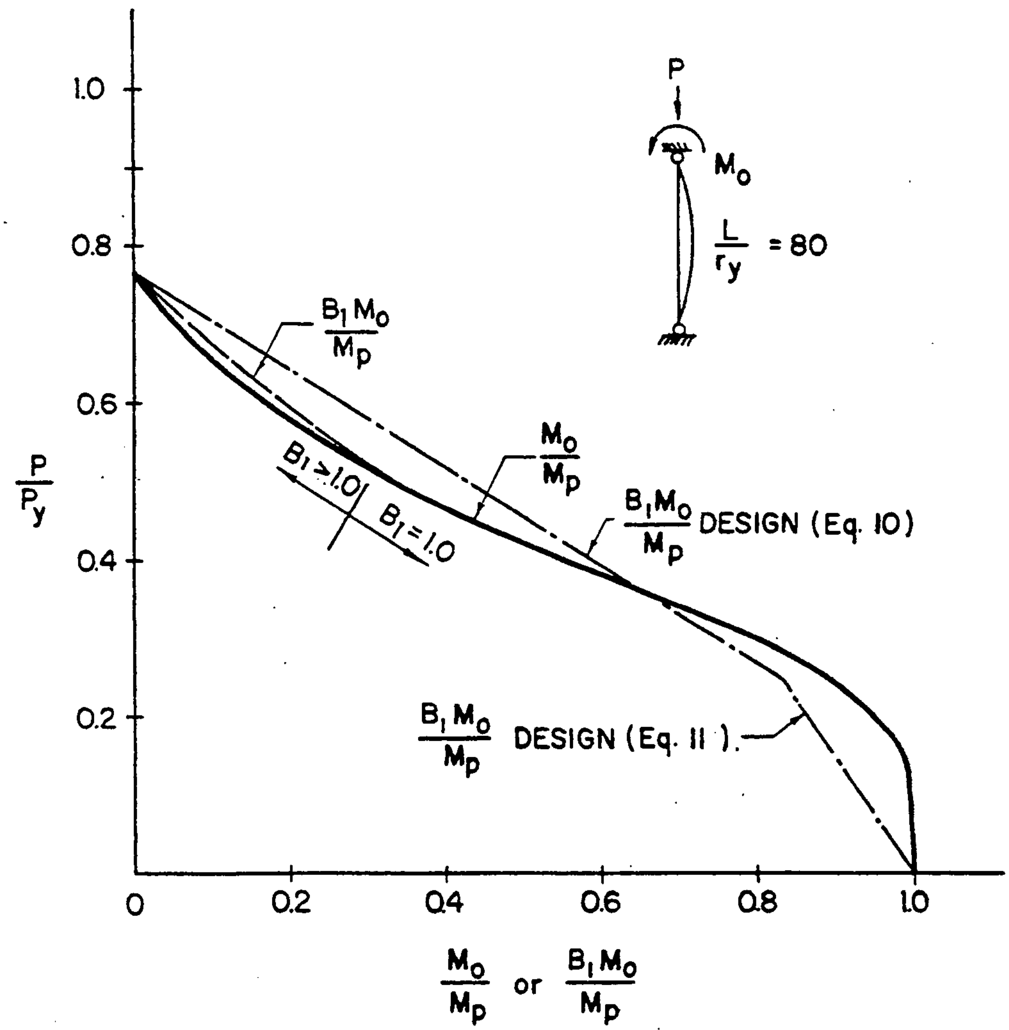

Fig. 7 Second-order moment in column subjected to moment at one end

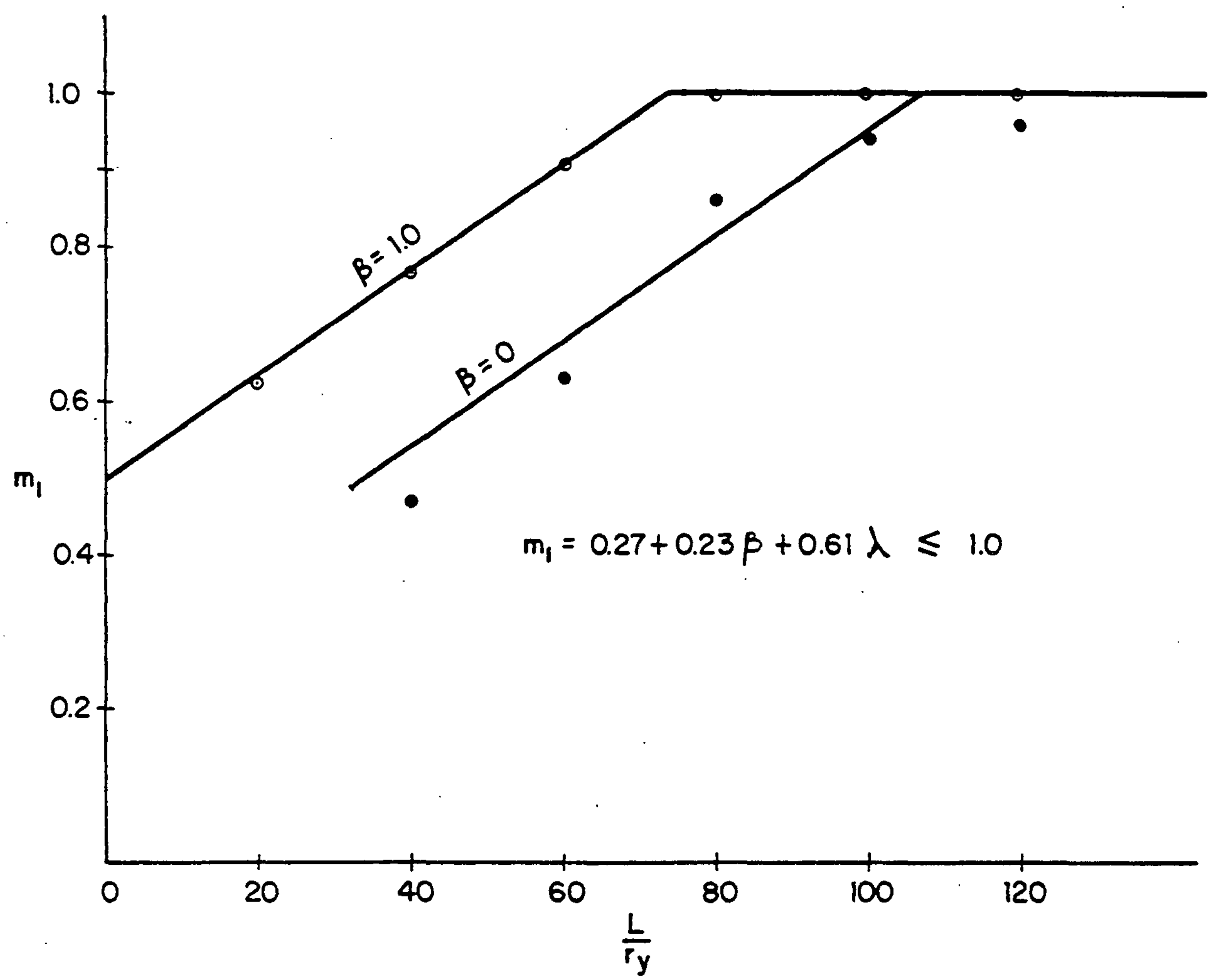

Fig. 8 Determination of coefficient m_1 in Eq. (10)

