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Low Cycle Fatigue

ANALYSIS OF ERROR IN DETERMINING FATIGUE CRACK GROWTH RATES

by

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ABSTRACT

The error associated with determining the crack growth rate is examined. Five methods of computing the rate of growth were studied. The error of each method was evaluated by integrating the rate of growth and comparing the results with the measured crack length and cycle data.

The error was also examined when the rates determined by these five methods were fitted to two crack growth equations. The error associated with these equations was also found by integration and comparison with the measured data.

The results of the error analysis revealed that considerable error is accumulated in the reduction of crack growth data.

1. INTRODUCTION

One trend in the study of the fatigue of structures has been to examine the fatigue crack propagation behavior of various materials in the laboratory. This basic information is then used to estimate the fatigue life of structures, components and details.[1,2] The degree of correlation between estimated and actual lives of structures depends upon many variables; one of which is how well the empirical crack growth relationships that are obtained from the experimental data actually predict the observed specimen behavior. The objective of this study is to investigate the methods used to analyze fatigue crack growth data and to determine which method best describes the observed behavior.

The primary data from a crack growth experiment is the crack length, a_i, observed at cycle N_i and the applied loads. Currently, interpretation of this data has focused upon the crack growth rate which is defined as the rate of extension of the crack with the applied load cycles. Determination of the growth rate requires an evaluation of the slope of the measured crack length-cycle data at various discrete points.

This study is concerned with the numerical determination of the crack growth rate from the measured data and the fitting of this data to crack growth relationships which use the fracture mechanics stress intensity factor. Methods of determining the degree of fit to the original crack length-cycle data are investigated.

2. REVIEW OF PROBLEM

2.1 Measurement of Crack Length, a_i, at Cycle N_i

The measured variables in a crack growth experiment are the crack length, a_i, the cycles of application of load N_i and the applied loads. There are numerous methods used to obtain this data. One common method is to use visual observations of the crack tip and measure the crack length by optical methods.[3,6] Other methods used to monitor the crack length make use of known relationships between measured parameters and crack length. Examples of these indirect methods are ultra-sonic detection, voltage potential monitoring and electrical crack followers.[4,5] Each of these methods requires that the measuring system be calibrated for a particular specimen geometry.

Some of the indirect methods provide continuous output of the monitored signals with time. However, in most cases, the output must be converted to crack length through a calibration equation. Consequently, the crack length-cycle relationship is not continuous but made up of a series of discrete points. The crack length is often plotted as a function of the applied cycles as shown in Fig. 1. The data shown is for 2024 aluminum and was obtained by visual methods.[6]

2.2 Determination of Rate of Crack Growth from Primary Data

The data shown in Fig. 1, is used to determine crack growth rate. The methods used to determine crack growth rate are numerous although not often described by investigators. The transformation of crack length measurements into crack growth rates can lead to substantial errors. When indirect measurements are used to monitor crack length, the rate of crack extension is often determined from the rate of change of the monitored parameter. However, the problem of data reduction is similar.

The methods used to determine the rate of crack growth can be divided into two categories. One involves graphical methods and the other numerical methods. Graphical methods have been used by some investigators.[3] This method requires skill and judgement by the person doing the work. It is also normal to select only the areas of the crack length-cycle life relationship which exhibit a continuous curve for evaluation. The resulting slopes may be biased by the proficiency and judgement of the person performing the data reduction.

Numerical methods provide a means of determining an unbiased estimate of the rate of crack growth. The bias which may occur is dependent on the numerical method employed. Several methods are examined in this paper.

2.3 Fitting of Crack Growth Rate to Crack Growth Equations

After the determination of growth rates from the crack length-cycle data the usual practice is to correlate the behavior with an applied stress variable. The fracture mechanics stress intensity factor has gained widespread acceptance as the variable providing the best degree of correlation. This is reasonable since the stress intensity factor describes the state of stress at the crack tip.

Two basic methods have been employed to fit the growth rates to the empirical relationships defined by stress-related variables. One method is a visual-graphical interpretation sometimes tempered with numerical judgement. This method is crude and simple but may lead to erroneous conclusions.[6,8]

Most investigators have used the method of least squares to fit their data to the model selected. Least squares provides a rational and unbiased method to fit these models. It was employed in this study to correlate the crack growth rates with two crack growth relationships.

2.4 Error Analysis

The error involved in determining crack growth relationships is often ignored. Normally the error in crack length measurements is discussed. However, errors that result from determining the growth rate from the primary data and then fitting that rate to a crack growth equation are not often considerd. Errors in measuring crack lengths are usually quite small when meticulous measurements are made. However, errors resulting from the analysis of the data may be great.[10]

The error criterion and selection of data reduction method should be based on how well the reduced data fits the primary data (the crack length-cycle data). Since the reduction of data requires a differentiation of the primary data the error can be determined by re-integrating the reduced data. The computed crack growth rates and equations fitted to the growth rates may both be integrated and the resulting crack length-cycle data compared to the measured primary data.

3. DETERMINATION OF CRACK GROWTH RATE

Five different numerical procedures were investigated as possible ways to determine the crack growth rate from crack length-cycle data. Three of the methods can be characterized as difference methods. The other two methods utilize second and third order polynominals fitted by least squares.

The data chosen for analysis was from a crack growth experiment performed by Broek and Schijve on a 2024 aluminum alloy.[6] The data consisted of the average crack length and cycle data for three replicates within each cell of a factorial experiment design. A total of nine average crack length-cycle data sets were analyzed.

Figures 2, 3, and 4 summarize the three difference methods used to determine the crack growth rate da/dN. The secant method was used to estimate the slope at the mid-point $\bar{a}_i$ and $\bar{N}_i$ by connecting a straight line between two adjacent points as illustrated in Fig. 2. The first central difference method estimated the slope by fitting a second order curve through a central point "i" and two adjacent points as illustrated in Fig. 3. The slope was estimated from the first derivative of the fitted second order curve.[7] A modified difference method was developed in an attempt to provide a better estimate of the slope. The method is an extension of the central difference method to include more points in the slope determination as shown in Fig. 4. The derivation of this relationship is given in Appendix A.

The other methods used least square techniques to fit second and third order polynomials to the measured data. The growth rate was determined by the value of the first derivative of the fitted curves at the central point. Figures 5 and 6 show schematically the methods and the applicable equations. The rates for points at the data extremes were determined by central difference techniques when the number of points on either side was less than that used in fitting the polynominal.

The error associated with the various methods used to determine growth rates was estimated by numerically integrating the rates to determine their estimation of N_i at the measured value of a_i. The integration was performed using Simpson's one third rule. This required the cycles to be estimated at the odd numbered crack lengths since three points were used in the integration scheme. The last increment was integrated using the trapezoidal rule when the total number of points was even.

The error was determined by taking the difference of the measured number of cycles between the crack lengths at the limits of the integration increments and comparing them to the number of cycles estimated by integrating the crack growth rate. The difference between the measured number of cycles and the number estimated from the integration process was divided by the measured number of cycles to give a percentage incremental error. The absolute value of this incremental error was summed for each data set and divided by the number of integration steps to determine an average incremental error. The average incremental error, $\bar{\epsilon}$, can be expressed as:

$$\bar{\epsilon} = \frac{\Sigma_1^n \; |\epsilon_i|}{n} \qquad (1)$$

where ϵ_i, the percentage incremental error, is

$$\epsilon_i = (\Delta N_i^* - \Delta N_i)*100/\Delta N_i \qquad (2)$$

where ΔN_i is the measured cyclic increment, ΔN_i^* is the increment estimated from the integration and n is the total number of integration increments.

