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SEQUENCE DETECTION IN NON-GAUSSIAN NOISE WITH INTERSYMBOL INTERFERENCE USING THE EM ALGORITHM

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Abstract

Sequence detection is studied for communication channels with intersymbol interference and non-Gaussian noise using a novel adaptive receiver structure. The receiver adapts itself to the noise environment using an algorithm which employs a Gaussian mixture distribution model and the expectation maximization algorithm. Two alternate procedures are studied for sequence detection. These are a procedure based on the Viterbi algorithm and a symbol-by-symbol detection procedure. The Viterbi algorithm minimizes the probability the sequence is in error and the symbol-by-symbol detector minimizes symbol error rate, which are different.

Chapter 1

Introduction

The problem of transmitting information in the presence of intersymbol interference (ISI) and additive Gaussian noise has received considerable attention [1, 2, 3]. Non-Gaussian noise cases have received much less attention. Recently reported results have confirmed that impulsive noise is present in many indoor [4] and outdoor communication environments [5] due to a variety of sources. Recent studies [6] indicate that in some frequency bands, impulsive noise appears to be a much more severe problem than thermal noise. It would be desirable to develop a receiver which could adapt itself to the type of noise which is actually present. The approach taken here is based on modeling the additive noise and interference using a mixture of Gaussian distributions and using the expectation maximization model to find the parameters of this distribution. We use this distribution to perform sequence detection using one of two algorithms typically employed for Gaussian noise cases. The Viterbi Algorithm (VA) was first proposed in 1967 [7] as a method for decoding convolutional codes. Since then it has become extremely popular. In particular, it has been recognized as an attractive solution to a variety of digital communication problems. The symbol-by-symbol algorithm we study was first proposed by Abend and Fritchman in [2]. While the Viterbi algorithm is more commonly used, the algorithm in [2] is optimum if the criterion is to minimize the probability of a symbol error. While simulations have shown that the two algorithms often provide similar performance for Gaussian noise cases, the difference in performance between these algorithms is not known for non-Gaussian noise cases. Our approach is applicable to several cases of practical interest including channels which produce intersymbol interference and to cases where some popular coding schemes, for example convolutional codes, are employed.

Chapter 2

Intersymbol Interference Model

Consider an m-ary digital communication system where B_k denotes the symbol transmitted at time t=kT, k=1, 2,...,N. B_k takes on one of the values $b_0, b_1, \ldots, b_{m-1}$. Denote the received signal sample, after matched filtering, at the i^{th} antenna by $R_{k,i}$. The additive noise which corrupts $R_{k,i}$ is denoted by $W_{k,i}$ Assume the system contains U receiving antennas and that no more than L_s successive symbols interfere at the channel output so that

$$R_{k,i} = \sum_{j=k-L_s+1}^{k} B_j g_{k-j,i} + W_{k,i} \tag{2.1}$$

for $k = 1, \ldots, N, i = 1, \ldots, U$. In (2.1) the sequence g describes the ISI and is obtained from a linear model of the channel and from the pulse shaping employed in the communications. If the channel between the transmitter and the matched filter output at the i^{th} antenna is linear and time invariant with impulse response $h_i(t)$ and the pulse shape is $p(t)$ then

$$g_{k,i} = \int_{\tau=0}^{\infty} p(kT-\tau)h_i(\tau)d\tau, k = 1, \ldots, N, i = 1, \ldots, U. \tag{2.2}$$

The noise samples can be collected in a vector $(W_{1,1}, \ldots, W_{N,U})$ which has components that are independent and identically distributed with each component modeled as an L-term mixture of Gaussian random variables with probability density function (pdf)

$$\mathbf{f_W}(w_{k,i}) = \sum_{l=1}^{L} \lambda_l \frac{1}{2\pi\sigma_l^2} \exp\left(-\frac{|w_{k,i}|^2}{2\sigma_l^2}\right). \tag{2.3}$$

Thus the probability of a received sample conditioned on all previously transmitted symbols and the corresponding observations is $P(R_{k,i}|B_1,\ldots,B_k,R_{1,i},\ldots,R_{k-1,i}) =$

$$P(R_{k,i}|B_{k-L_s+1}...B_k) = \mathbf{f_W}(R_{k,i} - \sum_{j=k-L_s+1}^{k} B_j g_{k-j,i}) \tag{2.4}$$

where $\mathbf{f_W}$ is defined in (2.3). The result in (2.4) can be employed with the techniques in [8], which provide techniques for efficiently estimating the unknown parameters of $\mathbf{f_W}$ from (2.3) using the Expectation Maximization algorithm, to perform sequence detection using the Viterbi and symbol-by-symbol algorithms.