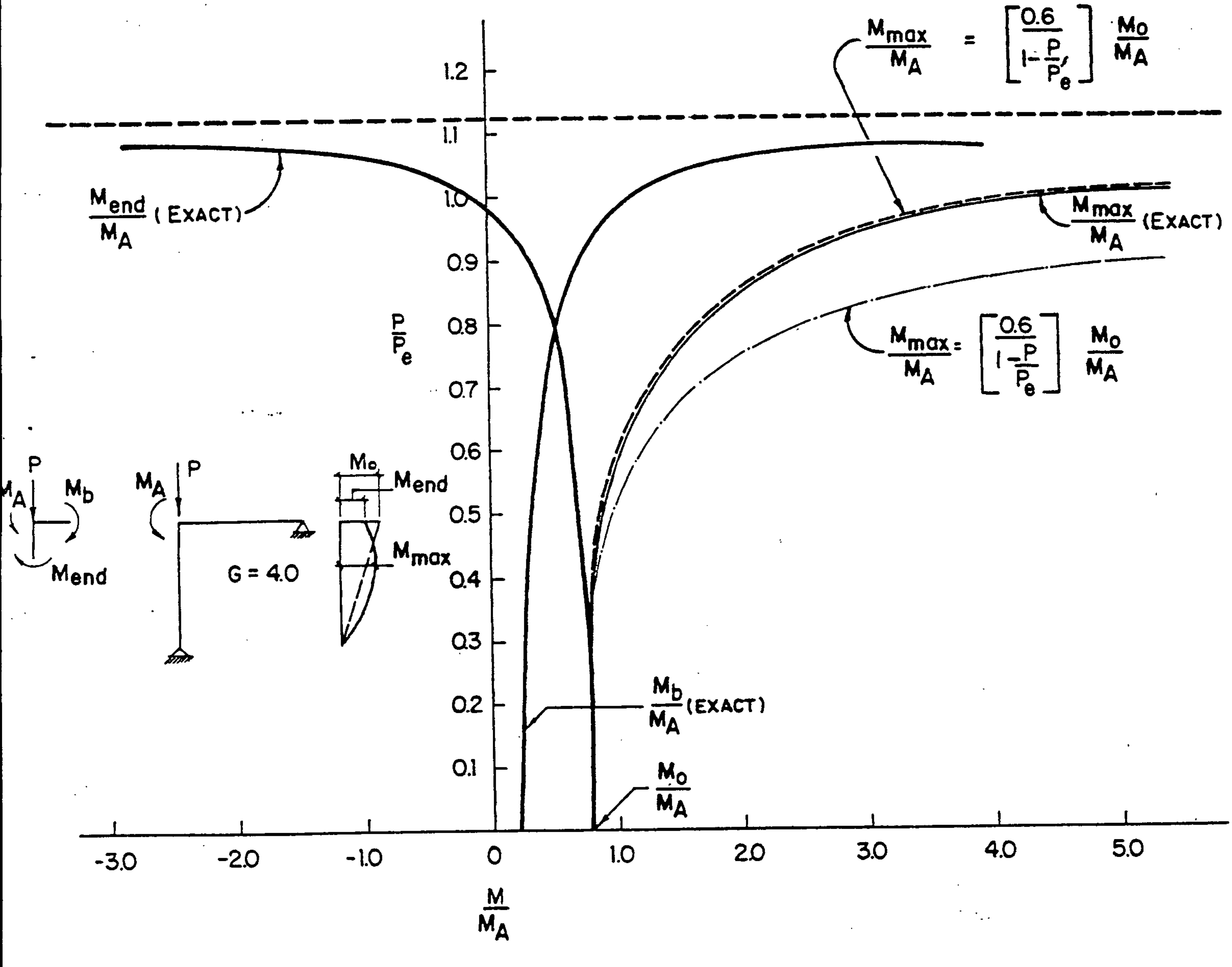

Fig. 9 Elastic second-order analysis of non-sway restrained column

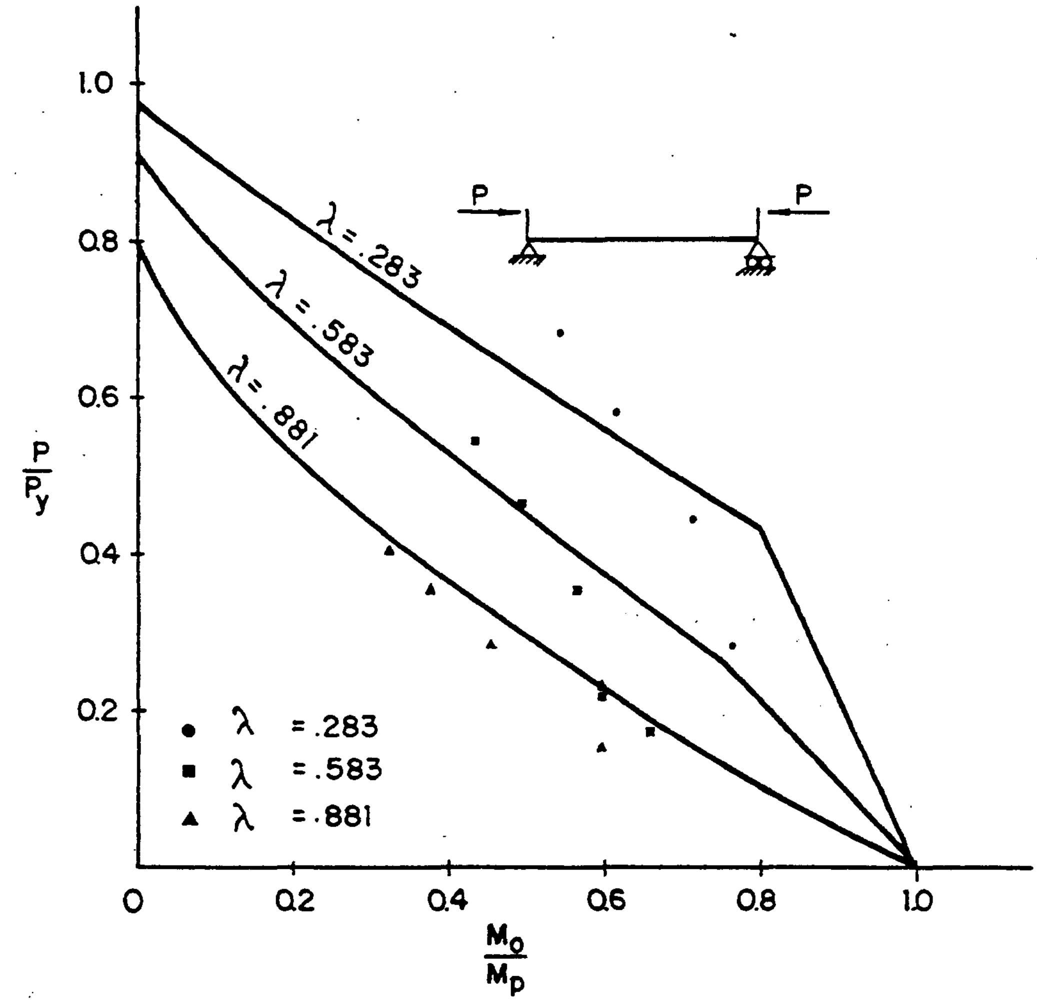

Fig. 10 Comparison of proposed interaction equations for non-sway columns with test results