Table 1 summarizes the average incremental errors for each method of determining the growth rate for the test data. Also given is the average error for all tests for each method. The secant method is seen to yield the smallest error and the second order least square polynominal the largest error. The modified difference method yielded an error comparable to the secant method and the third order polynominal gave an error comparable to that of the second order polynominal The central difference method yielded the median error.

Figure 7 compares the measured crack length-cycle data given in Fig. 1 with the computed results obtained from the growth rates determined by the secant, modified difference and second order polynominal method. The difference between the cycles estimated from integrating the crack growth rate and the measured cycles, is seen to vary with crack length. Hence, an error criterion based on the number of cycles estimated for a particular crack length would not be consistent since it would be a function of the crack length selected.

The computed points for the secant method are seen to lie between the measured values of crack length in Fig. 7. These crack lengths correspond to the average lengths shown in Fig. 2. The incremental error was determined by using these average points. The error determined for the secant method is consequently not based on the actual measured values of crack length-cycle data.

The secant and modified difference methods yielded the smallest average error and are considered to be the best methods for estimating crack growth rates. The modified difference method has the advantage of determining the rates at measured values of crack length and applied load cycles rather than at the average points that are used for the secant method.

4. FITTING OF CRACK GROWTH RATE TO CRACK GROWTH EQUATIONS

The purpose of a crack growth experiment is to determine the influence of the applied stress upon the crack growth rate for a particular material and test environment. The observed growth rate is then related to an applied stress variable so the results of the experiment can be used to estimate the fatigue behavior of other components. The range of the fracture mechanics stress intensity factor, ΔK, has been shown to provide the best correlation with experimental results.[8] The elastic range of the stress intensity factor is the product of the nominal stress range and a geometric factor for the specimen. The relationship between ΔK, the range of the stress intensity factor, and da/dn, the crack growth rate, is usually expressed as:

$$da/dn = f[\Delta K^n, C, K_c] \quad (3)$$

where n and C are constants determined from the experimental data and K_c is the value of the stress intensity factor at onset of rapid fracture or unstable crack growth.

All of the functional relationships used to relate da/dn with ΔK are empirical. Attempts to use more rigorous approaches for predicting the relationship between these variables has led to equations which only fit a small range of data for a particular test.[8,9,11]

Two equations where fitted to the crack growth rates that were determined from the data in Ref. 6. The two equations were the power law proposed by Paris[8]

$$da/dn = C\,\Delta K^n \qquad (4)$$

and the equation suggested by Forman[12]

$$da/dn = \frac{C}{K_c/K_{max} - 1}\,\Delta K^n \qquad (5)$$

These equations were fitted to the crack growth rates that were developed by the five methods described earlier. All K values were calculated using the formula

$$K = \sigma\,(\pi a)^{\frac{1}{2}}[\sec \pi a/2b]^{\frac{1}{2}} \qquad (6)$$

where "σ" is the nominal applied stress, "a" the crack length (the half length for this particular specimen geometry), and "b" the half width of the plate.[13] No correction for plasticity at the crack tip was used.

Equation 4 was fitted by the method of least squares. The equation was linearized by making a logarithmic transformation of the equation. This provided a linear equation of the form

$$\text{Log } da/dN = \text{Log } C + n \text{ Log } \Delta K. \qquad (7)$$

Equation 7 was fitted by least squares and the values of C and n determined. The growth rate, da/dN, was taken as the dependent variable and ΔK was taken as the independent variable in the least squares analysis.

Table 2 summarizes the values of n and C that were determined from the different sets of growth rate data. It is apparent that the values of C and n vary with the methods used to determine the growth rate. Also, the values of these coefficients vary for each set of data.

Table 3 summarizes the average incremental error variation that results when Eq. 7 is fitted to the growth rates calculated by the five methods. Since the fitted equation provides an analytical expression for the growth rate, the integration to determine the estimated cyclic intervals, ΔN_i^*, was performed using the Runge-Kutta method.

The variation in error due to the different fitting methods is small. However, the error is much larger than the error associated with the growth rate (see Table 1). A comparison of the error variation among the specimens in Tables 1 and 3 shows that the magnitude of the error varied similarly.

Although the secant method provided the best estimate of the growth rate, (see Table 1), it yielded the largest error when the exponential model was fitted to the crack growth rate.

Equation 5 was fitted by using a modified least squares procedure. The equation was first transformed into a quasi-linear form which yielded:

$$\log da/dN - \log \left(\frac{1}{K_c/K_{max} - 1}\right) = \log C + n \log \Delta K \quad (8)$$

The left side of Eq. 8 was considered as a single dependent variable. The values of n and C were determined from Eq. 8 by the method of least squares for each value of K_c. The value of K_c was determined by the method of interval halving so that the sum of squares computed from the original non-linear form of the equation was a minimum.

Equation 8 was fitted to all computed growth rates for each method since the equation was supposed to account for variations in the growth rate with different mean stress levels. Table 4 summarizes the values of K_c, n, and C that were obtained for the different methods of determining da/dN. Table 5 summarizes the error variation for each stress level. Comparison of Tables 4 and 5 shows maximum and minimum error occurred at the same stress levels for both relationships. Also, the best fits to the growth rate (the secant and modified difference) yielded the largest errors.

Figure 8 summarizes the average error variation between measured and predicted crack length-cycle data for the two crack growth equations. Also shown is the error attributable to the various methods of determining the growth rate. It is apparent that Forman's equation improved the fit to the test data, although the error was still in excess of 20%. The error variation for a particular crack growth equation is seen to vary inversely with the error associated with the various methods of obtaining da/dN. This error variation is also much less than the error associated with the methods of determining the growth rate.

Figure 9 shows the computed crack length-cycle relationships determined from Eqs. 4 and 5 after the coefficients were evaluated by least squares from the growth rate computed by the modified difference and second order polynominal methods. The solid line represents the measured data and is for the specimen shown in Figs. 1 and 7. The variation due to the method of determining the growth rate is seen to be small for each of the equations. The computed crack length-cycle curves for the two models are seen to be in considerable error during the last 200,000 cycles.

Figure 10 shows the same data plotted in a non-dimensional form. It is easily seen that the shape of the curve resulting from Eq. 5 is in better agreement with the experimental curve than Eq. 4. The error in total life shown in Fig. 9 is due to the error in estimation of the growth rates when the crack is small. Eq. 5 overestimates this rate and since the largest portion of the specimen's life is in this region small errors have a large effect on life.