Chapter 3

Viterbi Algorithm

The Viterbi algorithm can be viewed as an efficient method for maximum a posterior probability detection of the signal sequence generated from a finite-state discrete-time Markov process observed in memoryless noise [7, 9]. The ISI model we have outlined can be shown to be a special case of this. For such a process, the state s_k at any time k can be given by the L_s most recently transmitted symbols (or in some equivalent way)

$$s_k = (B_{k-1}, B_{k-2}, ..., B_{k-L_s+1}) \tag{3.1}$$

where by convention $B_k = 0$ for $k \leq 0$. From (3.1) there are m^{L_s-1} possible states. Also, from (3.1) it is clear that there is a one-to-one correspondence between the sequence of states and the sequence of symbols transmitted. The Viterbi algorithm [9] will produce an estimate $\hat{B}(k)$ of the transmitted sequence that maximizes the posterior probability $p(R(k)|B(k) = \hat{B}(k))$ or equivalently minimizes $-\ln p(R(k)|B(k) = \hat{B}(k)) =$

$$-\sum_{\ell=1}^{k}\sum_{i=1}^{U} \ln P(R_{\ell,i}|B_{\ell-L_s+1} = \hat{B}_{\ell-L_s+1}, \ldots, B_\ell = \hat{B}_\ell) \tag{3.2}$$

which can be calculated using (2.4). For exact optimality using the Viterbi algorithm we may need to wait until the entire sequence arrives at the receiver to make the decision on all bits sent. In practice [10] a decision is made prior to this in order to limit delay and memory. Here we choose a time delay interval of D symbols ($time = DT$) and after the $(k+D)^{th}$ transmitted symbol is received, a decision is made on what was transmitted at time $t = kT$.

Chapter 4

Symbol-by-Symbol Algorithm

The symbol-by-symbol algorithm [2] is another recursive procedure for detecting symbols in channels with ISI and additive noise. The decision about B_k will be made on the basis of the received noisy and ISI distorted version of B_{k+D} and all of the prior symbols. Thus there is again a delay of D symbols before a decision is made. The symbol-by-symbol decision procedure yields minimum symbol error probability among all procedures which produce $\hat{B}_k$ based only on $R_{1,1}, \ldots, R_{k+D,U}$. Thus it chooses $\hat{B}_k$ equal to the value of B_k that maximizes the posterior probability $p(B_k|R_{1,1} \ldots R_{k+D,U})$. If $p(B_k = b_i|R_{1,1}, \ldots, R_{k+D,U}) > p(B_k = b_j|R_{1,1}, \ldots, R_{k+D,U})$ for all $j \neq i$, the symbol-by-symbol algorithm will make a decision that $\hat{B}_k = b_i$.

Since $p(B_k|R_{1,1}, \ldots, R_{k+D,U}) = p(B_k, R_{1,1}, \ldots, R_{k+D,U}) \,/ p(R_{1,1}, \ldots, R_{k+D,U})$ the symbol-by-symbol decision rule chooses the value of B_k that maximizes the values of $p(B_k, R_{1,1}, \ldots, R_{k+D,U})$. Using the law of total probability, as in [2], the joint probability of $p(B_k, R_{1,1}, \ldots, R_{k+D,U})$ can be written in a recursive manner allowing efficient computation [2].