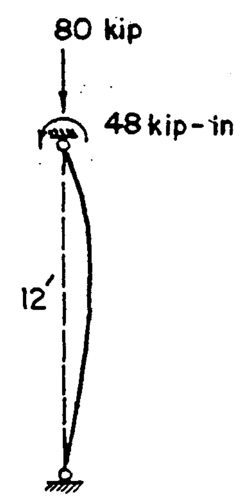

Fig. 11 Design example 1

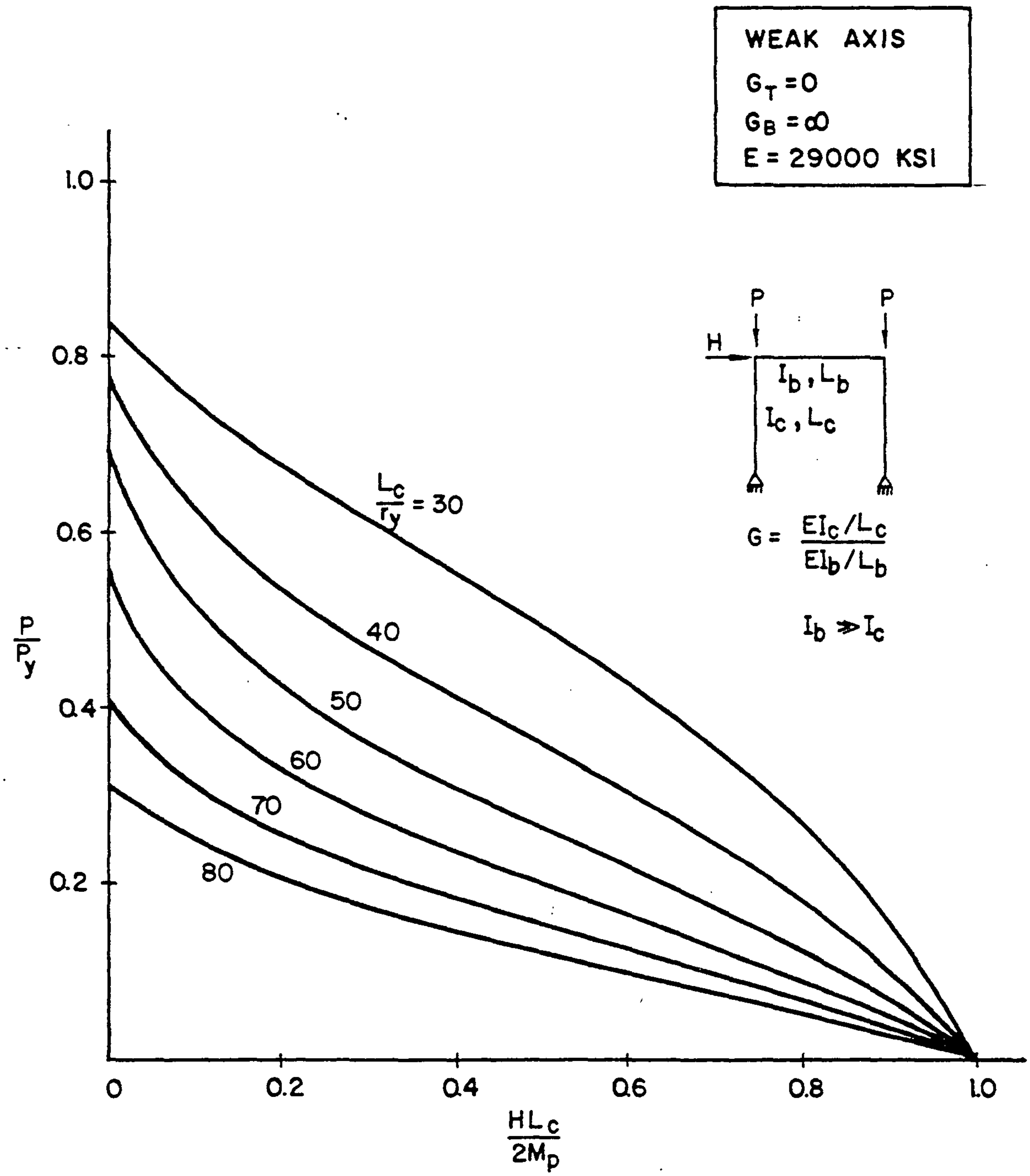

Fig. 12 Ultimate strength of columns in sway frames (both joints rigid)