The lack of correslation in the errors of the fitted equation and the error associated with the method of obtaining da/dN is probably due to the inability of the empirical crack growth equations to express the true form of the functional relationship that actually exists between the variables. This hypothesis was tested by generating crack length-cycle data from as assumed crack growth relationship and then performing a data reduction on it. The crack growth relationship proposed by Paris was used:[8]

$$da/dN = C\,\Delta K^4 \qquad (9)$$

The data was assumed to have been generated by a center cracked panel. The stress intensity K was calculated using the tangent correction to allow the equation to be integrated in closed form.[14] The value of da/dN, crack length, and number of cycles could all be calculated directly for a given crack length. Substituting the tangent formula for K into Eq. 9 yields

$$da/dN = 4C\,S_r^4\,b^2\,\tan^2 \pi a/2b \qquad (10)$$

Integrating this equation between the limits N_1, a_1 and N_2 a_2 yields

$$N_2 - N_1 = [2b/\pi\,(\mathrm{ctn}\,\pi a_1/2b - \mathrm{ctn}\,\pi a_2/2b) + a_1 - a_2]/A \qquad (11)$$

where

$$A = 4\,c\,b^2\,S_r^4$$

The growth constant C was chosen as 4.0×10^{-10} and b, the half width, as 3.15 in. The initial crack length was selected as 0.1 in. and the increment of crack extension was taken as 0.04 in. until a crack length of 0.94 in. was attained. The increment was then doubled to 0.08 in. until a terminal crack length of 2.46 in. was reached. The specimen size, initial crack length, and the increments of crack length were selected to match those used in Ref. 6.

Two methods of estimating da/dN were used to analyze the data generated from Eq. 11, the first central difference method and the modified difference method. The error analysis was performed in the same manner as was used with the experimental data. The growth rates obtained from the two methods were integrated numerically to determine their error. These crack growth rates were then fitted to Eq. 4 by least squares and the resulting equations were again integrated numerically to determine their error.

Figure 11 summarizes the results of the error analysis and the values of the coefficients C and n obtained from the least squares it. The error variation is consistent in the two stages of analysis. The error obtained from the integration of the calculated growth rate and the fitted equations is least for the modified difference method. The coefficients obtained from fitting Eq. 4 to the modified difference data also provide a better estimate of the exact values then those obtained from central differences.

The error shown in Fig. 11 is indicative of the error associated with the numerical methods employed. The error in Fig. 11 is much smaller than the errors shown in Tables 1, 3, and 5 which are from actual experimental data. The larger error associated with the analysis of the experimental data is most likely due to three causes. The first is experimental error in obtaining the crack length cycle data. The second is that the crack extension in the specimen is most likely not a continuous process and therefore the crack growth rate may not be unique. The last contribution to the error is the inability of the crack growth equations to model the crack extension behavior. This factor appears responsible for the divergence in the error associated with determining the crack growth rate and that of the crack growth equations fitted to these growth rates.

Equation 10 provides an analytical expression for the crack growth rate. Therefore the crack growth rates determined by the central and modified difference methods from the crack length-cycle data could be checked directly. The value of the growth rate for each increment of crack length for these two methods was compared to the value calculated from Eq. 10. A percentage error for each crack length, a_i, given by

$$\epsilon_i = \left[\frac{da/dN^* - da/dN}{da/dN}\right]_i \times 100 \qquad (12)$$

was calculated where da/dN* is the value of the growth rate estimated by the difference methods and da/dN the growth rate determined from Eq. 10. The absolute value of this error was summed and divided by the number of increments to provide an average error.

Figure 11 shows this error for the two difference methods. This error is seen to be consistent with the error found by integrating the crack growth rate. Consequently, the method of integrating the estimated crack growth rates and comparing these estimates with the crack length-cycle data provides a means of estimating the error in the crack growth rate.

The peculiar behavior of the error analysis of the experimental data seems to be attributable to the fact that the functional relationship between ΔK and da/dN as expressed by the empirical Eqs. 4 and 5 does not describe the true relationship between da/dN and ΔK.

5. SUMMARY AND CONCLUSIONS

1. The secant and modified difference methods provided the best estimate of the crack growth rate of the five numerical methods investigated.

2. Equation 5 provided a better estimate of the crack growth rate-ΔK relationship than Eq. 4. However, both equations were in considerable error.

3. The error analysis presented in this paper provides a rational way to evaluate the error in determining the crack growth rate and of the equations fitted to the growth rate data.

4. The error in the prediction of a specimen's or structure's total life is very sensitive to the estimation of the slow crack growth rate.

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7. TABLES AND FIGURES

TABLE 1

ERROR ANALYSIS - METHODS OF DETERMINING da/dN

S_{min} (ksi)	Average Incremental Error-Percent				
	Secant	Central Diff.	Modified Diff.	2nd Order Poly.	3rd Order Poly.
0.71	2.07	4.13	2.00	7.78	6.66
1.85	2.17	6.24	3.39	8.09	9.40
2.84	1.89	5.32	1.67	8.34	7.86
3.55	1.63	3.40	2.11	7.60	5.38
7.11	1.80	4.44	1.96	7.64	6.20
7.82	2.06	5.11	2.24	9.31	5.61
9.24	1.11	5.64	5.06	9.06	7.89
11.38	2.71	4.56	2.90	8.00	4.76
13.51	2.30	4.14	1.86	7.19	4.81
Average	1.97	4.78	2.58	8.11	6.51

S_{min} (ksi)	Secant $C \times 10^{-9}$	Secant n	Central Diff. $C \times 10^{-9}$	Central Diff. n	Modified Diff. $C \times 10^{-9}$	Modified Diff. n	2nd Order Poly. $C \times 10^{-9}$	2nd Order Poly. n	3rd Order Poly. $C \times 10^{-9}$	3rd Order Poly. n
0.71	2.482	3.0613	3.436	2.9615	2.711	3.0432	2.245	3.1180	1.971	3.1568
1.85	1.274	3.4506	1.610	3.3855	1.210	3.4950	1.372	3.4531	1.210	3.5034
2.84	1.895	3.3774	2.234	3.3365	2.046	3.3545	1.832	3.4263	1.643	3.4685
3.55	0.3874	3.8169	0.7070	3.6443	0.7130	3.6291	0.2331	4.0005	0.2493	3.9684
7.11	0.9094	3.7388	1.424	3.5948	1.211	3.6499	0.7631	3.8270	0.6819	3.8619
7.82	0.2094	4.1767	0.4725	3.9355	0.3926	3.9829	0.1017	4.4328	0.1194	4.3661
9.24	1.339	3.8159	1.680	3.7517	0.9528	4.0224	1.181	3.9119	1.113	3.9306
11.38	1.650	3.6385	2.768	3.4654	2.290	3.5270	1.216	3.7749	1.288	3.7348
13.51	2.990	3.5821	4.036	3.4735	3.354	3.5495	3.053	3.6080	3.009	3.5955
Average	1.4596	3.6287	2.0408	3.5054	1.6533	3.5837	1.3330	3.7282	1.2529	3.7282

TABLE 2 VALUES OF C AND n

TABLE 3 ERROR ANALYSIS OF EQUATION 4

S_{min} (ksi)	Secant	Central Diff.	Modified Diff.	2nd Order Poly.	3rd Order Poly.
0.71	31.82	30.28	30.89	31.03	31.86
1.85	35.39	33.60	34.26	33.61	33.89
2.84	23.60	22.73	22.95	22.42	22.78
3.55	26.20	26.08	27.11	25.78	25.65
7.11	33.74	32.99	33.04	33.00	33.45
7.82	27.75	27.76	27.81	30.38	29.16
9.24	51.03	49.39	48.66	48.71	49.11
11.38	26.24	26.19	25.55	26.75	26.75
13.51	20.87	19.75	20.03	20.39	20.64
Average	30.74	29.86	30.03	30.23	30.37