Chapter 5

Numerical Results

Here we provide Monte Carlo simulation results to compare the bit error rates (BERs) provided by the Viterbi and symbol-by-symbol methods. In all cases we take D to be the same for both algorithms. In our investigations we considered many different specific cases. We show only a few representative results here. First, consider the case of a fixed ISI channel with BPSK signals. The simulation results of Fig. 1 are based on 10,000,000 runs, where each run corresponds to the fixed set of $g_{k,i}$ coefficients indicated in Fig. 1. We choose the noise pdf parameters in equation (2.3) as L=2, $\lambda_2 = 0.025$, $\sigma_2^2/\sigma_1^2 = 100$, $\lambda_1\sigma_1^2 + \lambda_2\sigma_2^2 = 2$. We also choose $L_s = 3$ and $D = 2$. In Fig. 1, we observe that the Viterbi algorithm provides nearly identical BER performance as the symbol-by-symbol algorithm when the SNR is small. For larger SNR, the symbol-by-symbol algorithm provides slightly better performance than the Viterbi algorithm. Secondly we see that spatial antenna diversity can greatly improve the performance of both algorithms. This is true even for nonGaussian noise with the proper nonlinear processing. Increasing λ_2 with $\lambda_1\sigma_1^2 + \lambda_2\sigma_2^2 = 2$ will at least initially tend to increase the impulsiveness of the noise. This is illustrated in Fig. 2 for a case with SNR of $5dB$. As λ_2 is increased from 0.025 the advantage of the symbol-by-symbol algorithm over the Viterbi algorithm increases slightly. The peak difference appears to occur for that λ_2 that simultaneously gives the worst performance for both algorithms. Increasing λ_2 beyond this value appears to drive the two curves back together again. For different values of the $g_{k,i}$ coefficients and for different SNRs, generally similar results are obtained.

If the time delay interval D is increased, it is reasonable that the BER of both algorithms will decrease. This is illustrated in Table 1. The noise pdf parameters for the cases of Table 1 are the same as Fig. 1 except for D and the average SNR, which is $3dB$. From the results of Table 1, we also see the symbol-by-symbol algorithm gives better performance than the Viterbi algorithm for all D. However, the performance is very close for all cases in Table 1. As D approaches ∞ we expect the performance of the two algorithms approach one another and this is consistent with the results Table 1.

Table 5.1: BER PERFORMANCE WITH DIFFERENT D

Ant	U=1		U=2	
D	sym-by-sym	Viterbi	sym-by-sym	Viterbi
2	2.5296e-2	2.5472e-2	2.2176e-3	2.3611e-3
3	1.3668e-2	1.4037e-2	1.0140e-3	1.0327e-3
4	1.0354e-2	1.0406e-2	7.7830e-4	8.0189e-4
5	9.2853e-3	9.4231e-3	6.7143e-4	6.9524e-4
6	8.9048e-3	9.1571e-3	6.6872e-4	6.9231e-4

We studied the BER performances of both algorithms for cases with fading channels. In order to model the fading channel, we assume the $g_{k,i}$ coefficients are taken as time varying complex Gaussian random variables with zero mean and unit variance which were constant over several bits (100 bits) so that the $g_{k,i}$ coefficients can be estimated. To simplify matters we assume perfect estimates. Some flat fading cases where the $g_{k,i}$ are estimated are considered in [8]. We obtained the probability of error performance using Monte Carlo simulations and as one might expect the results were quite similar to those in Fig. 1 and Fig. 2.

Finally, we considered the performance of the two algorithms for cases with convolutional coding, ISI, and nonGaussian noise. In these cases the observed signal sequence can still be generated by a finite-state discrete-time markov process so the approach we discussed previously for receiving signals in ISI and nonGaussian