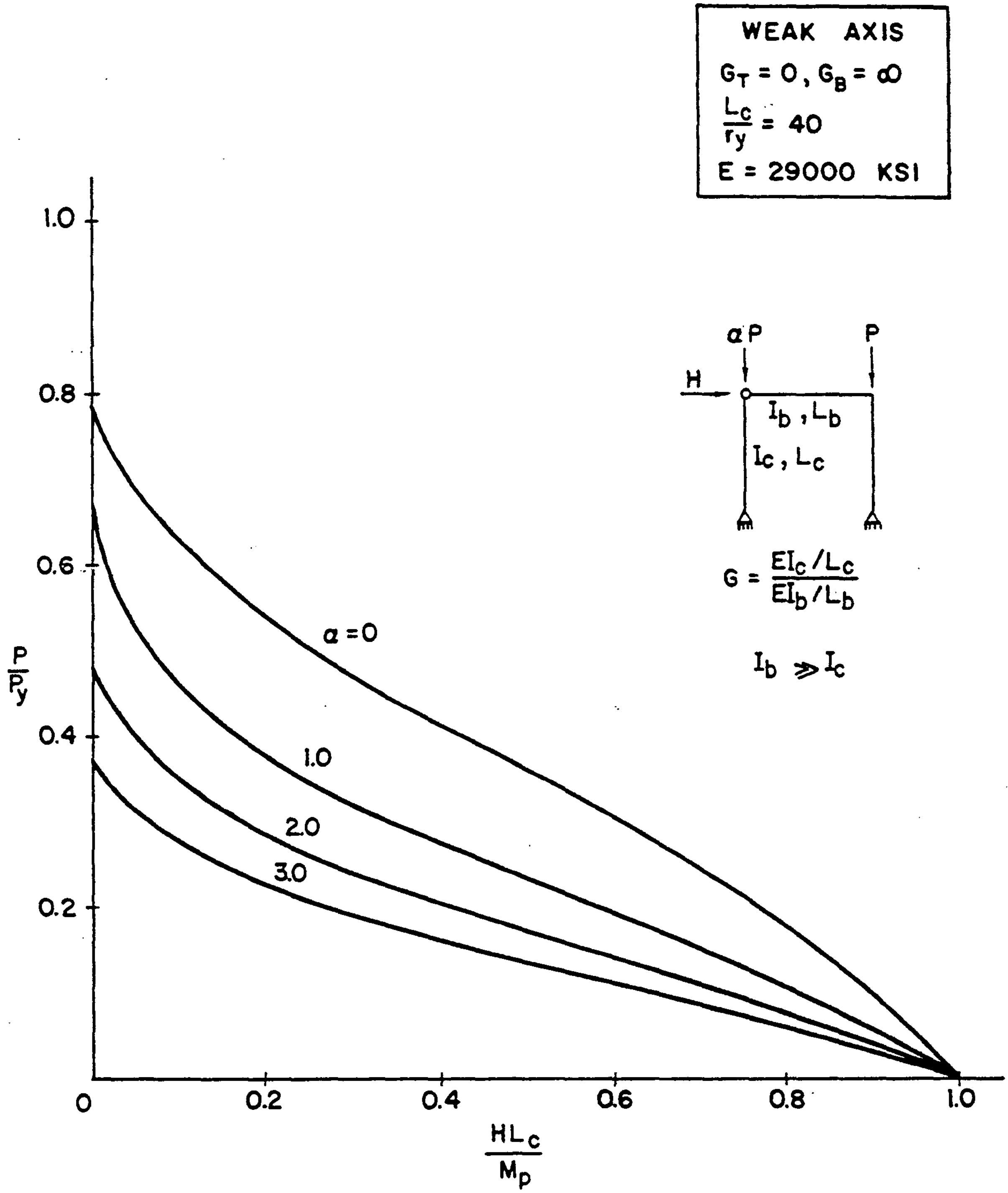

Fig. 13 Ultimate strength of columns in sway frames (one joint hinged)