TABLE 4 VALUES OF C, n and K_c

Method of Determining da/dN	Secant	Central Diff.	Modified Diff.	2^{nd} Order Poly.	3^{rd} Order Poly.
$C \times 10^{-7}$	1.838	2.159	1.949	1.711	1.684
n	1.8881	1.8099	1.9396	1.9244	1.9084
K_c (ksi in)	64.5	60.5	70.0	62.0	61.0

TABLE 5 ERROR ANALYSIS OF EQUATION 5

S_{min} (ksi)	Average Incremental Error-Percent: Secant	Central Diff.	Modified Diff.	2^{nd} Order Poly.	3^{rd} Order Poly.
0.71	26.75	25.27	27.59	28.35	27.93
1.85	28.33	26.38	29.24	26.26	26.36
2.84	19.96	17.31	19.62	17.90	18.46
3.55	24.35	24.69	22.87	23.90	24.08
7.11	21.10	17.16	23.69	18.63	18.11
7.82	24.40	22.61	28.19	18.70	20.34
9.24	26.33	22.29	33.09	24.69	22.55
11.38	15.35	19.21	14.34	20.92	20.88
13.51	18.82	26.15	15.76	23.75	23.58
Average	22.82	22.34	23.82	22.56	22.48

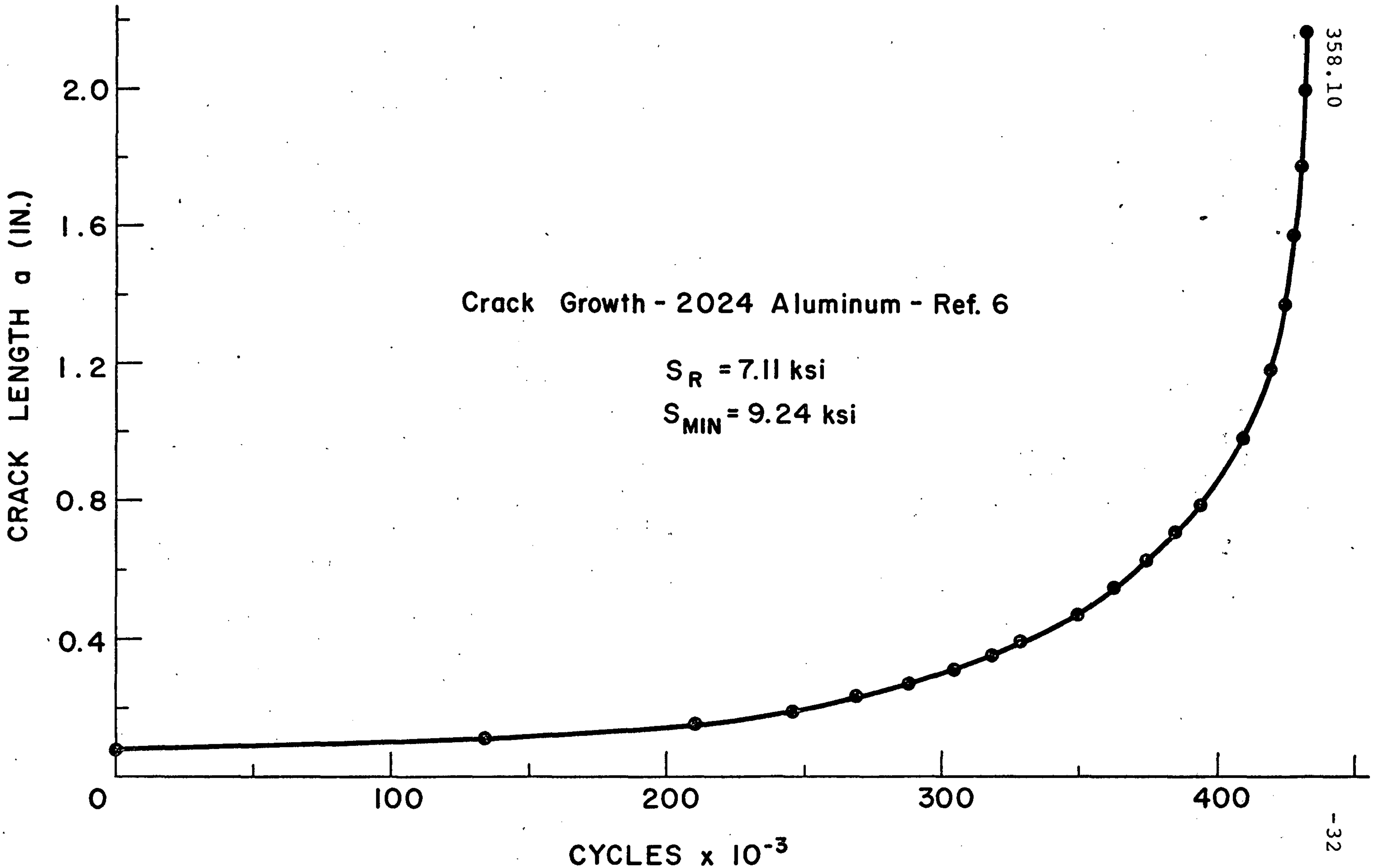
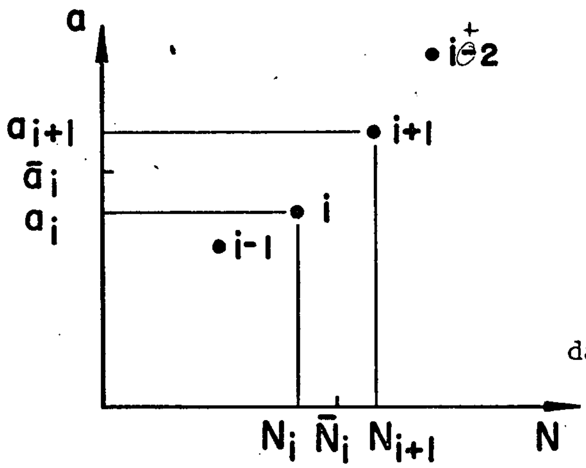


Fig. 1 Crack Length-Cycle Plot

358.10-32

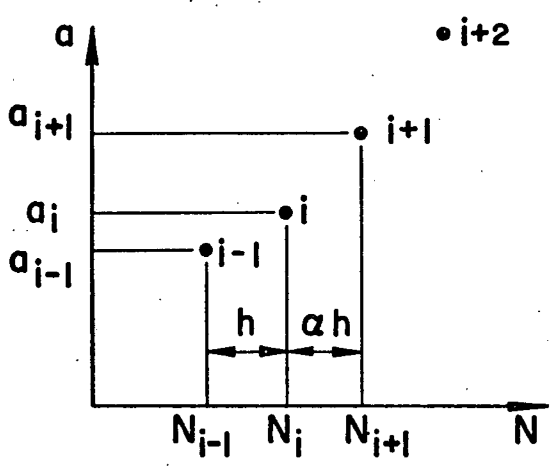


$$\bar{a}_i = (a_i + a_{i+1})/2$$

$$\bar{N}_i = (N_i + N_{i+1})/2$$

$$da/dN = \frac{(a_{i+1} - a_i)}{(N_{i+1} - N_i)} \text{ AT } \bar{a}_i \ \& \ \bar{N}_i$$