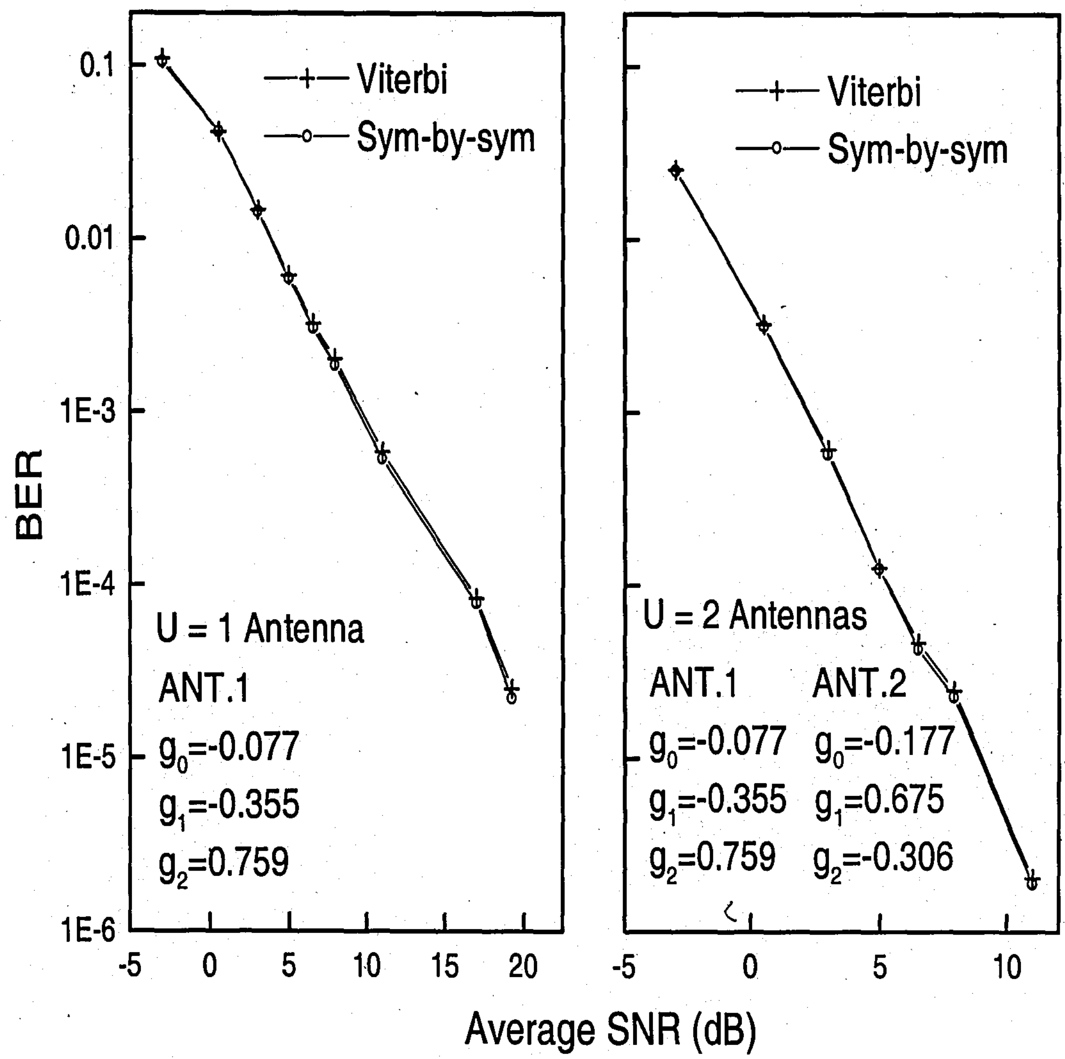

Figure 5.1: BER performance under various SNR (D=2)