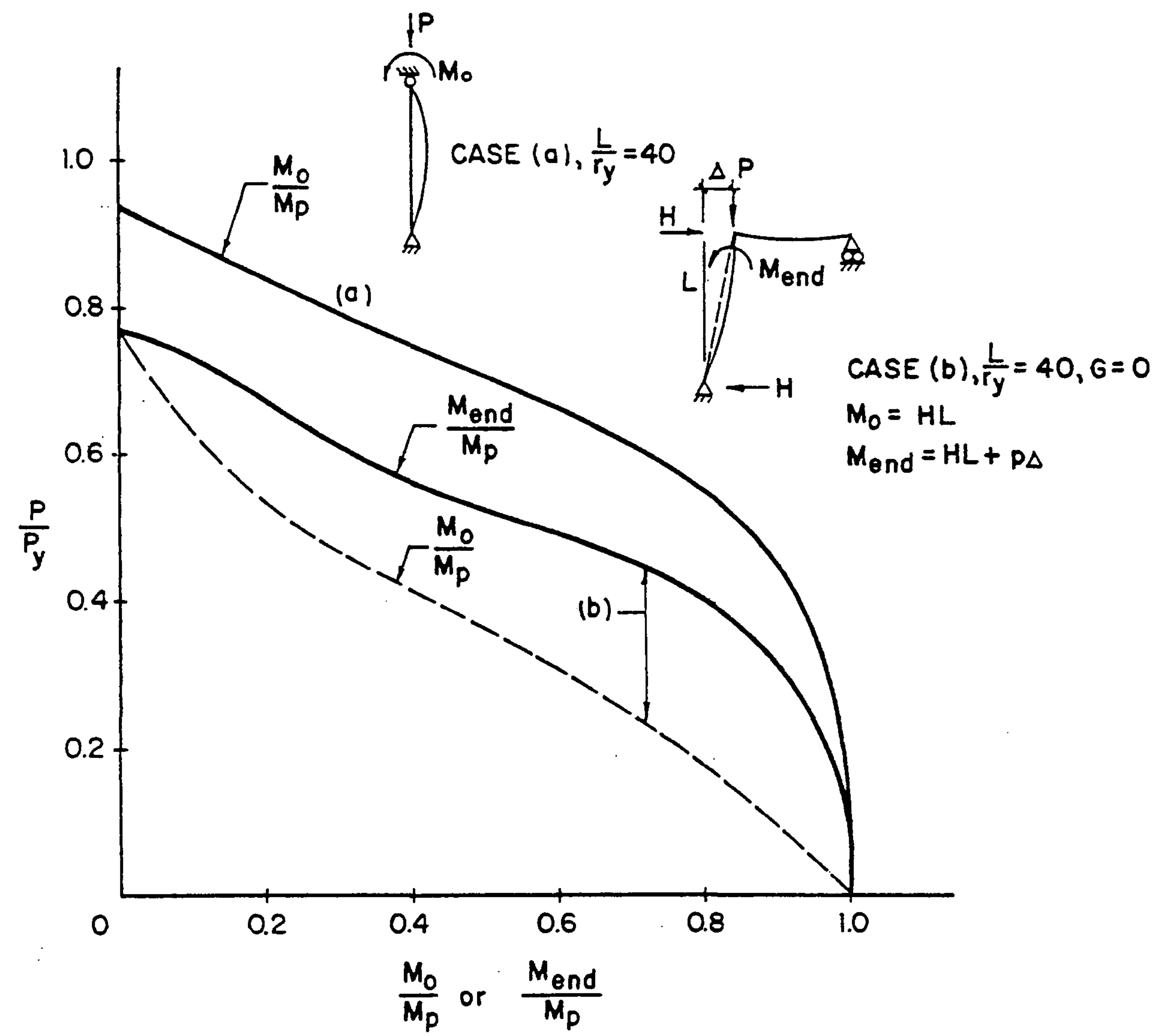

Fig. 14 Comparison of maximum end moments in sway and non-sway columns

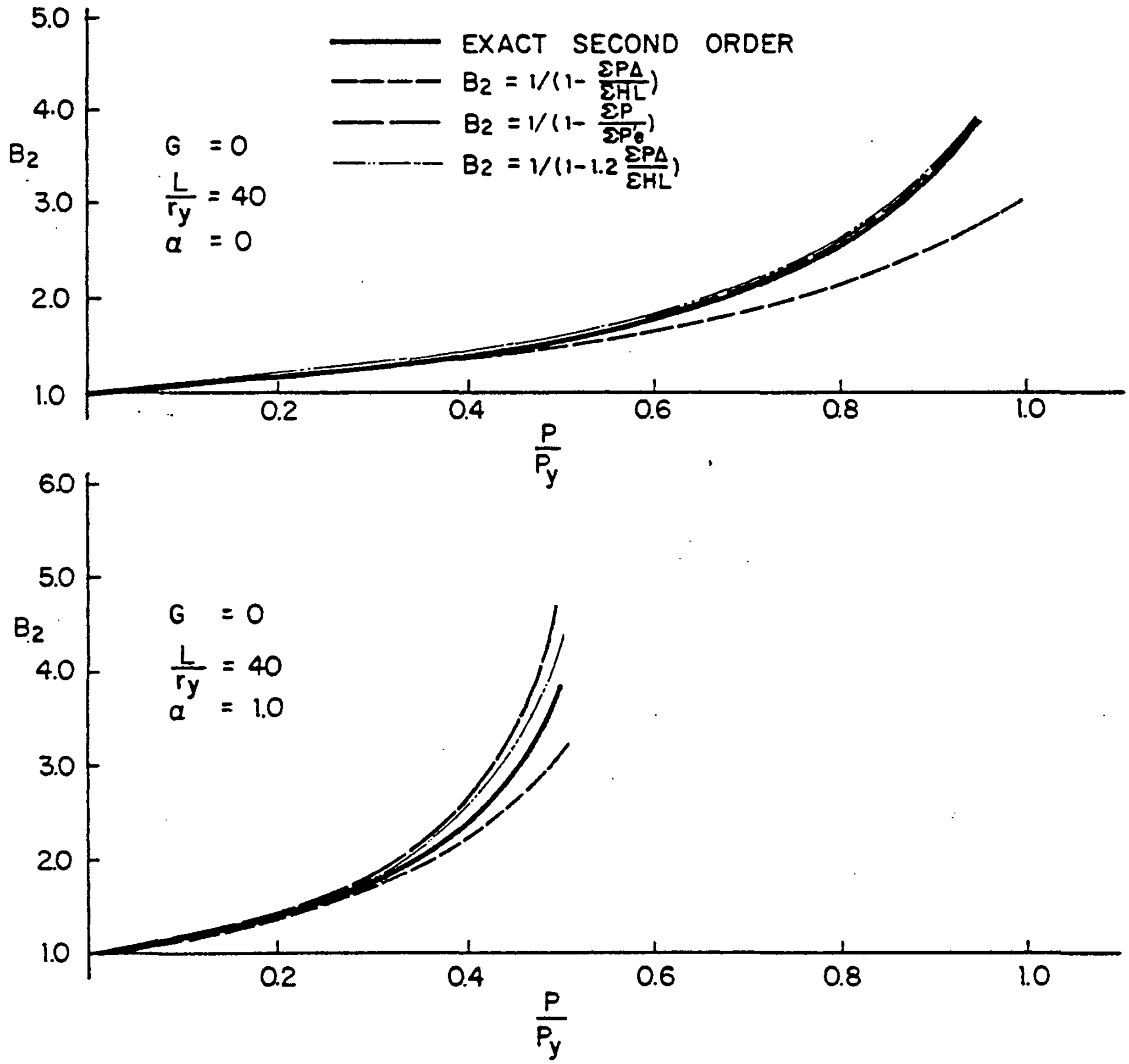

Fig. 15 Methods to account for effect of frame instability