Figure 2 SECANT METHOD



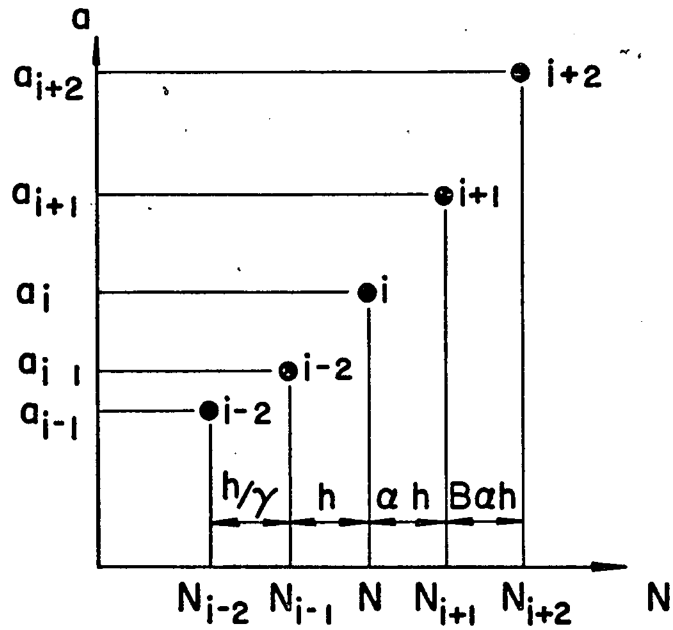
$$h = N_i - N_{i-1}$$

$$\alpha h = N_{i+1} - N_i$$

$$da/dN = \frac{a_{i-1} - a_i(1-\alpha^2) - \alpha^2 a_{i-1}}{\alpha(1+a)h}$$

AT a_i & N_i

Figure 3 FIRST CENTRAL DIFFERENCE



$$h/\gamma = N_{i-1} - N_{i-2}$$

$$h = N_i - N_{i-1}$$

$$\alpha h = N_{i+1} - N_i$$

$$\beta\alpha h = N_{i+2} - N_{i+1}$$

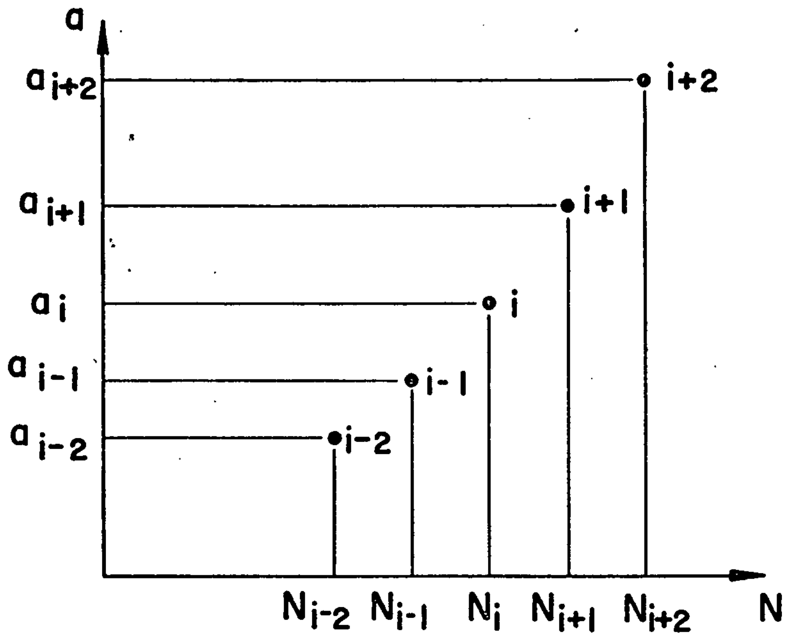
$$da/dN = \frac{1}{hA_2}\left(-a_{i+2}\frac{A_3}{B_0} + a_{i+1}\left[\frac{1}{A_0} + \frac{A_3 B_2}{B_0}\right] + a_i\left[\frac{A_3 B_1}{B_0} - \frac{A_1}{C_0}\right] + a_{i+1}\left[\frac{A_1 C_2}{C_0} - \frac{1}{A_0}\right] + a_{i+2}\frac{A_1 C_1}{C_0}\right)$$

Where: $A_0 = (\alpha+1)/6\alpha$, $A_1 = \alpha(2-\alpha)$, $A_2 = (\alpha+1)^2$, $A_3 = 2\alpha-1$

$B_0 = \alpha\beta(1+\beta)$, $B_1 = \beta^2$, $B_2 = 1-\beta^2$

$C_0 = (1+\gamma)$, $C_1 = \gamma^2$, $C_2 = 1-\gamma^2$

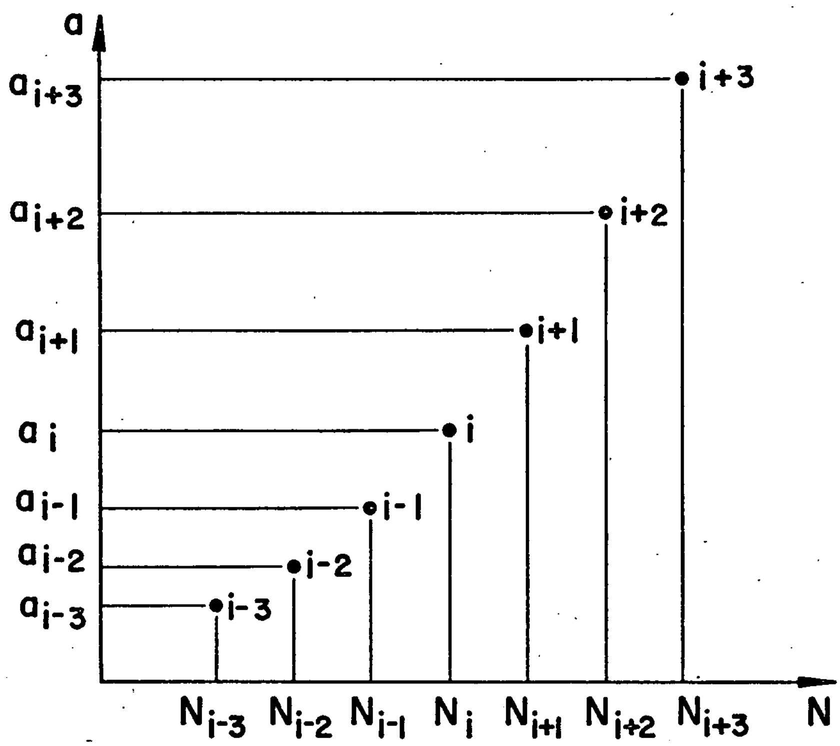
Figure 4 MODIFIED DIFFERENCE



Fitted Equation: $\text{Log } N = B_1 + B_2 a + B_3 a^2$ Fitted to Points i-2 through i+2