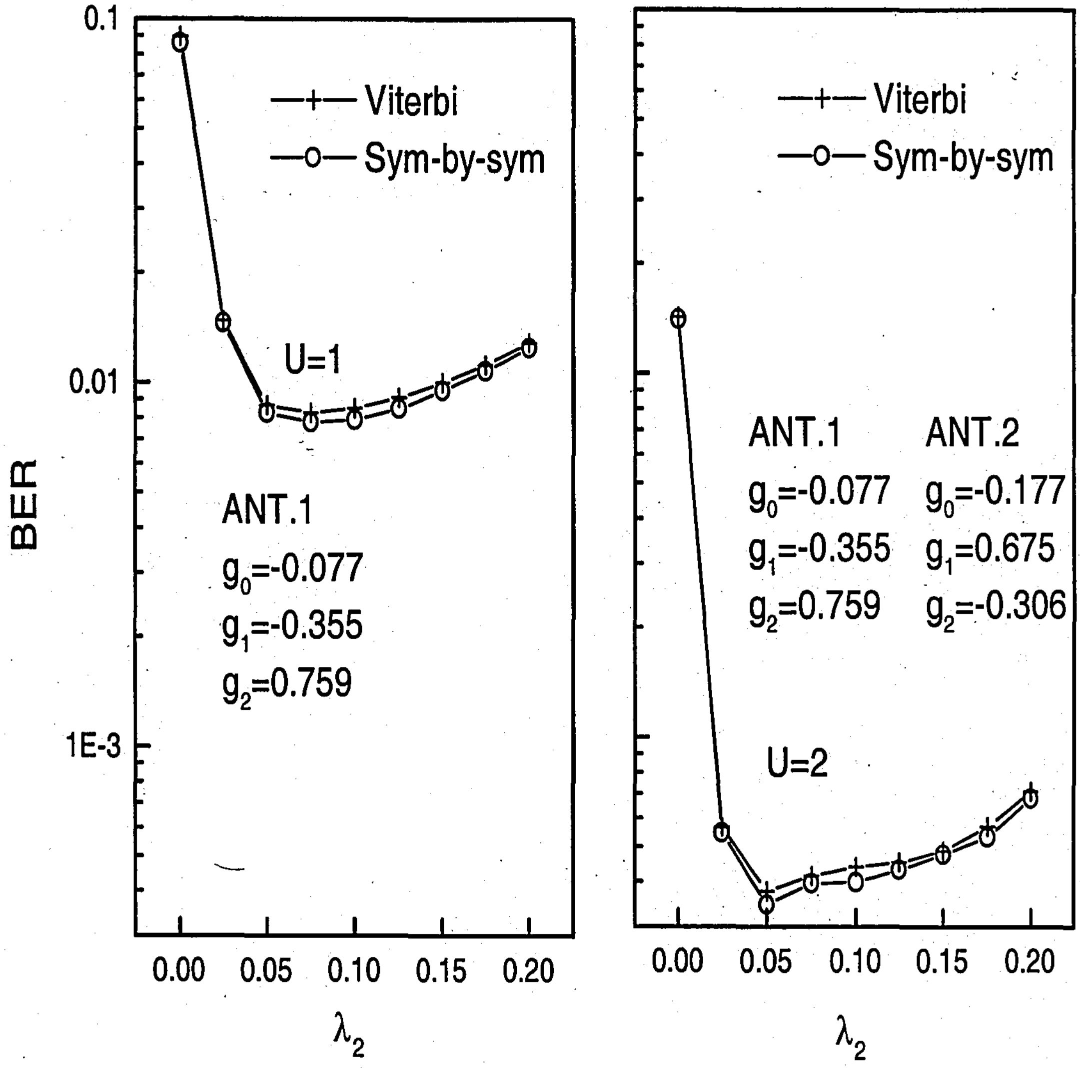

Figure 5.2: BER performance for fixed channel

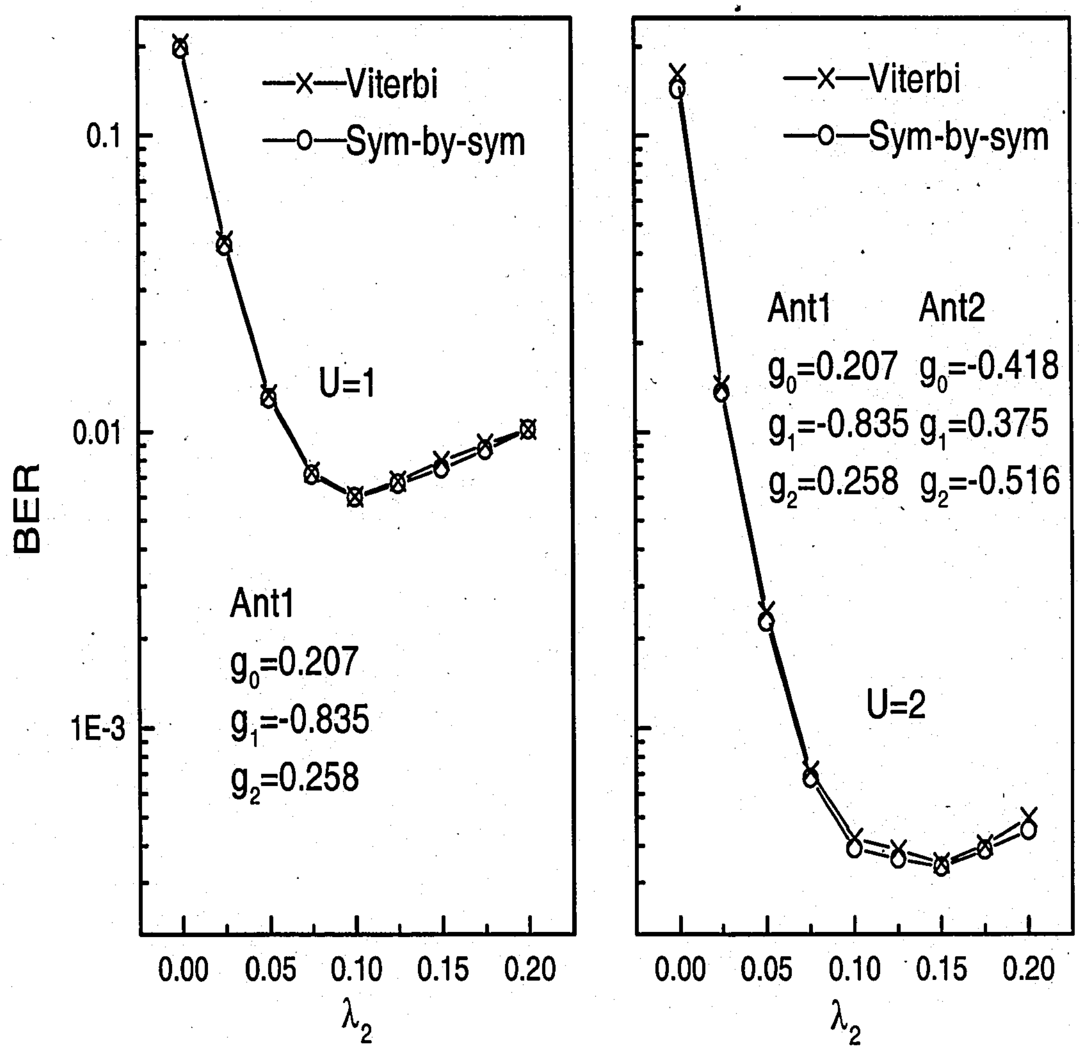

Figure 5.3: BER performance for fixed channel with coding

noise is still applicable [7, 9]. Here we present results for the Hagelbarger code described in Fig. 1 through Fig. 3 of [11]. One simulation result for this code is presented in Fig. 3. Here the noise pdf parameters in equation (2.3) are taken as L=2, $\sigma_2^2/\sigma_1^2 = 100$, $\lambda_1\sigma_1^2 + \lambda_2\sigma_2^2 = 2$ with λ_2 varied from 0.0 to 0.2. The ISI model assumes L_s equals to 3, D equals to 2 and the average SNR$=-3dB$. Fig. 3 shows the performance of the two algorithms is again similar.

Chapter 6

Conclusion

A nonlinear adaptive receiver is suggested for cases with ISI, convolutional coding and nongaussian noise. The receiver uses a mixture of Gaussian pdfs to model the noise and the Expectation Maximization algorithm to estimate the parameters of this model. The Viterbi algorithm and the symbol-by-symbol algorithm are tested to perform the sequence detection. The performance of the two algorithms is typically quite close, especially for small SNR cases when the noise is not too impulsive. For large SNR and highly impulsive noise cases, the symbol-by-symbol algorithm provides slightly smaller probability of bit error, when decisions are generated using a fixed, relatively small delay.

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Vita

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