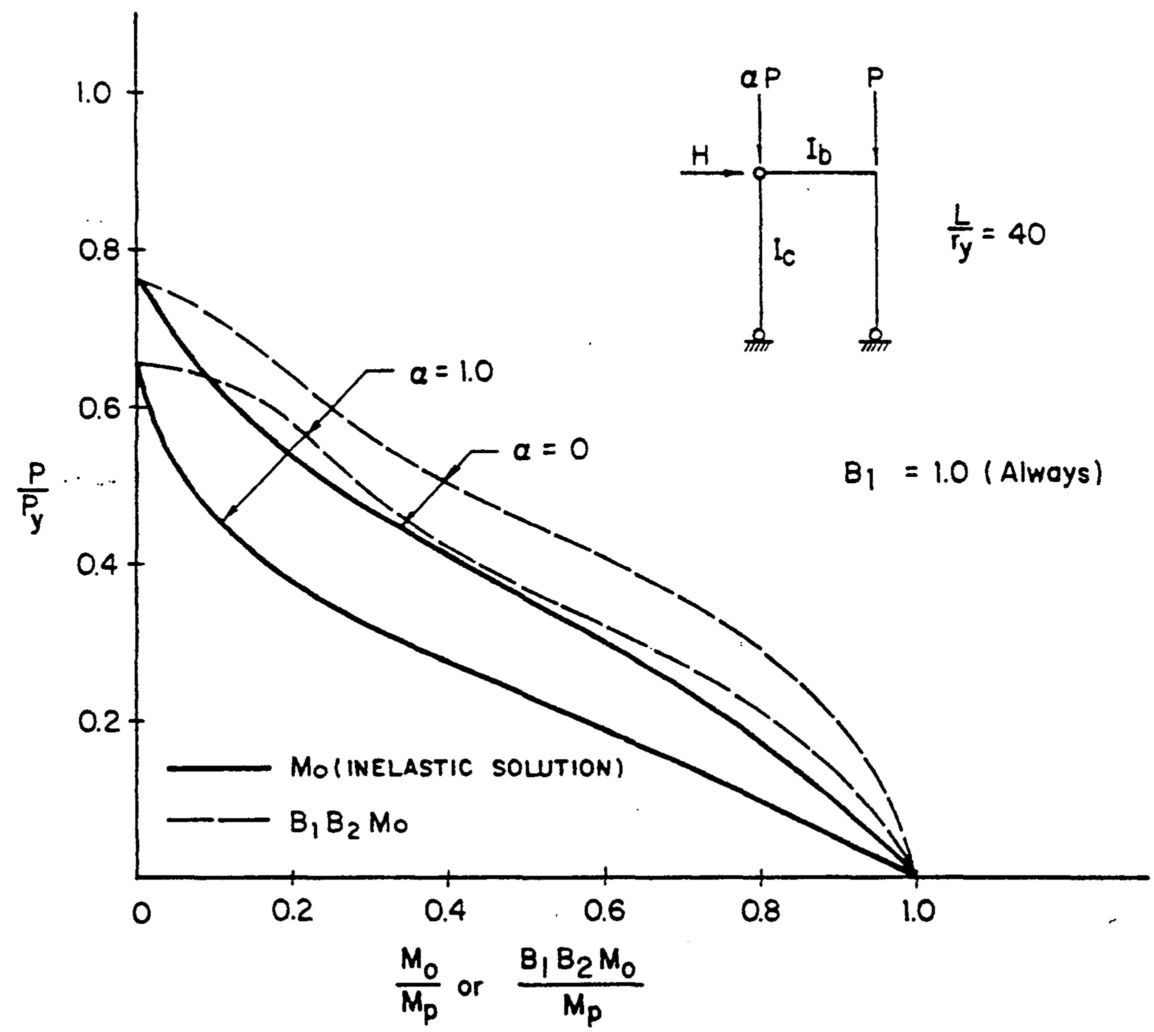

Fig. 16 Second-order moment in column in sway frame

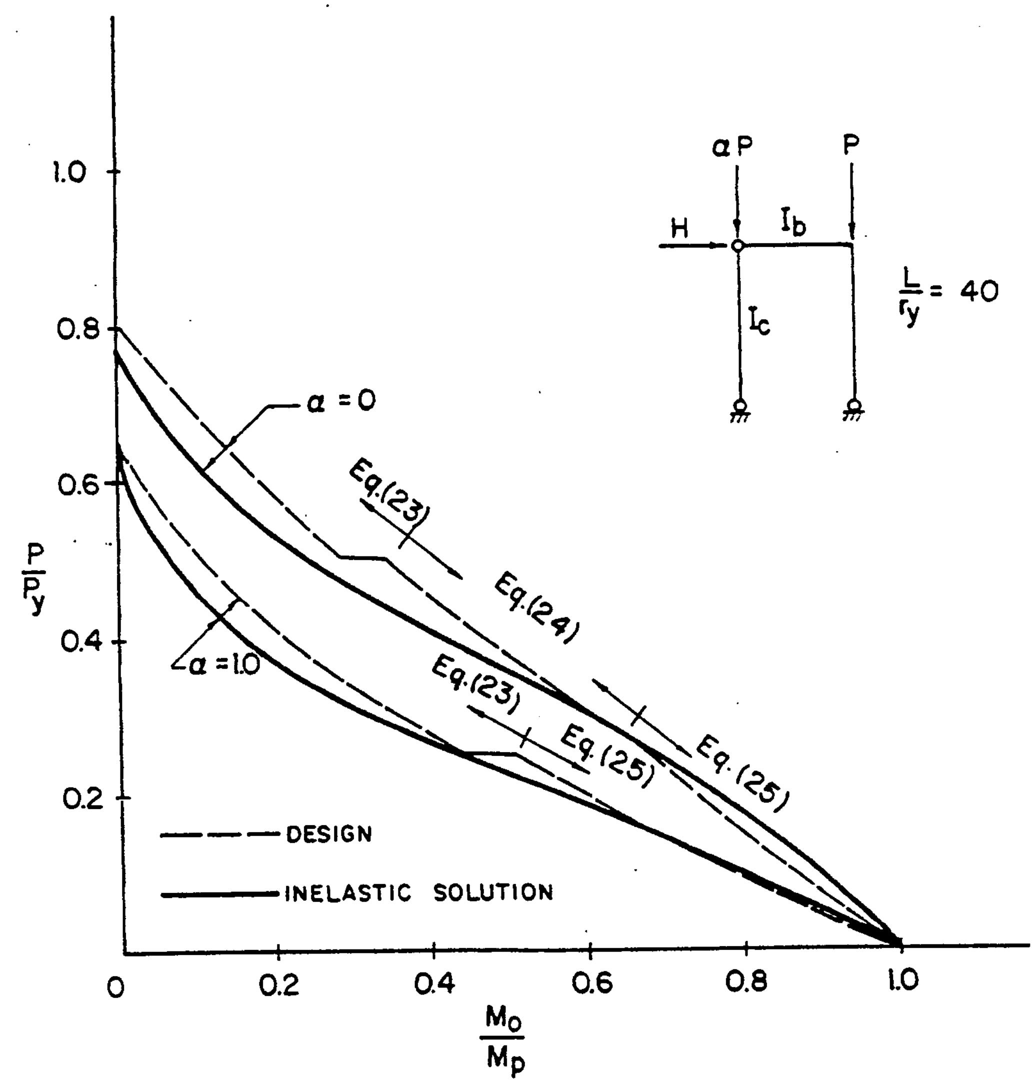

Fig. 17 Comparison of proposed interaction equations for sway columns with analytical solution