$$da/dN = B_2 + 2B_3 a_i$$

Figure 5 SECOND ORDER POLYNOMINAL



Fitted Equation: $\text{Log } N = B_1+B_2 a+B_3 a^2+B_4 a^3$ Fitted to Points i-3 through i+3

$$da/dN = B_2+2B_3 a_i+3B_4 a_i^2$$

Figure 6 THIRD ORDER POLYNOMINAL

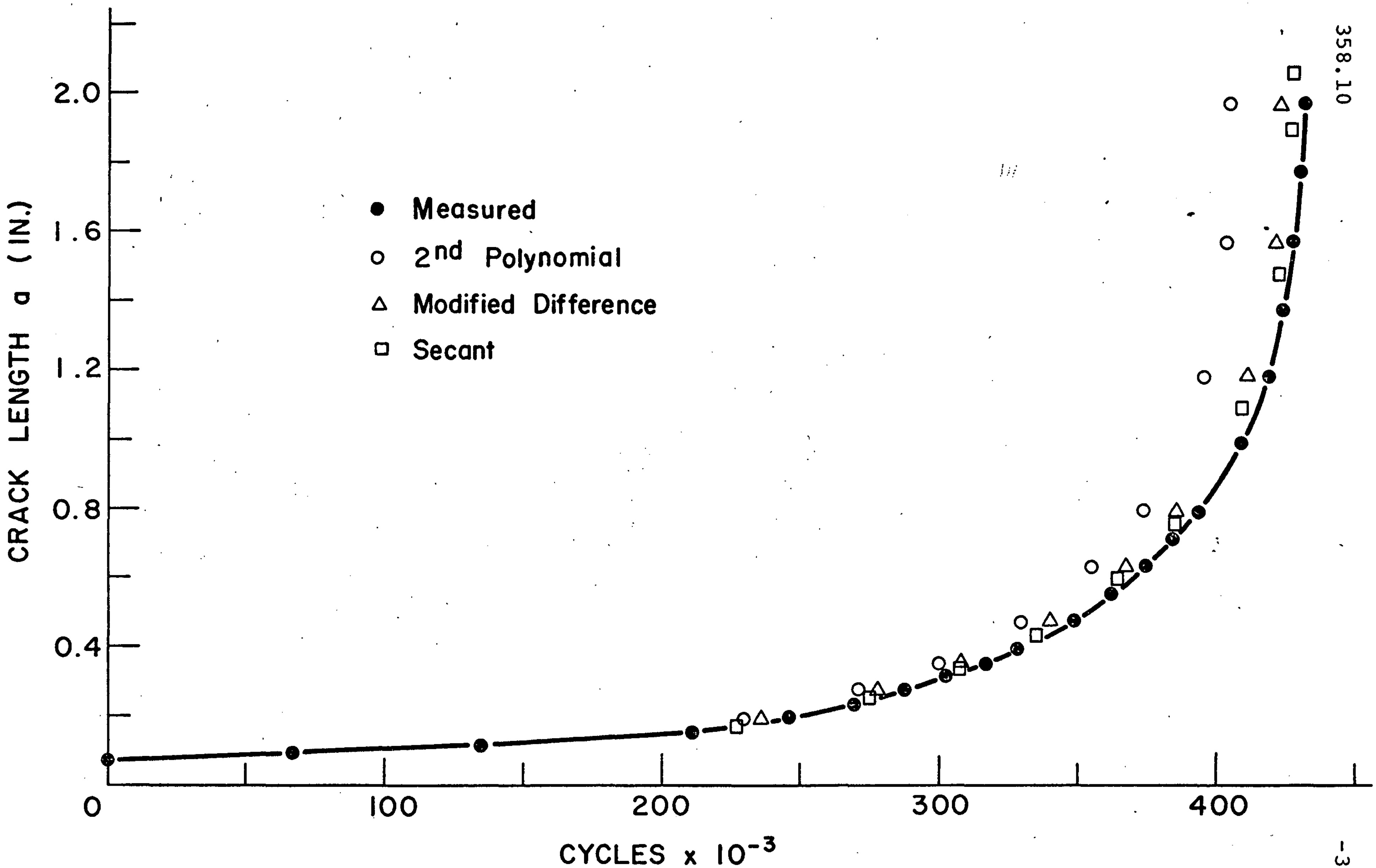


Fig. 7 Results of Integration of Growth Rates

358.10
-36

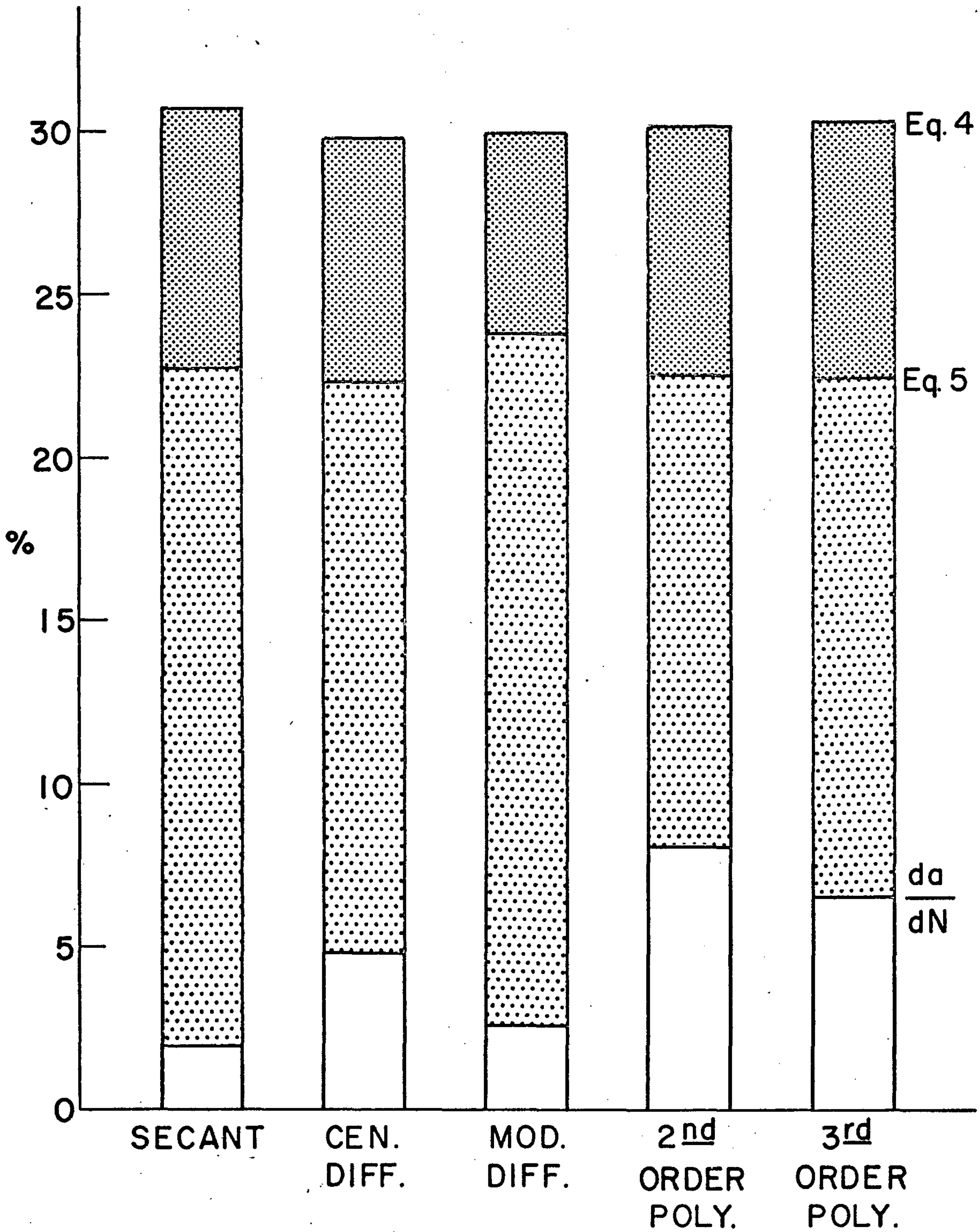


Fig. 8 Average Error Summary

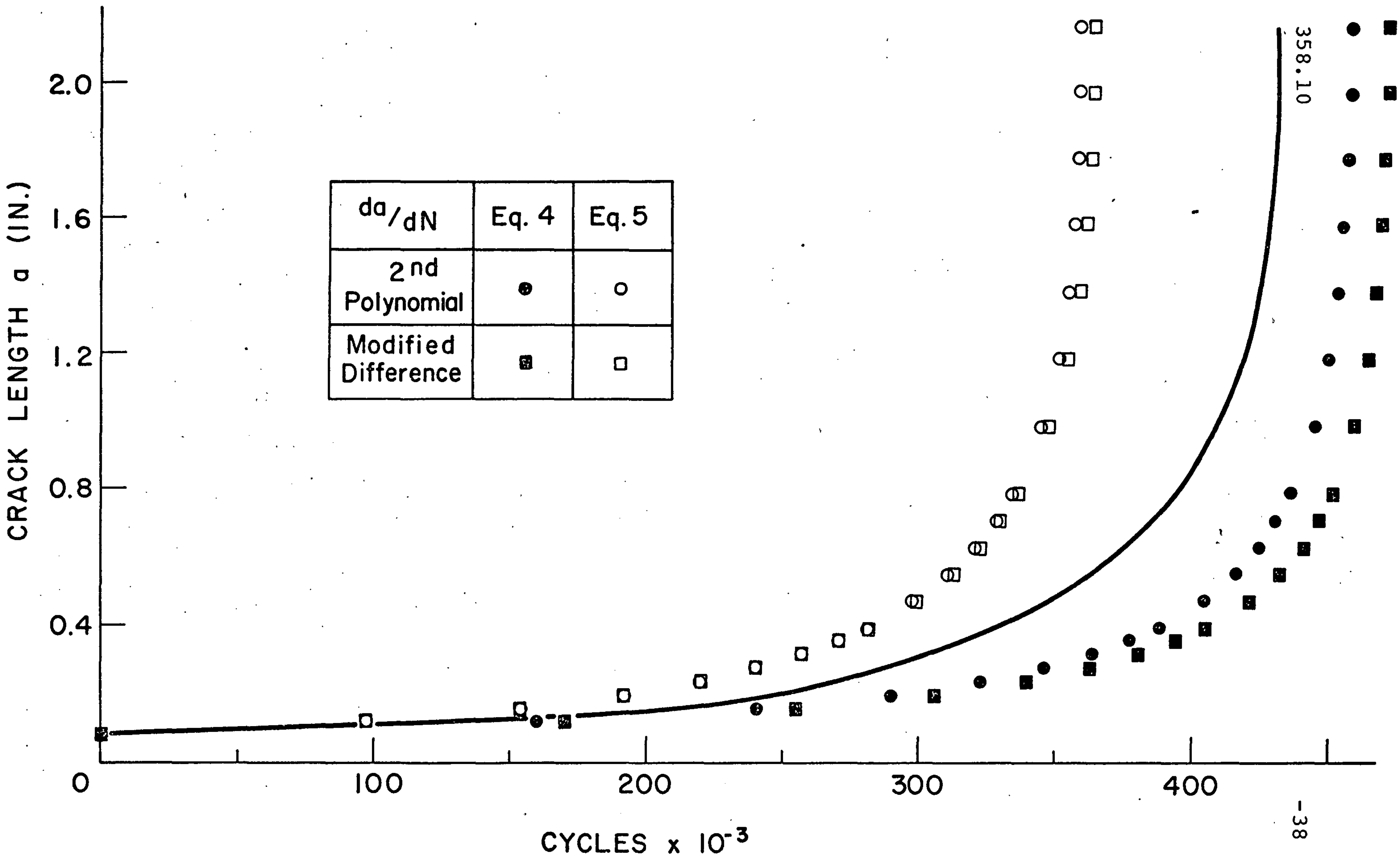


Fig. 9 Results of Integration of Equations Fitted to Growth Rates

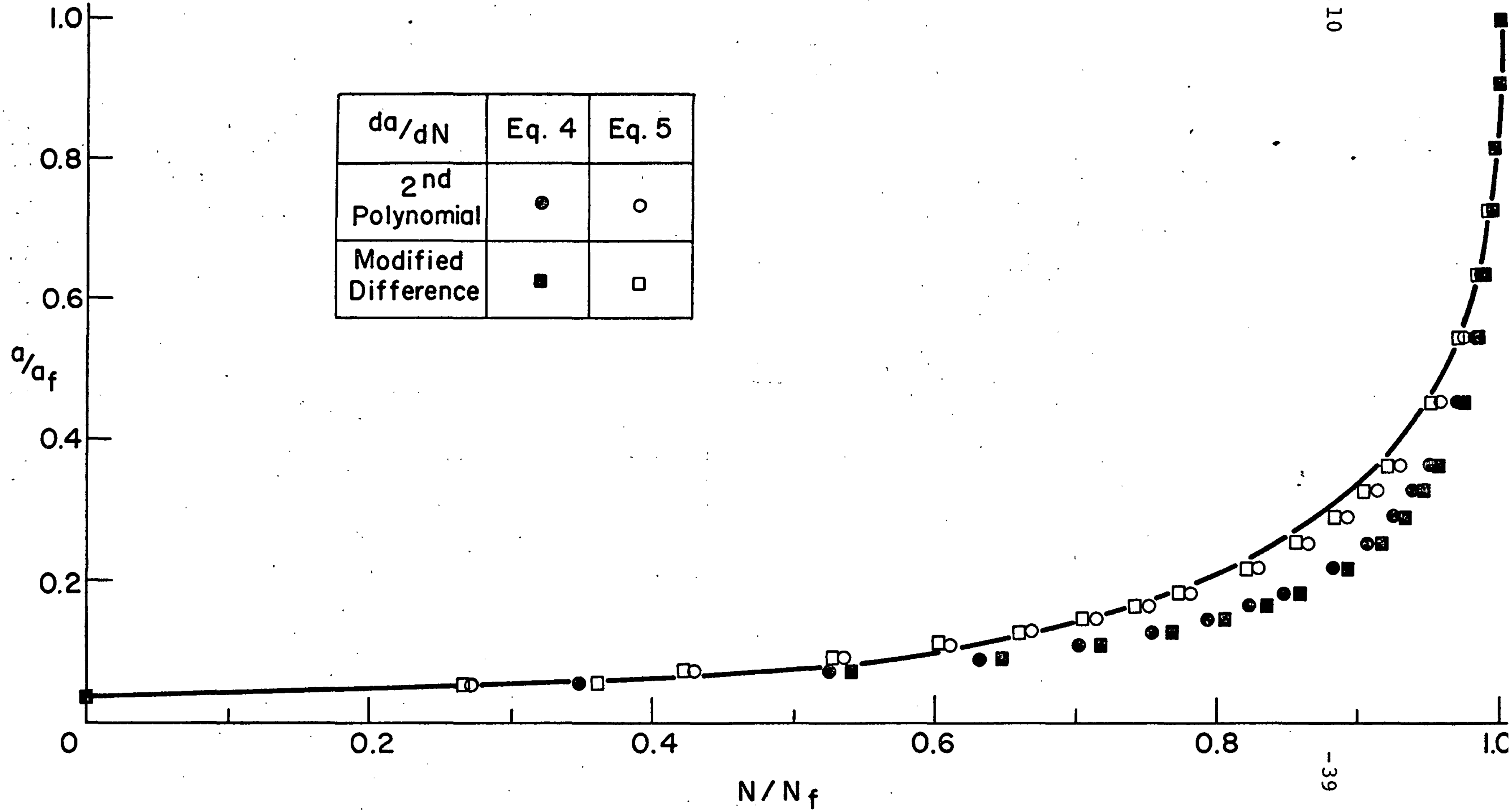


Fig. 10 Non-Dimensional Plot of Integration of Fitted Equations

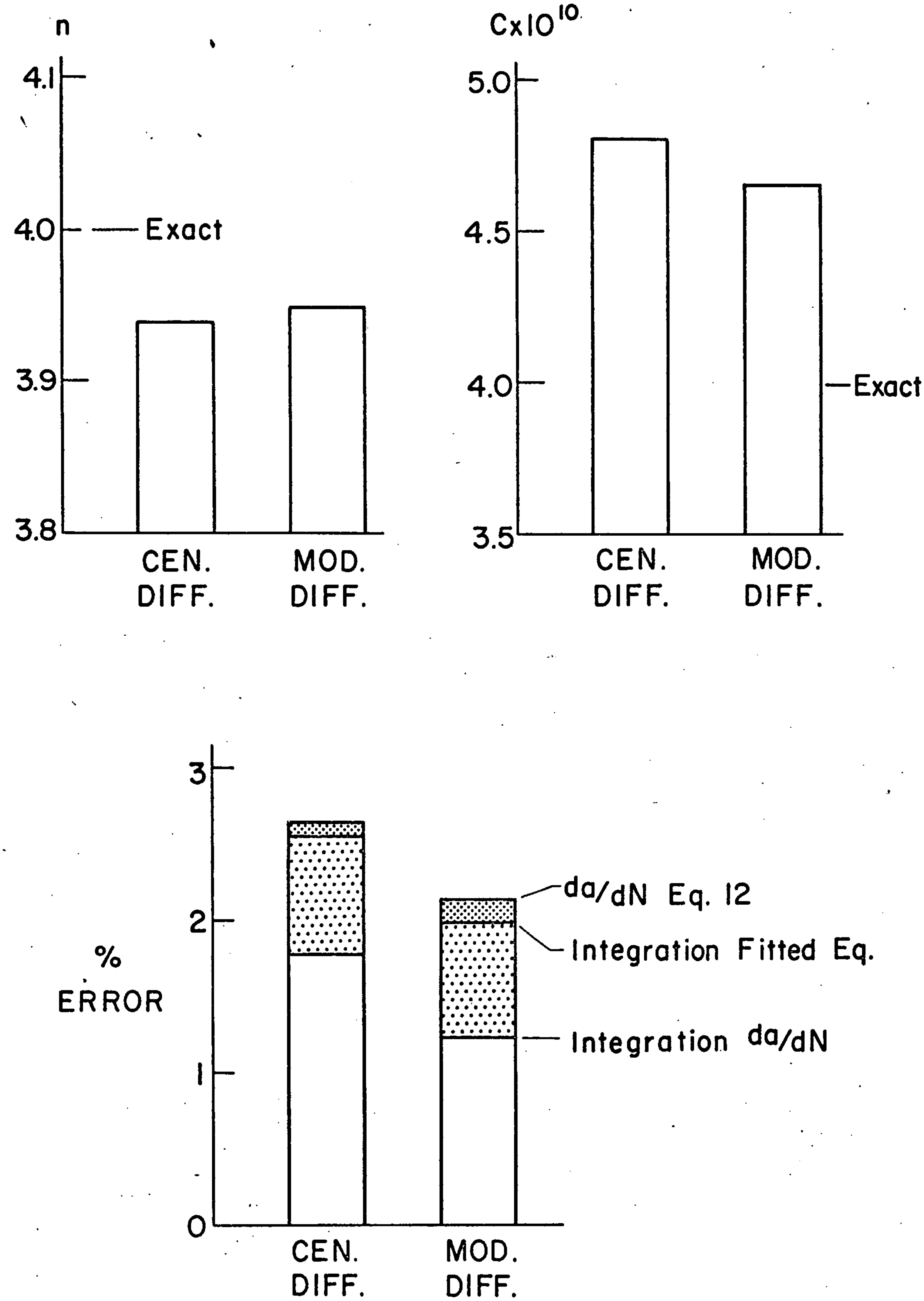


Fig. 11 Error of Reduction from Exact Data

8. APPENDIX A

DERIVATION OF MODIFIED DIFFERENCE OPERATOR

The modified difference operator used in the determination of the crack growth rate was based on the use of Simpson's rule for integration and first central differences. It was assumed that point a_{n+1} in Fig. 4 could be written as

$$a_{n+1} = h/3 \left(\frac{\alpha+1}{2\alpha}\right) [(2\alpha-1)\ a'_{n+1} + (\alpha+1)^2\ a'_n + \alpha\ (2-\alpha)\ a'_{n-1}] + a_{n-1} \qquad \text{(A1)}$$

This corresponds to Simpson rule for variably spaced data. a'_{n+1}, a'_n and a'_{n-1} are the first derivatives at points a_{n+1}, a_n, and a_{n-1}. The derivatives for a_{n+1} and a_{n-1} were estimated using central differences as

$$a'_{n+1} = [a_{n+2} - (1-\beta^2)a_{n+1} - \beta^2 a_n]/[\alpha\beta(1+\beta)h] \qquad \text{(A2)}$$

and

$$a'_{n-2} = [a_n - a_{n-1}\ (1-\gamma^2) - \gamma^2\ a_{n-2}]/h(1+\gamma) \qquad \text{(A3)}$$

Substituting these values for the derivatives into Eq. A1 yields the modified difference operator which is given in Fig. 4.

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