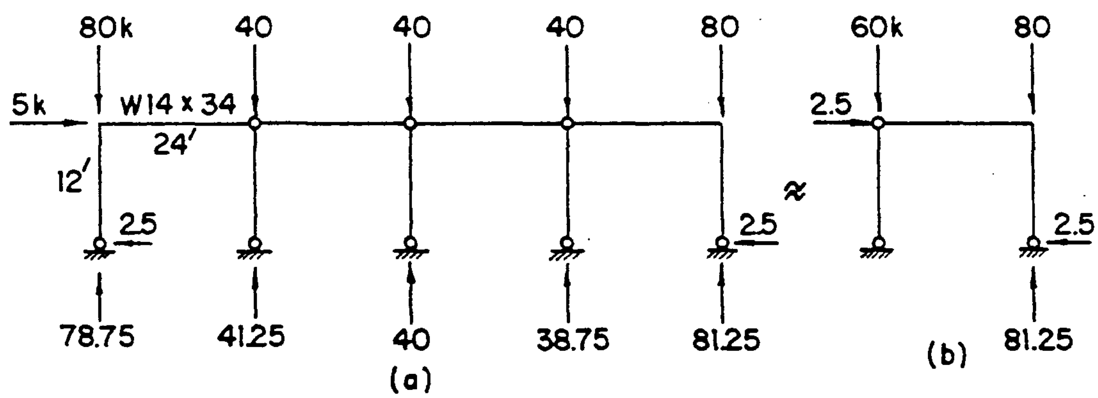

Fig. 18 Design example 2

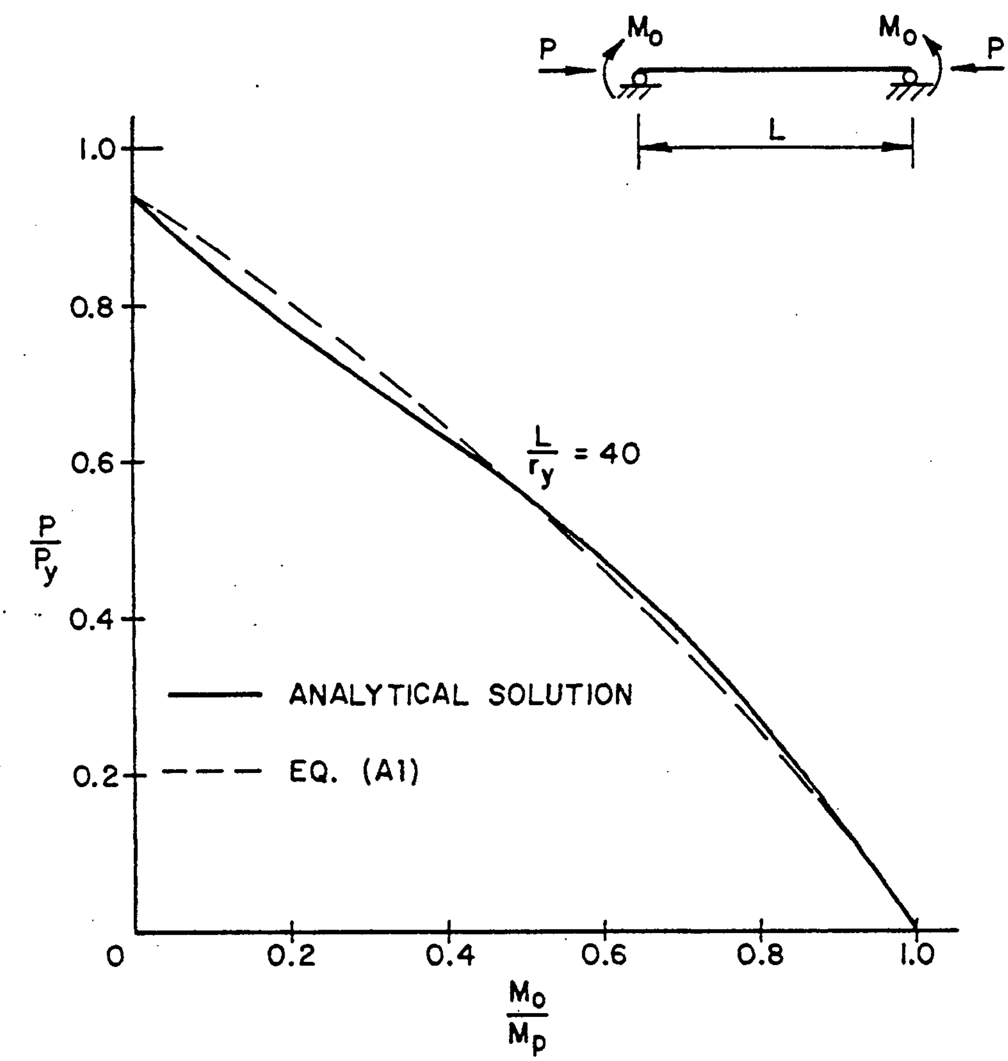

Fig. A1 Comparison of analytical solution with Eq. (A1) -- equal end moment case

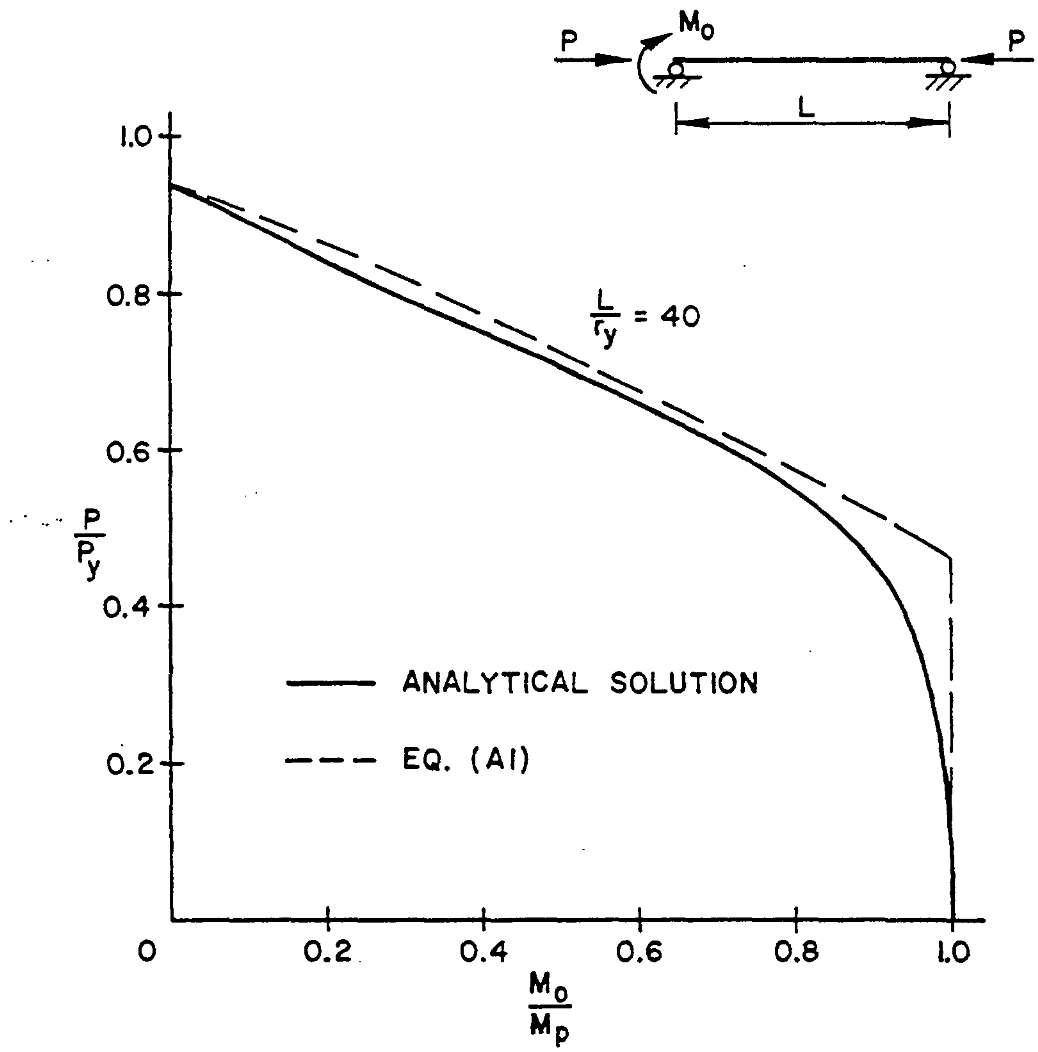

Fig. A2 Comparison of analytical solution with Eq. (A1) -- one end moment case

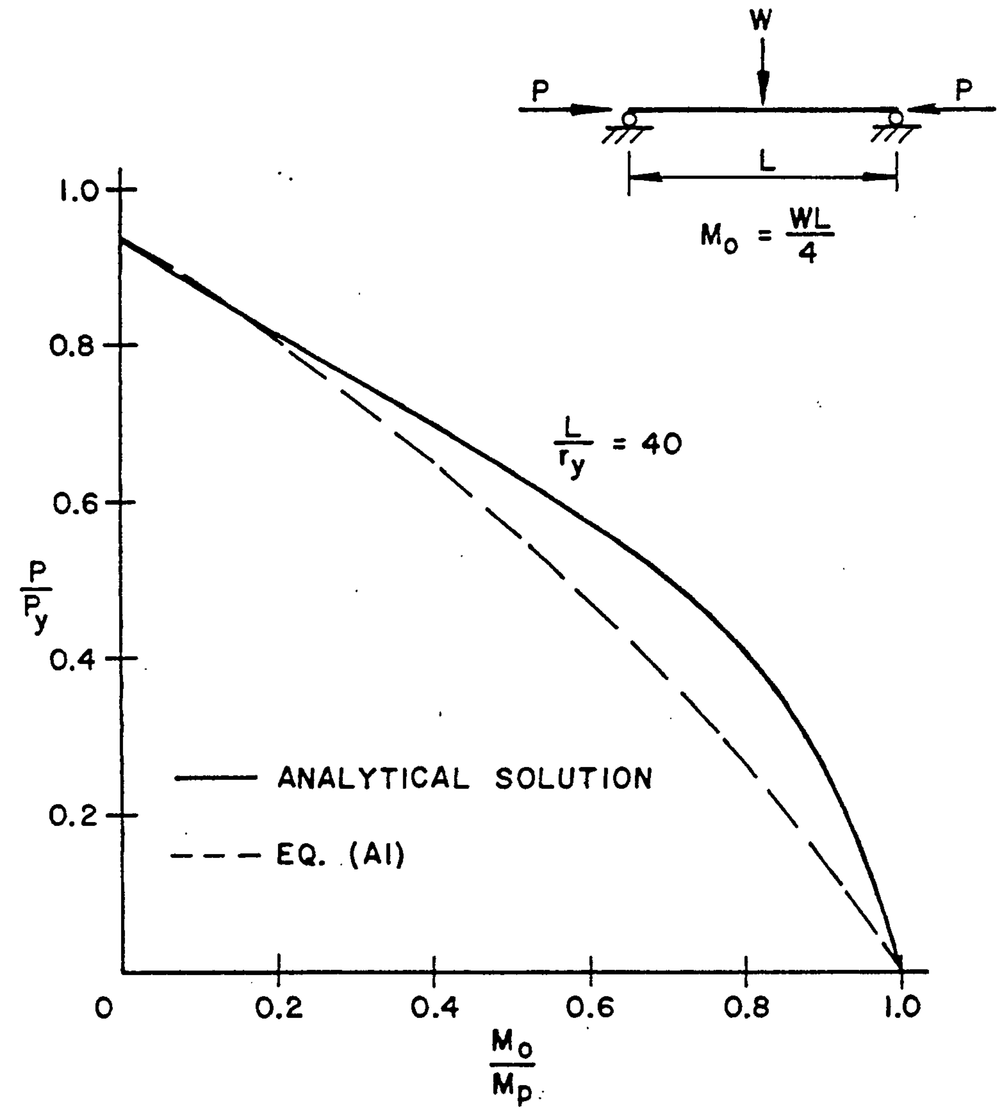

Fig. A3 Comparison of analytical solution with Eq. (A1) -- lateral load case